Credible Delta-Gamma-Normal Value-at-Risk for European Call Option Risk Valuation

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Abstract—**This paper formulates a new risk measure called as credible delta-gamma-normal Value-at-Risk (CredDGN). CredDGN is a generalization of credible Value-at-Risk (CredVaR), which determines risk by combining CredVaR with delta-gamma-normal VaR. This novel method is proposed as an appropriate tool for measuring European call option portfolio risk because it considers the nonlinear dependence of the market risk factors that determine a European call option value based on the Black-Scholes Formula. We apply this method to evaluate simulated financial data representing the profit/loss of several assets over ten investment periods. The new method is also utilized to analyze the risk of a portfolio composed of the active stocks which trade the options. Based on Kupiec's backtesting results, the performance of CredDGN effectively measures the risk of an option portfolio at 80%, 90%, and 95% confidence levels even when the profit/loss (*P/L*) is non-normally distributed.**

Index Terms—**nonlinear, greek, taylor-approximation, derivative, portfolio.**

I. Introduction

VALUE AT RISK (VaR) has rapidly become a standard quantitative benchmark for measuring the risk exposure of a portfolio. VaR gives an upper bound of the potential loss of a portfolio at a determined time horizon and confidence level, whereby a higher confidence level indicates a smaller probability that this loss may be exceeded [1].

VaR calculation of an option requires information about the profit/loss (P/L) of options that are not available on the capital market [2]. In addition, unlike stock prices, which are linearly dependent on market risk factors, the value of an option has a nonlinear dependence on the market risk factor [3]. This nonlinearity emerges in portfolios that include options, so VaR cannot be calculated directly from the market risk factors [4].

Many methods have been developed for determining nonlinear VaR, such as delta-normal VaR and delta-gamma-normal VaR. Delta-normal VaR uses the first-order Taylor series. In contrast, delta-gamma-normal VaR using a quadratic approximation of the asset value [5] utilizes the second-order Taylor series to approximate the P/L of the underlying assets [6]. Both methods have been studied by many researchers, including Mina and Ulmer [7], Britten-Jones and Schaefer [8], Feuerverger and Wong [9], Castellacci and Siclari [10], Cui, Zhu, Sun, and Li [11], Date and Bustreo [5], Ortiz-Gracia and Oosterlee [4], and Wang, Xie, Jiang, Wu, and He [12]. However, previous studies have not discussed nonlinear VaR from the credibility theory perspective, so there will be no consideration of the shared-risk concept among assets constructing a portfolio.

The credibility theory introduced by Bühlmann [13] combines two concepts, namely individual risk and group risk, which are useful in determining the credibility of insurance premiums. Credibility theory has been utilized in the insurance and financial field. In insurance, credibility theory has been applied in numerous studies. One of them was executed by Diao and Weng [14]. Meanwhile, in financial research, credibility theory has been used to develop a risk measure of a portfolio in terms of the fuzzy/nonfuzzy concept such as Chen, Liu, and Chen [15], Georgescu and Kinnunen [16], Vercher and Bermudez [17], Wang, Chen, and Liu [18], Pitselis [19] and Liu et al. [20]. Nevertheless, these researches omitted to explore derivative assets, such as options.

In Pitselis [21], credibility theory was used to explain how quantiles can be incorporated into the classical credibility model developed by Bühlmann [13]. Pitselis [19] then presented a novel risk measure, known as credible Value at Risk (CredVaR), which combines the credibility methodology with the popular risk measure, VaR. Pitselis [19] claimed that CredVaR is more informative than VaR because it can capture the risk of an individual insurer's contract (or the return of an individual asset). Moreover, this measure can capture the portfolio risk of some similar (but not identical) contracts (or the returns of a portfolio that comprises similar assets), which are gathered in a portfolio to share the risk. Credibility theory has been developed by many researchers in finance applications, but the application did not focus on the option risk measure. Therefore, the risk measures have not considered the nonlinear dependency aspects between the value of derivative assets such as options and their risk factors in the capital market.

The availability of a risk measurement tool for option portfolios is increasingly important because options significantly impact a portfolio's performance in respect of controlling risk in the market [22]. For these reasons, we propose a method for measuring European call option portfolio risk that incorporates delta-gamma-normal VaR under the Black-Scholes Formula with credible VaR.

The paper is organized as follows. In Section II, we present the theoretical aspects utilized to develop the proposed VaR model. In the third section, we introduce a new risk-measurement model, namely credible delta-gamma normal Value-at-Risk (CredDGN). In Section IV, we apply

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the proposed model to option P/L data generated under the normality assumption. In Section V, we utilize the model to measure the risk of options traded on the financial market. Finally, in Section VI, we give a conclusion and comments for future research.

II. Nonlinear Value-at-Risk using Delta-Gamma-Normal Approach

Option is a financial derivative product that enables the investor to conduct an effective risk management [23]. The European call option value at time t, expressed by a multivariable function, $C_t = f(S_t, K, r, t, \sigma)$ can be written as the following equation:

$$C_t = S_t N(d_1) - Ke^{-r(T-t)} N(d_2), \tag{1}$$

where $S_t, K, r, t,$ and σ are respectively asset price at time t, exercise price, the interest rate of a free risk asset, the expiration date, and asset volatility [24]. Meanwhile, $d_1 = \dfrac{ln(S_t/K) + (T-t)(r + \sigma^2/2)}{\sigma\sqrt{(T-t)}}$ and $d_2 = \dfrac{ln(S_t/K) + (T-t)(r - \sigma^2/2)}{\sigma\sqrt{(T-t)}} = d_1 - \sigma\sqrt{(T-t)}$ [25].

Based on Equation (1), various indicators are utilized to estimate option risk. The method used to estimate these risks is formulated using the Greeks. One of the Greeks is delta, δ, which measures the change in the option value/price in response to a change in the underlying asset price.

Delta, δ, for the Black–Scholes European call option can be written as in Equation (2).

$$\delta = N(d_1) + \frac{1}{\sigma\sqrt{T-t}} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}d_1^2}[A], \tag{2}$$

where $A = S_t - Ke^{-r(T-t)+d_1\sigma\sqrt{T-t}-\frac{1}{2}\sigma^2(T-t)}$. The power of the exponential in Equation (2), namely $-r(T-t) + d_1\sigma\sqrt{T-t} - \frac{1}{2}\sigma^2(T-t)$ can be expressed as $\ln(S_t/K)$. Therefore, the following equation can be obtained [26]:

$$\delta = N(d_1) + \frac{1}{\sigma\sqrt{T-t}} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}d_1^2}[B] = N(d_1), \tag{3}$$

where $B = S_t - Ke^{\ln(\frac{S_t}{K})}$.

Besides the delta Greek δ, there is also gamma Greek, γ, that measures the risk of a change in a delta option due to an asset-value change. Gamma Greek γ for the European call option can be expressed as in Equation (4) [26]:

$$\gamma = \frac{\partial \delta}{\partial S_t} \equiv \frac{\partial^2 C_t}{\partial S_t^2} = \frac{\partial N(d_1)}{\partial S_t} = N^{'}(d_1)\frac{1}{S_t\sigma\sqrt{T-t}}. \tag{4}$$

Below, we utilize delta δ and γ Greeks, respectively expressed in Equations (3) and (4), to formulate the nonlinear credible VaR using the delta-gamma-normal approach. By assuming that option value, C_t in Equation (1) is influenced only by its underlying asset while K, r, t and σ are constants, the value of an option can be expressed as

$$C_t \approx f(S_t). \tag{5}$$

Some assumptions must be made to measure the risk of an option using VaR based on a delta-gamma-normal approximation. The first assumption is that there is a nonlinear relationship between the P/L of a stock and a change in the option price [6]. The second assumption is that the P/L of the stock underlying the option has a normal distribution with volatility $\sigma_{\triangle S_{t+\triangle t}}$ and a zero mean [6].

On this basis, the nonlinear VaR using the delta-gamma-normal approach will approximate the risk of an option by considering the changing value of its underlying asset (in this research, we focus on stocks). The delta-gamma-normal approach (DGNA) requires a second-order Taylor series to approach a P/L option value around S_t as follows [26]:

$$\begin{aligned} \triangle C_t &\approx f(S_t + \triangle S_t) - f(S_t) \\ &\approx \left(\frac{\partial C_t}{\partial S_t}\right) \triangle S_t + \frac{1}{2}\left(\frac{\partial^2 C_t}{\partial S_t^2}\right)(\triangle S_{t+\triangle t})^2. \end{aligned} \tag{6}$$

$\left(\dfrac{\partial C_t}{\partial S_t}\right)$ and $\left(\dfrac{\partial^2 C_t}{\partial S_t^2}\right)$ are expressed respectively in Equations (3) and (4) as the delta Greek (δ) and gamma Greek (γ) of an option, so Equation (6) can be written as shown in Equation (7) below:

$$\triangle C_{t+\triangle t} \approx \delta \triangle S_{t+\triangle t} + \frac{\gamma}{2}(\triangle S_{t+\triangle t})^2. \tag{7}$$

We can write VaR based on a delta-gamma-normal approximation, called by delta-gamma-normal VaR in Equation (8) as follows [6]

$$VaR_{DG} \approx Z_\alpha \sigma_{\triangle S_{t+\triangle t}} \sqrt{\delta^2 + \frac{\gamma^2 \sigma^2_{\triangle S_{t+\triangle t}}}{4}}. \tag{8}$$

III. Credible Delta-Gamma-Normal Value-at-Risk for Option Risk Measurement

In this section, we briefly explain credibility theory and credible VaR. The concept of credibility theory was developed by Bühlmann [13] and Bühlmann and Straub [27]. Bühlmann Credibility is a model used in credibility theory to determine the premium of an insurance product. A claim made by the policyholder can be considered as a risk that must be anticipated by insurance assessor. In an effort to anticipate a bad situation (i.e., the number of claims is higher than the available funds owned by assessor), Bühlmann [13] created Bühlmann Credibility as a tool that can be utilized to estimate the claims for the next period.

In 2016, Pitselis [19] developed a new risk measure that combines the credibility theory concept proposed by Bühlmann [13] with a popular existing risk measurement tool used in the financial and insurance fields, i.e., VaR. This new risk measure tool is known as credible VaR.

In this paper, we use the concept developed by Pitselis [19] to propose a new risk-measurement technique for stock-option portfolios, which has not been addressed in other research papers. This novel risk-measurement approach, inspired by Pitselis [19], is derived by combining credible

VaR with nonlinear VaR, namely delta-gamma-normal VaR, which has been proven to perform well in approximating nonlinear VaR in financial research.

If we consider a portfolio of several assets, let $X_j^{'} = (X_{1,j}, X_{2,j}, \ldots, X_{n,j})$ as the observed P/L of a stock option $j = 1, 2, \ldots, m$ in period $i = 1, 2, \ldots, n$. Then, let v acts as a risk variable that explains the risk characteristics of j stock option P/L, which is unobservable. Next, we introduce a random vector $\varpi_j^{'}$ that represents the delta-gamma-normal VaR, $\eta_{i,j}$, of the j^{th} asset in period $i = 1, 2, \ldots, n$ where $j = 1, 2, \ldots, m$ is expressed as follows:

$$\varpi_j^{'} = (\eta_{1,j} \ldots, \eta_{n,m}). \tag{9}$$

Then, a random variable $\eta_{1,j} \ldots, \eta_{n,m}$ is assumed to be identically distributed with mean $E(\eta_{i,j}) = \mu$ and variance $Var(\eta_{i,j}) = \sigma^2$. Moreover, the risk of the stock-option P/L in a portfolio is represented by a random variable ϑ, which has an unknown distribution. A random variable $\eta_{1,j} \ldots, \eta_{n,m}$ is assumed to be conditionally independent and identically distributed (i.i.d) for a fixed ϑ with:

$$E(\eta_{i,j} \mid \vartheta = v) = \mu(v) \tag{10}$$

and

$$Var(\eta_{i,j} \mid \vartheta = v) = \tau(v) \tag{11}$$

for $i = 1, 2, \ldots, n$.

Next, by adopting credible VaR based on Bühlmann's Credibility for risk estimation, the VaR of the j^{th} asset in the next period is estimated: i.e., the delta-gamma-normal VaR (CredDGN), represented by Ψ_{DG}.

Theorem 1. Based on the assumptions mentioned above, a linear estimator of CredDGN of the j^{th} asset constituting a portfolio can be written as follows:

$$\Psi_{DG}(v) = E[\eta_j] Z_{DG} + (1 - Z_{DG})\mu(v), \tag{12}$$

where $E[\eta_j]$ is the mean of delta-gamma-normal VaR for the j^{th} asset during the entire observed period; $\mu(v)$ is the entire portfolio of the delta-gamma-normal-VaR mean; and Z_{DG} is the credible risk factor of CredDGN, which is defined as follows:

$$Z_{DG} = \frac{nVar(\mu(v))}{E[\tau(v)] + nVar[\mu(v)]}, \tag{13}$$

where $Var[\mu(v)]$ is the variance of the delta-gamma-normal VaR mean that explains the portfolio heterogeneity and $E[\tau(v)]$ is the mean of the delta-gamma-normal VaR variance reflecting all the variability of a portfolio.

Risk estimation of an asset using CredDGN requires more information, namely $E[\eta_j], \mu(v), E[Var(\eta_{i,j} \mid v)] = E[\tau(v)]$, and $Var[E(\eta_{i,j} \mid v)] = Var[\mu(v)]$. Based on the previous assumptions in which the distribution of a random variable ϑ is unknown, then $\mu(v), E[\tau(v)]$, and $Var[\mu(v)]$ are estimated by the mean sample formula of the data.

TABLE I
RECURSIVE PROCESS OF CREDDGN DERIVATION

Stage	Process
1	Determine the number of assets (m) constructing a portfolio.
2	Determine the observed period $n, n \geq 2$.
3	Calculate the estimated delta-gamma-normal VaR, $\eta_{i,j}$, for the j^{th} asset at period i, where $i = 1, 2, \ldots, n$ and $j = 1, 2, \ldots, m$.
4	Calculate the estimated mean of delta-gamma-normal VaR for the j^{th} asset, during n observed period, $\widehat{E[\eta_j]}$.
5	Calculate the estimated mean of delta-gamma-normal VaR for the m assets over n observed periods, symbolized by $\widehat{E[\mu(v)]}$.
6	Calculate the estimated mean of delta-gamma-normal-VaR variance, $\widehat{E[\tau(v)]}$.
7	Calculate the estimated variance of delta-gamma-normal-VaR mean, $\widehat{Var[\mu(v)]}$.
8	Calculate the estimated value of $n\widehat{Var[\mu(v)]}$.
9	Calculate the estimated risk credibility factor, Z_{DG}, for each asset.
10	Calculate the estimated value of CredDGN, $\Psi_{DG}(v)$.

The estimators will be a good estimator of $E[\eta_j], \mu(v), E[\tau(v)]$, and $Var[\mu(v)]$ when they are unbiased. The unbiased estimator of $E[\eta_j], \mu(v), E[\tau(v)]$, and $Var[\mu(v)]$ are given successively by Equation (14), Equation (15), Equation (16), and Equation (17).

$$\widehat{E[\eta_j]} = \frac{1}{n}\sum_{i=1}^{n} \eta_{i,j}, \tag{14}$$

$$\widehat{\mu(v)} = \frac{1}{mn}\sum_{j=1}^{m}\sum_{i=1}^{n} \eta_{i,j}, \tag{15}$$

$$\widehat{E[\tau(v)]} = \frac{1}{m(n-1)}\sum_{i=1}^{n}\sum_{j=1}^{m} (\eta_{i,j} - \bar{\eta_j})^2, \tag{16}$$

$$\widehat{Var[\mu(v)]} = \frac{1}{m-1}\sum_{j=1}^{m} (\bar{\eta_j} - \widehat{E[\tau(v)]})^2 - \frac{\widehat{E[\tau(v)]}}{n}. \tag{17}$$

This paragraph gives a recursive process that includes several stages to derive CredDGN and credible factors. The recursive process is given in Table I.

IV. Application of Credible Delta-Gamma-Normal Value-at-Risk using Data Simulation

Herein, we apply the theory presented in the previous section to evaluate five P/L portfolios. Portfolio I comprised two assets (asset A and asset B); Portfolio II comprised three assets (asset A, asset B, and asset C); Portfolio III comprised by four assets (asset A, asset B, asset C, and asset D); Portfolio IV comprised five assets (asset A, asset B, asset C, asset D, and asset E); and Portfolio V comprised five assets (asset B, asset C, asset D, asset E, and asset F). The P/L data were generated ten times using a simulation procedure to obtain ten periods of delta-gamma-normal VaR

TABLE II
$\sigma_{\triangle S_{t+\triangle t}}$, $\triangle t$, AND S_t OF EACH ASSET

Asset	$\sigma_{\triangle S_{t+\triangle t}}$	S_t	K
A	0.5	10	8
B	1	20	10
C	1.5	25	7
D	2	12	4
E	2.5	15	6
F	3	17	9

for each asset. These ten-period data were regarded as ten-year data fabricated to meet the assumptions for the delta-gamma-normal VaR approximation.

Based on the assumption that the P/L of the options' underlying assets were normally distributed with zero mean and volatility $\sigma_{\triangle S_{t+\triangle t}}$, we determined the zero mean and standard deviation of each asset for each period, as shown in Table II. Table II also provides the specified asset price at time t (S_t), time to maturity $(\triangle t)$, and exercise price (K) of each asset for each period needed to calculate the CredDGN. The interest rate of risk-free assets (r) was 0.0175 for each asset.

To estimate CredDGN for each asset with a determined confidence level (cl) for each period (period one until period ten), we used a recursive process comprising Stage 1 until Stage 10, which are listed in Table I. Delta-gamma-normal VaR for asset A, B, C, D, and E with a 95% confidence level for each period (period one until period ten), are calculated by Equation (8) and presented in Table III. Then, the delta-gamma-normal VaR for confidence levels of 99%, 90%, and 80% for the five assets can be determined using the same procedure. Delta-gamma-normal VaR for asset F with specified confidence levels can be counted by the same equation, but the calculation results are not given in this paper for brevity.

From Table III, we can interpret and conclude that the estimated potential losses with a 95% confidence level in the first period within a day holding period of asset A, asset B, asset C, asset D, and asset E were 0.835 dollars, 1.695 dollars, 2.522 dollars, 3.052 dollars, and 3.835 dollars, respectively, relative to its asset price on the preceding day. Delta-gamma-normal VaR for asset A, asset B, asset C, asset D, and asset E for 99%, 90%, and 80% confidence levels can be calculated and interpreted analogously. The calculation results of delta-gamma-normal VaR for the three determined confidence levels are also given in Table III while the estimators of the three parameters required in the CredDGN calculation for each asset in the portfolio, namely μ, τ, α, and Z_{DG} at the specified confidence levels, are counted and presented in Table IV. Table IV shows that $\widehat{Z_{DG}}$ for the analyzed portfolios is closer to one. Therefore, each asset's CredDGN VaR constructing the five portfolios tends to be equal with the estimated mean of delta-gamma-normal VaR for each asset at the corresponding confidence levels. The three parameters that are calculated based on Equation (15), Equation (16), and Equation (17) were utilized to estimate the three parameters required in the CredDGN calculation for each asset in the portfolio. The estimators of μ, τ, and α, at Portfolio I, Portfolio II, Portfolio III, Portfolio IV, and Portfolio V for 99%, 95%, 90%, and 80% confidence levels are summarized in Table IV. Based on these results, we can summarize that the average of the maximum potential loss at 80%, 90%, 95%, and 99% confidence levels (cl) for each asset in Portfolio I were, respectively, 0.634 dollars, 0.965 dollars, 1.238 dollars, and 1.751 dollars. In addition,

TABLE III
ESTIMATED DELTA-GAMMA-NORMAL VAR OF EACH ASSET

cl=80%

Period (i)	$\eta_{i,A}$	$\eta_{i,B}$	$\eta_{i,C}$	$\eta_{i,D}$	$\eta_{i,E}$
1	0.427	0.868	1.290	1.562	1.962
2	0.411	0.868	1.223	1.648	1.918
3	0.410	0.872	1.159	1.652	1.989
4	0.418	0.859	1.316	1.563	2.113
5	0.409	0.854	1.166	1.702	1.929
6	0.415	0.800	1.245	1.639	2.191
7	0.424	0.899	1.317	1.807	2.016
8	0.404	0.877	1.317	1.738	1.995
9	0.401	0.846	1.295	1.594	2.058
10	0.398	0.813	1.385	1.782	2.251
$\widehat{E[\eta_j]}$	0.412	0.855	1.271	1.669	2.042

cl=90%

Period (i)	$\eta_{i,A}$	$\eta_{i,B}$	$\eta_{i,C}$	$\eta_{i,D}$	$\eta_{i,E}$
1	0.650	1.321	1.965	2.378	2.988
2	0.625	1.321	1.863	2.510	2.921
3	0.624	1.328	1.764	2.515	3.025
4	0.637	1.308	2.003	2.381	3.217
5	0.622	1.299	1.775	2.592	2.937
6	0.632	1.218	1.896	2.497	3.336
7	0.646	1.368	2.005	2.751	3.069
8	0.615	1.335	2.006	2.646	3.038
9	0.610	1.288	1.972	2.428	3.133
10	0.606	1.238	2.109	2.714	3.428
$\widehat{E[\eta_j]}$	0.627	1.303	1.936	2.541	3.109

cl=95%

Period (i)	$\eta_{i,A}$	$\eta_{i,B}$	$\eta_{i,C}$	$\eta_{i,D}$	$\eta_{i,E}$
1	0.835	1.695	2.522	3.052	3.835
2	0.803	1.696	2.391	3.222	3.748
3	0.800	1.705	2.391	3.228	3.882
4	0.817	1.679	2.571	3.055	4.129
5	0.799	1.668	2.278	3.327	3.770
6	0.812	1.564	2.899	3.205	4.282
7	0.829	1.756	2.574	3.531	3.940
8	0.789	1.714	2.575	3.396	3.899
9	0.783	1.653	2.531	3.116	4.022
10	0.778	1.589	2.707	3.484	4.399
$\widehat{E[\eta_j]}$	0.805	1.672	2.531	3.262	3.991

cl=99%

Period (i)	$\eta_{i,A}$	$\eta_{i,B}$	$\eta_{i,C}$	$\eta_{i,D}$	$\eta_{i,E}$
1	1.181	2.398	3.566	4.317	5.424
2	1.135	2.398	3.381	4.556	5.301
3	1.132	2.411	3.203	4.565	5.491
4	1.156	2.374	3.637	4.321	5.840
5	1.129	2.359	3.222	4.705	5.332
6	1.148	2.212	3.442	4.532	6.057
7	1.173	2.484	3.640	4.994	5.572
8	1.117	2.424	3.641	4.803	5.514
9	1.107	2.339	3.579	4.407	5.689
10	1.101	2.247	3.828	4.927	6.222
$\widehat{E[\eta_j]}$	1.138	2.365	3.514	4.613	5.644

TABLE IV
ESTIMATORS OF $\widehat{\mu(v)}, \widehat{E[\tau(v)]}, \widehat{Var[\mu(v)]}$ AND Z_{DG} AT THE SPECIFIED CONFIDENCE LEVELS

	cl	$\widehat{\mu(v)}$	$\widehat{E[\tau(v)]}$	$\widehat{Var[\mu(v)]}$	$\widehat{Z_{DG}}$
Portfolio I	99%	1.751	0.004	0.752	0.999
	95%	1.238	0.002	0.376	0.999
	90%	0.965	0.001	0.228	0.999
	80%	0.634	0.000	0.098	0.999
Portfolio II	99%	2.339	0.016	1.410	0.999
	95%	1.669	0.013	0.744	0.999
	90%	1.288	0.005	0.428	0.999
	80%	0.846	0.002	0.185	0.999
Portfolio III	99%	2.907	0.026	2.232	0.999
	95%	2.067	0.017	1.129	0.999
	90%	1.611	0.008	0.677	0.999
	80%	1.052	0.003	0.292	0.999
Portfolio IV	99%	3.455	0.039	3.169	0.999
	95%	2.452	0.023	1.586	0.999
	90%	1.903	0.012	0.962	0.999
	80%	1.249	0.005	0.415	0.999
Portfolio V	99%	4.640	0.050	3.328	0.999
	95%	3.290	0.028	1.645	0.999
	90%	2.556	0.015	1.010	0.999
	80%	1.679	0.007	0.436	0.999

the mean of delta-gamma-norma-VaR variance at 80%, 90%, 95%, and 99% confidence levels for each asset in Portfolio I were 0.000, 0.001, 0.002, and 0.004 respectively, while the variances of the delta-gamma-normal-VaR mean for each asset at Portfolio I at 80%, 90%, 95%, and 99% confidence levels were 0.098, 0.228, 0.376, and 0.752, respectively.

Next, using the information from the preceding tables, the estimated CredDGN risk factors, which were calculated by Equation (12) and Equation (13) for Portfolio I, Portfolio II, Portfolio III, Portfolio IV, and Portfolio V were, respectively, 0.999, 0.999, 0.999, 0.999, and 0.999. The CredDGN values at the 99%, 95%, 90%, and 80% confidence levels for the four assets in Portfolio I-V are listed in Table V.

A method of risk measurement is regarded to become well-specified when it meets the required properties of theoretical statistics [28]. Whether or not a method fulfills this requirement can be assigned by verifying the proportion of the P/L values of assets that are greater than the VaR values of the proposed method. In this section, we analyze the performance of the CredDGN method using Kupiec backtesting [29]. The backtesting method involves the examination of how many times a risk measurement is exceeded over a given time interval. Table VI presents the backtesting results of CredDGN for Portfolio I, Portfolio II, Portfolio III, and Portfolio IV, for the determined confidence levels where NOL is Number of Outliers and POL is Percentage of Loss. The backtesting results of CredDGN for Portfolio V is not presented here for brevity.

As evident in Table VI, the P-values of the CredDGN method were greater than (1-cl) at the specified confidence levels. So, CredDGN can be regarded as a well-specified risk measure.

TABLE V
CREDDGN OF EACH ASSET

Portfolio I

	A	B			
$\Psi_{DG,99\%}$	1.138	2.364			
$\Psi_{DG,95\%}$	0.805	1.672			
$\Psi_{DG,90\%}$	0.627	1.302			
$\Psi_{DG,80\%}$	0.412	0.855			

Portfolio II

	A	B	C		
$\Psi_{DG,99\%}$	1.139	2.365	3.513		
$\Psi_{DG,95\%}$	0.806	1.672	2.530		
$\Psi_{DG,90\%}$	0.628	1.303	1.935		
$\Psi_{DG,80\%}$	0.412	0.855	1.271		

Portfolio III

	A	B	C	D	
$\Psi_{DG,99\%}$	1.140	2.365	3.513	4.611	
$\Psi_{DG,95\%}$	0.807	1.673	2.530	3.260	
$\Psi_{DG,90\%}$	0.628	1.303	1.935	2.540	
$\Psi_{DG,80\%}$	0.412	0.856	1.271	1.668	

Portfolio IV

	A	B	C	D	E
$\Psi_{DG,99\%}$	1.141	2.366	3.514	4.611	5.642
$\Psi_{DG,95\%}$	0.807	1.673	2.531	3.260	3.989
$\Psi_{DG,90\%}$	0.628	1.303	1.935	2.540	3.108
$\Psi_{DG,80\%}$	0.412	0.856	1.271	1.668	2.041

Portfolio V

	B	C	D	E	F
$\Psi_{DG,99\%}$	2.368	3.516	4.613	5.643	7.061
$\Psi_{DG,95\%}$	1.675	2.532	3.262	3.990	4.992
$\Psi_{DG,90\%}$	1.305	1.937	2.541	3.109	3.890
$\Psi_{DG,80\%}$	0.857	1.272	1.669	2.041	2.554

V. APPLICATION OF CREDIBLE DELTA-GAMMA-NORMAL VALUE-AT-RISK USING REAL FINANCIAL DATA

In this section, CredDGN was applied to Portfolio I, Portfolio II, Portfolio III, and Portfolio IV. The four portfolios contained real financial data from the active stocks at https://finance.yahoo.com/ on July 23^{rd} 2020: namely, General Electric Company (GE), Advanced Micro Devices Inc (AMD), Ford Motor Company (F), Bank of America Corp (BAC), The Coca-Cola Company (KO), and Walmart Inc. (WMT). Portfolio I comprised KO and WMT; Portfolio II comprised GE, AMD, and F; Portfolio III comprised AMD, BAC, KO, and WMT, while Portfolio IV comprised AMD, BAC, GE, and F. The analyzed data ranged from July 23^{rd} 2010 to July 23^{rd} 2020. Then, the data were divided into ten periods of risk analysis. These ten-period data tabulated in Table VII cover ten-year daily data.

Before conducting the analysis, the ten-period data of stocks' close prices were transformed into P/L data. The summary statistics of the P/L are given in Table VIII.

After that, the estimations of delta-gamma-normal VaR using Equation (8) and its mean for each asset with confidence levels 0.80, 0.90, 0.95, and 0.99 for each period

TABLE VI
RESULTS OF KUPIEC BACKTESTING FOR CREDDGN

	Asset	cl(%)	NOL	POL	P-Value
Portfolio I	A	80	516	20.476	0.266
	B	80	472	18.730	0.943
	A	90	244	9.682	0.688
	B	90	236	9.365	0.848
	A	95	118	4.683	0.752
	B	95	114	4.524	0.854
	A	99	21	0.833	0.766
	B	99	23	0.913	0.622
Portfolio II	A	80	516	20.476	0.266
	B	80	472	18.730	0.943
	C	80	495	19.643	0.663
	A	90	243	9.643	0.712
	B	90	236	9.365	0.848
	C	90	245	9.722	0.664
	A	95	117	4.643	0.779
	B	95	114	4.524	0.854
	C	95	112	4.444	0.893
	A	99	21	0.833	0.766
	B	99	23	0.913	0.622
	C	99	23	0.913	0.622
Portfolio III	A	80	516	20.476	0.266
	B	80	472	18.730	0.943
	C	80	495	19.643	0.662
	D	80	510	20.238	0.371
	A	90	243	9.643	0.712
	B	90	236	9.365	0.848
	C	90	245	9.722	0.664
	D	90	263	10.437	0.221
	A	95	117	4.643	0.779
	B	95	114	5.524	0.854
	C	95	112	4.444	0.893
	D	95	125	4.960	0.513
	A	99	20	0.794	0.826
	B	99	23	0.913	0.622
	C	99	23	0.913	0.622
	D	99	26	1.032	0.386
Portfolio IV	A	80	516	20.476	0.266
	B	80	472	18.730	0.943
	C	80	495	19.643	0.662
	D	80	510	20.238	0.371
	E	80	516	20.476	0.265
	A	90	243	9.643	0.712
	B	90	236	9.365	0.848
	C	90	245	9.722	0.664
	D	90	263	10.437	0.221
	E	90	241	9.563	0.756
	A	95	116	4.603	0.806
	B	95	114	5.524	0.854
	C	95	112	4.444	0.893
	D	95	124	4.921	0.549
	E	95	119	4.722	0.721
	A	99	20	0.794	0.826
	B	99	23	0.913	0.622
	C	99	23	0.913	0.622
	D	99	26	1.032	0.386
	E	99	23	0.913	0.622

TABLE VII
DATA PERIODS

Period	Duration
1	1 July 2010-30 June 2011
2	1 July 2011-30 June 2012
3	1 July 2012-30 June 2013
4	1 July 2013-30 June 2014
5	1 July 2014-30 June 2015
6	1 July 2015-30 June 2016
7	1 July 2016-30 June 2017
8	1 July 2017-30 June 2018
9	1 July 2018-30 June 2019
10	1 July 2019-30 June 2020

TABLE VIII
DATA PERIODS

Stock	Min	Mean	Max	Skewness	Kurtosis
GE	-1.452	-0.003	2.673	0.166	5.084
AMD	-6.690	0.017	4.930	-0.383	24.320
BAC	-3.779	0.004	3.650	-0.567	15.049
F	-2.520	-0.002	1.010	-0.883	9.749
KO	-5.049	0.008	3.110	-1.176	15.935
WMT	-10.670	0.028	12.500	0.704	30.526

TABLE IX
MEAN, $\sigma_{\triangle S_{t+\triangle t}}$, S_t, AND K OF EACH REAL ASSET'S PROFIT/LOSS

Asset	Mean	$\sigma_{\triangle S_{t+\triangle t}}$	S_t	K
GE	-0.003	0.289	7.060	1
AMD	0.017	0.624	61.790	30
BAC	0.004	0.388	24.310	15
F	-0.002	0.215	6.840	1
KO	0.007	0.478	48.160	39.5
WMT	0.028	1.041	130.70	65

were counted. The calculation of the estimated delta-gamma-normal VaR for ten periods for the five stocks required a long recursive process (listed in Table I) to be completed manually, so we built an R program to alleviate the complex computation.

The estimations of the delta-gamma-normal VaR, counted using the information on Table IX, for GE, AMD, BAC, KO, and WMT for the ten periods at 80% confidence level are summarized in Table X. Meanwhile, the delta-gamma-normal VaR of the five assets for 90%, 95%, and 99% confidence levels are also given in Table X and can be counted using the same procedures as used for obtaining the delta-gamma-normal VaR at an 80% confidence level. For brevity, the estimation of delta-gamma-normal VaR for F is not presented in Table X.

Next, prominent information included in Table X was utilized to estimate the three parameters required in the CredDGN calculation for each asset in Portfolios I, II, III, and IV. The estimators of $\widehat{\mu(v)}$, $\widehat{E[\tau(v)]}$, and $\widehat{Var[\mu(v)]}$ for Portfolios I, II, III, and IV are written in Table XI.

Based on Table XI, it can be summarized that the average of maximum potential losses at 80% confidence level of each asset for Portfolio I, Portfolio II, Portfolio III, and Portfolio IV, respectively, were 0.566 dollars, 0.257 dollars, 0.445

TABLE X
ESTIMATED DELTA-GAMMA-NORMAL VAR OF EACH ASSET

cl=80%

Period (i)	$\eta_{i,GE}$	$\eta_{i,AMD}$	$\eta_{i,KO}$	$\eta_{i,BAC}$	$\eta_{i,WMT}$
1	0.207	0.156	0.203	0.223	0.355
2	0.255	0.185	0.316	0.244	0.561
3	0.200	0.106	0.339	0.168	0.561
4	0.202	0.092	0.269	0.175	0.462
5	0.243	0.075	0.329	0.183	0.695
6	0.307	0.125	0.343	0.262	0.813
7	0.205	0.367	0.264	0.263	0.571
8	0.271	0.308	0.304	0.336	1.133
9	0.218	0.850	0.415	0.364	0.948
10	0.273	3.954	0.733	0.698	1.706
$\widehat{E[\eta_j]}$	0.238	0.357	0.352	0.291	0.781

cl=90%

Period (i)	$\eta_{i,GE}$	$\eta_{i,AMD}$	$\eta_{i,KO}$	$\eta_{i,BAC}$	$\eta_{i,WMT}$
1	0.315	0.238	0.310	0.339	0.541
2	0.388	0.281	0.481	0.371	0.854
3	0.305	0.161	0.517	0.256	0.855
4	0.307	0.140	0.409	0.266	0.703
5	0.370	0.113	0.502	0.279	1.058
6	0.467	0.190	0.523	0.398	1.238
7	0.312	0.557	0.402	0.401	0.869
8	0.412	0.469	0.463	0.511	1.725
9	0.332	1.292	0.631	0.554	1.443
10	0.416	1.988	1.116	1.063	2.598
$\widehat{E[\eta_j]}$	0.362	0.543	0.535	0.444	1.189

cl=95%

Period (i)	$\eta_{i,GE}$	$\eta_{i,AMD}$	$\eta_{i,KO}$	$\eta_{i,BAC}$	$\eta_{i,WMT}$
1	0.404	0.305	0.397	0.435	0.695
2	0.499	0.361	0.617	0.476	1.096
3	0.392	0.207	0.663	0.328	1.097
4	0.394	0.179	0.525	0.341	0.903
5	0.476	0.146	0.644	0.358	1.358
6	0.599	0.243	0.671	0.511	1.589
7	0.401	0.715	0.516	0.514	1.116
8	0.529	0.603	0.594	0.656	2.215
9	0.426	1.659	1.432	0.711	1.853
10	0.534	2.551	0.687	1.364	3.335
$\widehat{E[\eta_j]}$	0.465	0.697	0.687	0.570	1.526

cl=99%

Period (i)	$\eta_{i,GE}$	$\eta_{i,AMD}$	$\eta_{i,KO}$	$\eta_{i,BAC}$	$\eta_{i,WMT}$
1	0.572	0.432	0.562	0.615	0.982
2	0.705	0.511	0.873	0.674	1.550
3	0.554	0.292	0.938	0.465	1.552
4	0.558	0.254	0.742	0.482	1.278
5	0.673	0.206	0.911	0.507	1.921
6	0.848	0.344	0.949	0.723	2.248
7	0.567	1.011	0.730	0.728	1.578
8	0.749	0.853	0.840	0.928	3.132
9	0.603	2.346	1.146	1.006	2.620
10	0.756	3.608	2.026	1.929	4.717
$\widehat{E[\eta_j]}$	0.658	0.986	0.972	0.806	2.158

dollars, and 0.266 dollars, relative to the asset price at the previous day. In addition, the means of the delta-gamma-normal-VaR variance at an 80% confidence level for each asset at Portfolio I, Portfolio II, Portfolio III, and Portfolio IV, respectively, were 0.091 dollars, 0.056 dollars, 0.092 dollars, and 0.048 dollars, and the variances of delta-gamma-normal-VaR mean at 80% confidence level for Portfolio I, Portfolio II, Portfolio III, and Portfolio IV subsequently were 0.083, 0.003, 0.042, and 0.001. The estimators of $\widehat{\mu(v)}, \widehat{E[\tau(v)]}$, and $\widehat{Var[\mu(v)]}$ would be utilized to count an estimated risk factor of CredDGN, namely $\widehat{Z_{DG}}$, based on Equation (13). From the latter calculation, we can infer that the estimated risk factor, $\widehat{Z_{DG}}$, for Portfolio I, Portfolio II, Portfolio III, and Portfolio IV, respectively, were 0.902, 0.327, 0.818, and 0.175. The estimators of $\widehat{\mu(v)}, \widehat{E[\tau(v)]}, \widehat{Var[\mu(v)]}$, and $\widehat{Z_{DG}}$ for the other confidence levels (90%, 95%, and 99%) are written in Table XI. Moreover, when we compare the estimation of loss for every asset using delta-gamma-normal VaR given in Table X and $\widehat{\mu(v)}$ in Table XI, it can be noted that the estimation of maximum potential loss using CredDGN for the two assets in Portfolio I at specified confidence levels tended to approach the estimated mean of delta-gamma-normal VaR for every asset. This is caused by an estimated risk factor of CredDGN, $\widehat{Z_{DG}}$, at 0.902, which is relatively high. So, based on Equation (12) in Theorem 1, the weight given to $\widehat{\mu(v)}$ is smaller than the weight given to the mean of delta-gamma-normal VaR for each asset over the ten periods. This tendency can also be observed for Portfolio III with a risk factor of CredDGN, at 0.818. The relatively large value of the credibility factor (0.902 in Portfolio I and 0.818 in Portfolio III) at the 90% confidence level, in Portfolio I and III is because of the estimated variance of large delta-gamma-normal VaR expectations. In other words, the variance between assets in the portfolio is large enough so that it is considered the risk of assets in the corresponding portfolio is different.

TABLE XI
ESTIMATORS OF $\widehat{\mu(v)}, \widehat{E[\tau(v)]}, \widehat{Var[\mu(v)]}$ AND Z_{DG} AT THE SPECIFIED CONFIDENCE LEVELS

cl	$\widehat{\mu(v)}$	$\widehat{E[\tau(v)]}$	$\widehat{Var[\mu(v)]}$	$\widehat{Z_{DG}}$
Portfolio I				
99%	1.565	0.692	0.634	0.902
95%	1.106	0.346	0.317	0.902
90%	0.862	0.210	0.192	0.902
80%	0.566	0.091	0.083	0.902
Portfolio II				
99%	0.712	0.426	0.020	0.327
95%	0.503	0.213	0.010	0.327
90%	0.392	0.129	0.006	0.327
80%	0.257	0.056	0.003	0.327
Portfolio III				
99%	1.230	0.707	0.318	0.818
95%	0.869	0.354	0.159	0.818
90%	0.678	0.215	0.097	0.818
80%	0.445	0.092	0.042	0.818
Portfolio IV				
99%	0.735	0.367	0.008	0.175
95%	0.520	0.183	0.004	0.175
90%	0.405	0.111	0.002	0.175
80%	0.266	0.048	0.001	0.175

Conversely, for Portfolio II and IV, there are relatively small risk factors of CredDGN, at 0.327 and 0.175. The estimation of

TABLE XII
CREDDGN OF EACH ASSET

Portfolio I	CredDGN	KO	WMT		
	$\Psi_{DG,80\%}$	0.373	0.759		
	$\Psi_{DG,90\%}$	0.567	1.156		
	$\Psi_{DG,95\%}$	0.728	1.484		
	$\Psi_{DG,99\%}$	1.030	2.099		
Portfolio II	Asset	GE	AMD	F	
	$\Psi_{DG,80\%}$	0.251	0.289	0.231	
	$\Psi_{DG,90\%}$	0.382	0.441	0.352	
	$\Psi_{DG,95\%}$	0.491	0.566	0.452	
	$\Psi_{DG,99\%}$	0.694	0.801	0.639	
Portfolio III	Asset	BAC	AMD	KO	WMT
	$\Psi_{DG,80\%}$	0.319	0.373	0.369	0.719
	$\Psi_{DG,90\%}$	0.486	0.567	0.561	1.096
	$\Psi_{DG,95\%}$	0.624	0.728	0.720	1.406
	$\Psi_{DG,99\%}$	0.883	1.030	1.019	1.989
Portfolio IV	Asset	BAC	AMD	F	GE
	$\Psi_{DG,80\%}$	0.270	0.282	0.250	0.261
	$\Psi_{DG,90\%}$	0.412	0.429	0.381	0.398
	$\Psi_{DG,95\%}$	0.528	0.551	0.489	0.510
	$\Psi_{DG,99\%}$	0.747	0.779	0.692	0.722

maximum potential loss using CredDGN for the assets constructing Portfolio II and IV at specified confidence levels tended to approach the estimated mean of delta-gamma-normal VaR for the entire portfolio. In simple terms, we can summarize that the variance between assets in Portfolio II and IV is quite small, so it can be presumed that the risks for these assets are almost similar. Therefore, greater weight is assigned to $\widehat{\mu(v)}$ as the overall delta-gamma-normal average of the VaR assets in Portfolio II and IV. Furthermore, the small credibility risk factor results from a small variance estimation of the delta-gamma-normal VaR mean.

Next, using the precedential information, CredDGN and its risk factor of the j^{th} asset for the four portfolios at several confidence levels can be calculated directly using Equation (12) and Equation (13), and the results are given in Table XII. The information in Table XII suggests that maximum potential losses for the holder of Portfolio II in the next period of investment, when Portfolio II was held for one day with confidence level 80% for asset GE, AMD, and F successively, were 0.251 dollars, 0.289 dollars, and 0.231 dollars. From Table XII, it can be inferred that the greater the confidence level value, the greater the CredDGN value. Consequently, the greater specified level of confidence (cl), the greater the portfolio risk that must be tackled of the investors. In addition, the greater the level of confidence (cl) that is specified, the greater the allocation of capital required to cover losses from the investment.

In this section, we also analyze the performance of the CredDGN method using Kupiec backtesting, as proposed by Kupiec [29]. Table XIII presents the backtesting results of CredDGN for Portfolio I, Portfolio II, Portfolio III, at the determined confidence levels, where NOL is Number of Outliers and POL is Percentage of Loss. The backtesting results of CredDGN for Portfolio IV at the determined confidence levels is not presented here for brevity.

The results of Kupiec backtesting in Table XIII show that CredDGN performance at 80%, 90%, and 95% confidence levels was effective in measuring the risk. Meanwhile CredDGN's performance at a 99% confidence level was not effective in measuring the risk for this case. The results shown in Table XIII are in accord with Date and Bustreo [5], who stated that confidence levels higher than 95% are inappropriate for risk measures based on Gaussian

TABLE XIII
RESULTS OF KUPIEC BACKTESTING FOR CREDDGN

	Asset	cl(%)	NOL	POL	P-Value
Portfolio I	KO	80	322	12.803	1
	WMT	80	297	11.809	1
	KO	90	175	6.958	1
	WMT	90	152	6.044	1
	KO	95	98	3.897	0.995
	WMT	95	103	4.095	0.981
	KO	99	45	1.789	0.000
	WMT	99	47	1.869	0.000
Portfolio II	GE	80	332	13.201	1
	AMD	80	167	6.640	1
	F	80	238	9.463	1
	GE	90	171	6.799	1
	AMD	90	70	2.783	1
	F	90	105	4.175	1
	GE	99	37	1.471	0.740
	AMD	99	8	0.318	1
	F	99	19	0.755	0.874
Portfolio III	BAC	80	292	11.587	1
	AMD	80	105	4.167	1
	KO	80	333	13.214	1
	WMT	80	321	12.738	1
	BAC	90	151	5.992	1
	AMD	90	33	1.309	1
	KO	90	175	6.944	1
	WMT	90	168	6.667	1
	BAC	95	97	3.849	0.996
	AMD	95	13	0.516	1
	KO	95	98	3.889	0.995
	WMT	95	116	4.603	0.806
	BAC	99	46	1.825	0.000
	AMD	99	4	0.159	0.999
	KO	99	47	1.865	0.000
	WMT	99	59	2.341	0.000
Portfolio IV	BAC	80	357	14.195	1
	AMD	80	293	11.650	1
	F	80	219	8.708	1
	GE	80	332	13.201	1
	BAC	90	187	7.435	1
	AMD	90	199	7.913	1
	F	90	99	3.936	1
	GE	90	171	6.799	1
	BAC	95	134	5.328	0.210
	AMD	95	143	5.686	0.054
	F	95	62	2.465	1
	GE	95	88	3.499	1
	BAC	99	64	2.545	0.000
	AMD	99	98	3.897	0.000
	F	99	14	0.557	0.989
	GE	99	35	1.392	0.024

distribution.

VI. Conclusion

In this paper, we combine the quantile credibility developed by Pitselis [19] with nonlinear Value-at-Risk, namely delta-gamma-normal VaR, to obtain a new risk measure for European call option portfolios. The performance of the new method is evaluated using Kupiec backtesting, and the results indicate that CredDGN can be utilized as an alternative to measuring both the option portfolio risk and the risk of an individual option in a portfolio comprising similar asset P/L. Based on the results of the analysis, the new risk measure is also effective in measuring the risk of P/L assets, which are non-normally distributed. We hope that CredDGN can give a significant mark and can initiate other credible delta-gamma VaR based on nonnormal distribution.

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