Robust Optimization Model Using Ellipsoidal and Polyhedral Uncertainty Sets for Spatial Land-Use Allocation Problem

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Abstract—**Land-use planning becomes an important thing to do because some land-use types can impact the environment and life quality. Land-use planning is generally an activity that involves the allocation of activities in a particular land. Spatial Optimization can be applied in land-use planning activity. The optimization model for the land-use allocation problem aims to determine the percentage of land-use changes that can maximize the comprehensive index and compactness index. In land-use planning, there are several uncertainty factors. In this paper, the robust optimization model for the land-use allocation problem is discussed as a multi-objective function. There are two objective functions to maximize the comprehensive index and to maximize the density index. The uncertainties are assumed are in benefit and acquisition cost when a land planning is changed to another land-use type. In order to get a robust counterpart (RC) formulation, the uncertain parameter is assumed to lie within an ellipsoidal and a polyhedral uncertainties set. A study for the numerical experiment is done for Jatinangor District in Kabupaten Sumedang, Indonesia, as an educational area. In this case study, two scenarios are discussed, i.e., the first scenario allows a policy for changing all types of land to be other land types. The second scenario preserves a condition that a type of land cannot be changed to another type. To handle the multi-objective optimization function, the lexicographic method is employed. It is shown that the computational tractability of the RC is gained with ellipsoidal and polyhedral uncertainty set. Thus, the robust optimal solutions are achieved.**

Index Terms—**robust optimization, spatial land-use allocation problem, ellipsoidal, polyhedral, uncertainty set.**

I. Introduction

As an essential component of human living, land is important to help human needs. Referring to [1], land-use involves the manner how land is modified, managed, maintained and the intended use. The land-use arrangement is basic for biosphere function because several land-uses such as residential, industry, agriculture, and green-land have a huge impact on the environment and life quality.

Land-use planning commonly involves the allocation of land-use activities to a particular plot of land. Spatial Optimization connecting Geographic Information System (GIS) and mathematical modeling has been increasingly applied to support the evaluation of Land-use planning activities [1]. Church [2] introduced a basic land-use planning problem with the knapsack model and threshold model. The model was developed by Yao and Murray [3] in which this model considers the types of land-use.

In Xiaoli *et al.* [4] developed a spatial optimization model for land-use planning to maximize the comprehensive index and density index. Chaerani *et al.* [5] discussed a multi-objective optimization model for land-use allocation problem with spatial data analysis. The problem is considered as an integer linear programming problem.

In the land-use allocation problem, uncertainty factors are affecting the allocation of the land-use system. Thus, an optimization technique is needed considering the uncertainty factors such that a resistance optimal solution is obtained agains uncertainty. One of the optimization models which can handle uncertainty factors is Robust Optimization (RO). As mentioned in Szer and Thiele [6], the first step towards achieving the RO method was carried out by A.L. Soyster in 1973, followed by Mulvey, Vanderbei and Zenios in [7] also Ben-Tal and Nemirovskii in [8].

The general formulation for Robust Optimization is given as follows.

$$\begin{aligned} \min_{x_0,x}\ & x_0, \\ s.t\ & f_0(x,\zeta) \le 0, \\ & f_i(x,\zeta) \le 0, i = 1,\dots,m, \\ & \forall \zeta \in \mathcal{U}. \end{aligned} \tag{1}$$

where x is a design vector, f_0 is the objective function, f_i is constraint functions, ζ stands for the data specifying a particular problem instance, and $\mathcal{U}$ is the uncertain set.

According to Ben-Tal and Nemirovskii in [8], the RO paradigm is mainly play the role by considering that all decisions variable $x \in \mathcal{R}^n$ represent here-and-now decisions. The decision variable should get a specific numeral values when the problem is solved. When the actual data is assumed within the an uncertainty $\mathcal{U}$ the decision must be taken. The constraints are hard, this means that all constraints must be occupied when the data is in the uncertainty set $\mathcal{U}$. Without loss of generality, the assumptions are the objective and the vector of right-hand side are certain. The set $\mathcal{U}$ is a compact-convex set and the uncertainty is constraint-wise. The main challenge in uncertain optimization is in determining how and when the problem becomes a computationally tractable problem as stated by Ben-Tal [9].

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and Brown in [10] introduced a methodology for constructing a polyhedral uncertainty set for RO in linear programming model. Ben-Tal *et al.* [11] presents a robust counterpart formulation for non-linear inequalities with uncertain parameters. Gorissen *et al.* [12] compiled practical guidelines for RO. The problem is solved by considering uncertainty data located in an uncertainty set. There are several types of uncertainty sets i.e. box uncertainty, ellipsoidal uncertainty, and polyhedral uncertainty.

A short survey on optimization methods for solving the land-use allocation problem is presented. The survey is done by doing a bibliometrics mapping using Publish or Perish from Harzing [13] and VosViewer software (see Van Eck & Waltman, 2010 in [14]). The literature search employed the Google Scholar (GS) databases with 1000 papers and Scopus with 200 papers. We restricted the search to articles published in international peer-reviewed journals that were written in English and published during 2010-2020. The keywords ("Optimization") AND ("Land-use Allocation Problem") are used in literature search. The result can be seen in Figure 1.

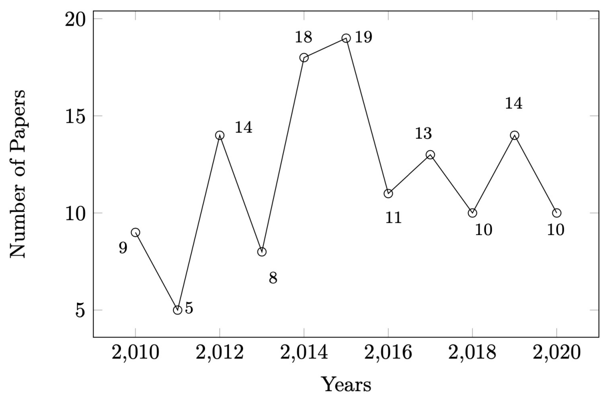

Fig. 1. Total Number of Papers with Keyword ("Optimization") AND ("Land-use Allocation Problem) from 2010 to 2020

The summary of the optimization methods used to solve the land-use allocation problem is presented in Table I. The most cited paper on optimization land-use allocation problem can be seen in in Table II.

TABLE I
SOLVING LAND-USE ALLOCATION PROBLEM WITH VARIOUS OPTIMIZATION METHODS (2010-2020)

Optimization Methods	**Number of Papers**
Integer Programming	76
Multi-objective Optimization	48
Robust Optimization	7

Based on literature search using Publish or Perish, Table III presents the recent papers with topic robust optimization and land-use allocation problem. Recent research on robust optimization for uncertain land-use allocation problem with box uncertainty is Romhadhoni *et al.* [20]. Same as Romhadhoni *et al.*, in this paper, the certain spatial optimization model for land-use allocation is formulated by combining the model of [3] and [4]. The difference is that the robust optimization model in this paper are performed using ellipsoidal and polyhedral uncertainty set. Assume that the uncertainty data are benefit and acquisition cost. The robust counterpart is formulated when the uncertain benefit and acquisition cost lie within two uncertainty sets, i.e., ellipsoidal uncertainty or polyhedral uncertainty.

TABLE II
THE MOST CITED PAPER ON OPTIMIZATION LAND-USE ALLOCATION PROBLEM

Cites	**Author**	**Title**
326	X Liu *et al.* [15]	A future land-use simulation model (FLUS) for simulating multiple land-use scenarios by coupling human and natural effects
223	K Cao *et al.* [16]	Sustainable land-use optimization using boundary-based fast genetic algorithm
205	K Cao *et al.* [17]	Spatial multi-objective land-use optimization: extensions to the non-dominated sorting genetic algorithm-II
146	X Liu *et al.* [18]	Classifying urban land-use by integrating remote sensing and social media data
126	Arciniegas *et al.* [19]	Spatial decision support for collaborative land-use planning workshops

TABLE III
PAPERS ON THE TOPIC ROBUST OPTIMIZATION AND LAND-USE ALLOCATION PROBLEM

Cites	Authors	Title
49	M Zhou [21]	An interval fuzzy chance-constrained programming model for sustainable urban land-use planning and land-use policy analysis
31	H Wang *et al.* [22]	Sustainable transportation network design with stochastic demands and chance constraints
4	E Reith *et al.* [23]	How Much Agroforestry Is Needed to Achieve Multifunctional Landscapes at the Forest Frontier Coupling Expert Opinion with Robust Goal Programming
2	Ramezanian *et al.* [24]	Integrated framework of system dynamics and meta-heuristic for multi-objective land-use planning problem
1	P Romhadhoni *et al.* [20]	Robust Optimization Model for Spatial Land-Use Allocation Problem in Jatinangor Subdistrict, Indonesia
1	CD Palma [25]	Robust optimization for forest resources decision-making under uncertainty

II. MATERIALS AND METHOD

In this section, the materials and method used in this paper is discussed. A brief discussion of model of [3] and [4], theory of Robust Optimization, Lexicographic Method and short description of case studies can be seen in this section.

A. Optimization Model for Land-use Allocation Problem

The following formulation is the land-use allocation optimization model that is introduced by Yao and Murray [3].

$$\min \sum_{i=1}^{N} \sum_{k=1}^{K} \overline{c_{ik}} x_{ik}, \tag{2}$$

$$\max \sum_{i=1}^{N} \sum_{k=1}^{K} a_{ik} x_{ik}, \tag{3}$$

$$s.t. \sum_{i=1}^{N} \overline{c_{ik}} x_{ik} \leq \theta_k, \forall k, \tag{4}$$

$$\sum_{i=1}^{N} a_{ik} x_{ik} \geq L_k, \forall k, \tag{5}$$

$$\sum_{k=1}^{K} x_{ik} = 1, \forall i, \quad (6)$$

$$\sum_{i=1}^{N} s_i x_{ik} \geq LS_k, \forall k, \quad (7)$$

$$\sum_{i=1}^{N} s_i x_{ik} \leq US_k, \forall k, \quad (8)$$

$$x_{ik} \in \{0,1\}, \forall i, k, \quad (9)$$

where the decision variable is declared as follows:

$$x_{ik} = \begin{cases} 1 \text{ if land parcels } i \text{ is used for land-use type } k, \\ \\ 0 \text{ otherwise.} \end{cases}$$

In $(2)-(8)$, consider that the total number of land parcels is denoted by N, total number of land-use types is denoted by K, the benefit if land parcel i is used for land-use type k is noted by a_{ik}., $\overline{c_{ik}}$ is acquisition cost if land parcel i is used for land-use type k, the total budget for acquisition of land-use type k is $_k$, the minimum benefit desired for land-use type k is L_k, US_k is upper bound of area for land-use type k, and LS_k is lower bound of area for land-use type k. The objective functions (2) and (3) are to minimize the total of acquisition cost and to maximize the total of benefit. Constraint (4) limit the total acquisition cost for each land-use type. Constraint (5) require as minimum level of benefit for each land-use type. Constraint (6) restrict only one land-use assigned to each land parcel. The lower and upper bounds on the total area for each land-use type is presented in Constraint (7) and (8). The decision variables to be binary as stated in Constraints (9).

The optimization model for land-use allocation that is introduced by Xiaoli *et al.* [4] is written in the following formulation.

$$\max Z = \sum_{k=1}^{K} \sum_{i=1}^{n} z_{ik} x_{ik}, \quad (10)$$

$$\max R_k = \sum_{i=1}^{n} r_{ik} x_{ik}, \quad (11)$$

$$s.t. \;\; B_{1k} \leq \sum_{i=1}^{n} a_i x_{ik} \leq N_{2k}, \forall k = 1, 2, ..., K, \quad (12)$$

$$\sum_{k=1}^{K} x_{ik} = 1, \forall i = 1, 2, ..., n, \quad (13)$$

$$x_{ik} \in \{0,1\}, \forall i, k, \quad (14)$$

where z_{ik} is comprehensive index if the planning unit i is arranged to land-use type k, r_{ik} is density index if the planning unit i is arranged to land-use type k, B_{1k} is lower bound on the total area for land-use type k, B_{2k} is upper bound on the total area for land-use type k, a_i is the area of planning unit i. The decision variables in Xiaoli *et al.* [4], x_{ik}, is defined as planning unit instead of land parcel as defined in Yao and Murray [3]. The notations as follows.

$$x_{ik} = \begin{cases} 1 \text{ if planning unit } i \text{ is arranged to land-use type } k, \\ \\ 0 \text{ otherwise.} \end{cases}$$

In Xiaoli *et al.*[4], the objective functions (equation (10) and (11)) are to maximize the comprehensive index and density index. Constraint (12) present the lower and upper bounds on each land-use type's total area. Constraint (13) restrict only one land-use type assigned to every planning unit. Constraint (14) require the decision variables to be binary.

B. Determination of Comprehensive Index and Density Index

The spatial weight matrix is W_{nxn} matrix with each element w_{ij} indicating the proximity measurement values between location i and location j which is observed based on the neighbourhood relations between locations. The closeness is determined based on contiguity. If location i is adjacent to or directly adjacent to location j, then the element (i,j) is given a value of 1. If location i is not adjacent to or directly adjacent to location j, then the element (i,j) is given a value of 0.

According to Le Sage [26], some methods to determine contiguity are rook contiguity, bishop contiguity, and queen contiguity. The contiguity matrix is standardized using this following formula:

$$w_{ij} = \frac{p_{ij}}{p_i} \quad (15)$$

where

$$p_i = \sum_{i=1}^{n} p_{ij} \quad (16)$$

with p_i is the number of values in the row i, $i = 1, 2, \ldots, n$ and p_{ij} is the value in the row i and column j with $j = 1, 2, \ldots, n$.

The comprehensive index and density index in the objective function is determined by a spatial weight matrix that states the correlation between land locations. A comprehensive index can be determined by determining the spatial weight matrix for land combinations. For example, the spatial weight matrix is stated in the following form:

$$W = \begin{pmatrix} w_{11} & w_{12} & \ldots & w_{1n} \\ w_{21} & w_{22} & \ldots & w_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ w_{m1} & w_{m2} & \ldots & w_{mn} \end{pmatrix} = \begin{pmatrix} w_1 \\ w_2 \\ \vdots \\ w_m \end{pmatrix}. \quad (17)$$

The vector z which represents a comprehensive index of land-use change from type i to type j is determined based on the spatial weight matrix, it can be stated as follows:

$$z^T = \left(w_1^T \; w_2^T \; \ldots w_m^T \right). \quad (18)$$

The density index matrix R represents the incidence matrix where the rows of the matrix represent the transition land planning from type i to type k and the column of the matrix represents type j. Thus, the elements of the matrix can be determined as follows:

$$R_{(i,k),j} = \begin{cases} z_{ij} & \text{if } k = j, \\ \\ 0 \text{ otherwise.} \end{cases} \quad (19)$$

C. Robust Optimization

In this section, a short discussion robust optimization paradigm and robust counterpart formulation is presented.

1) Robust Optimization Paradigm: According to Ben-Tal and Nemirovski [8], Robust Optimization is a method for solving optimization problems with uncertainty data and the data are only known in uncertainty set. In case for Robust Linear Programming with $c \in R^n$, $b \in R^m$ and $A \in R^{mxn}$, the formulation for Uncertain Linear Programming is presented as follows.

$$\begin{aligned} \min\ & c^T x \\ s.t\ \ & Ax \leq b, \\ & x \geq 0, \\ & (c, A, b) \in \mathcal{U}, \end{aligned} \tag{20}$$

where c, A, b are uncertainty data and $\mathcal{U}$ is an uncertainty set.

The basic Robust Optimization paradigm is based three assumptions as stated by Gorissen *et al.* in [12] as follows. First, all decision variables represent here and now decision, the decision variables should get specific numerical values as a result of solving the problem before the actual data reveals itself. Second, the decision maker is fully responsible for consequences of the decisions to be made when, and only when, the actual data is within the prespecified uncertainty set $\mathcal{U}$. Third, the constraints of the uncertain problem in question are hard. The decision maker cannot tolerate the violations of constraints when data is in uncertainty set $\mathcal{U}$. In addition to basic assumptions of Robust Optimization, it is assumed without loss of generality that the objective function is certain, the right-hand side is certain, the uncertainty is constraint-wise and $\mathcal{U}$ is a convex and compact set.

2) Solving Robust Counterpart (RC): Refers to Gorissen *et al.* in [12]), when $c \in R^n$ and $b \in R^m$ are certain, then the RC formulation from (20) is as follows.

$$\begin{aligned} \min\ & c^T x \\ s.t\ \ & A(\zeta)x \leq b, \\ & \forall \zeta \in Z, \end{aligned} \tag{21}$$

with $Z \in R^L$ denotes the user specified primitive uncertainty set. A solution $x \in R^n$ is called robust feasible if it satisfies the uncertain constraints $A(\zeta)x \leq b$ for all realizations of $\zeta \in Z$. For a single constraint of (21)

$$a(\zeta) = \bar{a} + P\zeta, \tag{22}$$

where $\bar{a} \in R^n$ is nominal values vector and $P \in R^{nxL}$ is perturbation matrix. Substitute equation (22) to equation (21), thus a new robust counterpart formulation is obtained as follows:

$$(\bar{a} + P\zeta)^T x \leq b, \forall \zeta \in Z. \tag{23}$$

The challenge of Robust Optimization is to find an uncertainty set $\mathcal{U}$ that can be formulated into a computationally tractable problem and determine an approximation that has already been proven to be computationally tractable.

According to Ben-Tal and Nemirovski [27], Chaerani & Ross [28] and Gorissen *et al.* [12], the RC computational tractability can be analyzed by representing RC into Linear Programming (LP), Conic Quadratic Programming (CQP), or Semidefinite Optimization (SDO) as can be seen in Table IV.

TABLE IV
TRACTABLE REFORMULATION FOR DIFFERENT TYPES OF UNCERTAINTY SETS (GORISSEN, *et al.* [12])

Uncertainty Set	Z	Robust Counterpart	Tractability
Box	$\lVert\zeta\rVert_\infty \leq 1$	$a^T x + \lVert P^T x \rVert_1 \leq b$	LP
Ellipsoidal	$\lVert\zeta\rVert_2 \leq 1$	$a^T x + \lVert P^T x \rVert_2 \leq b$	CQP
Polyhedral	$D\zeta + q \geq 0$	$\begin{cases} a^T x + q^T y \leq b \\ D^T y = -P^T x \\ y \geq 0 \end{cases}$	LP

D. Multi-objective Optimization

In Linear Programming (LP) problems, not all problems have one objective function, but there are some LP problems has two or more objective functions (this also called as multi-objective functions). There are several methods that can be used to find solutions to problems with two or more objective functions, one of which is the Lexicographic Method (LM). According to Rao [29], the LM ranks objective functions based on priority interests as desired. Consider the following multi-objective linear programming problem.

$$\begin{aligned} \min\ & \left\{ z_1 = c_1^T x, \ldots, z_k = c_k^T x \right\}, \\ s.t\ \ & Ax \leq b, \\ & x \geq 0. \end{aligned} \tag{24}$$

The procedure for solving (24) can be describe as follows.
Step 1: Solve the first priority objective function

$$\begin{aligned} \min\ & z_1 = c_1^T x, \\ s.t\ \ & Ax \leq b, \\ & x \geq 0, \end{aligned} \tag{25}$$

such that the optimal solution of (25) is obtained as x_1^* with objective function value $z_1^* = c_1^T x_1^*$.
Step 2: Solve the second priority objective function, add the optimal solution of (25) as new constraint.

$$\begin{aligned} \min\ & z_2 = c_2^T x, \\ s.t\ \ & c_1^T x_1^* = z_1^*, \\ & Ax \leq b, \\ & x \geq 0, \end{aligned} \tag{26}$$

such that the optimal solution of (26) is obtained as x_2^* with objective function value $z_2^* = c_2^T x_2^*$.
Step k: Repeat the procedure such that in the last step the problem to be solved is the following.

$$\begin{aligned} \min\ & z_k = c_k^T x, \\ s.t\ \ & c_1^T x_1^* = z_1^*, \\ & \vdots \\ & c_{k-1}^T x_{k-1}^* = z_{k-1}^*, \\ & Ax \leq b, \\ & x \geq 0. \end{aligned} \tag{27}$$

E. Case Studies

For study case, the model is implemented to one of education area in Indonesia, i.e., Jatinangor, a subdistrict located in Sumedang Regency, West Java, Indonesia. Jatinangor is an educational area, which is indicated by the existence of

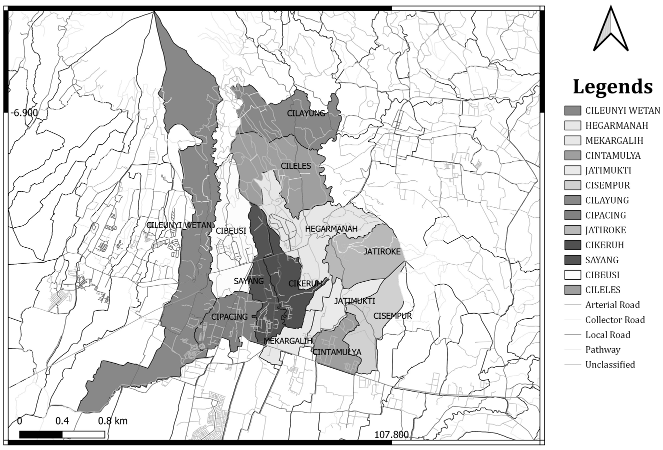

Fig. 2. Spatial Planning in Jatinangor [30]

four campuses in Jatinangor. According to the regulation of Sumedang Regency Number 4 Year 2018 (see [31]) concerning the spatial plan for Sumedang Regency in 2018 and 2038, Jatinangor is directed to become a residential area and high educational area that support the consolidation of Bandung Metropolitan Area. The spatial planning map in Jatinangor is presented in Figure 2. Because of the high mobility in Jatinangor, this implies that there are several land conversion in Jatinangor. Thus, it becomes an interesting topic to discuss whether there will be change in existing land-use types or not based on certain consideration. In the next section, a derivation of robust optimization for land-use allocation is presented. Two scenarios will be discussed as numerical experiment of the problem.

III. Results and Discussions

A. Formulation of Optimization Model for Land-use Allocation Problem (OMLAP)

The optimization model for the land-use allocation problem used in this research is a reformulation from the optimization model for land-use allocation problem introduced by Yao and Murray [3] and Xiaoli *et al.* [4]. The objective functions are based on optimization model from Xiaoli *et al.* [4], the constraints are based on optimization problem from Yao and Murray [3], and the decision variables are defined as the proportion of a land type i to change become another type k lies between 0 and 1. The optimization model for land-use allocation problem is as follows.

$$\max Z = \sum_{k=1}^{K}\sum_{i=1}^{N} z_{ik}x_{ik}, \tag{28}$$

$$\max R_k = \sum_{i=1}^{N} r_{ik}x_{ik}, \tag{29}$$

$$s.t. \ \sum_{i} \overline{c_{ik}}x_{ik} \le \theta_k, \forall k, \tag{30}$$

$$\sum_{i=1}^{N} a_{ik}x_{ik} \ge L_k, \forall k, \tag{31}$$

$$\sum_{k=1}^{K} x_{ik} = 1, \forall i, \tag{32}$$

$$\sum_{i=1}^{N} s_i x_{ik} \ge LS_k, \forall k, \tag{33}$$

$$\sum_{i=1}^{N} s_i x_{ik} \le US_k, \forall k, \tag{34}$$

$$0 \le x_{ik} \le 1, \forall i, k. \tag{35}$$

Notes that z_{ik} is comprehensive index, r_{ik} is density index, N is total number of planning unit, K is total number of land-use types, a_{ik} is benefit if planning unit i is changed to land-use type k, $\overline{c_{ik}}$ is acquisition cost if planning unit i is changed to land-use type k, θ_k is total budget for acquisition of land-use type k, s_i is the area of planning unit i, L_k is minimum benefit desired for land-use type k, US_k is upper bound of area for land-use type k, and LS_k is lower bound of area for land-use type k.

The objective function (28) is to maximize the comprehensive index and the objective function (29) is to maximize the density index. Constraints (30) and (31) impose lower and upper bounds on the total area for each land-use type. Constraint (32) restrict the sum of percentage change from planning unit i to land-use type k equal to 1. Constraint (33) limit the total acquisition cost for each land-use type. Constraint (34) requires a minimum level of benefit for each land-use type. Constraint (35) must be a decision variables between 0 and 1. The decision variables declare the percentage change from planning unit i to land-use type k.

B. Uncertainty Model for OMLAP

Assume that the uncertainties data for OMLAP are the benefit and acquisition cost when a land planning is changed to a land-use type. Thus, in this model, the uncertainty parameters are benefited if planning unit i is changed to land-use type k or a_{ik} and acquisition cost if planning unit i is changed to land-use type k or $\overline{c_{ik}}$. The uncertainty parameter $\overline{c_{ik}}$ is defined as follows:

$$\overline{c_{ik}} = \overline{\overline{c_{ik}}} + P_{ik}\zeta, \forall \zeta \in Z, \tag{36}$$

where $\overline{\overline{c_{ik}}}$ is nominal value vector for acquisition cost, P_{ik} is perturbation matrix, and ζ is primitive uncertainty vector. Then, the uncertainty parameter a_{ik} is defined as follows:

$$a_{ik} = \overline{a_{ik}} + B_{ik}\zeta, \forall \zeta \in Z, \tag{37}$$

where $\overline{a_{ik}}$ is nominal value vector for the benefit if planning unit i is changed to land-use type k, B_{ik} is perturbation matrix, and ζ is primitive uncertainty vector.

The uncertainty parameters are in the first constrain and the second constraint of the model, so the objective function is certain. $\overline{c_{ik}}$ and a_{ik} are in the left-hand side of the model, so the certain right-hand side assumption is fulfilled. Substitute (36) and (37) to (30) and (31). Thus, the uncertain model for OMLAP is defined as follows:

$$\max\ Z = \sum_{k=1}^{K}\sum_{i=1}^{N} z_{ik}x_{ik}, \tag{38}$$

$$\max\ R_k = \sum_{i=1}^{N} r_{ik}x_{ik}, \tag{39}$$

$$s.t.\ \sum_{i=1}^{N}(\overline{\overline{c_{ik}}} + P_{ik}\zeta)x_{ik} \le \theta_k, \forall k, \tag{40}$$

$$\sum_{i=1}^{N}(\overline{a_{ik}} + B_{ik}\zeta)x_{ik} \ge L_k, \forall k, \tag{41}$$

$$\sum_{k=1}^{K} x_{ik} = 1, \forall i, \tag{42}$$

$$\sum_{i=1}^{N} s_i x_{ik} \ge LS_k, \forall k, \tag{43}$$

$$\sum_{i=1}^{N} s_i x_{ik} \le US_k, \forall k, \tag{44}$$

$$0 \le x_{ik} \le 1, \forall i, k, \forall \zeta \in Z. \tag{45}$$

C. Robust Counterpart Formulation with Ellipsoidal Uncertainty Set for OMLAP

Assumed that the uncertain parameters in uncertain model for OMLAP are in ellipsoidal uncertainty set. It is known that ellipsoidal uncertainty set is defined as follows:

$$Z = \{\zeta : \|\zeta\|_2 \le 1\}. \tag{46}$$

Robust counterpart formulation for ellipsoidal uncertainty set is as follows:

$$a^T x + \left\|P^T x\right\|_2 \le b, \tag{47}$$

where $\bar{a} \in R^n$ is nominal value vector, $P \in R^{nxL}$ is perturbation matrix, and $\zeta \in R^L$ is uncertainty primitive vector. Assume that the uncertainty parameters located in ellipsoidal uncertainty set, then constraint (40) can be reformulated as follows.

$$\begin{aligned}\sum_{i=1}^{N}\overline{c_{ik}}x_{ik} &= \sum_{i=1}^{N}(\overline{\overline{c_{ik}}} + P_{ik}\zeta)x_{ik},\\ &= \sum_{i=1}^{N}\overline{\overline{c_{ik}}}x_{ik} + \max_{\zeta:\|\zeta\|_2\le 1}(\sum_{i=1}^{N}P_{ik}x_{ik}\zeta).\end{aligned}$$

Take ζ as a unit vector then

$$\zeta = \frac{P_{ik}x_{ik}}{\|P_{ik}x_{ik}\|}, \tag{48}$$

thus, the following holds.

$$\sum_{i=1}^{N}\overline{c_{ik}}x_{ik} = \sum_{i=1}^{N}\overline{\overline{c_{ik}}}x_{ik} + \sum_{i=1}^{N}P_{ik}x_{ik}\frac{P_{ik}x_{ik}}{\|P_{ik}x_{ik}\|} \le \theta_k.$$

Thus we have

$$\sum_{i=1}^{N}\overline{c_{ik}}x_{ik} \le \theta_k \forall k \tag{49}$$

which equivalent with

$$\sum_{i=1}^{N}\overline{\overline{c_{ik}}}x_{ik} + \|P_{ik}x_{ik}\|_2 \le \theta_k. \tag{50}$$

This means that (40) is can be replaced by (50). Using the same way, for constraint (41) the following holds.

$$\begin{aligned}\sum_{k=1}^{K}\sum_{i=1}^{N}a_{ik}x_{ik} &= \sum_{i=1}^{N}(\overline{a_{ik}} + B_{ik}\zeta)x_{ik}, \forall k,\\ &= \sum_{i=1}^{N}\overline{a_{ik}}x_{ik} + \max_{\zeta:\|\zeta\|_2\le 1}\sum_{i=1}^{N}B_{ik}x_{ik}\zeta, \forall k,\\ &= \sum_{i=1}^{N}\overline{a_{ik}}x_{ik} + \sum_{i=1}^{N}B_{ik}x_{ik}\frac{B_{ik}x_{ik}}{\|B_{ik}x_{ik}\|}, \forall k,\\ &= \sum_{i=1}^{N}\overline{a_{ik}}x_{ik} + \sum_{i=1}^{N}\sqrt{(B_{ik}x_{ik})^2}, \forall k.\end{aligned}$$

Thus (41) is equal to equation (51) as follows.

$$\sum_{i=1}^{N}\overline{a_{ik}}x_{ik} + \|B_{ik}x_{ik}\|_2 \ge L_k, \forall k. \tag{51}$$

The formulation of robust counterpart with ellipsoidal uncertainty set for land-use allocation problem as follows:

$$\max\ Z = \sum_{k=1}^{K}\sum_{i=1}^{N} z_{ik}x_{ik}, \tag{52}$$

$$\max\ R_k = \sum_{i=1}^{N} r_{ik}x_{ik}, \tag{53}$$

$$s.t.\ \sum_{i=1}^{N}\overline{\overline{c_{ik}}}x_{ik} + \|P_{ik}x_{ik}\|_2 \le \theta_k, \forall k, \tag{54}$$

$$\sum_{i=1}^{N}\overline{a_{ik}}x_{ik} + \|B_{ik}x_{ik}\|_2 \ge L_k, \forall k, \tag{55}$$

$$\sum_{k=1}^{K} x_{ik} = 1, \forall i, \tag{56}$$

$$\sum_{i=1}^{N} s_i x_{ik} \ge LS_k, \forall k, \tag{57}$$

$$\sum_{i=1}^{N} s_i x_{ik} \le US_k, \forall k, \tag{58}$$

$$0 \le x_{ik} \le 1, \forall i, k, \forall \zeta \in Z. \tag{59}$$

It can be seen that the form of robust counterpart formulation with ellipsoidal uncertainty for land-use allocation problem is conic quadratic programming, thus the computational tractability is obtained.

D. Robust Counterpart Formulation with Polyhedral Uncertainty Set for OMLAP

Assumed that the uncertain parameters in uncertain model for OMLAP are in polyhedral uncertainty set. It is known that polyhedral uncertainty set is defined as follows:

$$Z = \{\zeta : d - D\zeta \ge 0\}. \tag{60}$$

Robust counterpart formulation for polyhedral uncertainty set is as follows:

$$\exists y : \bar{a}^T x + d^T y \leq b, D^T y = P^T x, y \geq 0, \qquad (61)$$

where $D \in R^{mxL}$, $\zeta \in R^L$ and $d \in R^m$. Assume that the uncertainty parameters located in polyhedral uncertainty set, so constraint (40) is equal to equation (62).

$$\sum_{k=1}^{K}\sum_{i=1}^{N} \overline{c_{ik}} x_{ik} = \sum_{i=1}^{N} (\overline{\overline{c_{ik}}} x_{ik} + P_{ik}\zeta) x_{ik} \leq \theta_k, \forall k,$$

which equivalent with

$$\sum_{i=1}^{N} \overline{\overline{c_{ik}}} x_{ik} + \max_{d - D\zeta \geq 0} (\sum_{i=1}^{N} P_{ik} x_{ik} \zeta) \leq \theta_k, \forall k.$$

Refers to [12] the maximal problem can be solved by solving its dual. Thus constraint (40) it can be written as (62).

$$\sum_{i=1}^{N} \overline{\overline{c_{ik}}} x_{ik} + \left\{ \begin{array}{ll} \min & {d_k}^T y_k, \\ \text{s.t} & {D_k}^T y_k = \sum_{i=1}^{N} {P_{ik}}^T x_{ik}, \\ & y_k \geq 0 \end{array} \right\} \leq \theta_k. \quad (62)$$

Using the same way, constraint (41) can be proceed as follows.

$$\begin{aligned} \sum_{i=1}^{N} a_{ik} x_{ik} &= \sum_{i=1}^{N} (\overline{a_{ik}} + B_{ik}\zeta) x_{ik} \geq L_k, \forall k, \\ &= \sum_{i=1}^{N} \overline{a_{ik}} x_{ik} + \max_{d1 - D1\zeta \geq 0} (\sum_{i=1}^{N} B_{ik} x_{ik} \zeta) \geq L_k. \end{aligned}$$

Thus constraint (41) it can be written as (63).

$$\sum_{i=1}^{N} \overline{a_{ik}} x_{ik} + \left\{ \begin{array}{ll} \min & {d1_k}^T y1_k \\ \text{s.t} & {D1_k}^T y1_k = \sum_{i=1}^{N} {B_{ik}}^T x_{ik} \\ & y1_k \geq 0 \end{array} \right\} \geq L_k. \quad (63)$$

The formulation of robust counterpart with polyhedral uncertainty set for land-use allocation problem as follows:

$$\max Z = \sum_{k=1}^{K}\sum_{i=1}^{N} z_{ik} x_{ik}, \qquad (64)$$

$$\max R_k = \sum_{i=1}^{N} r_{ik} x_{ik}, \qquad (65)$$

$$s.t. \quad \sum_{i=1}^{N} \overline{\overline{c_{ik}}} + {d_k}^T y_k \leq \theta_k, \forall k, \qquad (66)$$

$$\sum_{i=1}^{N} \overline{a_{ik}} + {d1_k}^T y1_k \geq L_k, \forall k, \qquad (67)$$

$$\sum_{k=1}^{K} x_{ik} = 1, \forall i, \qquad (68)$$

$$\sum_{i=1}^{N} s_i x_{ik} \geq LS_k, \forall k, \qquad (69)$$

$$\sum_{i=1}^{N} s_i x_{ik} \leq US_k, \forall k, \qquad (70)$$

$${D_k}^T y_k = \sum_{i=1}^{N} {P_{ik}}^T x_{ik}, \qquad (71)$$

$${D1_k}^T y1_k = \sum_{i=1}^{N} {B_{ik}}^T x_{ik}, \qquad (72)$$

$$0 \leq x_{ik} \leq 1, \forall i, k. \qquad (73)$$

It can be seen that the form of robust counterpart formulation with polyhedral uncertainty for land-use allocation problem is linear programming, thus the computational tractability is achieved in this case.

E. Pseudocodes for calculating Robust OMLAP

The robust optimal solution for Robust OMLAP is calculated by following the steps on the pseudocode that is presented in Algorithm 1.

Algorithm 1 Algorithm for Calculating Robust OMLAP.

```
for i = 1, 2, ..., N do
    for k = 1, 2, ..., M do
        Input: a_ik, c_ik, θ_k, L_k, US_k, LS_k, z_ik, r_ik, s_i;
               Matrix P_ik, P1_ik, D1_k, d_k, d1_k
        Define the objective functions for RC-OMLAP
        Define the constraints functions for RC-OMLAP
        Compute RC-OMPLAP
        as a multi-objective Optimization Problem
        using Lexicographic Method.
        CodeTools:
        Usage(Optimization[LPSolve]
        (objective function, constraint function,
        assume=nonnegative, maximize));
        CodeTools:
        Usage(Optimization[NLPSolve]
        (objective function, constraint function,
        assume=nonnegative, maximize))
    end for
end for
Output: Optimal x_ik, proportion of land type i
        change to be another type k.
```

IV. Numerical Experiments

A. Data: Jatinangor Subdistrict Indonesia

Case studies that are used for the numerical experiment are spatial planning in Jatinangor Subdistrict. The data used in this paper are secondary data from the Indonesian Statistics 2018, observational data, and illustrative data. The data used for the numerical experiment is the data of Jatinangor District. Data on the area of the village according to its use in 2017 was obtained from the Jatinangor District catalog in 2018 numbers compiled by the Central Statistics Agency of Sumedang Regency (see [32]).

Jatinangor District has 26.20 km square (2,620 Ha) with 12 villages, 56 hamlets, and 503 families. According to the data, there are four land-use types in Jatinangor, i.e. rice field, residential, forest, and other types. Other types are used for educational area, industry and warehousing, trade area, governmental area, water catchment area, and green open area. Most of the Jatinangor area is used for settlements covering an area of 1,168 Ha (44.6 percentage). In comparison, the area of other land-uses is 371 Ha of rice fields (14.2 percentage), 755 Ha of forest (28.8 percentage), 326 Ha of other uses (12.4 percentage). Other uses are used for education, industrial and warehousing areas, trade, government, water catchment areas, and green open spaces. The land-use allocation problem discussed in this paper is to determine whether there will be a change in existing land-use types or not to maximize the comprehensive index and density index. The lower and upper bound are hypothetically determined according to the conditions. The Illustrative data about land area, lower and upper bound for each land-use are shown in Table V.

TABLE V
ILLUSTRATIVE DATA ABOUT LAND AREA, LOWER, AND UPPER BOUND FOR EACH LAND-USE

Land-use Type	Land Area (Ha)	Lower Bound (Ha)	Upper Bound (Ha)
Rice Field	371	0	371.65
residential	1,168	1,000	1,200
Forest	755	754.3	756
Other Types	326	325	1000

The benefit of each land-use type is observational data obtained based on the amount of the Land and Building Tax and considering each land-use type's land area. Data about each land-use type's acquisition costs is hypothetical data considering the land area and the 2018 Regional Budget of Sumedang Regency. The benefits and acquisition cost for a land-use type are shown in Table VI.

TABLE VI
ILLUSTRATIVE DATA OF BENEFIT AND ACQUISITION COST

Land-use Type	Benefit (Millions IDR)	Acquisition Cost (Millions IDR)
Rice Field	29,340	1,518.5
residential	110,620	1,917
Forest	27,000	1,710.5
Other Types	33,398.88	1,496

The illustrative scheme for the problem of land-use allocation in Jatinangor can be seen in Figure 3. The spatial weight

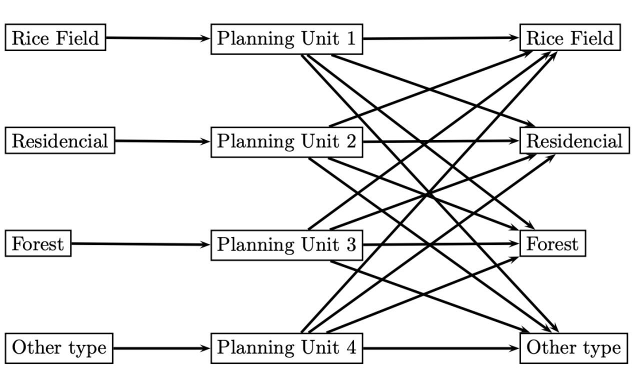

Fig. 3. Land-use Flow for Land-use Allocation Problem in Jatinangor

matrix can be obtained using the position between land-use type as shown in Figure 3.

Thus, the spatial weight matrix can be presented as (74).

$$W = \begin{bmatrix} 0 & 0.5 & 0 & 0.5 \\ 0.5 & 0 & 0 & 0.5 \\ 0 & 0 & 0 & 1 \\ 0.33 & 0.33 & 0.33 & 0 \end{bmatrix}. \tag{74}$$

The comprehensive index and density index determination are based on spatial weight matrix W in (74).

The comprehensive index and density index are shown in equation (75)-(79) and equation (80)-(84), respectively.

$$z^T = \begin{bmatrix} z_1 & z_2 & z_3 & z_4 \end{bmatrix}, \tag{75}$$

$$z_1 = \begin{bmatrix} 0 & 0.5 & 0 & 0.5 \end{bmatrix}, \tag{76}$$

$$z_2 = \begin{bmatrix} 0.5 & 0 & 0 & 0.5 \end{bmatrix}, \tag{77}$$

$$z_3 = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix}, \tag{78}$$

$$z_4 = \begin{bmatrix} 0.33 & 0.33 & 0.33 & 0 \end{bmatrix}, \tag{79}$$

$$R^T = \begin{bmatrix} R_1 & R_2 & R_3 & R_4 \end{bmatrix}, \tag{80}$$

$$R_1 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0.5 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.5 \end{bmatrix}, \tag{81}$$

$$R_2 = \begin{bmatrix} 0.5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.5 \end{bmatrix}, \tag{82}$$

$$R_3 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \tag{83}$$

$$R_4 = \begin{bmatrix} 0.33 & 0 & 0 & 0 \\ 0 & 0.33 & 0 & 0 \\ 0 & 0 & 0.33 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}. \tag{84}$$

For numerical experiment, assume that the minimum benefits for land-use types consisting of rice fields, residential, forests, and other types are respectively 0; 16,500; 11,000; and 5,500 (in millions IDR). The available budget for each land-use type are 1,500; 2,000; 1,720; and 10,000, respectively (in millions IDR).

B. Two Scenarios of Case Studies

There are two case studies presented in this numerical experiment. The first scenario allows a policy for changing all types of land to be other land types. The second scenario preserves a condition that the other types of land (see in

Table 3) cannot be changed to another type such as rice field, residential or forest. The experiment numeric is done using Maple 18. The calculation follows Algorithm 1. The result can be shown in the next subsection.

1) Scenario I: all land types can be changed to other land types: The calculation results for Scenario 1 for deterministic, RC-Ellipsoidal and RC-Polyhedral model shows that the land-use type in Jatinangor can be changed with optimal proportional decision as can be seen in Table VII. As shown in Table VII, the land-use conversion for deterministic, RC-Ellipsoidal and RC-Polyhedral Models are having the same scheme.

TABLE VII
OPTIMAL LAND-USE ALLOCATION FOR SCENARIO 1 WITH DETERMINISTIC, RC-ELLIPSOIDAL AND RC-POLYHEDRAL MODEL

Decision Variables	Optimal Solution Deterministic	Robust Optimal Ellipsoidal	Robust Optimal Polyhedral
x_{12}	0.3944	0.3059	0.1547
x_{14}	0.6056	0.6941	0.8453
x_{22}	0.7309	0.759	0.807
x_{23}	0.2691	0.241	0.193
x_{33}	0.4646	0.5289	0.6175
x_{34}	0.5354	0.4711	0.3825
x_{41}	0.7263	0.7746	0.8077
x_{43}	0.2737	0.2544	0.1923

This result can be illustrated as shown in Figure 4. The

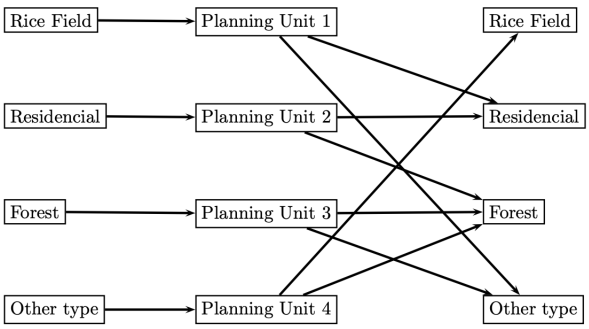

Fig. 4. The Optimal Land-use Conversion For Scenario 1 on Deterministic, RC-Ellipsoidal and Polyhedral Models

optimal land-use conversion for Scenario 1 shows that it is allowed to change the rice field become residential and forest; keep the residential area or change it become forest; the forest can be preserved as a forest of some of them become other types of land such as education, industrial and warehousing areas, trade, government, water catchment areas, and green open spaces. The last optimal option is to change the other types of land become rice field or forest.

For Scenario 1, the total of land area converted to another land-use type based on the result of deterministic, RC-Ellipsoidal and RC-Polyhedral model can be seen in Table VIII, Table IX and in Table X. In Table XI, is shown the objective function values from deterministic model, RC model with ellipsoidal uncertainty, RC model with polyhedral uncertainty for land-use allocation problem. Notes that Z^* is a comprehensive index, ${R_1}^*$ is a density index for

TABLE VIII
SCENARIO 1:TOTAL OF LAND AREA CONVERTED TO ANOTHER LAND-USE TYPE BASED ON THE RESULT OF DETERMINISTIC MODEL (HA)

Land-use Type	Rice Field	residential	Forest	Other Type
Rice Field	0	146.3224	0	224.6776
Residential	0	853.6912	314.3088	0
Forest	0	0	350.773	625.3472
Other Types	236.7738	0	89.2262	0

TABLE IX
SCENARIO 1: TOTAL OF LAND AREA CONVERTED TO ANOTHER LAND-USE TYPE BASED ON THE RESULT OF RC MODEL WITH ELLIPSOIDAL UNCERTAINTY (HA)

Land-use Type	Rice Field	residential	Forest	Other Type
Rice Field	0	113.4889	0	257.5111
residential	0	886.512	281.488	0
Forest	0	0	399.3195	399.3195
Other Types	252.5196	0	73.4804	0

TABLE X
SCENARIO 1: TOTAL OF LAND AREA CONVERTED TO ANOTHER LAND-USE TYPE BASED ON THE RESULT OF RC MODEL WITH POLYHEDRAL UNCERTAINTY (HA)

Land-use Type	Rice Field	residential	Forest	Other Type
Rice Field	0	57.3937	0	313.6063
residential	0	942.576	225.424	0
Forest	0	0	466.2125	288.7875
Other Types	264.31102	0	62.6898	0

rice field type, ${R_2}^*$ is a density index for residential type, ${R_3}^*$ is a density index for forest type, and ${R_4}^*$ is a density index for other types. The optimal solution of RC model with ellipsoidal uncertainty and polyhedral uncertainty are smaller than the optimal solution of deterministic model but the RC model has been made by considering the uncertainty factors that represent the worst possibility that might happen.

TABLE XI
OPTIMAL OBJECTIVE VALUE OF SCENARIO 1

Deterministic Model	RC Model with Ellipsoidal Uncertainty	RC Model with Polyhedral Uncertainty
$Z^* = 1.3654$	$Z^* = 1.3011$	$Z^* = 1.2125$
${R_1}^* = 0.2397$	${R_1}^* = 0.2556$	${R_1}^* = 0.2665$
${R_2}^* = 0.1972$	${R_2}^* = 0.1530$	${R_2}^* = 0.0773$
${R_3}^* = 0.0903$	${R_3}^* = 0.0744$	${R_3}^* = 0.0634$
${R_4}^* = 0.8382$	${R_4}^* = 0.8181$	${R_4}^* = 0.8051$

2) Scenario 2. Preserving the other land types such as education, industrial and warehousing areas, trade, government, water catchment areas, and green open spaces: In this case of scenario, assume that $x_{44} = 1$. This means that the land type such as education, industrial and warehousing areas, trade, government, water catchment areas, and green open spaces is preserved, cannot be changed to other types. This implies the optimal proportional result for the models of deterministic and RC-polyhedral is obtained and can be seen in Table XII.

As can be seen in Table XII, the land-use conversion for deterministic and RC-Polyhedral models are having the same scheme. This result can be illustrated as shown in Figure 5.

TABLE XII
OPTIMAL LAND-USE ALLOCATION FOR SCENARIO 2 WITH DETERMINISTIC AND RC-POLYHEDRAL MODEL

Decision Variables	Optimal Solution Deterministic	Robust Optimal Polyhedral
x_{12}	0.3944	0.1547
x_{14}	0.6056	0.8453
x_{22}	0.7309	0.807
x_{23}	0.2691	0.193
x_{33}	0.5828	0.7006
x_{34}	0.4172	0.2994
x_{44}	1	1

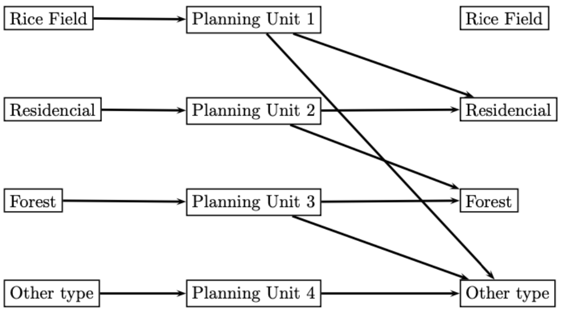

Fig. 5. The Optimal Land-use Conversion For Scenario 2 on Deterministic and Polyhedral Models

The optimal land-use conversion for Scenario 2 shows that it is allowed to change the rice field become residential and forest; keep the residential area or change it become forest; the forest can be preserved as a forest or some of them become other types of land such as education, industrial and warehousing areas, trade, government, water catchment areas, and green open spaces. When unit planning consider the changing of other land types, thus the optimal decision is to preserve the types of land such as education, industrial and warehousing areas, trade, government, water catchment areas, and green open spaces. For Scenario 2, the total of land area converted to another land-use type based on the result of deterministic and RC-Polyhedral model can be seen in Table XIII and in Table XIV.

TABLE XIII
SCENARIO 2:TOTAL OF LAND AREA CONVERTED TO ANOTHER LAND-USE TYPE BASED ON THE RESULT OF DETERMINISTIC MODEL (HA)

Land-use Type	Rice Field	residential	Forest	Other Type
Rice Field	0	146.3224	0	224.6776
residential	0	853.6912	314.3088	0
Forest	0	0	440.014	314.986
Other Types	0	0	0	326

The objective function values of Scenario 2 of deterministic model, RC model with polyhedral uncertainty for land-use allocation problem can be seen in Table XV.

C. Numerical Experiment Result Analysis

Based on West Java Regional Regulation [33] about Regional Spatial Plan for the Province of West Java Indonesia, 2009-2029, also refers to Spatial Regional Plan for Sumedang in 2018-2038 (see [31]), Jatinangor is allocated for urban settlement development and Higher Education Area Development. Based on the numerical experiment result presented for Scenario 1 and 2; thus, a recommendation to the local government is Scenario 2 can be one of the considerations for land-use allocation. This is based on the situation that the higher education area is preserved. This result is robust optimal since the model is already considered uncertain data. In this problem, the uncertain benefit and uncertain acquisition cost are taken into account.

In Scenario 2, the rice field and forest also can be changed to one of the other types of land such as education, industrial and warehousing areas, trade, government, water catchment areas, and green open spaces. As shown in Figure 4 and Figure 5, 4 and Figure 5, Scenario 1 recommends the other type of land, including the education area, to be rice field or forest, which is not supported by regional regulation. Thus, we may take Scenario 2 as a valid recommendation for the local government. For both scenarios, from a mathematical modeling point of view, it can be seen that the RC-Polyhedral model gives the best option. For the optimal proportion decision of land-use allocation, this fulfills the best worst-case scenario in maximizing comprehensive and density index.

TABLE XIV
SCENARIO 2: TOTAL OF LAND AREA CONVERTED TO ANOTHER LAND-USE TYPE BASED ON THE RESULT OF RC MODEL WITH POLYHEDRAL UNCERTAINTY (HA)

Land-use Type	Rice Field	residential	Forest	Other Type
Rice Field	0	57.3937	0	313.6063
residential	0	942.576	225.424	0
Forest	0	0	528.953	226.047
Other Types	0	0	0	326

TABLE XV
OPTIMAL OBJECTIVE VALUE OF SCENARIO 2

Deterministic Model	RC Model with Polyhedral Uncertainty
$Z^* = 0.9172$	$Z^* = 0,7994$
$R_1{}^* = 2.98 \times 10^{-10}$	$R_1{}^* = 2,08 \times 10^{-10}$
$R_2{}^* = 0.1972$	$R_2{}^* = 0.0773$
$R_3{}^* = 0,13 \times 10^{-10}$	$R_3{}^* = 1,19 \times 10^{-10}$
$R_4{}^* = 0.72$	$R_4{}^* = 0.7221$

V. CONCLUSION

Robust optimization model for spatial land-use allocation problem is shown as a multi-objective binary linear programming problem. By assuming that the benefit and acquisition cost are uncertain and lie within an ellipsoidal or polyhedral uncertainty set, it is shown that its robust counterpart is computationally tractable. Numerical experiments show a validation of the model using the data of Jatinangor District in Indonesia. The Lexicographic Method is used to handle the multi-objective function. The result shows that Jatinangor is can be preserved as education areas, industrial and warehousing areas, trade, government, water catchment areas, and green open spaces.

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