Determination of the Weights for the Ultimate Cross Efficiency Using Expert Scoring Method

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***Abstract*—The theoretical research on the DEA (data envelopment analysis) cross-efficiency evaluation mainly focused on how to select one unique set of inputs and outputs weights among multiple sets of optimal weights for each decision making unit (DMU), but payed little attention to how to aggregate efficiencies in cross-efficiency matrix. The commonly used aggregation method is to aggregate them with equal weights without considering their difference. This paper deals with how to aggregate efficiencies in cross-efficiency matrix reasonably and proposes the use of expert scoring method to aggregate them. This method views n efficiency values of each DMU in cross-efficiency matrix as its efficiency scores determined by different experts. Due to the distinction of different experts on the education background and work experience and other aspects, their efficiency scores to DMUs should be treated differently and allocated variant weights in the final overall assessment. The weight allocated to efficiency scores determined by one given expert is determined by its dissimilarity level to other experts. If the dissimilarity level of one given expert to other ones is huge, his authority will be questioned and it will be reasonable to allocate small weight to his efficiency scores. Finally, the numerical examples examine the validity of the proposed method.**

***Index Terms*—DEA Cross-efficiency evaluation, Cross-efficiency aggregation, Expert scoring method**

I. INTRODUCTION

DEA cross-efficiency evaluation proposed by Sexton et al. [1] can discriminate decision making units (DMUs) effectively. Unlike a DEA traditional model which is a self-appraisal model, DEA cross-efficiency model adopts both peer-evaluation and self-evaluation. DEA cross-efficiency evaluation will provide each DMU one self-evaluated efficiency value generated by itself favorable weights and $n-1$ peer-evaluated efficiency values produced by the favorable weights of other DMUs. Then all these n efficiency values for each DMU can be averaged into a value. In most cases, the DMUs can obtain unique rank orders through the final values. Due to its powerful discrimination ability, it has been widely applied in DMUs rank and selection issues such as the rank and choice of R&D projects [2] and so on.

However, each DMU probably has multiple optimal weights in a DEA traditional model and non-uniqueness of optimal weights damages the use of DEA cross-efficiency. To solve this problem, Sexton et al. [1] introduced secondary goal model. Inspired by this idea, many secondary goal models have been proposed to determine the weights uniquely for each DMU. The selection criteria of secondary goal models are different. The rank models [3, 4] consider that the DMUs will pay more attention to their rank orders than their individual scores. The weight-balanced model [5] focused on how to reduce the number of zero weights in inputs and outputs weights. Interested readers can acquire more secondary goal models in Wang and Chin [6], Wang, Chin and Jiang [7], Wang, Chin, and Luo [8], Liang et al. [9], Lim [10]. Among all the secondary goal models, the aggressive and benevolent models proposed by Doyle and Green [11] are most commonly used. When DMU uses aggressive model to select the unique set of inputs and outputs weights among multiple sets of optimal weights, the selected weights are hostile to other DMUs and minimize their average cross-efficiency value. The benevolent model adopts opposite selection strategy.

In DEA cross-efficiency evaluation research, how to determine the weights uniquely has been given considerable attention, but how to aggregate efficiencies in

Manuscript submitted December 15, 2020; revised February 1,2021. This work was substantially supported by Xi'an University of Finance and Economics Yanta Scholars Support Project.

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cross-efficiency matrix has been given little attention. The widely used method is to aggregate them with equal weights. Although this way is easy to calculate, it ignores the difference among efficiencies and that will result in the final aggregation result to be unreasonable [12]. Our literature review reveals that only few scholars adopt non-equal weight methods to aggregate them. Wu et al. [13-16] considered that average cross-efficiency determined by equal weights encounters a significant shortcoming that it is not a Pareto solution. To overcome this flaw, they determine the aggregation weights through the nucleolus solution and Shapley values in cooperative game and Shannon entropy. Except for the abstract reason that average cross-efficiency is not a Pareto solution, Wu's methods did not provide any concrete reasons to illustrate why we need to use non-equal weights to aggregate them [12]. Lianlian Song and Fan Liu [17] discovery that the aggregation weights generated by Shannon entropy [16] will break Zeleny's rule that if the scores of all evaluated objects on one given attribute are very similar, then such an attribute will be not important for decision maker because it cannot make some significant contribution to make a decision. So, it is reasonable to allocate a small weight to this attribute when making a decision [18]. To address this issue, they propose a variance coefficient method based on the Shannon entropy [17] for cross-efficiency aggregation. However, it still does not give a concrete reason to illustrate why using non-equal weights to aggregate efficiencies in cross-efficiency matrix. Wang and Chin [19] proposed the use of order weighted averaging (OWA) operator weights for cross-efficiency aggregation, which introduced the decision maker's optimism level to allocate the weights between self-evaluated and peer-evaluated efficiencies. The shortfalls of this method are that aggregation weights are determined by the DM's optimism level (characterized by orness degree α), and different DM's optimism level will lead to different results [16]. Moreover, it is difficult to measure the actual optimism level (orness degree value) of decision maker. YM Wang and S Wang [12] view the n efficiencies in cross-efficiency matrix for each DMU as results to be evaluated by n sets of inputs and outputs weights determined by n DMUs. They stated that n sets of inputs and outputs weights are from different points of view, so the cross-efficiencies generated by them should be allocated variant weights. Based on this idea, they propose three approaches which are weighted least-square dissimilarity approach, weighted least-square deviation approach, dissimilarity and deviation integrated approach [12] respectively for cross-efficiency aggregation. However, the n sets of inputs and outputs weights are selected by the same secondary goal model such as aggressive formulation or benevolent formulation. Each secondary goal model has fixed modeling idea, for example aggressive formulation's view is to minimize the average cross-efficiency value of other DMUs while keeping efficiency of DMU under evaluation at its CCR efficiency level. So n sets of inputs and outputs weights selected by the same secondary goal model are from the same viewpoint. That means the modeling idea of approaches proposed by YM Wang and S Wang [12] is not reasonable. Moreover, the three approaches usually generate different aggregation results and at same time they are very similar on modeling idea and other aspects. Each approach does not have distinct advantages to other ones resulting in that the three approaches have a significant drawback that it is not convenient for decision maker to use.

To overcome these flaws, we propose the use of expert scoring method for cross-efficiency aggregation. This method views the target DMUs in cross-efficiency matrix as external experts, so the efficiencies of each DMU in cross-efficiency matrix will be viewed as efficiency scores of each DMU determined by experts. Since the distinction of different experts on education background and work experience and other aspects, their efficiency scores to DMUs should be treated differently and allocated different weights. We use Euclidean distance in cluster analysis to measure the dissimilarity level of each expert to other experts. The weight allocated to efficiency scores of one given expert to DMUs is determined by his dissimilarity level to other experts. If the dissimilarity level of one given expert to other experts is huge, his authority will be questioned and it will be reasonable to allocate a small weight for his efficiency scores.

The remainder of paper is arranged as follows: The introduction to DEA cross-efficiency evaluation is shown in Section 2. Section 3 presents the details of the use of the expert scoring method for cross-efficiency aggregation. Section 4 uses two illustrative examples to present the potential applications of expert scoring method in cross-efficiency aggregation. Section 5 makes a conclusion of the paper.

II. Cross-Efficiency Evaluation and Aggregation

We suppose that n DMUs are needed to be evaluated and each DMU consumes m inputs to produce s outputs. The inputs and outputs values of $DMU_j\,(j=1,\cdots,n)$ are denoted by $x_{ij}\,(i=1,\ldots,m)$ and $y_{rj}\,(r=1,\ldots,s)$. The efficiency value of DMU_k under CCR model (a DEA traditional model) can be measured by the following model [1]:

$$\begin{aligned}
&Maximize \quad \theta_{kk}=\sum_{r=1}^{s}u_{rk}y_{rk}\\
&Subject\ to \quad \sum_{i=1}^{m}v_{ik}x_{ik}=1\\
&\qquad \sum_{r=1}^{s}u_{rk}y_{rj}-\sum_{i=1}^{m}v_{ik}x_{ij}\le 0,\ j=1,\ldots,n, \qquad (1)\\
&\qquad u_{rk},v_{ik}\ge 0,\ r=1,\ldots,s;\ i=1,\ldots,m
\end{aligned}$$

Where $DMU_k\in\{DMU_1\ldots,DMU_n\}$ is the decision making unit (DMU) under evaluation, $v_{ik}\,(i=1,\ldots,m)$ and $u_{rk}\,(r=1,\ldots,s)$ denote the inputs and outputs weights. If $u_{rk}^*\,(r=1,\ldots,s)$ and $v_{ik}^*\,(\mathrm{i}=1,\ldots,m)$ are the optimal weights solution to the above CCR model, the corresponding $\theta_{kk}^*=\sum_{r=1}^{s}u_{rk}^*y_{rk}$ will be treated as the CCR efficiency value of DMU_k. If θ_{kk}^* is equal to 1, the DMU_k is CCR efficient; Otherwise, it will be non-CCR efficient. $\theta_{jk}=\sum_{r=1}^{s}u_{rk}^*y_{rj}/\sum_{i=1}^{m}v_{ik}^*x_{ij}$ is treated to be cross-efficiency of DMU_j using the optimal weights of DMU_k, $j=1,\ldots,n,\ j\ne k$.

Each DMU can obtain its CCR-efficiency and one set of optimal weights by CCR model (1). Under n sets of weights, there will be n efficiency values including 1 self-evaluated efficiency value and $n-1$ peer-evaluated efficiency values for each DMU, which form a cross-efficiency matrix shown in table I. They are usually aggregated by equal weights to generate average cross-efficiency (ACE) value for each DMU, and usually DMUs can be fully ranked through ACE. However, ACE ignores the difference among efficiencies, to avoid it we need to use non-equal weights for cross-efficiency aggregation. Each DMU will obtain a weighted average cross-efficiency (WACE) score calculated by the following model (2):

$$\bar{\theta}_i=\sum_{k=1}^{n}w_k\theta_{ik},\ i=1,\ldots,n, \qquad (2)$$

where $w_1,\ldots,w_n$ are different weights which can be used to aggregate efficiencies in cross-efficiency matrix, satisfying $w_k\ge 0(k=1,\cdots,n)$ and $\sum_{k=1}^{n}w_k=1$.

We notice that each DMU maybe have multiple sets of optimal weights in CCR model that will damage the use of cross-efficiency evaluation. To handle this problem, Sexton et al. [1] introduced the use of secondary goal model to select one unique set of weights among multiple optimal weights solutions. Inspired by this idea, many secondary goal models have been proposed. Among them, the following aggressive and benevolent formulations (3) suggested by Doyle and Green [11] are most widely used.

Model (3) is aggressive formulation which minimizes the average cross-efficiency value of other DMUs while keeping CCR-efficiency of DMU under evaluation unchanged, whereas model (4) is known as benevolent formulation which aims to maximize the average cross-efficiency value of other DMUs. In practice, the aggressive formulation usually selects a set of inputs and outputs weights which includes too many zero weights, resulting in much inputs and outputs information to be ignored when calculating cross-efficiencies. To avoid this, we will use the cross-efficiency matrix determined by benevolent formulation to aggregate, but the proposed aggregation method is also applicable to aggregate efficiencies determined by other models.

III. Cross-Efficiency Aggregation Using Expert Scoring Method

$$\begin{aligned}
&Minimize \quad \sum_{r=1}^{s}u_{rk}\left(\sum_{j=1,j\ne k}^{n}y_{rj}\right)\\
&Subject\ to \quad \sum_{i=1}^{m}v_{ik}\left(\sum_{j=1,j\ne k}^{n}x_{ij}\right)=1\\
&\qquad \sum_{r=1}^{s}u_{rk}y_{rk}-\theta_{kk}^*\sum_{i=1}^{m}v_{ik}x_{ik}=0 \qquad (3)\\
&\qquad \sum_{r=1}^{s}u_{rk}y_{rj}-\sum_{i=1}^{m}v_{ik}x_{ij}\le 0,\quad j=1,\ldots,n;\ j\ne k,\\
&\qquad u_{rk}\ge 0,\quad r=1,\ldots,s,\\
&\qquad v_{ik}\ge 0,\quad i=1,\ldots,m,
\end{aligned}$$

$$
\begin{aligned}
&\text{Maximize} \quad \sum_{r=1}^{s} u_{rk}\left(\sum_{j=1, j\neq k}^{n} y_{rj}\right) \\
&\text{Subject to} \sum_{i=1}^{m} v_{ik}\left(\sum_{j=1, j\neq k}^{n} x_{ij}\right) = 1 \\
&\qquad \sum_{r=1}^{s} u_{rk} y_{rk} - \theta_{kk}^{*} \sum_{i=1}^{m} v_{ik} x_{ik} = 0 \qquad (4) \\
&\qquad \sum_{r=1}^{s} u_{rk} y_{rj} - \sum_{i=1}^{m} v_{ik} x_{ij} \le 0, \quad j = 1, \dots, n;\ j \neq k, \\
&\qquad u_{rk} \ge 0, \quad r = 1, \dots, s, \\
&\qquad v_{ik} \ge 0, \quad i = 1, \dots, m,
\end{aligned}
$$

TABLE I

CROSS-EFFICIENCY MATRIX AND WACE

DMUs	Target DMUs 1	2	$\cdots$	n	Weighted Average Cross-Efficiency (WACE)
1	θ_{11}	θ_{12}	$\cdots$	θ_{1n}	$\sum_{k=1}^{n} w_k \theta_{1k}$
2	θ_{21}	θ_{22}	$\cdots$	θ_{2n}	$\sum_{k=1}^{n} w_k \theta_{2k}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
n	θ_{n1}	θ_{n2}	$\cdots$	θ_{nn}	$\sum_{k=1}^{n} w_k \theta_{nk}$

There are two main issues to deal with before using expert scoring method to determine the weights for ultimate cross efficiency. One is to determine the external experts in cross efficiency matrix. The other one is to determine the weights of efficiency scores of different experts.

3.1 *The determination of external experts.* If we view target DMUs in cross-efficiency matrix shown in table I as external experts, the n efficiencies of each DMU in cross efficiency matrix will be viewed as its efficiency scores determined by n external experts shown in table II. Since the difference of experts on education background and work experience and so on, quite obviously their efficiency scores to DMUs should be treated differently and allocated different weights. So, the proposed aggregation approach provides the concrete reason why efficiencies in cross-efficiency matrix should be allocated different weights when aggregating them.

3.2 *The weights allocated to efficiency scores of different experts.* We view the efficiency scores of one expert to different DMUs as characteristic attributes values of one given expert shown in table III. To measure the dissimilarity level of one given expert to other ones, we refer to the methods which measure the spatial distance in coordinate system. The widely used methods are Euclidean distance, Manhattan distance, Chebyshev distance, Minkowski distance.

TABLE II

EFFICIENCY SCORES MATRIX OF n DMUS DETERMINED BY EXTERNAL EXPERTS

DMUs	Experts 1	2	$\cdots$	n
1	θ_{11}	θ_{12}	$\cdots$	θ_{1n}
2	θ_{21}	θ_{22}	$\cdots$	θ_{2n}
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
n	θ_{n1}	θ_{n2}	$\cdots$	θ_{nn}

Euclidean distance measures the straight-line distance in coordinate system. It can be measured by the following formulation: $d_{ij} = \left[\sum_{k=1}^{n} x_{ik} - x_{jk}{}^{2}\right]^{\frac{1}{2}}$, where $x_{ik}(k =,1\cdots,n)$ and $x_{jk}(k = 1,\cdots,n)$ denote the characteristic attributes values of x_i and x_j. Unlike the Euclidean distance, Manhattan distance measures the broken-line distance in coordinate system. It can be calculated by the following formulation: $d_{ij} = \sum_{k=1}^{n}\left|x_{ik} - x_{jk}\right|$. Chebyshev distance is also called "chessboard distance". It can be measured by the following formulation: $d_{ij} = \underset{k=1}{\overset{n}{Max}}\left|x_{ik} - x_{jk}\right|$. The formulation of Minkowski distance is defined as: $d_{ij} = \left[\sum_{k=1}^{n}\left|x_{ik} - x_{jk}\right|^{q}\right]^{\frac{1}{q}}$. Clearly if q is equal to 2, the Minkowski distance will be equal to Euclidean distance. Among the above methods which can measure the spatial distance in coordinate system, the Euclidean distance is most commonly used. Here, we use average Euclidean distance to measure the dissimilarity level of one given expert to other ones. The calculation formula is shown as: $D_G(p,q) = (1/lk)\sum_{i\in G_p}\sum_{j\in G_q} d_{ij}$, where l and k are the number of individuals in G_p and G_q respectively. The Euclidean distance matrix of n experts is shown in table IV. From table IV, we can gain the average Euclidean distance d_k of one given expert k to other experts. We can gain d_k through the following formula: $d_k = \sum_{i=1}^{n} d_{ki} \Big/ n-1$, $k \in (1,\cdots,n), i \neq k.$ If the dissimilarity level of one given expert to other ones is huge, the authority and ability of one given expert will be questioned and it is reasonable to allocate small weight to his efficiency scores to DMUs. Inspired by this idea and satisfying $\sum_{k=1}^{n} w_k = 1$ (w_k denotes the weight allocated to efficiency scores of expert k to DMUs), the w_k should be equal to $\left(1-d_k \Big/ \sum_{k=1}^{n} d_k\right) \Big/ (n-1)$ where d_k represents dissimilarity level of expert k to other experts.

IV. Merical Examples

In this section, we use one numerical examples to illustrate the potential application of expert scoring method in cross-efficiency aggregation.

TABLE III

CHARACTERISTIC ATTRIBUTES VALUES OF EXPERTS

Attributes	Experts			
	1	2	$\cdots$	n
1	θ_{11}	θ_{12}	$\cdots$	θ_{1n}
2	θ_{21}	θ_{22}	$\cdots$	θ_{2n}
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
n	θ_{n1}	θ_{n2}	$\cdots$	θ_{nn}

TABLE IV

EUCLIDEAN DISTANCE MATRIX OF EXPERTS

Experts	Experts			
	1	2	$\cdots$	n
1	d_{11}	d_{12}	$\cdots$	d_{1n}
2	d_{21}	d_{22}	$\cdots$	d_{2n}
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
n	d_{n1}	d_{n2}	$\cdots$	d_{nn}

Example: Five DMUs are needed to evaluate where two inputs are consumed to produce one normalized output [12]. Their inputs and outputs data and CCR-efficiency values are show in table V [12]. It clearly shows that CCR model cannot fully rank DMUs. DEA cross-efficiency evaluation can solve this problem effectively and provide a unique rank order for each DMU. The inputs and outputs weights of each DMU determined by benevolent model (4) are shown in table VI [12]. The corresponding cross-efficiency matrix and average cross-efficiency for each DMU are shown in table VII [12]. Based on ACE results, DMU_2 performs best. The table VIII shows the cross-efficiencies aggregation results determined by OWA. The different optimism level of DM will lead to different weights. They clearly show that different optimism level of DM (decision maker) leads to different results. Moreover, it is difficult to measure actual optimism level of DM. The aggregation results by dissimilarity, deviation and integrated approaches are shown in table IX-XI[12]. Although they provide same rank orders for DMUs, the efficiency aggregation results are not unique. Moreover, each of the three approaches does not have distinct advantages to other ones resulting in choice dilemma among them for DM.

TABLE V

INPUTS AND OUTPUTS DATA AND CCR EFFICIENCIES OF 5 DMUS

DMUs	Input1	Input2	Output	CCR Efficiency
1	2	12	1	1
2	2	8	1	1
3	5	5	1	1
4	10	4	1	1
5	10	6	1	0.75

TABLE VI

INPUTS AND OUTPUT WEIGHTS OF EACH DMU VIA BENEVOLENT FORMULATION

DMUs	Input1	Input2	Output
1	0.0370	0	0.0741
2	0.0185	0.0185	0.1852
3	0.0185	0.0185	0.1852
4	0.0057	0.0287	0.1724
5	0.0061	0.0305	0.1829

If we view the target DMUs in cross-efficiency matrix as external experts, the efficiencies of each DMU will be treated as its efficiency scores determined by experts. Then it shows that only first expert evaluates DMU_2 to be better than DMU_3 and gives a very lower efficiency score to DMU_4 and DMU_5. The efficiency scores determined by the first expert are very different from other experts. Allocating the equal weights to efficiency scores of different experts is not reasonable. Next, we use expert scoring method to generate relative importance weights. Table XII shows the efficiency scores matrix transformed by cross-efficiency matrix shown in table VII. We view the efficiency scores of one given expert to DMUs as its characteristic attributes values. The standardized characteristic attributes values of different experts are shown table XIII. Through the data shown in table XIII, the Euclidean distance matrix among different experts shown in table XIV can be calculated. The average Euclidean distance of one expert to other ones can reflect and measure his dissimilarity level to other ones. From table XIV, we can obtain that the dissimilarity level of different experts to other ones are 4.3410, 2.0740, 2.0740, 2.4566 and 2.4566 respectively. It clearly shows that the dissimilarity level of the first expert to other ones is maximal that accords to previous intuitive judgment. Via the formulation $\left(1-d_k\Big/\sum_{k=1}^{n}d_k\right)\Big/(n-1)$ where d_k represents dissimilarity level of expert k to other experts, we can obtain the weights allocated to efficiency scores of different experts. They are 0.1690, 0.2113, 0.2113, 0.2042 and 0.2042 respectively. The weight allocated to efficiency scores of first expert is minimum that is consistent with previous intuitive analysis. The final aggregation results calculated by the weights are shown in table XV. Different from ACE result, DMU_3 performs better than DMU_2 in table XV. This rank result is reasonable since only target DMU_1 (first expert) in table VII evaluated DMU_2 to perform better than DMU_3. At the same time, the proposed method produces unique aggregation weights and results. While using it, it does not need to measure the optimism level of decision maker (DM). Moreover, the proposed method will not make choice dilemma for DM and it is convenient for DM to use it.

V. Conclusions

This paper focused on cross-efficiency aggregation process. This paper focused on cross-efficiency aggregation process. We propose the use of expert scoring method for cross-efficiency aggregation considering the difference among efficiencies in cross-efficiency matrix.

Compared with existing non-equal aggregation approaches, the proposed model has clear modeling mechanism and provides the concrete reasons why cross-efficiency should be allocated different weights for aggregation. Meanwhile, using the proposed method it does not need to measure the optimism level of DM and the proposed approach produces the unique aggregation results. At the same time, it will not make choice dilemma for DM.

TABLE VII

BENEVOLENT CROSS-EFFICIENCY MATRIX AND AVERAGE CROSS-EFFICIENCY RESULTS

DMUs	Target DMUs					Average	Rank
	1	2	3	4	5	Cross-Efficiency	
1	1.0000	0.7143	0.7143	0.4839	0.4839	0.6793	4
2	1.0000	1.0000	1.0000	0.7143	0.7143	0.8857	1
3	0.4000	1.0000	1.0000	1.0000	1.0000	0.8800	2
4	0.2000	0.7143	0.7143	1.0000	1.0000	0.7257	3
5	0.2000	0.6250	0.6250	0.7500	0.7500	0.5900	5

TABLE VIII

CROSS-EFFICIENCY AGGREGATION RESULTS BY OWA OPERATOR WEIGHTS

DMUs	Optimism Level of the DM					
	α=1	α=0.9	α=0.8	α=0.7	α=0.6	α=0.5
1	1.0000(1)	0.8875(4)	0.8365(4)	0.7803(4)	0.7298(4)	0.6793(4)
2	1.0000(1)	1.0000(1)	0.9886(2)	0.9543(2)	0.9200(2)	0.8857(1)
3	1.0000(1)	1.0000(1)	1.0000(1)	0.9760(1)	0.9280(1)	0.8800(2)
4	1.0000(1)	0.9904(3)	0.9371(3)	0.8766(3)	0.8011(3)	0.7257(3)
5	0.7500(5)	0.7458(5)	0.7225(5)	0.6880(5)	0.6390(5)	0.5900(5)

TABLE IX

AGGREGATION RESULTS THROUGH DISSIMILARITY APPROACH

DMUs	Target DMUs					WACE	Rank
	1	2	3	4	5		
	0.0219	0.3639	0.3639	0.1252	0.1252		
1	1.0000	0.7143	0.7143	0.4839	0.4839	0.6629	4
2	1.0000	1.0000	1.0000	0.7143	0.7143	0.9285	2
3	0.4000	1.0000	1.0000	1.0000	1.0000	0.9869	1
4	0.2000	0.7143	0.7143	1.0000	1.0000	0.7746	3
5	0.2000	0.6250	0.6250	0.7500	0.7500	0.6470	5

TABLE X

AGGREGATION RESULTS THROUGH DEVIATION APPROACH

DMUs	Target DMUs					WACE	Rank
	1	2	3	4	5		
	0.0434	0.3259	0.3159	0.1624	0.1624		
1	1.0000	0.7143	0.7143	0.4839	0.4839	0.6519	4
2	1.0000	1.0000	1.0000	0.7143	0.7143	0.9072	2
3	0.4000	1.0000	1.0000	1.0000	1.0000	0.9740	1
4	0.2000	0.7143	0.7143	1.0000	1.0000	0.7848	3
5	0.2000	0.6250	0.6250	0.7500	0.7500	0.6472	5

TABLE XI

AGGREGATION RESULTS THROUGH INTEGRATED APPROACH

DMUs	Target DMUs					WACE	Rank
	1	2	3	4	5		
	0.0361	0.3280	0.3280	0.1539	0.1539		
1	1.0000	0.7143	0.7143	0.4839	0.4839	0.6537	4
2	1.0000	1.0000	1.0000	0.7143	0.7143	0.9121	2
3	0.4000	1.0000	1.0000	1.0000	1.0000	0.9784	1
4	0.2000	0.7143	0.7143	1.0000	1.0000	0.7837	3
5	0.2000	0.6250	0.6250	0.7500	0.7500	0.6481	5

TABLE XII

EFFICIENCY SCORES MATRIX

Experts	DMUs				
	1	2	3	4	5
1	1.0000	1.0000	0.4000	0.2000	0.2000
2	0.7143	1.0000	1.0000	0.7143	0.6250
3	0.7143	1.0000	1.0000	0.7143	0.6250
4	0.4839	0.7143	1.0000	1.0000	0.7500
5	0.4839	0.7143	1.0000	1.0000	0.7500

TABLE XIII

THE STANDARDIZED CHARACTERISTIC ATTRIBUTES VALUES OF EXPERTS

Experts	Characteristic Attributes				
	x_1	x_2	x_3	x_4	x_5
1	1.5050	0.7303	-1.7889	-1.6089	-1.7196
2	0.1643	0.7303	0.4472	-0.0350	0.1543
3	0.1643	0.7303	0.4472	-0.0350	0.1543
4	-0.9168	-1.0955	0.4472	0.8394	0.7055
5	-0.9168	-1.0955	0.4472	0.8394	0.7055

TABLE XIV

EUCLIDEAN DISTANCE MATRIX

Experts	Experts				
	1	2	3	4	5
1	0.0000	3.5760	3.5760	5.1060	5.1060
2	3.5760	0.0000	0.0000	2.3600	2.3600
3	3.5760	0.0000	0.0000	2.3600	2.3600
4	5.1060	2.3600	2.3600	0.0000	0.0000
5	5.1060	2.3600	2.3600	0.0000	0.0000

TABLE XV

AGGREGATION RESULTS VIA EXPERT SCORING METHOD

DMUs	Target DMUs (Experts)					WACE	Rank
	1	2	3	4	5		
1	1.0000	0.7143	0.7143	0.4839	0.4839	0.6685	4
2	1.0000	1.0000	1.0000	0.7143	0.7143	0.8833	2
3	0.4000	1.0000	1.0000	1.0000	1.0000	0.8986	1
4	0.2000	0.7143	0.7143	1.0000	1.0000	0.7440	3
5	0.2000	0.6250	0.6250	0.7500	0.7500	0.6042	5

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