Spatial-Time Relationships When Measuring the Range and the Velocity of Spacecrafts

Oleg V. Chernoyarov, Vladimir A. Ivanov, Tatiana I. Demina, Serguei Dachian, and Alexandra V. Salnikova

***Abstract*—The problem of determining the range and the radial velocity of space crafts in the middle and deep space by a monostatic radar is considered. The tested determination method is based on the calculations concerning the relayed signal reception moment and its delay relative to both the emission moment and the Doppler shift of the carrier frequency from the emitted signal frequency (active mode). The results of the determination of the radial velocity only by calculating the Doppler shift of the onboard generator carrier frequency (passive mode) are also examined. The analysis of the space-time relationships during the curvilinear movement of the radar in an inertial geocentric system is conducted and it allows testing the radar as an autonomous measuring device that measures the range to the spacecraft in a certain coordinate system and then matches the measurement result with the estimated time suggested by this system. It is shown that practically the same result is obtained in the station coordinate system for the mean time between the signal emission and the reception relative to the calculated point that does not coincide with the radar location but changes its position with the measured signal delay depending on the time of the day and time of the year. It is established that when the range and the velocity in the station coordinate system (active operation mode) are measured simultaneously, the radial velocity is calculated relative to the target point, while the measured value depends on the angular direction set for the spacecraft. In the passive mode of operation (one-sided emission), to calculate the direction and the components of the velocity vector accurately, one needs a knowledge of the spacecraft coordinates and the scalar of orbital velocity.**

***Index Terms*— Space-time relationships, spacecraft, range, velocity, monostatic radar**

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I. INTRODUCTION

Today, the problem of a deep space exploration is becoming more and more relevant [1]-[3]. A necessary condition for its solution is a reliable navigation and ballistic support of the long-range spacecraft flights [4]. To measure the current navigation parameters of spacecraft movement, laser and radar facilities are used presently. The modern procedure for analyzing space-time relationships in radar measurements of spacecraft motion parameters is based on the use of the basic principles of the theory of relativity [5]-[9]. Within the limits of near space, as a rule, the methodology of the special theory of relativity is sufficient [6]-[9]. However, for middle and deep space [10], it may be necessary to use the methodology of the general theory of relativity [5], [6].

The basic provision of the special theory of relativity is the principle of relativity. Under this, it is important to define an inertial system as a system that moves freely in the absence of any external influences (gravitational fields, accelerating forces, etc.). The general theory of relativity considers the mutual motion of systems that cannot be considered inertial. The practical conclusions of this theory relate to the motion with constant acceleration in a gravitational field or under the action of another accelerating force. An example of such a motion is a circular motion with a constant angular velocity (constant tangential acceleration) under the action of gravitational forces or due to a rigid mechanical connection between the two systems.

As it is well known [5], [6], in two inertial systems moving with relative velocity **v**, the coordinate systems $\{X,Y,Z\}$ and $\{X',Y',Z'\}$ can be introduced so that their axes X and X' are parallel to the vector **v**. The time origin, that is $t=0$, in each of the systems can be conditionally attributed to the moment when the planes $\{Y,Z\}$ and $\{Y',Z'\}$ coincide. In this case, the principle of relativity concerning the space and time relationships can be formulated in the form of two postulates [6]:

1) the velocity of light in free space is unchanged in each of the systems;

2) the time t' of an event at the point (x',y',z') in one of the systems is related to the time t and the coordinates (x,y,z) of the other system by the Lorentz transformations:

$$t=\frac{t'-vx'/c^2}{\sqrt{1-\beta^2}},\quad x=\frac{x'-vt'}{\sqrt{1-\beta^2}},$$
$$y=y'+y_0,\quad z=z'+z_0. \tag{1}$$

In (1), the notations are the following: v is the scalar of system relative motion velocity; c is the velocity of light in free space; $\beta=v/c$; y_0, z_0 is the position of the point $O'(x'=0,y'=0,z'=0)$ in the coordinate system $\{X,Y,Z\}$.

The movement of real physical objects within the solar system cannot be considered inertial in the general case [11], [12]. Therefore, the possibility of applying the Lorentz transformations as approximate relations should be carefully evaluated in each of the cases separately. The system $\{X,Y,Z\}$ should be considered inertial if its motion in the coordinates of another known inertial system is uniform and rectilinear. When approximating the motion of certain systems, the system related to the Sun should be referred to as the known inertial one, because accounting for any external influences on this system it is still impossible.

The real measurement results in a monostatic radar system are the arrival time t_r of the relayed signal, its delay Δt_d relative to the emission moment t_p and the Doppler shift Δf_d of the carrier frequency from the emitted signal frequency f_0 [13], [14].

Thus, two approaches to the solution of the tasks of the radar measurements can be considered:

1) the radar provides the values of t_r, Δt_d, Δf_d (with available corrections for the state of the propagation path), while the interpretation of the measurement results for the estimation of the spacecraft motion parameters is carried out at the stage of data processing;

2) the radar is considered as an autonomous measuring device that provides the values of the range and the radial velocity in a certain coordinate system and at a certain time.

The first approach is currently widely used for measuring the motion parameters in the near and middle space. It has been developed both in several monographs and practical approaches for processing trajectory measurement data [15]-[17].

In this paper, the second approach is tested, as it appears to be more natural in the context of formulating the tasks of radar trajectory control devices and providing the requirements for their efficiency.

II. The Space-Time Relations under Curvilinear Motion in an Inertial System

To begin with, one presents the motion of some system $\mathbf{O}'(X',Y',Z')$ in the inertial reference system $\mathbf{O}(X,Y,Z)$. The law of motion of the point O' that is the origin of coordinates of the system $\mathbf{O}'$ is considered specified in the form of the radius vector $\mathbf{r}(t)$ and the velocity vector $\mathbf{v}(t)$ (Fig. 1).

As the movement of the point O' in the coordinate

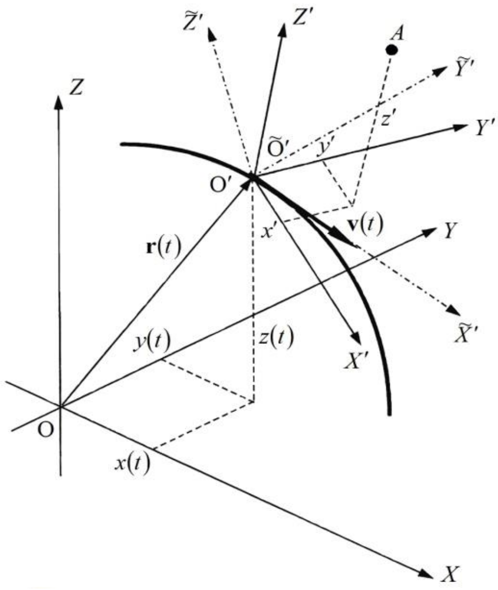


Fig. 1. The movement of the system $\mathbf{O}'$ in the inertial system $\mathbf{O}$.

system $\mathbf{O}$ is known, the one-to-one relationships can be determined between the readings of the clock set at the origin of coordinates of the system $\mathbf{O}$ (point O) and the one at the point O', and thus no resorting to the methods of the general theory of relativity is required. For example, this relation can be implemented by observing the clock at the point O' from the point O on the television channel and taking into account the delay that is known at each time moment.

In order to determine this relation, one studies the successive positions of the point O' separated by a sufficiently small interval Δt. This makes it possible to substitute the real law of motion by the approximate polygonal function with the nodes at the points t_i. Then, within each partial interval Δt, the system $\mathbf{O}'$ can be considered as an inertial one, and thus the interval Δt in this system corresponds to the interval

$$\Delta t_i'=\Delta t\sqrt{1-\beta_i^2},\qquad \beta_i=|\mathbf{v}(t_i)|/c.$$

Thus, the moment t_n in the coordinate system $\mathbf{O}$ corresponds to the moment

$$t_n'=t_0'+\Delta t\sum_{i=0}^{n}\sqrt{1-\beta_i^2}$$

at the point O', where t_0' is the fixed time shift (value of t' at $t=0$).

Moving to the limit, while $\Delta t\to 0$, one gets:

$$t'=t_0'+\int_0^t\sqrt{1-|\mathbf{v}(\tau)|^2/c^2}\,\mathrm{d}\tau. \tag{2}$$

In particular, in a circular motion with a constant angular velocity, the scalar of linear velocity remains unchanged:

$|\mathbf{v}(t)| = v = \text{const}$ and

$$t' = t_0' + t\sqrt{1-\beta^2}, \qquad \beta = v/c. \tag{3}$$

It must be emphasized that in a non-inertial system at different fixed points in space, the flow of time may not be the same. Further, the value of t' denotes the clock at the origin of the coordinates of the system $\mathbf{O}'$ (that is, at the point O').

It is presupposed that the event A occurs in the system $\mathbf{O}'$ at the moment t_c'. The direction of the X', Y', Z' axes in the system $\mathbf{O}'$ is set based on the terms of the task. The coordinates of the point of the event in this system are denoted as x', y', z'. Our task now is determining the parameters t_c, x, y, z of the same event in the system $\mathbf{O}$.

Thus one should consider the auxiliary coordinate system $\tilde{\mathbf{O}}'(\tilde{X}', \tilde{Y}', \tilde{Z}')$ obtained by rotating the $\tilde{X}'$, $\tilde{Y}'$, $\tilde{Z}'$ axes so that the $\tilde{X}'$ axis coincides with the direction of the current velocity vector $\mathbf{v}(t)$ corresponding to the moment t_c', while the $\tilde{Y}'$, $\tilde{Z}'$ axes together with $\tilde{X}'$ axis form a right-handed coordinate system (Fig. 1). The coordinates of the point of the event in the system $\tilde{\mathbf{O}}'$ are denoted as $\tilde{x}'$, $\tilde{y}'$, $\tilde{z}'$.

It is also useful to introduce an auxiliary system $\tilde{\mathbf{O}}$, the coordinate axes of which are obtained by rotating the axes of the system $\mathbf{O}$ with a parallel transfer of the origin of coordinates to the point $\tilde{\mathrm{O}}'$ corresponding to the moment t_c' so that the systems $\tilde{\mathbf{O}}$ and $\tilde{\mathbf{O}}'$ coincide at this moment. To pass from the system $\tilde{\mathbf{O}}'$ to the system $\tilde{\mathbf{O}}$, one should conditionally assume that, when they coincide, the time in each of the systems is equal to zero, that is, $\tilde{t}_0' = \tilde{t}_0 = 0$. It is obvious that the moment of the event A in the system $\tilde{\mathbf{O}}'$ is also zero:

$$\tilde{t}_c' = \tilde{t}_0 = 0, \tag{4}$$

and the current time $\tilde{t}$ in the system $\tilde{\mathbf{O}}$ differs from the current time t in the system $\mathbf{O}$ by the $\bar{t}$ value corresponding to the moment t_c' in the system $\mathbf{O}'$.

In this case, as it follows from (1), for the system $\tilde{\mathbf{O}}$ one can obtain:

1) the time of the event

$$\tilde{t}_c = v(\bar{t})\tilde{x}' / c^2\sqrt{1-\beta^2(\bar{t})}, \qquad v(\bar{t}) = |\mathbf{v}(\bar{t})|; \tag{5}$$

2) the space coordinates

$$\tilde{x} = \tilde{x}' / \sqrt{1-\beta^2(\bar{t})}, \qquad \tilde{y} = \tilde{y}', \qquad \tilde{z} = \tilde{z}'. \tag{6}$$

In the system $\mathbf{O}$, the moment of the event is

$$t_c = \tilde{t}_c + \bar{t}, \tag{7}$$

while the spatial coordinates x, y, z can be determined by the common formulas for transforming Cartesian coordinates.

It is assumed now that the moment of the event t_c and the coordinates of the point (x, y, z) are specified in the system $\mathbf{O}$. Then one defines the same parameters t_c', x', y', z' of the event in the system $\mathbf{O}'$. The solution to the problem may require focusing on the diagram presented in Fig. 2, where it is now necessary to carry out the transition from the known values $\tilde{x}$, $\tilde{y}$, $\tilde{z}$ to the desired ones that are x', y', z'. It should be noted that, in this case, the condition (4) must be satisfied, i.e. when recalculating the value $\tilde{t}_c$ in terms of the value $\tilde{t}_c'$, one should get zero:

$$\tilde{t}_c' = \left(\tilde{t}_c - v(\bar{t})\tilde{x}/c^2\right) / \sqrt{1-\beta^2} = 0 \text{ or } \tilde{t}_c - v(\bar{t})\tilde{x}/c^2 = 0. \tag{8}$$

There, according to (7),

$$\tilde{t}_c = t_c - \bar{t}. \tag{9}$$

Thus, to reduce the solution to the diagram presented in Fig. 2, it is necessary to choose the appropriate value of $\bar{t}$ that determines all the parameters of the relation (8).

In general case, the value of $\bar{t}$ cannot be analytically found from the condition (8). For this purpose, the iteration algorithm can be applied, for example, as follows. First, taking into account (9), the condition (8) can be presented in the form:

$$I(\bar{t}) = t_c - \bar{t} - v(\bar{t})x(\bar{t})/c^2 = 0.$$

At the initial iteration step ($n = 0$), it is assumed that

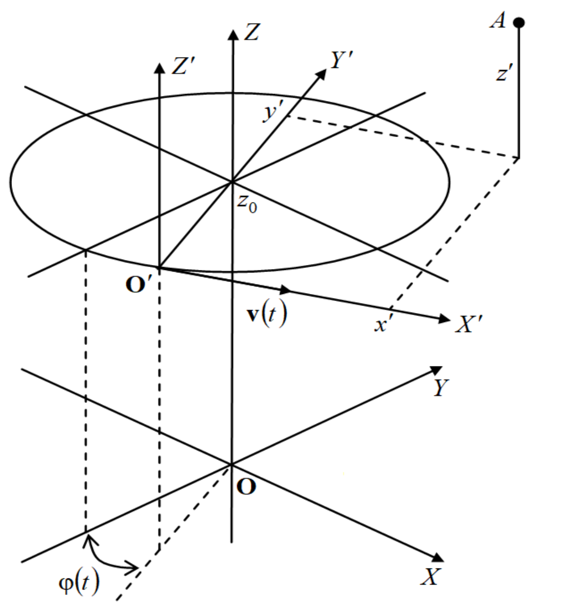


Fig. 2. The descriptive diagram for determining the relationships between the parameters of the event A in the systems **O** and **O′**.

$\bar{t} = t_c$. The order of calculations at the n-th step is presented as

$$\hat{t}_n = t_c - v(\bar{t}_{n-1})\tilde{x}(\bar{t}_{n-1})/c^2 = 0,$$
$$\Delta\bar{t}_n = I(\hat{t}_n)/[\mathrm{d}I(\bar{t})/\mathrm{d}\bar{t}]\big|_{\bar{t}=\hat{t}_n}, \quad \bar{t}_n = \hat{t}_n - \Delta\bar{t}_n. \tag{10}$$

The iterations continue until the required accuracy is achieved (for example, until the step at which the value of $\Delta\bar{t}_n$ becomes commensurate with the computer accuracy). It should be noted that iterative calculations are feasible to use when all the functional relations included in (10) can be expressed analytically.

The order of calculations after choosing the value of $\bar{t}$ is

$$\tilde{x}' = (\tilde{x} - v(\bar{t})\tilde{t}_c)\Big/\sqrt{1-\beta^2(\bar{t})},$$
$$\tilde{y}' = \tilde{y}, \quad \tilde{z}' = \tilde{z}. \tag{11}$$

The (x', y', z') coordinates of the point of the event in the system $\mathbf{O}'$ are determined by the angular rotation of the system $\tilde{\mathbf{O}}'$ axes, and the moment of the event t'_c corresponds to the moment $\bar{t}$ in the system $\mathbf{O}$.

As an example, one now introduce a circular motion with the constant angular velocity Ω around the Z axis in the $z = z_0$ plane of the system $\mathbf{O}$ (Fig. 2). There, for simplicity of relations, it is assumed that the systems $\mathbf{O}'$ and $\tilde{\mathbf{O}}'$ coincide, so that, instead of $\tilde{x}'$, $\tilde{y}'$, $\tilde{z}' = \tilde{z}$, one can directly examine the x', y', z' coordinates. Then, assuming that in (3) $t'_0 = 0$, the current time relation in the systems $\mathbf{O}$ and $\mathbf{O}'$ can be obtained in the form of $t' = t\sqrt{1-\beta^2}$, where $\beta = v/c$, $v = \Omega R$, and R is the radius of rotation.

When setting the parameters t'_c, x', y', z' of the event A in the system $\mathbf{O}'$, for the corresponding parameters in the systems $\tilde{\mathbf{O}}$, $\mathbf{O}$ one gets:

$$\bar{t} = t'_c\Big/\sqrt{1-\beta^2}.$$

The angle of sight of the point O' in the system $\mathbf{O}$ at the moment $\bar{t}$ is

$$\varphi = \Omega\bar{t} = \Omega t'_c\Big/\sqrt{1-\beta^2}. \tag{12}$$

Moreover, under (5), (6), the following relations hold:

$$\tilde{t}_c = vx'\Big/c^2\sqrt{1-\beta^2}, \quad \tilde{x} = x'\Big/\sqrt{1-\beta^2}, \quad \tilde{y} = y', \quad \tilde{z} = z'.$$

When passing from the system $\tilde{\mathbf{O}}$ to the system $\mathbf{O}$, taking into account (11), one can write

$$x = \tilde{x}\cos\varphi - (\tilde{y} - R)\sin\varphi, \quad y = \tilde{x}\sin\varphi + (\tilde{y} - R)\cos\varphi,$$
$$z = \tilde{z} + z_0, \quad t_c = \tilde{t}_c + \bar{t} = \left(t'_c + v\,x'/c^2\right)\Big/\sqrt{1-\beta^2}. \tag{13}$$

The inverse transformation of the coordinates of the event A from the system $\mathbf{O}$ to the system $\mathbf{O}'$ in the considered case can be carried out by the iteration method. Here the condition (8), taking into account that $v = \text{const}$, can be written as

$$t_c - \bar{t} - v\,\tilde{x}/c^2 = 0. \tag{14}$$

After application of (12), it follows from (13) that

$$t' = t'_0 + \int_0^t \sqrt{1 - |\mathbf{v}(\tau)|^2/c^2}\,\mathrm{d}\tau. \tag{15}$$

Then, by substituting (15) into (14), one gets the equation for $\bar{t}$ taking the form

$$I(\bar{t}) = t_c - \bar{t} - v\left(x\cos\Omega\bar{t} + y\sin\Omega\bar{t}\right)/c^2 = 0. \tag{16}$$

The equation (16) does not have an analytical solution, but it can be solved by the iteration method. As above, at the first iteration step ($n = 0$) it should be assumed that $\bar{t} = t_c$. Then the calculations produced at the n-th step are:

$$\hat{t}_n = t_c - v\left(x\cos\Omega\bar{t}_{n-1} + y\sin\Omega\bar{t}_{n-1}\right)/c^2,$$
$$\Delta\bar{t}_n = \frac{t - \hat{t}_n - v\left(x\cos\Omega\bar{t}_n + y\sin\Omega\bar{t}_n\right)/c^2}{\Omega v\left(x\sin\Omega\hat{t}_n - y\cos\Omega\hat{t}_n\right)/c^2 - 1}, \quad \bar{t}_n = \hat{t}_n - \Delta\bar{t}_n,$$

and the further course of transformations is:

$$\tilde{y} = -x\sin\Omega\bar{t} + y\cos\Omega\bar{t} + R, \quad \tilde{t}_c = \hat{t}_n - \Delta\bar{t}_n,$$
$$x' = (\tilde{x} - v\tilde{t}_c)\Big/\sqrt{1-\beta^2}, \quad y' = \tilde{y}, \quad t' = \bar{t}\sqrt{1-\beta^2}.$$

III. The Range Measurement

A. The General Provisions

The three coordinate systems are considered: the heliocentric (inertial) system $\mathbf{O}_g$, the geocentric (conditionally "frozen") system $\mathbf{O}_e$, and the coordinate system of the station $\mathbf{O}_s$ on the Earth's surface.

The origin of coordinates of the system $\mathbf{O}_g$ coincides with the center of the Sun, the X_g, Y_g axes of coordinates are within the ecliptic plane. The Z_e axis of the system $\mathbf{O}_e$ is directed from the center of the Earth to the north pole, while the X_e axis is perpendicular to Z_e within the ecliptic plane (and, besides, the X_g axis is chosen parallel to the X_e axis) and the Y_e axis forms a right-handed coordinate system with the X_e and Z_e axes. The Z_s axis of the system $\mathbf{O}_s$ is directed from the center of the Earth to the point where the station is located, while the X_s axis is collinear with the current vector of the station rotation around the Earth's axis, and the Y_s axis adds the system to the right-handed one.

The notations that should be introduced here are: Ω_e, R_e are the angular velocity and the radius of rotation of the Earth around the Sun; Ω_s, R_s are the angular velocity and the radius of rotation of the station around the Earth axis. The radar range measurement includes determining the probe signal delay and registration of the measurement result at a certain moment in time. According to the problem statement defined in Section 1, the measured value of the range and the moment of the time registration of the result should refer to the station coordinate system. And the procedure for recalculating the measurement results for other systems should be determined.

The station located on the Earth's surface is in a complex three-dimensional motion in the inertial heliocentric system $\mathbf{O}_g$. It should be noted that the moment of emission, the moment of reception as well as the spatial coordinates of the station are different in each of the three systems $\mathbf{O}_s$, $\mathbf{O}_e$, $\mathbf{O}_g$. The problem of measuring the range and the time registration in the system $\mathbf{O}_s$ can be solved through the examination of the electromagnetic wave propagation in a non-inertial system. This propagation appears anisotropic by its direction, and that causes the radio beam refraction in such a system. Therefore, it is more convenient to use another method.

If one recalculates the moments of emission and reception of the signal by the station and the spatial coordinates of the station at these moments for the selected inertial system, then the task solution in this system is not difficult. This is because under the assumed isotropic propagation medium the wave propagates isotropically at the group velocity v_g. Otherwise, one can use the value of the velocity of light in free space c for the case when the influence of the real propagation medium is taken into account by introducing an appropriate correction for the delay. The inverse recalculation of the results obtained in the inertial system into the station coordinate system finalizes the task solution.

It is obvious that in any inertial system, in which the movement of the station is specified, the coordinates of the points of emission and reception do not necessarily coincide and the delay interval defines the total range passed by the radio beam to the spacecraft and back. In this case, the geometrical locus of the points of the possible spatial position of the spacecraft is an ellipsoid with focuses at the points of emission and reception, and each point of the ellipsoid corresponds to its moment of time registration (the moment of “contact” between the spacecraft and the probe beam). Recalculation of the results for the station coordinate system determines a new geometrical locus of points in this system and, in general, new points of time registration.

B. Inertial Earth’s Motion

To start with, one addresses the problem of measuring the range, assuming the system $\mathbf{O}_e$ to be inertial. And besides, the heliocentric system $\mathbf{O}_g$ may not be considered. The radius of rotation of the point of station location near the Earth's axis R_s is determined by its latitude θ as follows: $R_s = R_l \cos\theta$, where R_l is the local Earth's radius. If the moment of coincidence of the directions of the axes X_s, X_e in both systems is taken as zero, then the motion of the station in the system $\mathbf{O}_e$ can be specified by the relations (Fig. 3):

$$x_{es} = R_s \sin\Omega_s t_e,\quad y_{es} = -R_s \cos\Omega_s t_e,\quad z_{es} = R_l \sin\theta. \quad (17)$$

It is assumed that the moment of reception of the response signal $t_r = t_{2s}$ and the delay $\Delta t_d = \Delta t_s$ are measured in the system $\mathbf{O}_e$, so that the moment of emission is $t_p = t_{1s} = t_{2s} - \Delta t_s$. The emission and reception occur at the same point O_s ($x_s = 0$, $y_s = 0$, $z_s = 0$). The next step is to determine the spatial coordinates of the points of emission and reception and the corresponding time moments in the system $\mathbf{O}_e$. According to the procedure presented in Section 2, in the system $\mathbf{O}_e$ one gets

$$t_{1e} = t_{1s}\Big/\sqrt{1-\beta^2},\ t_{2e} = t_{2s}\Big/\sqrt{1-\beta^2},\ \Delta t_e = \Delta t_s\Big/\sqrt{1-\beta^2}, \quad (18)$$

where $\beta = v/c$ and $v = \Omega_s R_s$.

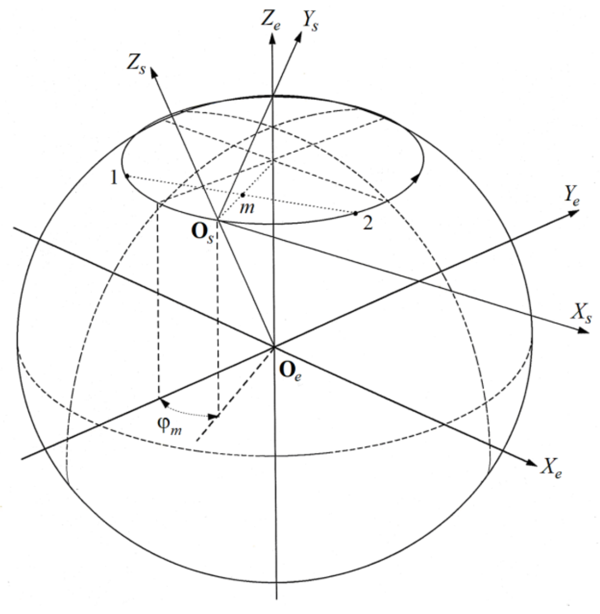


Fig. 3. The heliocentric coordinate system $\mathbf{O}_e$ and the coordinate system of station $\mathbf{O}_s$ on the Earth's surface.

Introducing the angles of sight of the emission and reception points φ_1 and φ_2, according to (17), in the plane $z_e = z_{es}$ one can write:

$$x_{1e} = R_s \sin\varphi_1, \quad x_{2e} = R_s \sin\varphi_2, \quad y_{1e} = -R_s \cos\varphi_1, \quad y_{2e} = -R_s \cos\varphi_2, \quad z_{1e} = z_{2e} = z_{es}. \tag{19}$$

Under $v_g = c$, the total range is determined as follows

$$r = c\Delta t_e. \tag{20}$$

Thus, the ellipsoid determining the possible position of the spacecraft in space has the focuses at the points corresponding to the coordinates (19), the center at the point

$$x_{0e} = (x_{1e} + x_{2e})/2, \quad y_{0e} = (y_{1e} + y_{2e})/2, \quad z_{0e} = z_{es},$$

and semi-major and semi-minor axes

$$a = r/2, \quad b = \sqrt{(r/2)^2 - R_s^2 \sin^2(\Omega_s \Delta t_e/2)}. \tag{21}$$

The moment of time registration corresponding to the point of the ellipsoid with the coordinates x_e, y_e, z_e is determined by the relation:

$$t_{eo} = t_{1e} + \sqrt{(x_e - x_{1e})^2 + (y_e - y_{1e})^2 + (z_e - z_{1e})^2} \Big/ c.$$

The transfer of the time registration moment t_{eo} and the spatial coordinates x_e, y_e, z_e from the system $\mathbf{O}_e$ to the system $\mathbf{O}_s$ is carried out for each point of the ellipsoid according to the procedure similar to the one presented in Section 2. This recalculation provides a sphere with the radius b and the center shifted from the origin. Each point of this sphere corresponds to its own moment of time registration t_{so}. Thus, it is generally impossible to determine the range from the station to the spacecraft at a certain point in time by the measurement results, since, depending on the angles of sight of the spacecraft, these values are different at the same moments of emission t_{1s} and reception t_{2s}.

In order to demonstrate the materiality of the noted condition, an example of measurement setting for the following parameters is suggested:

– the local radius $R_l = 6372$ km (mean Earth's radius),

– the period of Earth's rotation $T_s = 24$ hours (86400 s),

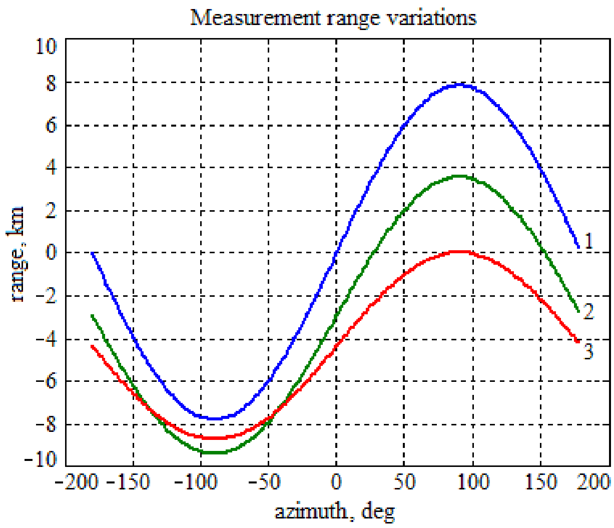
– the latitude of the station location $\theta = 56^\circ$ (latitude of Moscow),

– the station rotation radius $R_s = R_l \cos\theta$,

– the range from the Earth's center to the plane of rotation $z_{es} = R_l \sin\theta$,

– the moments of emission and reception $t_{1s} = -1000$ s, $t_{2s} = 1000$ s,

– the group propagation velocity $v_g = c = 3 \cdot 10^5$ km/s.



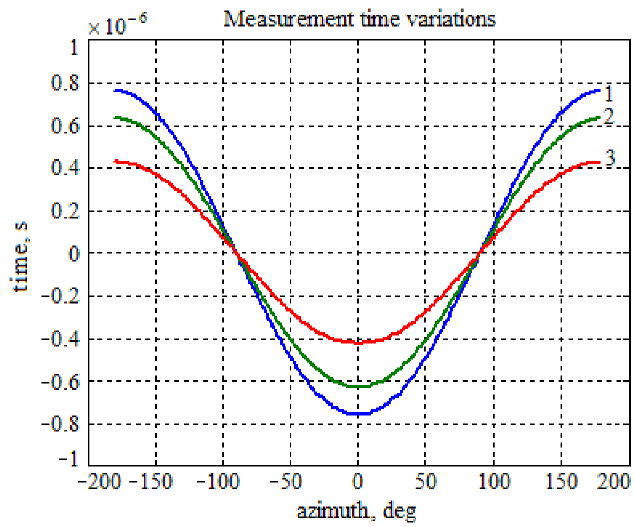


Fig. 4. The dependences of both the variations of the measured distance R_{so} between the station and the spacecraft (a) and the moments of registration of the measurement results t_{so} upon the spacecraft azimuth (b) (the inertial motion of the Earth): 1 – $\delta = 0$; 2 – $\delta = \pi/2 - \Theta$; 3 – $\delta = \Theta$.

The results of calculating the range R_{so} are presented in Fig. 4a in the form of graphs depending on the azimuth at the fixed elevation angles δ. Curve 1 corresponds to $\delta = 0$, curve 2 – to $\delta = \pi/2 - \theta$, curve 3 – to $\delta = \theta$. In Fig. 4b, the moments of registration of measurement results t_{so} corresponding to the same angular directions are shown.

It follows from Figs. 4 that, if the changes in the moment of registration in different directions can be considered insignificant, then the variations in the radial range reach about 17 km. To determine the real range, the angular target designation is required with an accuracy of tenths of an angular minute. However, it is possible to specify the point m against which both the radial range and the moment of registration of the measurement data do not depend on the angular direction at the set moments of emission and reception. Then one introduces the system $\mathbf{O}_m$ moving in the system $\mathbf{O}_e$ uniformly and rectilinearly and intersecting the points of emission 1 and reception 2 at times t_{1e}, t_{2e} (Fig. 3). It is obvious that its velocity in the system $\mathbf{O}_e$ with the space-time coordinates (18), (19) is

$$v_m = 2R_s \sin(\Omega_s \Delta t_e/2)/\Delta t_e .$$

Recalculation of both the points of the ellipsoid that determines the possible position of the spacecraft in the system $\mathbf{O}_e$ and the corresponding moments of registration for the system $\mathbf{O}_m$ provides, according to (21), the sphere with the radius

$$R_m = b = \sqrt{(c\Delta t_e/2)^2 - R_s^2 \sin^2(\Omega_s \Delta t_e/2)} \quad (22)$$

and the moment of registration t_{mo} which is constant for each point of the sphere.

All the values included in (22) are determined by the time of reception t_{2s} and the signal delay Δt_s measured at the station location that is described by the expressions (18), (19). Thus, the measurement result at the station determines the range R_m at the moment t_{mo} relative to the point *m* located in the plane of rotation (X_e, Y_e) and at the distance

$$d_m = R_s\left[1 - \cos(\Omega_s \Delta t_e/2)\right]$$

from the station in the direction of the axis of rotation.

The procedure for calculating the range measurement results by the observation data t_{2s} (moment of reception) and Δt_s (delay) at the station, in the general case, should be as follows:

– determination of the moment of registration of the measured value of the range as $t_{mo} = t_{2s} - \Delta t_s/2 = (t_{1s} + t_{2s})/2$;

– determination of the coordinates of the point of registration of the range x_{sm}, y_{sm}, z_{sm} in the station system $\mathbf{O}_s$ according to the algorithm described by the expressions:

$$R_s = R_l \cos\theta, \qquad \beta = \Omega_s R_s/c, \qquad \Delta t_e = \Delta t_s \big/ \sqrt{1-\beta^2},$$
$$\Delta\varphi = (\varphi_2 - \varphi_1)/2 = \Omega_s \Delta t_e/2 = \Omega_s \Delta t_s \big/ 2\sqrt{1-\beta^2}, \quad x_{sm} = 0,$$
$$y_{sm} = R_s(1 - \cos\Delta\varphi)\sin\theta, \qquad z_{sm} = -R_s(1-\cos\Delta\varphi)\cos\theta;$$

– calculation of the measured range value according to (22): $R_m = \sqrt{(c\Delta t_e/2)^2 - R_s^2 \sin^2 \Delta\varphi}$.

C. *Circular Earth's Motion*

It is assumed now that the center of the Earth makes a circular motion around the Sun with constant angular velocity $\Omega_e = 2\pi/31536000$ rad/s and radius $R_e = 1.5 \cdot 10^8$ km, while the axis of its own rotation moves parallel. Here the heliocentric system $\mathbf{O}_g$ should be considered as inertial, and the procedure presented in Section 2 can be used in the calculations for an arbitrary curvilinear motion (in this case, such a motion is the motion of the coordinate system of the station $\mathbf{O}_s$).

For further analysis, first of all, it is necessary to describe the law of motion of the station in the system $\mathbf{O}_g$. The movement of the station around the Earth's axis should still be specified in the system $\mathbf{O}_g$ following (17). For the generality of consideration, one can additionally introduce the phase φ_s of the station position at the moment t_e. Then the relations (17) take the form

$$x_{es} = R_s \sin(\Omega_s t_e + \varphi_s), \qquad y_{es} = -R_s \cos(\Omega_s t_e + \varphi_s),$$
$$z_{es} = R_l \sin\theta .$$

It is also convenient to introduce the auxiliary coordinate system $\mathbf{O}'_e$, for which the X'_e axis coincides with the X_e axis of the system $\mathbf{O}_e$ while the Z'_e axis is perpendicular to the ecliptic plane. If the angle of deviation of the Earth's axis from the normal to the ecliptic plane is denoted by ψ, then the coordinates of the station in the system $\mathbf{O}'_e$ are

$$x'_{es} = R_s \sin(\Omega_s t_e + \varphi_s),$$
$$y'_{es} = -R_s \cos(\Omega_s t_e + \varphi_s)\cos\psi + R_l \sin\theta \sin\psi ,$$
$$z'_{es} = R_s \cos(\Omega_s t_e + \varphi_s)\sin\psi + R_l \sin\theta\cos\psi .$$

It is assumed that the time in the systems $\mathbf{O}_e$ and $\mathbf{O}'_e$ is the same.

The motion of the center of the Earth in the system $\mathbf{O}_g$ can be written as

$$x_{ge} = R_e \sin(\Omega_e t_g + \psi_e),\ y_{ge} = R_e \sin(\Omega_e t_g + \psi_e),\ z_{ge} = 0,$$

where ψ_e is the phase of the Earth's position at the moment $t_g = 0$.

The task of recalculating the movement of the station from the system $\mathbf{O}'_e$ to the system $\mathbf{O}_g$ corresponds to the example of circular motion in the inertial system described in Section 2. In this case, "event" should be considered as the location of the station at a certain point in space and at a certain moment in time for each of the systems. The relation between the current times in the systems $\mathbf{O}'_e$ ($\mathbf{O}_e$) and $\mathbf{O}_g$ is determined by the ratio:

$$t_e = t_g \sqrt{1-\beta_e^2} ,$$

where $\beta_e = v_e/c$, $v_e = R_e \Omega_e$ is the linear velocity of the Earth around the Sun.

Following the procedure described in the Section 2, one gets:

1) the angle of sight of the center of the Earth in the system $\mathbf{O}_g$ at the time t_{es} fixing the position of the station in the system $\mathbf{O}'_e$ is

$$\varphi = \Omega_e t_{es} \Big/ \sqrt{1-\beta_e^2} + \varphi_e ; \tag{23}$$

2) the station coordinates in the auxiliary system $\tilde{\mathbf{O}}'_e$ (in Section 2, such the system is $\tilde{\mathbf{O}}'$) are

$$\begin{aligned} \tilde{x}'_{es} &= x'_{es}\cos\varphi - y'_{es}\sin\varphi, \\ \tilde{y}'_{es} &= x'_{es}\sin\varphi + y'_{es}\cos\varphi, \quad \tilde{z}'_{es} = z'_{es}; \end{aligned} \tag{24}$$

3) the time in the auxiliary system $\tilde{\mathbf{O}}_g$ (in Section 2, such the system is $\tilde{\mathbf{O}}$) is

$$\tilde{t}_{gs} = v_e \tilde{x}'_{es} \Big/ c^2\sqrt{1-\beta_e^2} \;; \tag{25}$$

4) the station coordinates in the system $\tilde{\mathbf{O}}_g$ are

$$\tilde{x}_{gs} = \tilde{x}'_{es} \Big/ \sqrt{1-\beta_e^2} \,, \qquad \tilde{y}_{gs} = \tilde{y}'_{es} \,, \qquad \tilde{z}_{gs} = \tilde{z}'_{es} ; \tag{26}$$

5) the time in the system $\mathbf{O}_g$ is

$$t_{gs} = t_{es} \Big/ \sqrt{1-\beta_e^2} + \tilde{t}_{gs} ; \tag{27}$$

6) the station coordinates in the system $\mathbf{O}_g$ are

$$\begin{aligned} x_{gs} &= \tilde{x}_{gs}\cos\varphi + \left(\tilde{y}_{gs} + R_e\right)\sin\varphi, \\ y_{gs} &= -\tilde{x}_{gs}\sin\varphi + \left(\tilde{y}_{gs} + R_e\right)\cos\varphi, \quad z_{gs} = \tilde{z}_{gs}. \end{aligned} \tag{28}$$

Thus, sequential recalculation $\mathbf{O}_e \to \mathbf{O}'_e \to \tilde{\mathbf{O}}'_e \to \tilde{\mathbf{O}}_g \to \mathbf{O}_g$ makes it possible to relate the coordinates of the station x_{es}, y_{es}, z_{es} at the time t_{es} of the geocentric system $\mathbf{O}_e$ with the coordinates of the station x_{gs}, y_{gs}, z_{gs} at the time t_{gs} of the heliocentric system $\mathbf{O}_g$.

The next step is to determine the relation between the current time t_s at the station, that is, in the system $\mathbf{O}_s$, and the current time t_g in the heliocentric system $\mathbf{O}_g$. According to (2), calculating the time dependence of the scalar of the station motion vector in the system $\mathbf{O}_g$ is required for it. Using the sequential transitions (23), (24), (26), (28), the coordinates x_{gs}, y_{gs}, z_{gs} can be directly expressed in terms of the time t_{es} in the system $\mathbf{O}_e$. In turn, the relation between t_{es} and t_{gs} is specified by the expression (27). Then, for the components of the velocity of the station movement v_x, v_y, v_z in the system $\mathbf{O}_g$, one can write:

$$\begin{aligned} v_x &= \frac{\mathrm{d}x_{gs}}{\mathrm{d}t_{gs}} = \frac{\mathrm{d}x_{gs}}{\mathrm{d}t_{es}}\frac{\mathrm{d}t_{es}}{\mathrm{d}t_{gs}}, \quad v_y = \frac{\mathrm{d}y_{gs}}{\mathrm{d}t_{gs}} = \frac{\mathrm{d}y_{gs}}{\mathrm{d}t_{es}}\frac{\mathrm{d}t_{es}}{\mathrm{d}t_{gs}}, \\ v_z &= \frac{\mathrm{d}z_{gs}}{\mathrm{d}t_{gs}} = \frac{\mathrm{d}z_{gs}}{\mathrm{d}t_{es}}\frac{\mathrm{d}t_{es}}{\mathrm{d}t_{gs}}, \end{aligned} \tag{29}$$

and the scalar of the velocity vector is $v_{gs} = \sqrt{v_x^2 + v_y^2 + v_z^2}$. The derivatives (29) can be easily calculated and are therefore omitted here.

The relations provided allow making point calculations of the station motion parameters in the system $\mathbf{O}_g$ at a specified time point t_{es} in the system $\mathbf{O}_e$. However, for the calculations to be effective, the expression of these parameters is directly required as a function of the current time t_g in the system $\mathbf{O}_g$. To do this, relevant approximations of the dependences $x_{gs}(t_g)$, $y_{gs}(t_g)$, $z_{gs}(t_g)$, $v_x(t_g)$, $v_y(t_g)$, $v_z(t_g)$ can be used. They can be obtained by the t_e point values within the required time interval, for example, using power polynomials. It should be noted that for the convenience of integration in (2) the approximation of the expression $J(t_g) = \sqrt{1-\left|\mathbf{v}_g(t_g)\right|^2 \big/ c^2} = \sqrt{1 - v_{gs}^2(t_g) \big/ c^2}$ is required. In so doing, the dependence $t_s(t_g)$ and the inverse dependence $t_g(t_s)$ can be also obtained.

If all the necessary relations are presented analytically, then, following the general procedure presented in Section 2, the times of signal emission t_{1s} and reception t_{2s} by the station can be recalculated in the corresponding times t_{1g}, t_{2g} of the system $\mathbf{O}_g$. Both of these events occur at the same point of the system $\mathbf{O}_e$, namely, at its origin. In the system $\mathbf{O}_g$, the points of emission and reception are determined by the radius vectors $\mathbf{r}_{1g} = \mathbf{r}_{gs}(t_{1g})$, $\mathbf{r}_{2g} = \mathbf{r}_{gs}(t_{2g})$ based on the analytical approximations which are obtained within the required time interval.

As in the case of the inertial motion of the Earth (Subsection 3.B), the data obtained determines the geometrical locus of points of the possible position of the spacecraft in the system $\mathbf{O}_g$. This locus is an ellipsoid of revolution with focuses on the points of emission and reception and the total range $r_g = c(t_{2g} - t_{1g}) = c\Delta t_g$. Semi-major and semi-minor axes, similarly to (21), are determined by the expressions $a = r_g/2$, $b = \sqrt{(r_g/2)^2 - (\Delta r_g/2)^2}$, where Δr_g is the range between the points of emission and reception in the system $\mathbf{O}_g$. And the moments of time registration of ellipsoid points are

$$t_{go} = t_{1g} + \sqrt{(x_{go} - x_{1g})^2 + (y_{go} - y_{1g})^2 + (z_{go} - z_{1g})^2} \,/ c\,,$$

where x_{go}, y_{go}, z_{go} are the coordinates of the points of the ellipsoid.

Similarly to Subsection 3.B, the conditional system $\mathbf{O}_m$ can be introduced that moves along the semi-major axis of the ellipsoid with the velocity $v_{gm} = \Delta r_g / \Delta t_g$ and sequentially coincides at the corresponding times with the points of emission and reception. For this system, at the moment $t_{0g} = (t_{1g} + t_{2g})/2$ of coincidence of its origin with the center of the ellipsoid $\mathbf{r}_{0g} = (\mathbf{r}_{1g} + \mathbf{r}_{2g})/2$, the geometrical locus of points of the possible position of the spacecraft is a sphere with the radius $R_m = b$ and the fixed moment of registration of the measurement results t_{mo}. It determines the coordinate system in which the range is calculated based on the moments of emission and reception measured by the station.

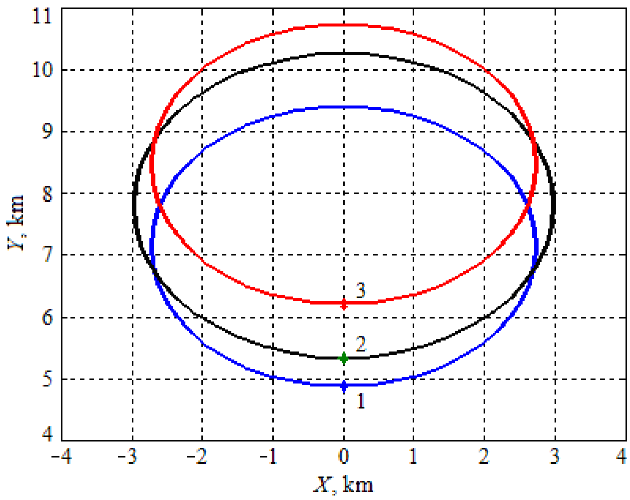
The coordinates of the center of the ellipsoid (point *m*) can be recalculated into the coordinate system of the station $\mathbf{O}_s$ according to the procedure presented in Section 2 (where "event" refers to the coincidence of the origin of coordinates of the system $\mathbf{O}_m$ with the center of the ellipsoid of the system $\mathbf{O}_g$ at the time t_{0g}). Calculations demonstrate that, at the same time moments t_{1s}, t_{2s}, these coordinates depend on both the time of day and the time of year.

In Figs. 5, there are shown examples of such a calculation in the form of parametric graphs of the coordinates x_{sm}, y_{sm} (Fig. 5a) and x_{sm}, z_{sm} (Fig. 5b) of the point *m* in the station system for the delay of $\Delta t_d = 2000$ s. The parameter is the current daily time t_s. The points mark the noon ones. On the *X*, *Y* curves, the time passing corresponds to the counter-clockwise direction, while on the *X*, *Z* curves – to the clockwise direction.

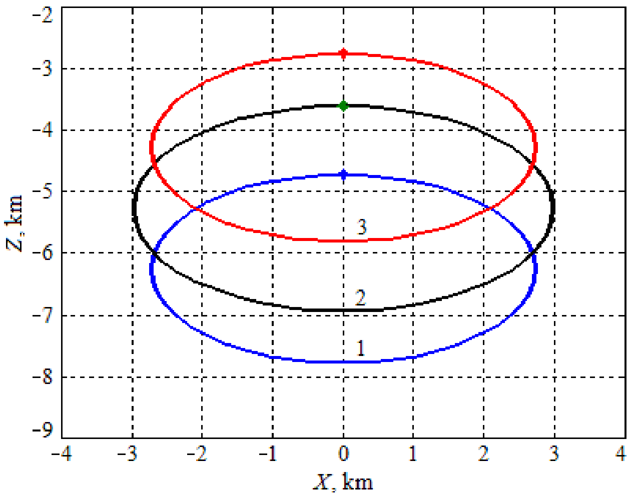
The graphs are plotted based on the location of the station at the latitude of Moscow for different times of the year. Blue lines correspond to the middle of winter, black lines – to spring or autumn (equinox), and red lines – to the middle of summer. As one can see from Figs. 5, the initial point of reference of the range by the measured delay is significantly shifted depending on the time of day and time of year. It results from the change in the law of motion of the station in the inertial heliocentric system.

Although, in general, the system $\mathbf{O}_s$ does not coincide with the system $\mathbf{O}_m$, however, the calculations under the conditions specified above demonstrate that the difference between the times of registration of the range measurement result R_m and the value $t_{0s} = (t_{1s} + t_{2s})/2$ does not exceed about 1 μs in all cases. At the same time, the change in the value of R_m is not reliably determined.

further course of transformations is:



a)



b)

Fig. 5. The parametric graphs of the coordinates x_{sm}, y_{sm} (a) and x_{sm}, z_{sm} (b) of the point *m* in the station coordinate system under the delay Δt_d = 2000 s (the circular motion of the Earth): 1 – winter; 2 – equinox; 3 – summer.

IV. The Range Measurement

A. *Preliminaries*

Concerning the problem of estimating the velocity of a spacecraft by a monostatic radar, the objective result of observation is the current value of the carrier frequency of the received signal. In so doing, the space-time relations in the radio channel should provide an appropriate interpretation of the measurement results, commonly referred to as Doppler shift. It is obvious that in this case, the result of radio observation can provide only one of the components of the motion vector. As a rule, this term refers to a radial component, i.e. the projection of the velocity vector on the radial direction relative to the station.

Determination of the radial component of the velocity vector by the measurement data of the Doppler shift is unambiguous for the inertial motion of the station. However, it is necessary to clarify the interpretation of the measurement results for the conditions of radio observation in middle and deep space, when the moments of emission and reception are separated by time intervals, during which the non-inertia of the system related to the station can be significantly felt.

Below, one of the possible interpretations of the relation between the measured value of the frequency of the signal received by the station and the components of the spacecraft velocity is provided.

B. Repeated Doppler Shift in an Inertial System in Retransmitting (Active Mode)

It is suggested that the spacecraft arbitrarily moves in an inertial system with a certain law of range variation $r(t)$. The station is located at the origin of the coordinates of the system and emits a harmonic signal with the frequency ω_0 so that the phase of the emitted signal is $\varphi_0(t)=\omega_0 t$. The signal phase at the point of the current position of the spacecraft is

$$\varphi_1(t)=\varphi_0(t-\tau_1(t))=\omega_0[t-\tau_1(t)], \tag{30}$$

where $\tau_1(t)=r(t)/c$ is the current signal delay.

If when retransmitting the frequency is transformed according to the ratio $q=m/n$, then the retransmitted signal possesses a phase (to within a constant shift) $\overline{\varphi}_1(t)=q\varphi_1(t)$. In turn, the phase of the signal received by the station is $\overline{\varphi}_2(t)=\varphi_1(t-\tau_2(t))$, where $\tau_2(t)$ is determined by the relation

$$\tau_2(t)=r(t-\tau_2(t))/c\,. \tag{31}$$

Thus, the signal received by the station has the phase

$$\overline{\varphi}_2(t)=q\omega_0[t-\tau_2(t)-r(t-\tau_2(t))/c].$$

The phase derivative (instantaneous frequency) is

$$\begin{aligned}\overline{\omega}_2(t)&=q\omega_0[t-\tau_2'(t)-v_r(t-\tau_2(t))(1-\tau_2'(t))/c]=\\&=q\omega_0[c-v_r(t-\tau_2(t))][1-\tau_2'(t)]/c.\end{aligned} \tag{32}$$

In (32), $v_r(t)$ is the spacecraft radial velocity (range derivative); $\tau_2'(t)$ is the retransmission delay derivative.

From (31), it can be written $\tau_2'(t)=v_r(t-\tau_2(t))[1-\tau_2'(t)]/c$, and from here one gets

$$\tau_2'(t)=v_r(t-\tau_2(t))/[c+v_r(t-\tau_2(t))]. \tag{33}$$

Thus, after substituting (33) into (32), one produces

$$\overline{\omega}_2(t)=q\omega_0\frac{c-v_r(t-\tau_2(t))}{c+v_r(t-\tau_2(t))}. \tag{34}$$

According to (34), the value of the received frequency at time t is determined by the radial velocity of the spacecraft at the time of retransmission $t_r=t-\tau_2(t)$. Thus, when measuring the radial velocity, it is necessary to know the range to the spacecraft at the time of retransmission (31). The required accuracy of knowing the range (delay in the retransmission channel) depends on the expected value of the radial acceleration.

C. Doppler Shift in the Inertial System with One-Sided Emission (Passive Mode)

The measurement of the radial velocity with one-sided emission by the spacecraft is hampered by inaccurate knowledge of the initial frequency of the onboard signal. However, in some cases, while retaining the parameters of the spacecraft's free motion (state vector) over significant time intervals, the influence of frequency deviations from the nominal value can be significantly reduced by refining the state vector. Such a study has been carried out, in particular, during the experiments with the spacecraft "Granat" [18]. Therefore, consideration of space-time relations for this case is of practical interest.

When the signal emission is one-sided, it is necessary to consider two independent systems: the inertial system **O**, in the center of which the receiving station is located, and the system $\mathbf{O}'$ that is non-inertial one in the general case and is related to the spacecraft.

The relation between the current time t' in the system $\mathbf{O}'$ and the current time t in the system **O** specified by the relation (2) one denotes as $t'=\mathrm{T}(t)$. In so doing, the current phase of the onboard harmonic signal $\varphi_0(t')=\omega_0 t'$ in the system **O** takes the form $\varphi_1(t)=\varphi_0(\mathrm{T}(t))=\omega_0\mathrm{T}(t)$.

Similarly to (30), the phase at the receiving point (origin O) is $\varphi_2(t)=\varphi_1(t-\tau_2(t))=\omega_0\mathrm{T}(t-\tau_2(t))$ while the instantaneous frequency is $\omega_2(t)=\omega_0\mathrm{T}'(t-\tau_2(t))[1-\tau_2'(t)]$, where $\mathrm{T}'(t)=\mathrm{d}\mathrm{T}(t)/\mathrm{d}t$.

It follows from (2) that $\mathrm{T}'(t-\tau_2(t))=\sqrt{1-|\mathbf{v}(t-\tau_2(t))|^2/c^2}$, where $|\mathbf{v}(t)|$ is the scalar of the spacecraft relative velocity vector at time t. Therefore, according to (33), one gets

$$\omega_2(t)=\frac{\omega_0 c\sqrt{1-|\mathbf{v}(t-\tau_2(t))|^2/c^2}}{c+v_r(t-\tau_2(t))}. \tag{35}$$

As it can be seen from (35), the measurement of the radial velocity in this mode presupposes the assignment of the scalar of the spacecraft orbital velocity. The last multiplier in the numerator of (35) captures the so-called "transverse Doppler effect" (frequency shift at $v_r(t)=0$). Formally neglecting this term can lead to significant errors. However, under conditions of inaccurate knowledge of the frequency ω_0 in separate measurement sessions, it should be considered acceptable to refer to the frequency deviation from the nominal value. It is also obvious that the value of the spacecraft range (delay $\tau_2(t)$ in the emission channel) should be obtained using other data.

D. Doppler Shift in an Active Mode of Operation with the Circular Motion of the Earth

Now it is time to consider the motion of two objects in some inertial system (for example, a heliocentric one) such as a measuring station and a spacecraft. As the specified conditions, one takes the two-dimensional function $r(\theta_1,\theta_2)$ determining the range between the point of spacecraft position at the moment θ_1 and the point of station position at the moment θ_2. It should be noted that in Subsections 4.B, 4.C this function degenerates into the one-dimensional one since the position of the station is assumed to be unchanged and it is the origin of the coordinates of the system. And besides, the function $r(\theta_1,\theta_2)$ can be related to the time function

$$\tau(\theta_1,\theta_2)=r(\theta_1,\theta_2)/c\,. \tag{36}$$

If a harmonic signal with the frequency ω_0 is emitted in the system $\mathbf{O}_s$ related to the station so that the signal phase is

$$\varphi_0(t_s)=\omega_0 t_s\,, \tag{37}$$

then in the heliocentric system $\mathbf{O}_g$ it corresponds to the frequency

$$\tilde{\omega}_0(t_s)=\omega_0\mathrm{T}'(t_g), \tag{38}$$

where $\mathrm{T}'(t_g)=\mathrm{dT}(t_g)/\mathrm{d}t_g$, and the function $\mathrm{T}(t_g)$ establishes the relation between the current times t_s and t_g:

$$t_s=\mathrm{T}(t_g). \tag{39}$$

It should be noted that there is a relation between the laws of change in the signal phase at the points of emission, retransmission and reception, taking into account the current values of the signal delay $\tau_1(t)$, $\tau_2(t)$ in the request and retransmission paths, but now in the heliocentric system, where the current time notation t is temporarily used instead of t_g.

When calculating $\tau_1(t)$, one should take $\theta_1=t$, $\theta_2=t-\tau_1(t)$ for the current moment t. Accordingly, when calculating $\tau_2(t)$ it can be written $\theta_1=t-\tau_2(t)$, $\theta_2=t$. Therefore, taking into account (36)

$$\tau_1(t)=\tau(t,t-\tau_1(t)),\qquad \tau_2(t)=\tau(t-\tau_2(t),t). \tag{40}$$

The next step is to determine the signal phases $\varphi_1(t)$ and $\varphi_2(t)$ at the points of retransmission and reception. Taking into account (36)-(40), for $\varphi_1(t)$ and $\varphi_2(t)$ it can be written

$$\varphi_1(t)=\tilde{\omega}_0\left[t-\tau_1(t)\right],\qquad \varphi_2(t)=\varphi_1(t-\tau_2(t)). \tag{41}$$

From here, the current derivative of the phase at the receiving point is determined as

$$\tilde{\omega}_2(t)=\varphi_1'(t-\tau_2(t))\left[1-\tau_2'(t)\right],$$

while according to (41)

$$\varphi_1'(t)=\tilde{\omega}_0\left[1-\tau_1'(t)\right].$$

To simplify the subsequent expressions, one denotes

$$\partial_1(t_a,t_b)=\left.\frac{\partial\tau(\theta_1,\theta_2)}{\partial\theta_1}\right|_{\substack{\theta_1=t_a\\ \theta_2=t_b}},\qquad \partial_2(t_a,t_b)=\left.\frac{\partial\tau(\theta_1,\theta_2)}{\partial\theta_2}\right|_{\substack{\theta_1=t_a\\ \theta_2=t_b}}.$$

As in each case, the arguments θ_1, θ_2 (36) in (40) are the functions of the current time t, one should introduce $\theta_1'(t)=\mathrm{d}\theta_1(t)/\mathrm{d}t$, $\theta_2'(t)=\mathrm{d}\theta_2(t)/\mathrm{d}t$. And besides, taking into account (40), one can write

$$\tau_1'(t)=\partial_1(t,t-\tau_1(t))\,\theta_1'(t)+\partial_2(t,t-\tau_1(t))\,\theta_2'(t)=$$
$$=\partial_1(t,t-\tau_1(t))+\partial_2(t,t-\tau_1(t))\left[1-\tau_1'(t)\right].$$

From here, it follows that

$$\tau_1'(t)=\frac{\partial_1(t,t-\tau_1(t))+\partial_2(t,t-\tau_1(t))}{1+\partial_2(t,t-\tau_1(t))}.$$

Similarly, one can get

$$\tau_2'(t)=\frac{\partial_1(t-\tau_2(t),t)+\partial_2(t-\tau_2(t),t)}{1+\partial_1(t-\tau_2(t),t)}.$$

The notations here are the following: $t_2=t$ is the reception time, $t_0=t-\tau_2(t)$ is the retransmission time, $t_1=t-\tau_1(t)-\tau_2(t)$ is the request time. Then, finally, one can obtain

$$\tilde{\omega}_2(t_2,t_0,t_1)=\tilde{\omega}_0\,\frac{1-\partial_1(t_0,t_1)}{1+\partial_1(t_0,t_2)}\,\frac{1-\partial_2(t_0,t_2)}{1+\partial_2(t_0,t_1)}. \tag{42}$$

Otherwise, the expression (42) can be written as

$$\tilde{\omega}_2(t_2,t_0,t_1)=\tilde{\omega}_0\,\frac{c-v_a(t_0,t_1)}{c+v_a(t_0,t_2)}\,\frac{c-v_s(t_0,t_2)}{c+v_s(t_0,t_1)}. \tag{43}$$

In (43), the notations are: $v_a(t_0,t_1)$ is the radial velocity of the spacecraft at the time of retransmission t_0 relative to the point of the station position at the time of emission t_1; $v_a(t_0,t_2)$ is the radial velocity of the spacecraft at the time of retransmission t_0 relative to the point of the station position at the time of reception t_2; $v_s(t_0,t_1)$ is the radial velocity of the station at the time of emission t_1 relative to the point of the spacecraft position at the time of retransmission t_0; $v_s(t_0,t_2)$ is the radial velocity of the station at the time of reception t_2 relative to the point of the spacecraft position at the time of retransmission t_0.

It should be noted that all the functions and variables are considered in the heliocentric system $\mathbf{O}_g$. Therefore, the current times of reception and emission in the systems $\mathbf{O}_s$, $\mathbf{O}_g$ are related by the relations (39), that is,

$$t_{1g} = \mathrm{T}^{-1}(t_{1s}), \qquad t_{2g} = \mathrm{T}^{-1}(t_{2s}), \tag{44}$$

while the frequency $\omega_2(t_{2s})$ measured at the station corresponds to the frequency $\tilde{\omega}_2(t_{2g}) = \omega_2(t_{2s})\mathrm{T}'(t_{2g})$. And, accordingly, for the time of emission one gets $\tilde{\omega}_0(t_{1g}) = \omega_0 \mathrm{T}'(t_{1g})$.

E. Determination of the Velocity Vector Components during the Circular Motion of the Earth

According to (43), the frequency of the received signal is determined by four parameters of the station and spacecraft motion in the system $\mathbf{O}_g$. These parameters can be transformed into a more convenient characteristic of the spacecraft motion in the system $\mathbf{O}_g$ that is the projection of the velocity vector onto a certain direction determined by three direction cosines or, otherwise, a certain component of the velocity vector.

If one introduces the ratio of the frequencies of the signals emitted and received by the station as $k = \omega_2(t_{2s})/\omega_0$, then the relation (43) can be written in the form

$$k = \frac{\mathrm{T}'(t_{2g})}{\mathrm{T}'(t_{1g})} \frac{c - v_a(t_{0g}, t_{1g})}{c + v_a(t_{0g}, t_{2g})} \frac{c - v_s(t_{0g}, t_{2g})}{c + v_s(t_{0g}, t_{1g})}.$$

As the movement of the station in the system $\mathbf{O}_g$ is known, the values $v_s(t_{0g},t_{1g})$, $v_s(t_{0g},t_{2g})$ can be calculated for a specified position of the spacecraft $\mathbf{r}_{ag} = [x_{ag}, y_{ag}, z_{ag}]$. To calculate the values $v_a(t_{0g},t_{1g})$, $v_a(t_{0g},t_{2g})$, one uses the following ratio:

$$K = \frac{c - v_a(t_{0g}, t_{1g})}{c + v_a(t_{0g}, t_{2g})} = k \frac{\mathrm{T}'(t_{1g})}{\mathrm{T}'(t_{2g})} \frac{c + v_s(t_{0g}, t_{1g})}{c - v_s(t_{0g}, t_{2g})}. \tag{45}$$

The values $v_a(t_{0g},t_{1g})$, $v_a(t_{0g},t_{2g})$ are the projections of the vector $\mathbf{v}_{ag} = [v_{xg}, v_{yg}, v_{zg}]$ of the spacecraft velocity to the direction "station-spacecraft" at the appropriate times. The unit vectors of these directions are denoted as $\mathbf{n}_1(t_{0g},t_{1g}) = [n_{x1}, n_{y1}, n_{z1}]$, $\mathbf{n}_2(t_{0g},t_{2g}) = [n_{x2}, n_{y2}, n_{z2}]$. Thus,

$$v_a(t_{0g},t_{1g}) = \mathbf{v}_{ag}\mathbf{n}_1(t_{0g},t_{1g}) = v_{xg}n_{x1} + v_{yg}n_{y1} + v_{zg}n_{z1},$$

$$v_a(t_{0g},t_{2g}) = \mathbf{v}_{ag}\mathbf{n}_2(t_{0g},t_{2g}) = v_{xg}n_{x2} + v_{yg}n_{y2} + v_{zg}n_{z2}.$$

In this case, by applying (45), one can obtain

$$v_{xg}(Kn_{x2} + n_{x1}) + v_{yg}(Kn_{y2} + n_{y1}) + v_{zg}(Kn_{z2} + n_{z1}) = \\ = c(1-K). \tag{46}$$

Denoting

$$M_g = \sqrt{(Kn_{x2} + n_{x1})^2 + (Kn_{y2} + n_{y1})^2 + (Kn_{z2} + n_{z1})^2}$$

and dividing both sides of (46) by M_g, one gets

$$v_{xg}l_g + v_{yg}m_g + v_{zg}n_g = c(1-K)/M_g. \tag{47}$$

The left side of the relation (47) is the component of the velocity vector of the spacecraft with the direction cosines $l_g = (Kn_{x2} + n_{x1})/M_g$, $m_g = (Kn_{y2} + n_{y1})/M_g$, $n_g = (Kn_{z2} + n_{z1})/M_g$, while the measured value of the frequency $\omega_2(t_{2s})$ determining this component belongs to the right side of (47). In vector form, the expression (47) can be represented as follows

$$\mathbf{n}_g^{+}\mathbf{v}_g = V_g. \tag{48}$$

In further calculations, one implies the joint measurement of the range and the velocity, so that the times of emission t_{1s} and reception t_{2s} are fixed directly at the station and, according to (44), can be recalculated into the values t_{1g}, t_{2g} in the system $\mathbf{O}_g(t_{1g},t_{2g})$.

The retransmission time t_{0g} and the unit vectors $\mathbf{n}_1$, $\mathbf{n}_2$ are specified by choosing a point on the surface of the ellipsoid, which determines the possible positions of the spacecraft at the fixed times of emission and reception (see Section 3). It should be noted that

$$\mathbf{n}_1(t_{0g},t_{1g}) = [\mathbf{r}_{go}(t_{0g}) - \mathbf{r}_{gs}(t_{1g})] / |\mathbf{r}_{go}(t_{0g}) - \mathbf{r}_{gs}(t_{1g})|,$$

$$\mathbf{n}_2(t_{0g},t_{1g}) = [\mathbf{r}_{go}(t_{0g}) - \mathbf{r}_{gs}(t_{2g})] / |\mathbf{r}_{go}(t_{0g}) - \mathbf{r}_{gs}(t_{2g})|,$$

where $\mathbf{r}_{go}(t)$, $\mathbf{r}_{gs}(t)$ are the radius vectors of the spacecraft and the station at the corresponding times.

To recalculate the measured components of the velocity vector from the system $\mathbf{O}_g$ to other systems, it is necessary to introduce general relations for transforming velocities in inertial systems. From the Lorentz transformation (1), it follows that

$$\frac{dx}{dt}=\frac{dx}{dt'}\frac{dt'}{dt}=\frac{dx'/dt'-v}{1-\left(v/c^2\right)dx'/dt'},$$
$$\frac{dy}{dt}=\frac{dy}{dt'}\frac{dt'}{dt}=\frac{dy'}{dt'}\frac{\sqrt{1-\beta^2}}{1-\left(v/c^2\right)dx'/dt'},\qquad(49)$$
$$\frac{dz}{dt}=\frac{dz}{dt'}\frac{dt'}{dt}=\frac{dz'}{dt'}\frac{\sqrt{1-\beta^2}}{1-\left(v/c^2\right)dx'/dt'}.$$

Using other notation, the expression (49) can be written in the form

$$v_x=\frac{v_x'-v}{1-vv_x'/c^2},\quad v_y=v_y'\frac{\sqrt{1-\beta^2}}{1-vv_x'/c^2},\quad v_z=v_z'\frac{\sqrt{1-\beta^2}}{1-vv_x'/c^2}.\qquad(50)$$

It should be noted that in (49), (50), the relative velocity v is an algebraic value, i.e. it can take both positive and negative values.

It is time to recalculate the direction of the measured component of the velocity vector and its value in its relation to the system $\mathbf{O}_m$ (see Subsection 3.C).

Sequential recalculation should be based on the relation (47). Expressing the orthogonal components of the velocity vector of the reference system in terms of the same components of the transformed system, one can obtain a new relation, the corresponding normalization of which will determine the direction cosines of the measured component in this system (left side) and its value (right side).

The order of conversion into the system $\mathbf{O}_m$ should be as follows:

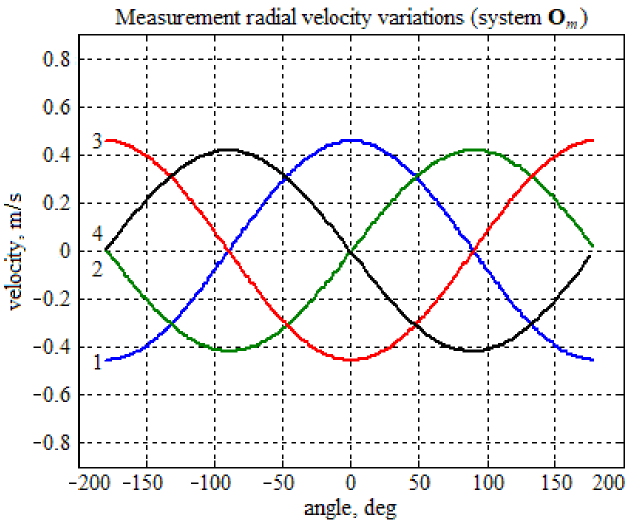
1) recalculating the relation (47) into the system $\mathbf{O}_{ge}$ related to the ellipsoid of the spacecraft's possible positions and obtained by the angular rotation and parallel translation of the coordinate axes of the system $\mathbf{O}_g$ in the following way:

$$\mathbf{v}_g=\mathbf{N}_{ge}^{+}\mathbf{v}_{ge},$$

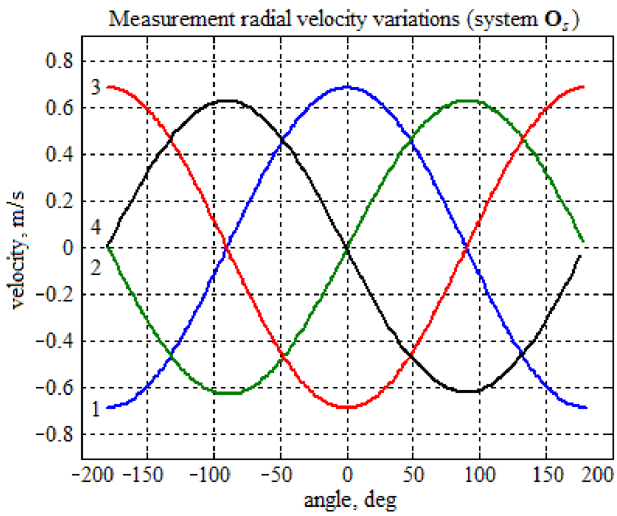
where $\mathbf{N}_{ge}$ is the angular coordinate transformation matrix, $\mathbf{v}_{ge}$ is the vector of the measured velocity component in the system $\mathbf{O}_{ge}$, while

$$\mathbf{n}_{ge}^{+}\mathbf{v}_{ge}=V_g,\qquad \mathbf{n}_{ge}=\mathbf{N}_{ge}\mathbf{n}_g;\qquad(51)$$

2) recalculating the relation (50) into the system $\mathbf{O}_m$ applying (49), (51) and taking into account that the velocity of the system $\mathbf{O}_{ge}$ relative to the system $\mathbf{O}_m$ is negative:



a)



b)

Fig. 6. The dependences of the change in the results of measuring the radial velocity upon the direction to the spacecraft in the coordinate systems $\mathbf{O}_m$ (a) and $\mathbf{O}_s$ (b) (the circular motion of the Earth, active mode): 1 – noon; 2 – evening; 3 – midnight; 4 – morning.

$$v_{xge}=\frac{v_{xm}+v_{gm}}{1+v_{gm}v_{xm}/c^2},\qquad v_{yge}=v_{ym}\frac{\sqrt{1-\beta_m^2}}{1+v_{gm}v_{xm}/c^2},$$
$$v_{xm}\left(n_{xge}-V_g v_{gm}/c^2\right)+v_{ym}n_{yge}\sqrt{1-\beta_m^2}+$$
$$+v_{zm}n_{zge}\sqrt{1-\beta_m^2}=V_g-v_{gm}n_{xge},$$

$$M_m=\sqrt{\left(n_{xge}-V_g v_{gm}/c^2\right)^2+n_{yge}^2\left(1-\beta_m^2\right)+n_{zge}^2\left(1-\beta_m^2\right)},$$

$$\mathbf{n}_m^{+}\mathbf{v}_m=V_m,\qquad V_m=V_g/M_m,\qquad(52)$$

$$\mathbf{n}_m=M_m^{-1}\left[n_{xge}-\frac{V_g v_{gm}}{c^2},\,n_{yge}\sqrt{1-\beta_m^2},\,n_{zge}\sqrt{1-\beta_m^2}\right].$$

Calculations using the formulas (48), (51), (52) show that the velocity component measured in the system $\mathbf{O}_m$ is radial, i.e. its direction coincides with the line of sight of the spacecraft from the origin of the coordinates of the system. However, the value of this component depends on the chosen direction. This is because in the inertial system $\mathbf{O}_m$, the points of emission and reception, although they are located at the origin of coordinates, have derivatives by the time as the station is moving in this system, and the projections of movement on different radial directions differ.

In Fig. 6a, the graphs of the change in the measurement result are shown for the conditions specified in Section 3, Subsection B in the absence of the Doppler shift of the frequency measured at the station ($k=1$).The argument of the graph is the angle in some section of the ellipsoid of the system $\mathbf{O}_g$ passing through its semi-major axis. The season is mid-summer. The numbers indicate the following: 1 (blue curve) – noon; 2 (green curve) – evening; 3 (red curve) – midnight; 4 (black curve) – morning.

It can be seen from the presented graphs that the measured value of the radial velocity varies within ± 45 cm/s. This implies that the direction from the emitting point onto the spacecraft during the measurements at the time t_{mo} should be taken into account.

The recalculation of the measured velocity component into the system $\mathbf{O}_s$ related to the measuring station should be carried out based on the general procedure described in Section 2 for arbitrary curvilinear motion in the inertial system. For the times $\bar{t}$ calculated by this procedure, the relations (48), (51), (52) can be used in the same way as while transforming the measured component of the velocity vector in inertial systems, according to the scheme $\mathbf{O}_g \to \tilde{\mathbf{O}}_g \to \tilde{\mathbf{O}}_s \to \mathbf{O}_s$. The calculation results produced in the system $\mathbf{O}_s$ are shown in Fig. 6b. It is noteworthy that these calculations have been carried out under the same conditions as the ones which results are presented in Fig. 6a.

In the system $\mathbf{O}_s$, the measured value of the velocity component already changes within ± 68 cm/s. In addition, it appears to be not radial with respect to the point of the station position, but remains practically radial with respect to the point *m* in the coordinate system $\mathbf{O}_s$.

F. Measurement of the Velocity Component in the Passive Mode of Operation during the Circular Motion of the Earth

Now one can consider the task of measuring the velocity while one-sided signal emission from the spacecraft is implemented. It is assumed that the function $\tau(\theta_1, \theta_2)$ (36) is still pre-defined.

In this case, in the system $\mathbf{O}_g$ two points in time should be considered: the moment of signal reception by the station t_2 and the moment of emission from the spacecraft $t_0 = t_2 - \tau_2(t_2)$. The moment of reception that takes place in the system $\mathbf{O}_g$ is calculated directly by the moment of measuring the frequency of the signal received at the station.

To determine the moment of emission, it is necessary to solve the nonlinear equation of the form $\tau(t-\tau_2(t), t) = \tau_2(t)$ in view of the delay $\tau_2(t)$ in the transmission path.

The relation between the current time t_a at the emitting spacecraft and the current time t_g in the system $\mathbf{O}_g$ can be specified by the relation $t_a = \mathrm{T}_a(t_g)$. In the subsequent relations, the current time in the system $\mathbf{O}_g$, just as in Subsection 4.D, is denoted by *t*. Thus, the phase of the emitted harmonic signal with the frequency ω_0 is $\varphi_1(t) = \omega_0 \mathrm{T}_a(t)$, while the phase $\varphi_2(t)$ and the instantaneous frequency $\tilde{\omega}_2(t)$ of the signal received by the station are $\varphi_2(t) = \varphi(t-\tau_2(t)) = \omega_0 \mathrm{T}_a(t-\tau_2(t))$ and $\tilde{\omega}_2(t) = \omega_0 \mathrm{T}'_a(t-\tau_2(t))[1-\tau'_2(t)]$.

Similarly to Section 4.C, $\mathrm{T}'_a(t-\tau_2(t)) = \sqrt{1 - |\mathbf{v}_a(t-\tau_2(t))|^2 / c^2}$, where $|\mathbf{v}_a(t-\tau_2(t))|$ is the scalar of the spacecraft velocity vector in the system $\mathbf{O}_g$. In turn, using the notations introduced in Section 4.D, the derivate $\tau'_2(t)$ allows the representation

$$\tau'_2(t) = \frac{\partial_1(t-\tau_2(t), t) + \partial_2(t-\tau_2(t), t)}{1 + \partial_1(t-\tau_2(t), t)}.$$

From here, it can be written

$$\tilde{\omega}_2(t_2, t_0) = \omega_0 \frac{c - v_s(t_0, t_2)}{c + v_a(t_0, t_2)} \sqrt{1 - |\mathbf{v}_a(t_2)| / c^2}, \quad (53)$$

where $v_a(t_0, t_2)$ is the radial velocity of the spacecraft at the time of emission t_0 relative to the point of the station position at the time of reception t_2; $v_s(t_0, t_2)$ is the radial velocity of the station at the time of reception t_2 relative to the position of the spacecraft at the time of emission t_0.

The frequency of the received signal in the system $\mathbf{O}_g$ is related to the frequency $\omega_2(t_{2s})$ measured at the station by the relation $\tilde{\omega}_2(t_{2g}) = \omega_2(t_{2s}) \mathrm{T}'(t_{2g})$, where $\mathrm{T}'(t_{2g}) = \sqrt{1 - |v_a(t_{2g})|^2 / c^2}$. Then, taking into account (53), one can write:

$$K = \frac{c}{c + v_a(t_0, t_2)} = k \frac{c\sqrt{1 - |\mathbf{v}_s(t_2)|^2 / c^2}}{[c - v_s(t_0, t_2)]\sqrt{1 - |\mathbf{v}_a(t_0)|^2 / c^2}},$$

$$k = \omega_2(t_2)/\omega_0,$$

while the value of the measured component of the velocity vector can be presented as $v_a(t_0,t_2)=c(1-K)/K$.

The angular direction of the component coincides with the direction from the point of the station position at the time of reception t_{2g} to the point of the spacecraft position at the time of emission t_{0g}. Recalculation of the measurement results into the station system $\mathbf{O}_s$ can be carried out in the same way as in Subsection 4.E.

As an example, the calculation results are provided below for a fixed value of the delay in the emission path $\tau_2=1000\,\text{s}$ determining the geometrical locus of points of the possible position of the spacecraft in the system $\mathbf{O}_g$ in the form of a sphere with a radius of about $3\cdot10^8$ km.

In Fig. 7, the change is shown in the measured component of the spacecraft velocity relative to the station in the central section of the sphere parallel to the *XY* plane of the system $\mathbf{O}_g$, while Doppler shift is zero and the orbital velocity of the spacecraft is $v=20$ km/s. The graph argument is the angle of sight of the spacecraft in the cross-section of the sphere from the point of the station location in the system $\mathbf{O}_g$. The measurement time corresponds to the middle of winter. The numbers denote the times of day during measurements: 1 (blue curve) – noon; 2 (green curve) – evening; 3 (red curve) – midnight; 4 (black curve) – morning.

As it follows from the presented graphs, the result of measuring the velocity vector component at zero frequency shift measured at the station significantly depends upon the angle of sight of the spacecraft. The direction of the measured component does not coincide with the radial one and, as calculations show, the deviation from the radial direction reaches about 10^{-4} rad.

In Fig. 8, for the same angular directions, there are plotted the graphs of changes in the times of registration of the measurement results that also demonstrate a significant dependence of these times upon the direction to the spacecraft.

Thus, the performed consideration shows that in order to accurately measure the velocity vector component in the passive mode of operation, the knowledge of the spacecraft coordinates is even more necessary. And besides, the one-sided emission method requires data on the orbital velocity module of the spacecraft.

V. Conclusion

The conducted analysis of the space-time relations for the curvilinear motion of the station in the inertial heliocentric system allows considering the radar as an autonomous measuring instrument that provides the measurement of the range to the spacecraft in the certain coordinate system $\mathbf{O}_m$ and the assignment of the measurement result to the calculated time t_{mo} in this system.

Almost the same result is obtained in the coordinate system $\mathbf{O}_s$ related to the station for the mean time between

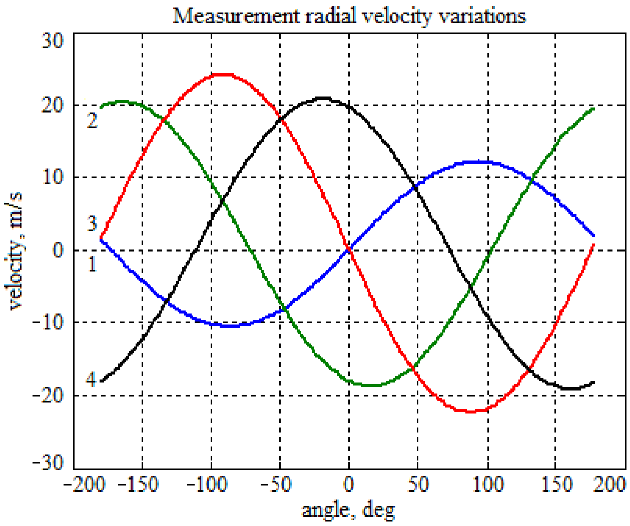


Fig. 7. The dependences of the variations of the measured component of the spacecraft velocity upon the angle of sight of the spacecraft (the circular motion of the Earth, passive mode): 1 – noon; 2 – evening; 3 – midnight; 4 – morning.

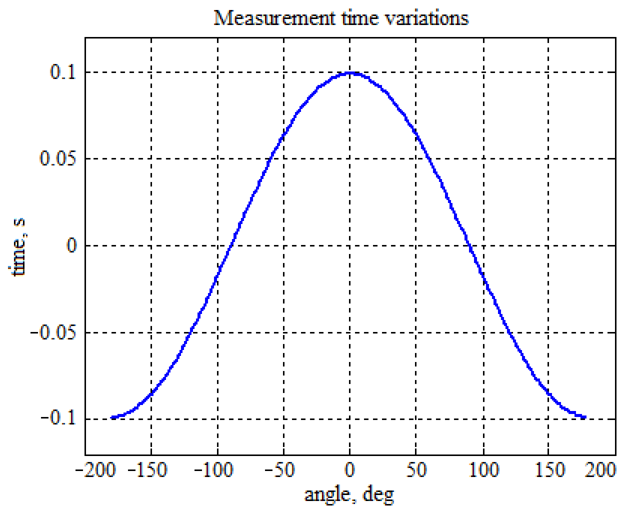


Fig. 8. The dependence of the variations of the moments of registration upon the angle of sight of the spacecraft (the circular motion of the Earth, passive mode).

the emission and reception of the signal relative to the calculated point *m* which does not coincide with the origin (the station location point) and changes its position with a measured signal delay of several kilometers depending on the time of day and time of the year.

With the simultaneous measurement of the range and the velocity (active operation mode) in the system $\mathbf{O}_m$, the radial component of the spacecraft velocity vector is determined. Its value depends upon the angle of sight of the spacecraft at the same Doppler shift measured at the station. In the station coordinate system, the radial velocity relative to point *m* is measured, and the measured value also depends upon the angular direction of the spacecraft.

In the passive mode of operation (one-sided emission), to accurately measure the direction and value of the velocity vector component, the knowledge of the coordinates and the scalar of the orbital velocity module of the spacecraft are required.

The transition to an inertial heliocentric system allows recalculating the measurement result to any specified reference system.

It should also be noted that in all the cases of the above analysis of space-time relations, time is considered as an objective category. However, in reality, time samples are produced by one or another "clock" and, since physical processes in non-inertial systems proceed, generally speaking, differently than in the inertial ones, the readings of various "clocks" (quartz, molecular) may differ.

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