A Time-Varying Scaling Approach to Global Fixed-Time Stabilization of Switched Nonlinear Systems in p-Normal Form

Bing Zhang, Deheng Hou and Yanling Shang

Abstract—**In this paper, we discuss the problem of global fixed-time stabilization(FxTS) for a class of switched nonlinear systems(SNSs) in p-normal form. The constructive design procedure for state feedback control is developed using the addition of the time-varying scaling transform (TVST) and the common Lyapunov function technique. The proposed controller is shown that it drives the states of the closed-loop system (CLS) to zero at any finite time under arbitrary switchings(ASs). Finally, simulation results are given to confirm the effectiveness of the proposed method.**

Index Terms—**Arbitrary switchings(ASs), fixed-time stabilization(FxTS), SNSs in p-normal form, time-varying scaling transform.**

During the past years, switched systems have attracted much attention by extensive applications in various fields, such as, gene regulatory networks, multi-agent systems, mechanical systems, robotic and switching power converters [1]. Different properties, especially analysis and design of switched systems, were considered under arbitrary or constrained switchings, for example refer to [2-4].

By comparison, the stability under ASs is more desirable for switching systems than stability under constrained switchings because of its theoretical and practical importance. Although the existence of a common Lyapunov function for all subsystems is enough to ensure the asymptotic stability of switch systems under ASs [5], finding a common Lyapunov function for SNSs is often difficult and challenging. Fortunately, the typical system structures are sometimes available. Recently, as the important kind of the SNSs, p-normal SNSs has gained considerable attention [6–9].

On the flip side, the research on finite-time control has proved to be popular recently because of the superior properties of finite-time stable system, such as fast response, good robustness and disturbance rejection [10–17]. As for SNSs under ASs, some remarkable results on finite-time stabilization under ASs have been also reported [18–20]. However, the settling time function derived in the finite time control design depends on the initial state of the research system, which prohibits the practical application to some extent. Naturally, the following questions are raised. *For a SNS, is it possible to design a fixed-time stabilizing controller? Under what conditions and how can we achieve this object?* To our best knowledge, these questions have not been well-answered in the literature, which motivates our present work.

Concretely, for a class of SNSs in p-normal form, we first introduce a time-varying scaling transformation (TVST) [21, 22] to converts the original FxTS problem into the asymptotic stabilization problem of transformed system. Then, by employing the based adding a power integrator technique, we develop a common state feedback control design procedure to render the states of CLS to zero in any prescribed finite time under ASs.

Throughout this paper, the following notations are adopted. $\mathbb{R}_+$ denotes the set of all nonnegative real numbers and $\mathbb{R}^n$ denotes the real n-dimensional space. $\mathbb{R}_{\geq 1}$:={$p \in \mathbb{R}_+ \cap [1, +\infty)$. For any $a \in \mathbb{R}_+$ and $x \in \mathbb{R}$, the function $[x]^a$ is defined as $[x]^a = \text{sign}(x)|x|^a$.

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I. Problem Formulation and Preliminaries

Consider the following SNSs in p-normal form

$$\begin{aligned} \dot{x}_i &= [x_{i+1}]^{p_i} + f_{i,\sigma(t)}(\bar{x}_i), \\ i &= 1, \ldots, n-1, \\ \dot{x}_n &= [u_{\sigma(t)}]^{p_n} + f_{n,\sigma(t)}(\bar{x}_n), \end{aligned} \tag{1}$$

where $x = (x_1, \ldots, x_n)^T \in \mathbb{R}^n$ is the state vector, $\bar{x}_i = (x_1, \ldots, x_i)^T$, $i = 1, \ldots, n$. $\sigma(t)\colon [0, +\infty) \to \mathbb{M} = \{1, 2, \ldots, m\}$ is the switching signal. $p_i \in \mathbb{R}_{\geq 1}$, $i = 1, \ldots, n$ are the power orders. For each $k \in \mathbb{M}$, $u_k \in \mathbb{R}$ is the control input of the kth subsystem. For any $k \in \mathbb{M}$ and $i = 1, \ldots, n$, the functions $f_{i,k}\colon \mathbb{R}^i \to \mathbb{R}$ are continuous *w.r.t.* their arguments and satisfy $f(0) = 0$. Moreover, the states of system (1) not jumping at the switching instants is considered here.

The following assumptions are imposed on system (1), which are commonly used in the literature (e.g., [19, 21, 23]).

Assumption 1: The switching signal $\sigma(t)$ has finite number of switching on every bounded time interval.

Assumption 2: For $i = 1, \ldots, n$, $k \in \mathbb{M}$, there exist C^1 functions $\varphi_{i,k}(\bar{x}_i) \geq 0$ such that

$$|f_{i,k}(\bar{x}_i)| \leq \varphi_{i,k}(\bar{x}_i) \sum_{j=1}^{i} |x_j|^{\frac{r_i+\tau}{r_j}}, \tag{2}$$

for $\tau > -1/(\sum_{l=1}^{n} p_1 \cdots p_{l-1})$ and r_i's defined as

$$r_1 = 1,\ p_i r_{i+1} = r_i + \tau,\ i = 1, \ldots, n. \tag{3}$$

Definition 1: [23] Consider

$$\dot{x} = f(t,x), \ \ f(t,0) = 0, \ \ x(0) = x_0 \in \mathbb{R}^n, \tag{4}$$

where $f : \mathbb{R}_+ \times \mathbb{R}^n \to \mathbb{R}^n$. The origin of system (4) is referred to be globally fixed-time stable if it is globally finite-time stable and there exists a bounded settling-time function $T(x_0)$ to make sure that the solution $x(t,x_0)$ of (4) satisfies $x(t,x_0) = 0$, $\forall t \geq T(x_0)$.

Lemma 1: [24] For any x, $y \in \mathbb{R}$, and a constant $a \geq 1$, one has

$$|x+y|^a \leq 2^{a-1}|x^a + y^a|;$$

$$(|x|+|y|)^{1/a} \leq |x|^{1/q} + |y|^{1/a} \leq 2^{(a-1)/a}(|x|+|y|)^{1/a}.$$

Lemma 2: [24] If c, d are positive constants, then for any real-valued function $\delta(u,v) > 0$, one has

$$|u|^c|v|^d \leq \frac{c}{c+d}\delta(u,v)|u|^{c+d} + \frac{d}{c+d}\delta^{-c/d}(u,v)|v|^{c+d}.$$

Lemma 3: [25] If $0 < p \leq 1$ and $a > 0$, then for any $x, y \in \mathbb{R}$, one has

$$|[x]^{ap} - [y]^{ap}| \leq 2^{1-p}|[x]^a - [y]^a|^p.$$

II. Global fixed-time stabilization

In this section, we propose a constructive design mechanism of state feedback controller which can stabilize system (1) within any prescribed finite time $T > 0$. The design procedure is proceeded in two cases: $0 \leq t < T$ and $t \geq T$.

Case 1: $\mathbf{0 \leq t < T}$. Introduce the following TVST

$$z_i = L^{\lambda_i} x_i, \ \ i = 1, \ldots, n, \tag{5}$$

where

$$L = \frac{T}{T-t}$$

and

$$\lambda_i = 1 + p_i\lambda_{i+1}, \ \ i = 1, \ldots, n-1, \ \ \lambda_n = 1.$$

Then, switched system (1) is rewritten as

$$\begin{aligned} \dot{z}_i &= L\left([z_{i+1}]^{p_i} + \tilde{f}_{i,\sigma(t)}(\bar{z}_i)\right), \ \ i = 1, \ldots, n-1, \\ \dot{z}_n &= L\left([u_{\sigma(t)}]^{p_n} + \tilde{f}_{n,\sigma(t)}(\bar{z}_n)\right), \end{aligned} \tag{6}$$

where

$$\tilde{f}_{i,\sigma(t)}(\bar{z}_i) = \lambda_i z_i \frac{\dot{L}}{L^2} + L^{\lambda_i - 1} f_{i,\sigma(t)}(\bar{x}_i), \quad i = 1, \ldots, n, \tag{7}$$

Note that $L(0) = 1$, $L(T) = +\infty$ and $L(t)$ is monotonically increasing, we can check that for $i = 1, \ldots, n$, $k \in \mathbb{M}$, there are smooth functions $\tilde{\varphi}_{i,k}(\bar{z}_i) \geq 0$ such that

$$|\tilde{f}_{i,k}(\bar{z}_i)| \leq \tilde{\varphi}_{i,k}(\bar{z}_i)\sum_{j=1}^{i}|z_j|^{\frac{r_i+\tau}{r_j}}. \tag{8}$$

Then, by choosing ρ $\rho \geq \max_{1\leq i\leq n+1}\{r_i\}$, we introduce

$$\begin{aligned} \xi_1 &= [z_1]^{\frac{\rho}{r_1}}, \ z_{n+1}^* = u, \\ \xi_i &= [z_i]^{\frac{\rho}{r_i}} - [z_i^*]^{\frac{\rho}{r_i}}, \ i = 2, \ldots, n, \end{aligned} \tag{9}$$

where $z_i^* : \mathbb{R}^i \to \mathbb{R}$, $i = 2, \ldots, n+1$ are C^0 functions independent of k to be specified later.

With (5)–(9), a fixed-time state feedback stabilizing controller will be developed for the case of $0 \leq t < T$.

Step 1. Choose a common Lyapunov function candidate as follows:

$$V_1(z_1) = W_1(z_1) = \frac{r_1}{2\rho}|z_1|^{\frac{2\rho}{r_1}}. \tag{10}$$

For every subsystem k of (6), a simple calculation based on (8) gives

$$\begin{aligned} \dot{V}_1 \leq L\Big([\xi_1]^{\frac{2\rho-r_1}{\rho}}&\left([z_2]^{p_1} - [z_2^*]^{p_1}\right) \\ &+ [\xi_1]^{\frac{2\rho-r_1}{\rho}}[z_2^*]^{p_1} + |\xi_1|^{\frac{2\rho+\tau}{\rho}}\tilde{\varphi}_{1,k}\Big). \end{aligned} \tag{11}$$

It is clear that the virtual control

$$z_2^* = -\beta_1^{\frac{r_2}{\rho}}(z_1)[\xi_1]^{\frac{r_2}{\rho}}, \tag{12}$$

with $\beta_1(z_1) \geq \max_{k\in\mathbb{M}}(n+\tilde{\varphi}_{1,k})^{\rho/p_1r_2}$ being a C^1 function independent of k, renders

$$\dot{V}_1 \leq -nL|\xi_1|^{\frac{2\rho+\tau}{\rho}} + L[\xi_1]^{\frac{2\rho-r_1}{\rho}}\left([z_2]^{p_1} - [z_2^*]^{p_1}\right). \tag{13}$$

Step i ($i = 2, \cdots, n$)**.** Assume that at step $j-1$, there exists a common C^1 Lyapunov function V_{i-1} that is positive definite and proper, and a series of C^0 virtual controllers $z_2^*, \ldots, z_i^*$ defined by (9) such that

$$\begin{aligned} \dot{V}_{i-1} \leq &-(n-i+2)L\sum_{j=1}^{i-1}|\xi_j|^{\frac{2\rho+\tau}{\rho}} \\ &+L[\xi_{i-1}]^{\frac{2\rho-r_{i-1}}{\rho}}\left([z_i]^{p_{i-1}} - [z_i^*]^{p_{i-1}}\right). \end{aligned} \tag{14}$$

Then, for step i choose

$$V_i(\bar{z}_i) = V_{i-1}(\bar{z}_{i-1}) + W_i(\bar{z}_i), \tag{15}$$

where

$$W_i(\bar{z}_i) = \int_{z_i^*}^{z_i}\left[[s]^{\frac{\rho}{r_i}} - [z_i^*]^{\frac{\rho}{r_i}}\right]^{\frac{2\rho-r_i}{\rho}}\mathrm{d}s. \tag{16}$$

It can be shown that $V_i(\bar{z}_i)$ is C^1, positive definite and proper [24]. Furthermore, from (14) we have

$$\begin{aligned} \dot{V}_i \leq &-(n-i+2)L\sum_{j=1}^{i-1}|\xi_j|^{\frac{2\rho+\tau}{\rho}} + L[\xi_i]^{\frac{2\rho-r_i}{\rho}}[z_{i+1}]^{p_i} \\ &+L[\xi_{i-1}]^{\frac{2\rho-r_{i-1}}{\rho}}\left([z_i]^{p_{i-1}} - [z_i^*]^{p_{i-1}}\right) \\ &+L[\xi_i]^{\frac{2\rho-r_i}{\rho}}f_{i,k} + L\sum_{j=1}^{i-1}\frac{\partial W_i}{\partial z_j}\left([z_{j+1}]^{p_j} + f_{j,k}\right). \end{aligned} \tag{17}$$

To continue, the following estimates are needed.

First, one derives from Lemmas 2 and 3 that

$$\begin{aligned} [\xi_{i-1}]^{\frac{2\rho-r_{i-1}}{\rho}}&\left([z_i]^{p_{i-1}} - [z_i^*]^{p_{i-1}}\right) \\ &\leq \frac{1}{3}|\xi_{i-1}|^{\frac{2\rho+\tau}{\rho}} + |\xi_i|^{\frac{2\rho+\tau}{\rho}}c_{i,1}, \end{aligned} \tag{18}$$

where $c_{i,1}$ is a positive constant.

Second, from Lemmas 1 and 2, one has

$$[\xi_i]^{\frac{2\rho-r_i}{\rho}}f_{i,k} \leq \frac{1}{3}\sum_{j=1}^{i-1}|\xi_j|^{\frac{2\rho+\tau}{\rho}} + |\xi_i|^{\frac{2\rho+\tau}{\rho}}c_{i,2,k}, \tag{19}$$

where $c_{i,2,k} \geq 0$ are C^1 functions.

Finally, after some tedious but simple calculations based Lemmas 1-3, there holds

$$\begin{aligned}&[\xi_i]^{\frac{2\rho-r_i}{\rho}} f_{i,k} + \sum_{j=1}^{i-1} \frac{\partial W_i}{\partial z_j} \left([z_{j+1}]^{p_j} + f_{j,k}\right) \\ &\quad \le \frac{1}{3} \sum_{j=1}^{i-1} |\xi_j|^{\frac{2\rho+\tau}{\rho}} + |\xi_i|^{\frac{2\rho+\tau}{\rho}} c_{i,3,k},\end{aligned} \tag{20}$$

for C^1 functions $c_{i,3,k} \ge 0$.

By substituting (18) and (20) into (17), one has

$$\begin{aligned}\dot{V}_i \le& -(n-i+1)L \sum_{j=1}^{i-1} |\xi_j|^{\frac{2\rho+\tau}{\rho}} + L[\xi_i]^{\frac{2\rho-r_i}{\rho}} [z_{i+1}^*]^{p_i} \\ &+L[\xi_i]^{\frac{2\rho-r_i}{\rho}} \left([z_{i+1}]^{p_i} - [z_{i+1}^*]^{p_i}\right) \\ &+L|\xi_i|^{\frac{2\rho+\tau}{\rho}} (c_{i,1} + c_{i,2,k} + +c_{i,3,k}).\end{aligned} \tag{21}$$

Therefore, design the common controller z_{i+1}^* as

$$z_{i+1}^* = -\beta_i^{\frac{r_{i+1}}{\rho}} (\bar{z}_i)[\xi_i]^{\frac{r_{i+1}}{\rho}}, \tag{22}$$

where the function $\beta_i(\bar{z}_i)$ is C^1 independent of k and satisfies

$$\beta_i(\bar{z}_i) \ge \max_{k\in\mathbb{M}} (n-i+1+c_{i,1}+c_{i,2,k}++c_{i,3,k})^{\frac{\rho}{p_i r_{i+1}}}. \tag{23}$$

It is obtained that

$$\begin{aligned}\dot{V}_i \le& -(n-i+1)L \sum_{j=1}^{i} |\xi_j|^{\frac{2\rho+\tau}{\rho}} \\ &+L[\xi_i]^{\frac{2\rho-r_i}{\rho}} \left([z_{i+1}]^{p_i} - [z_{i+1}^*]^{p_i}\right).\end{aligned} \tag{24}$$

Step n. Take

$$\begin{aligned}V_n(z) &= \sum_{j=1}^{n} W_j(\bar{z}_j) \\ &= \frac{r_1}{2\rho}|z_1|^{\frac{2\rho}{r_1}} + \sum_{j=2}^{n} \int_{z_j^*}^{z_j} \left[[s]^{\frac{\rho}{r_j}} - [z_j^*]^{\frac{\rho}{r_j}}\right]^{\frac{2\rho-r_j}{\rho}} \mathrm{d}s.\end{aligned} \tag{25}$$

Following the above inductive steps, at this step there exists a continuous common state feedback controller

$$u = z_{n+1}^* = -\beta_n^{\frac{r_{n+1}}{\rho}} (\bar{z}_n)[\xi_n]^{\frac{r_{n+1}}{\rho}}, \tag{26}$$

such that

$$\dot{V}_n \le -L \sum_{j=1}^{n} |\xi_j|^{\frac{2\rho+\tau}{\rho}} \le -\sum_{j=1}^{n} |\xi_j|^{\frac{2\rho+\tau}{\rho}}. \tag{27}$$

Thus, it is completed the controller design for the case of $0 \le t < T$.

Case 2: t ≥ T. The common state feedback controller u can be simply taken as $u = 0$.

Theorem 1: For SNS (1) with Assumptions 1 and 2, there exists a switched common state feedback controller of form

$$u = \begin{cases} -\beta_n^{\frac{r_{n+1}}{\rho}} (\bar{z}_n)[\xi_n]^{\frac{r_{n+1}}{\rho}}, & 0 \le t < T, \\ 0, & t \ge T, \end{cases} \tag{28}$$

such that the origin of the CLS is globally fixed-time stable under ASs within given finite settling time T.

Proof. From (9), Lemma 4.3 in [26] and the fact that $V_n(z)$ is positive definite and proper, one knows that $\mathcal{K}_\infty$-class functions π_1, π_2 and π_3 exist such that

$$\pi_1(|z|) \le V_n(z) \le \pi_2(|z|), \tag{29}$$

$$\dot{V}_n \le -\pi_3(|z|). \tag{30}$$

which indicate that $z(t)$ is asymptotically convergent and is bounded on $[0, T)$.

On the other hand, the TVST (5) gives

$$x_i(t) = L^{-\lambda_i}(t) z_i(t) = \left(\frac{T-t}{T}\right)^{\lambda_i} z_i(t), \quad i = 1, \ldots, n. \tag{31}$$

Consequently, it is obtained that

$$\lim_{t\to T} x_i(t) = \lim_{t\to T} \left(\frac{T-t}{T}\right)^{\lambda_i} z_i(t) = 0, \quad i = 1, \ldots, n. \tag{32}$$

By the solution properties of existence and continuation, one has $x(T) = 0$. Furthermore, by $f_{i,k}$'s vanishing at the origin and $u(t) = 0$, $\forall\ t \ge T$, it is concluded that $x(t) = 0$ $\forall\ t \ge T$ and

$$\begin{cases} |x(t)| \le |z(t)| \le |z(0)| = |x(0)|, & 0 \le t < T, \\ 0 = |x(t)| \le |x(0)|, & t \ge T. \end{cases} \tag{33}$$

From Definition 1, it is obvious that the origin of the CLS is globally fixed-time stable under ASs within given finite settling time T. Therefore, the proof is completed.

III. Simulation Study

Consider a CSTR system with two modes feed stream as in [27], whose dynamic can be described as

$$\begin{aligned}\dot{C}_A &= \frac{q_\sigma}{V}(C_{Af_\sigma} - C_A) + a_0 \exp\left(-\frac{E}{RT}\right) C_A, \\ \dot{T} &= \frac{q_\sigma}{V}(T_{f_\sigma} - T) + a_1 \exp\left(-\frac{E}{RT}\right) C_A + a_1(T_c - T),\end{aligned} \tag{34}$$

where $\sigma(t) : [0, +\infty) \to \{1, 2\}$ and the physical meaning of the parameters for this system can be referred to [27]. Letting $x_1 = C_A - C_A^*$ and $x_2 = T - T^*$, after some simple calculations, the system can be expressed as

$$\dot{x}_1 = x_2 + f_{1,\sigma(t)}(x_1), \quad \dot{x}_2 = u, \tag{35}$$

with $f_{1,1}(x_1) = 0.5x_1$ and $f_{1,2}(x_1) = 2x_1$. For the above system, it is readily verified that all the assumptions are satisfied with $\tau = 0$, $\varphi_{1,1} = 0.5$, $\varphi_{1,2} = 2$ and $r_1 = r_2 = r_3 = 1$. By introducing the TVST $z_1 = L^2 x_1$ and $z_2 = Lx_2$, the system (35) is transformed into

$$\dot{z}_1 = L\left(z_2 + \tilde{f}_{i,\sigma(t)}(z_1)\right), \quad \dot{z}_2 = Lu, \tag{36}$$

where $\tilde{f}_{1,\sigma(t)}(z_1) = 2z_1\dot{L}/L^2 + Lf_{1,\sigma(t)}(x_1)$, which satisfies (8) with $\tilde{\varphi}_{1,1} = (2/T) + 0.5$, and $\tilde{\varphi}_{1,2} = (2/T) + 2$. Taking $\rho = 1$, by the design procedure in Section III, we can design a switched common fixed-time controller for system (35) as

$$u = \begin{cases} -\beta_2(\bar{z}_2)\xi_2, & 0 \le t < T, \\ 0, & t \ge T, \end{cases} \tag{37}$$

where $\xi_2 = z_2 + \beta_1 z_1$, $\beta_1 = (3.1 + 2/T)$ and $\beta_2 = 0.1 + \beta_1 + 0.5\beta_1^2(1 + \tilde{\varphi}_{1,2}^2)$.

By setting the prescribed time $T = 2$ and three cases of initial conditions: (i) $(x_1(0), x_2(0)) = (-1, 2)$, (ii) $(x_1(0), x_2(0)) = (-5, 10)$ and (iii) $(x_1(0), x_2(0)) = (-10, 20)$, Fig. 1 is obtained to exhibit the state responses of the CLS. From this Figure, we can clearly observe that the CSTR system can be stabilized in a given prescribed time independently of the initial conditions, which demonstrates the effectiveness of the control method proposed in this paper.

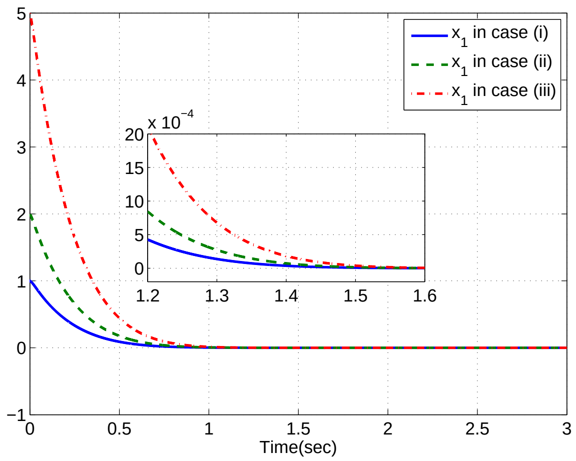

Fig. 1. Curves of the state x_1.

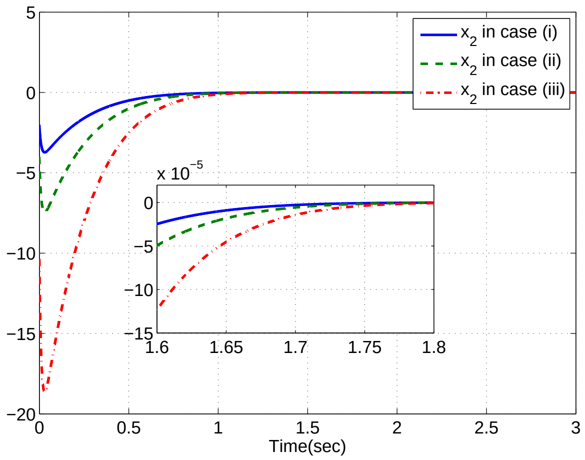

Fig. 2. Curves of the state x_2.

IV. Conclusion

In this paper, a novel TVST has been found to convert the original fixed-time stabilization problem into the asymptotic stabilization problem of transformed one, and a constructive solution to fixed-time stabilization of SNSs in p-normal form has also been provided. Some related problems remain open, e.g., for system (1), whether an output feedback stabilizing controller can be designed? For system (1) with state or input delay, can a stabilizing controller be designed under the similar assumptions? Recently, the time-varying powers case has been investigated for non-switched nonlinear systems. Thereupon, a naturally arose issue is to address the fixed-time stabilization when the SNSs contain the time-varying powers.

References

[1] H. Yang, B. Jiang, V. Cocquempot, and H. Zhang, "Stabilization of switched nonlinear systems with all unstable modes: application to multi-agent systems," *IEEE Transactions on Automatic Control*, vol. 56, no. 9, pp. 2230–2235, 2011.

[2] R. Ma, J. Fu, and T. Chai, "Dwell-time-based observer design for unknown input switched linear systems without requiring strong detectability of subsystems," *IEEE Transactions on Automatic Control*, vol. 62, no. 8, pp. 2230–2235, 2017.

[3] R. Ma and J. Zhao, "Backstepping design for global stabilization of switched nonlinear systems in lower triangular form under arbitrary switchings," *Automatica*, vol. 46, no.11, pp. 1819–1823, 2010.

[4] X. Zhao, X. Zheng, B. Niu, and L. Liu, "Adaptive tracking control for a class of uncertain switched nonlinear systems," *Automatica*, vol. 52, pp. 185–191, 2015.

[5] D. Liberzon, Switching in systems and control, Brikhauser, Boston, 2003.

[6] R. Ma, Y. Liu, S. Zhao, M. Wang, and G. Zong, "Global stabilization design for switched power integrator triangular systems with different powers," *Nonlinear Analysis: Hybrid Systems*, vol. 15, pp. 74–85, 2015.

[7] X. Lin, X. Li, C.C. Chen, and S. Li, "Smooth output feedback stabilization for a class of high-order switched nonlinear systems," *Nonlinear Analysis: Hybrid Systems*, vol. 29, pp. 34–53, 2018.

[8] X. Zhao, X. Wang, G. Zong, and X. Zheng, "Adaptive neural tracking control for switched high-order stochastic nonlinear systems," *IEEE Transactions on Cybernetics*, vol. 47, no. 10, pp. 3088–3099, 2017.

[9] S. Li, C. Ahn, and Z. Xiang, "Sampled-data adaptive output feedback fuzzy stabilization for switched nonlinear systems with asynchronous switching," *IEEE Transactions on Fuzzy Systems*, vol. 27, no. 1, pp. 200–205, 2019.

[10] S. P. Bhat and D. S. Bernstein, "Finite-time stability of continuous autonomous systems," *SIAM Journal on Control and Optimization*, vol. 38, no. 3, pp. 751–766, 2000.

[11] X. Huang, W. Lin, and B. Yang, "Global finite-time stabilization of a class of uncertain nonlinear systems," *Automatica*, vol. 41, no. 5, pp. 881–888, 2005.

[12] Z.Y. Sun, L.R. Xue, and K. Zhang, "A new approach to finite-time adaptive stabilization of high-order uncertain nonlinear system," *Automatica*, vol. 58, pp. 60–66, 2015.

[13] Y. L. Shang, J. C. Huang, X. X. Shi, and X. C. Zhu, "Global finite-time Stabilization for a class of upper-triangular systems subject to input saturation," *Engineering Letters*, vol. 28, no. 1, pp. 16–21, 2020.

[14] Y. L. Shang, J. C. Huang, H. S. Li, and X. W. Wen, "Finite-time control design for nonholonomic mobile robots subject to spatial constraint," *IAENG International Journal of Applied Mathematics*, vol. 48, no. 4, pp. 449–454, 2018.

[15] F. Z. Gao, X. C. Zhu, J. C. Huang, and X. L. Wen, "Finite-time state feedback stabilization for a class of uncertain high-order nonholonomic feedforward systems," *Engineering Letters*, vol. 27, no. 1, pp. 108–113, 2019.

[16] Y. L. Shang and J. C. Huang, "Fixed-time stabilization of spatial constrained wheeled mobile robot via nonlinear mapping," *IAENG International Journal of Applied Mathematics*, vol. 50, no. 4, pp791–796, 2020.

[17] Y. L. Shang, W. J. Tang, J. C. Huang, and X. X. Shi, "Global finite-time stabilization of stochastic nonlinear systems via output feedback," *Engineering Letters*, vol.

29, no. 1, pp. 82–86, 2021.
[18] Y. J. Liang, R. Ma, M. Wang, and J. Fu, "Global finite-time stabilisation of a class of switched nonlinear systems," *Int. J. Syst. Sci.*, vol. 46, no. 16, pp. 2897–2904, 2015.
[19] J. Fu, R. Ma, and T. Chai, "Global finite-time stabilization of a class of switched nonlinear systems with the powers of positive odd rational numbers," *Automatica*, vol. 54, pp. 360–373, 2015.
[20] M. Cai and Z. Xiang, "Global adaptive finite-time stabilization of a class of switched nonlinear systems with parametric uncertainty," *International Journal of Adaptive Control And Signal Processing*, vol. 30, no. 7, pp. 1002–1018, 2016.
[21] F. Gao, Y. Wu, and Z. Zhang, "Global fixed-time stabilization of switched nonlinear systems: a time-varying scaling transformation approach," *IEEE Transactions on Circuits and Systems II: Express Briefs*, vol. 66, no. 11, pp. 1890–1894, 2019.
[22] D. Hou and F. Gao, "Global prescribed-time controller design for p-normal switched nonlinear systems with dead-zone inputs," *IEEE Access*, vol. 9, pp. 11272–11281, 2021.
[23] A. Polyakov, "Nonlinear feedback design for fixed-time stabilization of linear control systems," *IEEE Trans. Autom. Control*, vol. 57, no. 8, pp. 2106–2110, 2012.
[24] C. Qian and W. Lin, "A continuous feedback approach to global strong stabilization of nonlinear systems," *IEEE Transactions on Automatic Control*, vol. 46, no. 7, pp. 1061–1079, 2001.
[25] S. Ding, W.H. Chen, K. Mei, and D. Murray-Smith, "Disturbance observer design for nonlinear systems represented by input-output models," *IEEE Transactions on Industrial Electronics*, vol. 67, no. 2, pp. 1222–1232, 2020.
[26] H. K. Khalil, *Nonlinear System*, 3rd ed. New Jersey: Prentice-Hall, 2002.
[27] M. B. Yazdi and M.R. Jahed-Motlagh, "Stabilization of a CSTR with two arbitrarily switching modes using modal state feedback linearization," *Chemical Engineering Journal*, vol. 155, no. 3, 838–843 2009.