A New GM(1,1) Based on Piecewise Rational Linear/linear Monotonicity-preserving Interpolation Spline

Fengyi Chen and Yuanpeng Zhu*

***Abstract*—The classical GM(1,1) model reduces the randomness of the original nonnegative sequence by cumulatively generating the original nonnegative sequence to get a monotone increasing 1-AGO sequence. The accuracy and usability of the model will be directly affected by the background construction method. Hence, reconstructing the model background value is of great research significance for improving its prediction accuracy. In this paper, we establish a new GM(1,1) model by using the C^1 continuous monotonic piecewise rational linear/linear interpolation spline, which provides a more reasonable formula for calculating the background value. Compared with the classic GM(1,1) model, the new GM(1,1) model has more advantages in data processing for their effectiveness and accuracy.**

***Index Terms*—Background value, GM(1,1) model, Grey theory, Monotonicity-preserving interpolation spline**

I. INTRODUCTION

Let an original nonnegative sequence be

$$X^{(0)} = \left\{x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(n)\right\}. \tag{1}$$

The classical grey prediction GM(1,1) model was firstly proposed by Deng in 1982 [1], whose main idea is to make an cumulated generating operation on the original nonnegative sequence, so as to reduce the randomization of the original data and obtain a monotonically increasing 1-AGO sequence $X^{(1)}$. Then establish a first-order grad prediction differential equation on the sequence $X^{(1)}$. Besides, use the least square method to solve the differential equation numerically to estimate the parameters. Finally, the original data is simulated and predicted by using the inverse cumulative generation operation showed by Deng and Wei [2].

The 1-AGO sequence $X^{(1)}$ is given as follows

$$X^{(1)} = \left\{x^{(1)}(1), x^{(1)}(2), \ldots, x^{(1)}(n)\right\}, \tag{2}$$

where

$$\begin{aligned} x^{(1)}(k) &= \sum_{i=0}^{k} x^{(0)}(k) \\ &= x^{(1)}(k-1) + x^{(0)}(k), k = 1, 2, \ldots, n. \end{aligned} \tag{3}$$

From Eq. (3), we can see that the 1-AGO sequence $X^{(1)}$ has the feature of monotonicity-increasing. Let's suppose that $x^{(1)}(t)$ meets the following first order grad forecasting differential equation

$$\frac{dx^{(1)}(t)}{dt} + ax^{(1)}(t) = b, \tag{4}$$

where the grey developmental coefficient a and the grey control parameter b are the parameters in the model to be estimated.

From the Eq. (4) and the initial condition $\widetilde{X}^{(1)}(1) = X^{(1)}(1)$, we get

$$x^{(1)}(t) = \left[x^{(1)}(1) - \frac{b}{a}\right] e^{-a(t-1)} + \frac{b}{a}. \tag{5}$$

Thus, to find the prediction model of the original data sequence, we need to identify the effect of the grey development coefficient a and the grey control parameter b in Eq. (4). In order to get this, we do the integral accumulation on both sides of Eq. (4) for $[k, k+1], k = 1, 2, \ldots, n-1$, then we can get

$$\int_k^{k+1} \frac{dx^{(1)}(t)}{dt} dt + a \int_k^{k+1} x^{(1)}(t) dt = b,$$

that is

$$x^{(1)}(k+1) - x^{(1)}(k) + a \int_k^{k+1} x^{(1)}(t) dt = b,$$

or

$$x^{(0)}(k+1) + a \int_k^{k+1} x^{(1)}(t) dt = b. \tag{6}$$

Let background value be $z^{(1)}(k+1) := \int_k^{k+1} x^{(1)}(t) dt$.

To find the result of the background value $z^{(1)}(k+1)$, we need to integrate $x^{(1)}(t)$, which requires the values of a and b to be given in advance. Besides, we can determine the values of a and b by the values of the original sequence and structure form of the background value from the Eq. (6). Thus, to estimate the values of a and b, we must use some methods to estimate the background value $z^{(1)}(k+1)$ in the first place, which is a key factor affecting the simulation error $\overline{\varepsilon}$ and the quality of the predicting model.

In the classical GM(1,1) model, the piecewise linear polynomial interpolation $L(t) := (k+1-t)\, x^{(1)}(k) + (t-k)\, x^{(1)}(k+1)$ will be used to approximate $x^{(1)}(t)$, then the estimated background value $z^{(1)}(k+1)$ will be found as follows

$$\begin{aligned} z^{(1)}(k+1) &= \int_k^{k+1} x^{(1)}(t) dt \\ &\approx \int_k^{k+1} L(t) dt \\ &= \tfrac{1}{2}\left[x^{(1)}(k) + x^{(1)}(k+1)\right]. \end{aligned} \tag{7}$$

Manuscript received July 10th, 2020; revised April 24th, 2021. The research was supported by the National Natural Science Foundation of China (Grant No. 61802129) and the Natural Science Foundation Guangdong Province, China (Grant No. 2018A030310381).

Fengyi Chen is a lecturer of the School of Management Administration, Guangdong Polytechnic industry College, Guangzhou 510330, PR China (e-mail: 2013110028@gdip.edu.cn).

Yuanpeng Zhu is an associated professor of the School of Mathematics, South China University of Technology, Guangzhou 510641, PR China (e-mail: ypzhu@scut.edu.cn).

At each interval $[k, k+1]$, $k = 1, 2, \ldots, n-1$, by substituting the estimated background value $z^{(1)}(k+1)$ into Eq. (6) and the use of the least square method in the further, the values of the parameters a and b can be estimated by the following formula

$$\begin{pmatrix} a \\ b \end{pmatrix} = (G^T G)^{-1} G^T X,$$

where

$$X = \begin{bmatrix} x^0(2) \\ x^0(3) \\ \vdots \\ x^0(n) \end{bmatrix}, \; G = \begin{pmatrix} -z^{(1)}(2) & 1 \\ -z^{(1)}(3) & 1 \\ \vdots & \vdots \\ -z^{(1)}(n) & 1 \end{pmatrix}.$$

At last, we can get the estimated solution from the differential Eq. (4) with the initial condition $\widetilde{X}^{(1)}(1) = X^{(1)}(1)$, as follows

$$\widetilde{x}^{(1)}(t) = \left[x^{(1)}(1) - \frac{b}{a}\right] e^{-a(t-1)} + \frac{b}{a}, \quad k = 1, 2, \ldots. \quad (8)$$

Therefore we obtain the following grey prediction equation

$$\widetilde{x}^{(0)}(k+1) = \widetilde{x}^{(1)}(k+1) - \widetilde{x}^{(1)}(k), \quad k = 1, 2, \ldots. \quad (9)$$

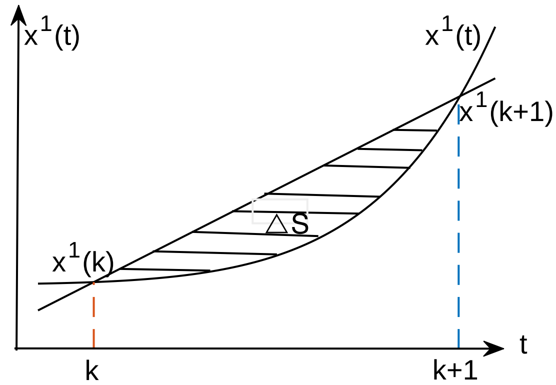

Fig. 1. Prediction error source diagram of conventional GM(1,1) model.

From Eq. (7), it is known that the average of adjacent values is used by the classical GM(1,1) model to construct the background value $z^{(1)}(k+1)$. Its geometric meaning is to replace the straight ladder region to the trapezoidal area which is based on the edge of exponential curve $x^{(1)}(t)$, as shown in Fig. 1. This method, however, has its own weakness. When the 1-AGO data sequence changes significantly, the prediction result comes out a large error (ΔS) which would increase exponentially. Hence, the classical GM(1,1) model has some limitations in actual applications. As indicated in [3], [4], [5], the background value's construction determines the prediction accuracy of the GM(1, 1) model directly. In [6], Li and Dai reconstructed $x^{(1)}(t)$ by a high-order Newton interpolation polynomial. Besides, the value of the background $z^{(1)}(k+1)$ was estimated by Newton-Cores integral. Nevertheless, the high-order Newton interpolation polynomial may have the Runge phenomenon when there are a mess of data and the truncation error become very large. Thus the numerical stability cannot be guaranteed when the approximate value of Newton-Cores was calculated integral. Tang and Xiang [7] reconstructed $x^{(1)}(t)$ on the interval $[k, k+1]$ by using the piecewise quadratic interpolation polynomial to estimate the background value $z^{(1)}(k+1)$. It shows that numerical stability is good and the calculation complexity is low. Wang et al. [8] used piecewise cubic interpolation spline to reconstruct $x^{(1)}(t)$, and found out the estimated background value $z^{(1)}(k+1)$. The advantage of this method is that it has a better approximation order which could avoid Runge phenomenon of high-order polynomial. However, all the methods that we have mentioned ignore the important of the increasing monotone characteristic of the curve $x^{(1)}(t)$ which is to be reconstructed. If the reconstructed curve loses the monotonicity-increasing characteristic of $x^{(1)}(t)$, a large error in the background value $z^{(1)}(k+1)$ will be accrued. Thus, a crucial way to exact the function of $x^{(1)}(t)$ approximately is to enhance the background value's estimation and to improve its effectiveness and accuracy. It is believed that constructing monotonicity-preserving interpolation splines has a great significance, meanwhile many methods have been proposed in recent years. Qin and his team are committed to building new splines, including rational polynomial interpolation spline, rational trigonometric interpolation spline and the piecewise bivariate rational interpolation spline; see [9], [10] and the references quoted therein.

Recently, many grey prediction models have been developed based on the analogous methods of the GM(1,1). For example, the FGM(1,1), the NGM model, INDGM, TDPGM(1,1), Grey polynomial model, seasonal GM(1,1), etc; Refer to [11], [12], [13], [14], and the references quoted therein. Since the classical GM(1,1) model has some limitations in practical application, lots of new methods concerning the improvement of the classical GM(1,1) model have been proposed by many researchers, for instance [15], [16], [17], [18], [19], [20], [21], [22], [23], [24].

In this paper, we use a C^1 continuous monotonicity-preserving piecewise rational linear/linear interpolation spline developed in [25] to reconstruct the curve $x^{(1)}(t)$ and present a new scheme to estimate the background value $z^{(1)}(k+1)$. It improves on the schemes in some ways:

(1) The Lagrange polynomial interpolation scheme developed in [6] may have Runge phenomenon. Whereas, the given monotonicity-preserving piecewise rational linear/linear interpolation spline method can avoid this situation, thus helping to reduce the error and improve the numerical stability.

(2) The existing methods developed in [5], [6], [7], [8] ignore the important monotone increasing characteristic of the curve $x^{(1)}(t)$, which is considered in this paper.

(3) Our paper refers to monotonicity-preserving piecewise rational linear/linear interpolation spline in [25] to reconstruct $x^{(1)}(t)$, which is based on function value only.

(4) Compared with the usage of piecewise rational quadratic/Quadratic interpolation spline in [24] to reconstruct $x^{(1)}(t)$, the usage of piecewise rational linear/linear interpolation spline to reconstruct $x^{(1)}(t)$ in this paper is more computational efficient.

(5) In the numerical examples, we compare the new model with the classical GM(1,1) model and other models in this paper. The prediction results show that our new model can

improve the prediction accuracy in practice.

The rest of this paper is organized as follows. In section II, the construction of C^1 monotonicity-preserving piecewise rational linear/linear interpolation spline is given. In section III, a new GM(1,1) model based on C^1 monotonicity-preserving rational linear/linear interpolation spline is constructed in detail. Several numerical examples are also given to prove the value of the new developed schemes. And section IV presents the conclusions.

II. C^1 MONOTONICITY-PRESERVING RATIONAL LINEAR/LINEAR INTERPOLATION SPLINE

According to Eq. (3), the 1-AGO sequence $X^{(1)}$ has a monotonically increasing feature, that is $x^{(1)}(k) \leq x^{(1)}(k+1)$, $\forall k$. And the fitting exponential curve $x^{(1)}(t)$ to the 1-AGO sequence $X^{(1)}$ also has a monotonicity-increasing feature and infinite smoothness. Hence, we use a C^1 monotonic-preserving rational linear/linear interpolation spline to interpolate the 1-AGO sequence, so as to reconstruct the curve $x^{(1)}(t)$.

For the 1-AGO monotone increasing sequence $\left(k, x^{(1)}(k)\right)$, $k = 1, 2, \ldots, n$, for $t \in [k, k+1]$, $1 \leq k \leq n-1$, a rational linear/linear interpolation spline is constructed as follows

$$S(t) = \frac{\lambda_k x^{(1)}(k)(1-s) + x^{(1)}(k+1)s}{\lambda_k(1-s)+s}, \tag{10}$$

where $s = t - k \in [0, 1]$.

From Eq. (10), by direct calculation, we have

$$\begin{aligned} S'(t) &= \frac{\lambda_k\left[x^{(1)}(k+1)-x^{(1)}(k)\right]}{[\lambda_k(1-s)+s]^2} \\ &= \frac{\lambda_k\left[x^{(0)}(k+1)\right]}{[\lambda_k(1-s)+s]^2} \geq 0, \end{aligned} \tag{11}$$

which indicates that $S'(t) \geq 0$ for any $t \in [k, k+1]$ on condition that $\lambda_k \geq 0$. Therefore, the interpolant $S(t)$ is monotone increasing on $[1, n]$ for all $\lambda_k \geq 0$.

From (10) and (11), we can also see that

$$\begin{aligned} &S_+(k) = x^{(1)}(k),\ S_-(k+1) = x^{(1)}(k+1), \\ &S'_+(k) = \tfrac{x^{(0)}(k+1)}{\lambda_k},\ S'_-(k+1) = \lambda_k x^{(0)}(k+1). \end{aligned}$$

Thus we can see that $S_+(k) = S_-(k)$. Moreover, if $\lambda_{k-1}\lambda_k = x^{(0)}(k+1)/x^{(0)}(k)$, we further have $S'_+(k) = S'_-(k+1)$, which implies that $S(t)$ is C^1 continuous at $t = k$.

From the above analysis, when dealing with the 1-AGO monotone increasing sequence $\left(k, x^{(1)}(k)\right)$, $k = 1, 2, \ldots, n$, we choose the parameter λ_k, $k = 1, 2, \ldots, n$ by the following way so as to generate a C^1 monotone increasing rational linear/linear interpolant $S(t)$

$$\begin{cases} \lambda_1 = 1, \\ \lambda_k = \frac{x^{(0)}(k+1)}{\lambda_{k-1}x^{(0)}(k)},\ k = 2, 3, \ldots, n-1. \end{cases}$$

III. ESTABLISH NEW GM(1,1) MODEL

For the original non-negative sequence $X^{(0)} = \left\{x^{(0)}(1), x^{(0)}(2), \cdots, x^{(0)}(n)\right\}$, we first calculate its 1-AGO sequence $X^{(1)} = \left\{x^{(1)}(1), x^{(1)}(2), \cdots, x^{(1)}(n)\right\}$. Then for the 1-AGO sequence $X^{(1)}$, we use the C^1 monotonicity-preserving piecewise rational linear/linear interpolation s-pline $S(t)$ to reconstruct the exponential curve $x^{(1)}(t)$. For the interval $[k, k+1]$, we estimate the background value $z^{(1)}(k+1) = \int_k^{k+1} x^{(1)}(t)dt$ by the following method

$$\begin{aligned} &z^{(1)}(k+1) = \int_k^{k+1} x^{(1)}(t)dt \\ &\approx \int_k^{k+1} S(t)dt \\ &= \int_0^1 \tfrac{\lambda_k x^{(1)}(k)(1-s)+x^{(1)}(k+1)s}{\lambda_k(1-s)+s}ds \\ &= x^{(1)}(k) + x^{(0)}(k+1)\int_0^1 \tfrac{s}{\lambda_k(1-s)+s}\mathrm{ds} \\ &= \begin{cases} \frac{x^{(0)}(k+1)}{2} + x^{(1)}(k), & if\ \lambda_k = 1, \\ x^{(1)}(k) + \frac{x^{(0)}(k+1)}{1-\lambda_k}\left[1 + \frac{\lambda_k}{1-\lambda_k}\ln\lambda_k\right], & if\ \lambda_k \neq 1, \end{cases} \end{aligned} \tag{12}$$

Then, the estimated background value was substituted into the grey differential equation Eq. (6). And we further use the least square method to solve Eq. (6). The formula is as follows

$$\begin{pmatrix} a \\ b \end{pmatrix} = \left(G^T G\right)^{-1} G^T X,$$

where

$$X = \begin{bmatrix} x^0(2) \\ x^0(3) \\ \vdots \\ x^0(n) \end{bmatrix}, G = \begin{pmatrix} -z^{(1)}(2) & 1 \\ -z^{(1)}(3) & 1 \\ \vdots & \vdots \\ -z^{(1)}(n) & 1 \end{pmatrix}.$$

Finally, we obtain the following estimated solution to the differential equation (4) with the initial condition $\widetilde{X}^{(1)}(1) = X^{(1)}(1)$ as follows

$$\widetilde{x}^{(1)}(t) = \left[x^{(1)}(1) - \frac{b}{a}\right]e^{-a(t-1)} + \frac{b}{a}.$$

We thus get the following grey prediction equation

$$\begin{aligned} x^{(0)}(k+1) &= x^{(1)}(k+1) - x^{(1)}(k) \\ &= (1-e^a)\left[x^{(1)}(1) - \tfrac{b}{a}\right]e^{-ak},\ k = 1, 2, \ldots. \end{aligned}$$

We shall give several numerical examples to show that the new GM(1,1) model based on C^1 monotonicity-preserving piecewise rational linear/linear interpolation spline improves prediction accuracy compared to the classical GM(1,1) model. In the following examples, the relative error is computed by

$$\varepsilon = \frac{\left|\overline{x}^{(0)}(k) - x^{(0)}(k)\right|}{x^{(0)}(k)}.$$

Example 1. *In this example, we consider the non-negative data $x^{(0)}(k)$, $k = 1, 2, \cdots, 7$ given in [19]. In addition, we compare the results predicted by our new GM(1,1) model with the GM(1,1) model and the method proposed in [19]. Table I and Fig. 2 give the numerical results. The results show that our model has the best prediction effect compared with the other two prediction models, and it performs very well in predicting data with the exponential growth trend.*

Example 2. *In this example, we consider the non-negative data $x^{(0)}(k)$, $k = 1, 2, \ldots, 12$ given in [8]. Similarly, we compare the results predicted by our new GM(1,1) model with the GM(1,1) model and the method proposed in [7]. Table II and Fig. 3 give the numerical results. The results turn out that the new GM(1,1) model still performs the best among the three prediction models. In addition, its prediction accuracy is significantly higher than the classical GM(1,1) model.*

TABLE I
NUMERICAL RESULTS FOR EXAMPLE 1.

$x^{(0)}$	Classical GM(1,1)		New GM(1,1)		The Model in [19]	
	Prediction data	Relative error ε (%)	Prediction data	Relative error ε (%)	Prediction data	Relative error ε (%)
2.9836	2.9836	0.0000	2.9836	0.0000	2.9836	0.0000
4.4511	4.3804	1.5884	4.4925	0.9292	4.4561	0.1123
6.6402	6.5006	2.1027	6.6826	0.6383	6.6132	0.4066
9.9061	9.6469	2.6161	9.9404	0.3465	9.8146	0.9237
14.7781	14.3162	3.1226	14.7865	0.0569	14.5657	1.4373
22.0464	21.2454	3.6331	21.9951	0.2327	21.6168	1.9486
32.8893	31.5285	4.1374	32.7180	0.5209	32.0812	2.4570
$\bar{\varepsilon}$ (%)		2.4576		0.3892		1.0408

TABLE II
NUMERICAL RESULTS FOR EXAMPLE 2.

$x^{(0)}$	Classical GM(1,1)		New GM(1,1)		The Model in [7]	
	Prediction data	Relative error ε (%)	Prediction data	Relative error ε (%)	Prediction data	Relative error ε (%)
110852	110852	0.00	110852	0.00	110852	0.00
135175	117980	12.72	130156	3.71	127821	5.41
153647	119117	22.47	128634	16.27	126664	17.66
120296	128264	6.62	127130	5.68	125830	4.68
96362	121422	26.27	125644	30.38	124380	29.23
90798	122592	35.01	124176	36.76	123253	35.70
102591	123773	20.65	122724	19.62	122137	19.11
150534	124965	16.99	121289	19.42	121031	19.63
175123	126168	27.95	119872	31.55	119934	31.52
127148	113383	10.83	118470	6.82	114848	9.76
102085	128610	25.98	117085	14.69	117772	15.47
97103	116705	20.19	115717	19.16	116705	20.21
$\bar{\varepsilon}$ (%)		19.92		17.01		17.37

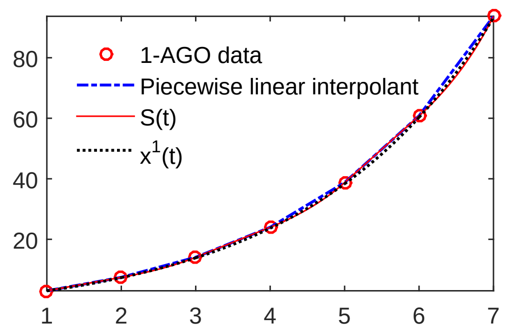

Fig. 2. Graphic results for example 1.

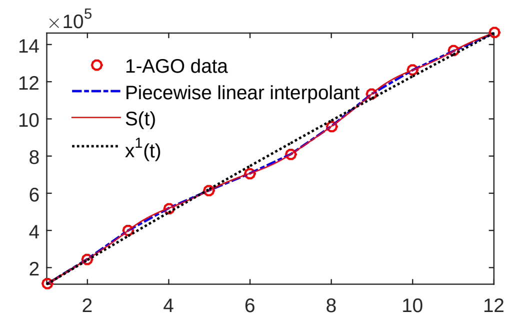

Fig. 3. Graphic results for example 2.

Example 3. *In this example, we consider the non-negative data $x^{(0)}(k)$, $k = 1, 2, \ldots, 9$ given in [20]. Table III and Fig. 4 give the numerical results.*

TABLE III
NUMERICAL RESULTS FOR EXAMPLE 3.

$x^{(0)}$	Classical GM(1,1)		New GM(1,1)	
	Prediction data	Relative error ε (%)	Prediction data	Relative error ε (%)
0.0200	0.0200	0.0000	0.0200	0.0000
0.0191	0.0192	0.9929	0.0192	1.0434
0.0176	0.0174	0.8486	0.0174	0.7981
0.0159	0.0157	0.7109	0.0157	0.6596
0.0144	0.0142	0.8202	0.0142	0.7681
0.0129	0.0129	0.1575	0.0129	0.2108
0.0117	0.0116	0.0979	0.0116	0.0439
0.0105	0.0105	0.7067	0.0105	0.7618
0.0095	0.0095	0.6958	0.0095	0.7517
$\bar{\varepsilon}$ (%)		0.5589		0.5597

Example 4. *In this example, we consider the non-negative data $x^{(0)}(k)$, $k = 1, 2, \ldots, 10$ given in [21]. Table IV and Fig. 5 give the numerical results.*

Example 5. *In this example, we consider the non-negative data $x^{(0)}(k)$, $k = 1, 2, \ldots, 14$ given in [22]. Table V and Fig. 6 give the numerical results.*

TABLE V
NUMERICAL RESULTS FOR EXAMPLE 5.

$x^{(0)}$	Classical GM(1,1)		New GM(1,1)		The model in [24]	
	Prediction data	Relative error ε (%)	Prediction data	Relative error ε (%)	Prediction data	Relative error ε (%)
64832.05	64832.05	0.000	64832.05	0.000	64832.05	0.000
71847.09	57476.77	20.001	57476.76	20.001	57600.77	19.829
78646.30	67165.21	14.598	67165.20	14.598	67327.40	14.392
86293.10	78486.76	9.046	78486.75	9.046	78696.50	8.803
93887.95	91716.70	2.312	91716.69	2.312	91985.41	2.026
105557.09	107176.71	1.534	107176.71	1.534	107518.32	1.858
125761.85	125242.71	0.413	125242.70	0.412	125674.17	0.070
143143.63	146353.96	2.243	146353.95	2.242	146895.86	2.621
168850.20	171023.78	1.287	171023.77	1.287	171701.11	1.688
198739.27	199852.01	0.560	199852.00	0.559	200695.05	0.984
245352.80	233539.60	4.815	233539.60	4.814	234584.98	4.389
278541.09	272905.57	2.023	272905.66	2.023	274197.66	1.559
334839.41	318907.39	4.758	318907.38	4.758	320499.45	4.283
386086.72	372663.28	3.477	372663.27	3.476	374619.89	2.970
$\bar{\varepsilon}$ (%)		4.791		4.650		4.677

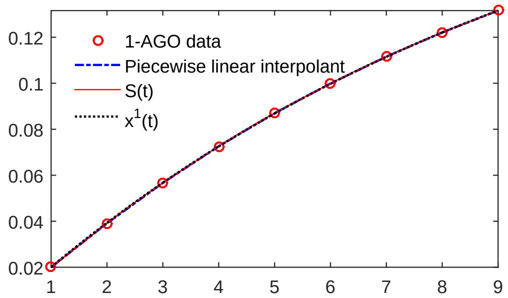

Fig. 4. Graphic results for example 3.

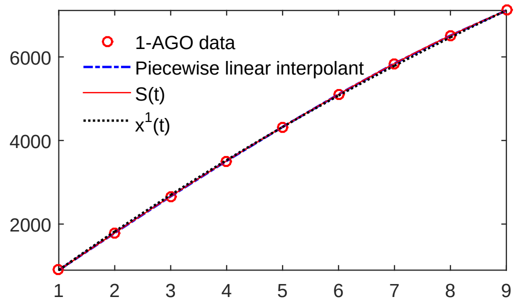

Fig. 5. Graphic results for example 4.

TABLE IV
NUMERICAL RESULTS FOR EXAMPLE 4.

$x^{(0)}$	Classical GM(1,1)		New GM(1,1)	
	Prediction data	Relative error ε (%)	Prediction data	Relative error ε (%)
897	897.00	0.0000	897.00	0.0000
897	939.52	4.7407	939.53	4.7414
890	898.25	0.9275	898.29	0.9320
876	858.80	1.9637	858.86	1.9555
848	821.07	3.1751	821.17	3.1634
814	785.01	3.5616	785.13	3.5463
779	750.53	3.6551	750.67	3.6362
738	717.56	2.7698	717.72	2.7470
669	686.04	2.5470	686.22	2.5749
600	655.90	9.3175	656.10	9.3513
$\bar{\varepsilon}$ (%)		3.2658		3.2648

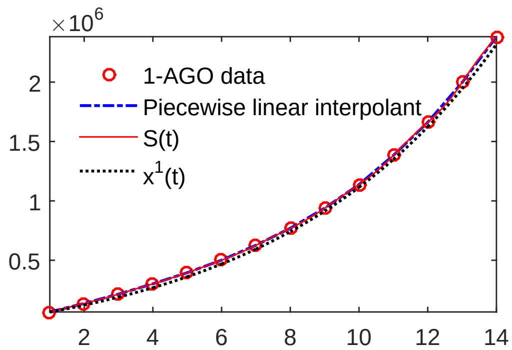

Fig. 6. Graphic results for example 5.

Example 6. *In this example, we consider the non-negative data $x^{(0)}(k)$, $k = 1, 2, \ldots, 7$ given in [23]. Table VI and Fig. 7 give the numerical results.*

Example 7. *In this example, we consider the non-negative data $x^{(0)}(k)$, $k = 1, 2, \ldots, 7$ given in Table VII. And Table VII give the numerical results.*

From the above examples 1-7, we can get the conclusion that the average relative error $\bar{\varepsilon}$ of the new GM(1,1) model is lower than that of the classical GM(1,1) model, which implies that the new GM(1,1) model can improve the quality of the forecasting model.

IV. CONCLUSION

In order to reconstruct the background value, we establish a new GM(1,1) model by using a C^1 monotonicity-

TABLE VII
NUMERICAL RESULTS FOR EXAMPLE 7.

$x^{(0)}$	Classical GM(1,1)		New GM(1,1)		The Model in [24]	
	Prediction data	Relative error ε (%)	Prediction data	Relative error ε (%)	Prediction data	Relative error ε (%)
2.8414	2.8414	0.0000	2.8414	0.0000	2.8414	0.0000
4.9092	4.7726	2.7831	4.7911	2.4061	4.7534	3.1741
6.1411	5.9365	3.3315	5.9630	2.8996	5.9358	3.3418
7.2431	7.3842	1.9468	7.4215	2.4624	7.4124	2.3366
9.0410	9.1849	1.5911	9.2368	2.1648	9.2562	2.3804
11.7205	11.4247	2.5237	11.4960	1.9158	11.5588	1.3803
$\bar{\varepsilon}$ (%)		2.0294		1.9748		2.1022

TABLE VI
NUMERICAL RESULTS FOR EXAMPLE 6.

$x^{(0)}$	Classical GM(1,1)		New GM(1,1)	
	Prediction data	Relative error ε (%)	Prediction data	Relative error ε (%)
21.1	21.1	0.0000	21.1	0.0000
26.6	21.4	19.4566	21.3	19.8219
36.1	32.7	9.3820	32.8	8.8849
52.3	49.9	4.4941	50.7	3.0036
80.1	76.2	4.7842	78.2	2.3247
126.8	116.4	8.1599	120.6	4.8393
196.3	177.8	9.4182	186.0	5.1982
$\bar{\varepsilon}$ (%)		7.9564		6.2961

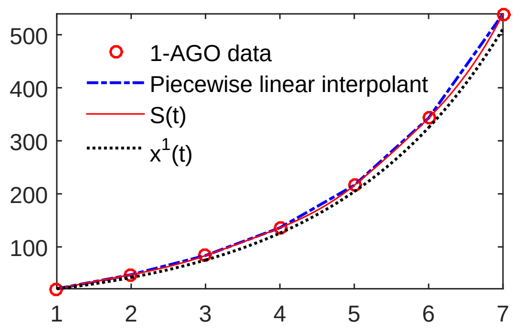

Fig. 7. Graphic results for example 6.

preserving piecewise rational linear/linear interpolation spline. Numerical examples show that the new GM(1,1) model has smaller prediction error than the classical one, especially in these aspects such as reliability and validity of the prediction. And the new model performs better for the original data with convexity in time series. The practical applications of the new GM(1,1) model both in industry and service area will be the next step of the study. And the applications in hotel revenue forecast and tourism market forecasting will be also our future work.

REFERENCES

[1] J.L. Deng, "Control problems of grey systems," *Systems and Control Letters,* vol. 5, pp. 288-294, 1982.

[2] J.L. Deng, "Grey control system," *Journal of Huazhong University of Science and Technology,* vol. 10, no. 3, pp. 9-18, 1982.

[3] G.J. Tan, "The structure method and application of background value in grey system GM(1,1) model (I)," *Systems Engineering-Theory and Practice,* vol. 20, no. 4, pp. 98-103, 2000.

[4] G.J. Tan, "The structure method and application of background value in grey system GM(1,1) model (II)," *Systems Engineering-Theory and Practice,* vol. 20, no. 5, pp. 125-132, 2000.

[5] G.J. Tan, "The structure method and application of background value in grey system GM(1,1) model (III)," *Systems Engineering-Theory and Practice,* vol. 20, no. 6, pp. 70-74, 2000.

[6] J.F. Li and W.Z. Dai, "A new approach of background value-building and its application based on data interpolation and Newton-Cores formula," *Systems Engineering-Theory and Practice,* vol. 24, no. 10, pp. 122-126, 2004.

[7] W.M. Tang and C.H. Xiang, "The improvements of forecasting method in GM(1,1) model based on quadratic interpolation," *Chinese Journal of Management Science,* vol. 14, no. 6, pp. 109-112, 2006.

[8] X.J. Wang, S.L. Yang, J. Ding, and H.J. Wang, "Dynamic GM(1,1) model based on cubic spline for electricity consumption prediction in smart grid," *China Communications,* vol. 7, no. 4, pp. 83-88, 2010.

[9] X.B. Qin, L. Qin, and Q. S. Xu, "C^1 positivity-preserving interpolation schemes with local free parameters," *IAENG International Journal of Computer Science,* vol. 43, no. 2, pp. 219-227, 2016.

[10] X.B. Qin and Q.S. Xu, "C^1 rational cubic/linear trigonometric interpolation spline with positivity-preserving property," *Engineering Letters,* vol. 25, no. 2, pp.152-159, 2017.

[11] H.M. Duan, X.P. Xiao, and L.L. Pei, "Forecasting the short-term traffic flow in the intelligent transportation system based on an inertia nonhomogenous discrete gray model," *Complexity,* vol. 2017, pp. 1-16, 2017.

[12] X. Ma and Z.B. Liu, "Application of a novel time-delayed polynomial grey model to predict the natural gas consumption in China," *Journal of Computational and Applied Mathematics,* vol. 324, pp. 17-24, Nov. 2017.

[13] B.L. Wei, N.M. Xie, and A.Q. Hu, "Optimal solution for novel grey polynomial prediction model," *Applied Mathematical Modelling,* vol. 62, pp. 717-727, 2018.

[14] Z.X. Wang, Q. Li, and L.L. Pei, "A seasonal GM(1,1) model for forecasting the electricity consumption of the primary economic sectors," *Energy,* vol. 154, pp. 522-534, 2018.

[15] Z.X. Wang, Y.G. Dang, and S.F. Liu, "An optimal GM(1,1) based on the discrete function with exponential law," *Systems Engineering-Theory and Practice,* vol. 2, pp. 61-67, 2008.

[16] P.Y. Chen and H.M. Yu, "Foundation settlement prediction based on a novel NGM model," *Mathematical Problems in Engineering,* vol. 2014, pp. 1-8, 2014.

[17] B. Zeng, H.M. Duan, Y. Bai, and W. Meng, "Forecasting the output of shale gas in China using an unbiased grey model and weakening buffer operator," *Energy,* vol. 151, pp. 238-249, 2018.

[18] J.Z. Wang, P. Du, H.Y. Lu, W.D. Yang, and T. Niu, "An improved grey model optimized by multi-objective ant lion optimization algorithm for annual electricity consumption forecasting," *Applied Soft Computing,* vol. 72, pp. 321-337, 2018.

[19] Y.H. Wang, Y.G. Dang, Y.Q. Li, and S.F. Liu, "An approach to increase prediction precision of GM(1,1) model based on optimization of the initial condition," *Expert Systems with Applications,* vol. 37, no. 8, pp. 5640-5644, 2010.

[20] J. Liu, X.P. Xiao, J.H. Guo, and S.H. Mao, "Error and its upper bound estimation between the solutions of GM(1,1) grey forecasting models," *Applied Mathematics and Computation,* vol. 246, pp. 648-660, 2014.

[21] T.L. Tien, "A research on the grey prediction model GM(1,n)," *Applied Mathematics and Computation,* vol. 218, no. 9, pp. 4903-4916, 2012.

[22] Z. He, Y. Shen, J.B. Li, and Y. Wang, "Regularized multivariable grey model for stable grey coefficients estimation," *Expert Systems with Applications,* vol. 42, no. 4, pp. 1806-1815, 2015.

[23] Y.H. Wang, Q. Liu, J.R. Tang, W.B. Cao, and X.Z. Li, "Optimization approach of background value and initial item for improving prediction

precision of GM(1,1) model," *Journal of Systems Engineering and Electronics,* vol. 25, no. 1, pp. 77-82, 2014.

[24] W.Q. Chen and Y.P. Zhu, "A new GM (1, 1) model based on piecewise rational quadratic monotonicity-preserving interpolation spline," *IAENG International Journal of Computer Science*, vol. 47, no. 1, pp. 130-135, 2020.

[25] Z. Liu, L. Zhang, J. Shi, and X.Y. Chen, "Rational linear interpolating spline based on function values," *Journal of Engineering Graphics,* vol. 30, no. 6, pp. 86-90, 2009.