Finite-time Backstepping Control for Fractional-order Hydro-turbine Governing System

Xiaomin Tian, Zhong Yang, Yan Shao, Lixin Zhai

Abstract—**This paper investigates a finite-time backstepping control for robust stabilization of fractional-order hydro-turbine governing system. We assume that the controlled system is perturbed by external disturbance, the bound of external disturbance to be unknown in advance. Through designing the virtual controllers step by step, then the appropriate actual controller is obtained. The fractional-order stability theory is used to shown the correctness of the proposed control strategy, and finally numerical simulations have been implemented to confirm the effectiveness and feasibility of the presented finite-time backsteping method.**

Index Terms—**finite-time control, backstepping method, fractional-order hydro-turbine governing system, stability analysis**

I. Introduction

THE hydro-turbine governing system is a highly nonlinear, multi-variable coupled and nonminimum phase system, hydro energy is the largest renewable energy source for global electricity generation, accounting for about 71% of total renewable energy generation in the world. Compared with wind and solar energy, hydroelectricity is a reliable, flexible, and cost-effective energy generation technology, with the advantages of high energy efficiency and easily stored in reservoir, and it can be used for frequency regulation, peak load shaving and emergency reserve in smart grid [1, 2]. Due to its great potentials on social, economic and environmental benefits, the modelling and control of hydro-turbine governing system is significant to ensure the system frequency stability and the stable operation of hydroelectric stations.

Fractional-order calculus derives from the end of 17th century, it is particularly suitable for describing the viscoelastic system [3], and the memory and hereditary properties of various materials and processes. Now, studying fractional-order systems has became an active research area. In particular, control and stabilization of the fractional-order systems have attracted much attention from various scientific fields. It has been proven that applying fractional-order controllers to fractional-order system can obtain a better control effect than integer-order controllers, such as fractional-order PID control [4], fractional-order sliding mode control [5], fractional fuzzy control [6], fractional-order finite-time control [7], and so on.

The backstepping method is a recursive approach for controller design, through designing virtual controllers and partial Lyapunov functions step by step, a common Lyapunov function of the whole system can be deduced from the above operations. This method can guarantee the global stability, tracking, and transient performance of nonlinear systems [8]. In view of the excellent performance of backstepping, an increasing number of researchers have focused on this potential problem. Many studies for the backstepping-based control and synchronization of fractional-order chaotic system have been reported. For example, Luo [9] researched the robust control and synchronization of a fractional-order system by adding one power integrator. Shukla [10,11] realized the stabilization.

However, the above mentioned approaches are only focus on the asymptotic stability of the controlled system, the finite-time stabilization of fractional-order nonlinear system based on backstepping method is rarely involved, besides, there are many research results about integer-order hydro-turbine governing system, while the finite-time control for fractional-order hydro-turbine governing system is seldom reported so far. Consequently, in view of the advantages of fractional-order models, it is still very challenging and essential to research the finite-time stabilization of fractional-order hydro-turbine governing system.

Motivated by the above discussions, in this paper, a finite-time backstepping control strategy is proposed to realize the stabilization of fractional-order hydro-turbine governing system with unknown bounded uncertainties. The structure of this paper is organized as follows. In section 2, relevant definitions, lemmas are given. Main results are presented in section 3. Simulation results are shown in section 4. Finally, conclusions are included in section 5.

II. Preliminaries

Firstly, we recall the basic definition with respect to Caputo fractional derivative, which is the most commonly used definition of fractional calculus.

Definition 1 The Caputo fractional derivative of order α of the function $f(t)$ is defined as

$$ {}_{t_0}D_t^{\alpha} f(t) = \begin{cases} \frac{1}{\Gamma(m-\alpha)} \int_{t_0}^{t} \frac{f^{(m)}(\tau)}{(t-\tau)^{\alpha-m+1}} d\tau, & m-1<\alpha<m \\ \frac{d^m}{dt^m} f(t), & \alpha = m \end{cases} \tag{1} $$

where $\Gamma(\cdot)$ is the Gamma function, m is the smallest integer number, larger than α. In the rest of this paper, we will use

Manuscript received February 3rd, 2021; revised September 14th, 2021. This work is supported by the Foundation of Jinling Institute of Technology (Grant No: jit-fhxm-2003 and jit-b-201706), the University-industry Collaboration Education Foundation of Ministry of Education (Grant No: 202002192004), the foundation of Jiangsu province modern education research (Grant No: 2019-R-80918), the Education Reform Project of Jinling Institute of Technology (Grant No: KCSZ2019-5).

X. M. Tian is a teacher of College of Intelligent Science and Control Engineering, Jinling Institute of Technology, Nanjing 211169, P.R. China, email: tianxiaomin100@163.com;

Z. Yang is a professor of College of Intelligent Science and Control Engineering, Jinling Institute of Technology, Nanjing 211169, P.R. China, email: yz@jit.edu.cn;

Y. Shao is a teacher of College of Intelligent Science and Control Engineering, Jinling Institute of Technology, Nanjing 211169, P.R. China, email: shaoyan@jit.edu.cn;

L. X. Zhai is an associate professor of College of Intelligent Science and Control Engineering, Jinling Institute of Technology, Nanjing 211169, P.R. China, email: lxzhai@jit.edu.cn;

D^α instead of ${}_0D_t^\alpha$.

Lemma 1 (see [12]) Let $x(t) \in R^n$ be a continuous and derivable function. Then, for any $t > 0$,

$$\frac{1}{2}D^\alpha x^2(t) \leq x(t)D^\alpha x(t) \tag{2}$$

Lemma 2 (see [13]) Assume a, b and $0 < r < 1$ are real numbers, then the following inequality holds:

$$(|a| + |b|)^r \leq |a|^r + |b|^r \tag{3}$$

III. Main Results

In this section, a finite-time backstepping control strategy is presented to achieve the stabilization of fractional-order hydro-turbine governing system, the effects of unknown bounded external disturbances and saturated nonlinear input are both considered. For deal with these uncertainties, the fractional version of unknown parameters update laws are given.

In order to derive the main theory results, the structure of strict feedback system should be first given. Strict feedback system [14] can be used to express different real world systems, which can be described as follows

$$\begin{aligned}
D^\alpha x_1 &= g_1(x_1,t)x_2 + \delta_1^T F_1(x_1,t) + f_1(x_1,t)\\
D^\alpha x_2 &= g_2(x_1,x_2,t)x_3 + \delta_2^T F_2(x_1,x_2,t) + f_2(x_1,x_2,t)\\
&\vdots\\
D^\alpha x_{n-1} &= g_{n-1}(x_1,x_2,...,x_{n-1},t)x_n + \delta_{n-1}^T F_{n-1}(x_1,x_2,\\
&\quad ...,x_{n-1},t) + f_{n-1}(x_1,x_2,...,x_{n-1},t)\\
D^\alpha x_n &= g_n(x_1,x_2,...,x_n,t)u + \delta_n^T F_n(x_1,x_2,...,x_n,t)\\
&\quad + f_n(x_1,x_2,...,x_n,t)
\end{aligned} \tag{4}$$

where δ_i is the system parameters vector of the i-th state equation, $g_i(\cdot)$, $F_i(\cdot)$, $f_i(\cdot)$ for $i = 1,2,...,n$ are known, smooth nonlinear functions. When consider the effects of external disturbance $d_i(t)$ and the saturated nonlinear input $sat(u(t))$, meantime, $g_1(\cdot)$, $g_2(\cdot)$, ..., $g_n(\cdot)$ are constants, then the system can be rewritten as

$$\begin{aligned}
D^\alpha x_1 &= k_1x_2 + \delta_1^T F_1(x_1,t) + f_1(x_1,t) + d_1(t)\\
D^\alpha x_2 &= k_2x_3 + \delta_2^T F_2(x_1,x_2,t) + f_2(x_1,x_2,t) + d_2(t)\\
&\vdots\\
D^\alpha x_{n-1} &= k_{n-1}x_n + \delta_{n-1}^T F_{n-1}(x_1,x_2,...,x_{n-1},t)\\
&\quad + f_{n-1}(x_1,x_2,...,x_{n-1},t) + d_{n-1}(t)\\
D^\alpha x_n &= k_nu(t) + \delta_n^T F_n(x_1,x_2,...,x_n,t)\\
&\quad + f_n(x_1,x_2,...,x_n,t) + d_n(t)
\end{aligned} \tag{5}$$

In this paper, the research object is fractional-order hydro-turbine governing system [15] with external disturbances $d_i(t)$, which is described as

$$\begin{aligned}
D^\alpha x_1 &= \omega_0x_2 + d_1(t)\\
D^\alpha x_2 &= \frac{1}{T_{ab}}\Big(x_3 - Fx_2 - \frac{E_q{}'V_s}{x_{d\Sigma}'}sinx_1 - \frac{V_s^2}{2}\frac{x_{d\Sigma}' - x_{q\Sigma}}{x_{d\Sigma}'x_{q\Sigma}} \times\\
&\quad sin2x_1\Big) + d_2(t)\\
D^\alpha x_3 &= \frac{1}{e_{qh}T_w}\Big(-x_3 + e_yx_4 + \frac{ee_yT_w}{T_y}x_4\Big) + d_3(t)\\
D^\alpha x_4 &= \frac{1}{T_y}\big(u(t) - x_4\big) + d_4(t)
\end{aligned} \tag{6}$$

Obviously, fractional-order hydro-turbine governing system is strict feedback system, where $\alpha = 0.98$, $\omega_0 = 314$, $E_q{}' = 1.35$, $x_{q\Sigma} = 1.474$, $x_{d\Sigma}' = 1.15$, $T_{ab} = 9$, $F = 2$, $T_w = 0.8$, $V_s = 1$, $T_y = 0.1$, $e_{qh} = 0.5$, $e = 0.7$, $e_y = 1$, respectively. $u(t)$ is the comprehensive actual controller to be designed later.

Assumption 1 It is assumed that the external disturbances $d_i(t)$, i=1, 2, 3, 4 are bounded by

$$|d_i(t)| \leq \rho_i \tag{7}$$

where ρ_i is an unknown positive constant.

Next, the main control strategy will be introduced in detail, through design the virtual controllers step by step, then an appropriate actual controller is determined. In order to deal with these unknown parameters, some fractional-order version of adaptive update laws are given. Finally, the fractional-order finite-time stability theory is applied to demonstrate the finite-time stability of the controlled system.

To deduce the actual controller, transformation variables should be assigned firstly as

$$\xi_1 = x_1,\ \xi_2 = x_2 - \tau_1,\ \xi_3 = x_3 - \tau_2,\ \xi_4 = x_4 - \tau_3 \tag{8}$$

where $\xi_i(i = 1,2,3,4)$ is transformation variable, $\tau_j(j = 1,2,3)$ is virtual controller and can be designed as

$$\begin{aligned}
\tau_1 &= \frac{1}{\omega_0}\Big[-m_1sgn(\xi_1) - \hat{\rho}_1sgn(\xi_1) - ||\widetilde{\rho}||\frac{\xi_1}{||\Xi||^2}\Big]\\
\tau_2 &= T_{ab}\Big(-m_2sgn(\xi_2) - \omega_0\xi_1 + D^\alpha\tau_1 - \hat{\rho}_2sgn(\xi_2)\\
&\quad -||\widetilde{\rho}||\frac{\xi_2}{||\Xi||^2}\Big) + Fx_2 + \frac{E_q'V_s}{x_{d\Sigma}'}sinx_1 + \frac{V_s^2}{2}\times\\
&\quad \frac{x_{d\Sigma}' - x_{q\Sigma}}{x_{d\Sigma}'x_{q\Sigma}}sin2x_1\\
\tau_3 &= \frac{e_{qh}T_wT_y}{e_yT_y + ee_yT_w}\Big(-m_3sgn(\xi_3) - \frac{1}{T_{ab}}\xi_2 + \frac{1}{e_{qh}T_w}x_3\\
&\quad + D^\alpha\tau_2 - \hat{\rho}_3sgn(\xi_3) - ||\widetilde{\rho}||\frac{\xi_3}{||\Xi||^2}\Big)
\end{aligned} \tag{9}$$

where $\rho = (\rho_1,\rho_2,\rho_3,\rho_4)^T$, $\hat{\rho}_i$ is the estimation of ρ_i, $\Xi = (\xi_1,\xi_2,\xi_3,\xi_4)^T$, $||\cdot||$ represents L2 norm. Denote $\widetilde{\rho}_i = \hat{\rho}_i - \rho_i$ for identifying these unknown parameters, the adaptive update laws are chosen as

$$D^\alpha\widetilde{\rho}_i = D^\alpha\hat{\rho}_i - D^\alpha\rho_i = \eta|\xi_i| \tag{10}$$

in which $i = 1,2,3,4$. η is positive adaptive gain.

Theorem 1 Consider the system (6) with saturated nonlinear input and external disturbances, the controller which leads to finite-time stabilization of system (6) is given below

$$\begin{aligned}
u(t) &= T_y\Big[-\frac{e_yT_y + ee_yT_w}{e_{qh}T_wT_y}\xi_3 - m_4sgn(\xi_4) - \hat{\rho}_4sgn(\xi_4)\\
&\quad + D^\alpha\tau_3 - ||\widetilde{\rho}||\frac{\xi_4}{||\Xi||^2}\Big] + x_4
\end{aligned} \tag{11}$$

Proof. Step 1: The first new subsystem can be obtain according to eqs. (6) and (8)

$$\begin{aligned}
D^\alpha\xi_1 &= D^\alpha x_1 = \omega_0x_2 + d_1(t)\\
&= \omega_0(\xi_2 + \tau_1) + d_1(t)
\end{aligned} \tag{12}$$

in order to demonstrate the stability of system (12), the following Lyapunov candidate function can be chosen as

$$V_1(t)=\frac{1}{2}\xi_1^2+\frac{1}{2\eta}\widetilde{\rho}_1^2 \tag{13}$$

taking the α-th derivative of $V_1(t)$, according to Lemma 1 and Assumption 1, we have

$$\begin{aligned} D^\alpha V_1(t) &\le \xi_1 D^\alpha\xi_1+\frac{1}{\eta}\widetilde{\rho}_1 D^\alpha\hat{\rho}_1 \\ &= \xi_1\big[\omega_0(\xi_2+\tau_1)+d_1(t)\big]+\frac{1}{\eta}\widetilde{\rho}_1 D^\alpha\hat{\rho}_1 \\ &= \omega_0\xi_1\xi_2+\omega_0\xi_1\tau_1+\xi_1 d_1(t)+\frac{1}{\eta}\widetilde{\rho}_1 D^\alpha\hat{\rho}_1 \\ &\le \omega_0\xi_1\xi_2+\omega_0\xi_1\tau_1+\rho_1|\xi_1|+\widetilde{\rho}_1|\xi_1| \end{aligned} \tag{14}$$

substituting τ_1 from the first equation of (9) into (14), it yields

$$\begin{aligned} D^\alpha V_1(t) &\le \omega_0\xi_1\xi_2-m_1|\xi_1|-||\widetilde{\rho}||\frac{\xi_1^2}{||\Xi||^2} \\ &= \omega_0\xi_1\xi_2-\sqrt{2}m_1\Big(\frac{\xi_1^2}{2}\Big)^{1/2}-||\widetilde{\rho}||\frac{\xi_1^2}{||\Xi||^2} \end{aligned} \tag{15}$$

if $\xi_2=0$, then ξ_1 and ρ_1 are both converge to zero.

Step 2: The Second new subsystem about ξ_2 can be constructed as

$$\begin{aligned} D^\alpha\xi_2 &= D^\alpha x_2-D^\alpha\tau_1 \\ &= \frac{1}{T_{ab}}\Big[(\xi_3+\tau_2)-Fx_2-\frac{E_q{}'V_s}{x'_{d\Sigma}}sinx_1-\frac{V_s^2}{2}\times \\ &\quad \frac{x'_{d\Sigma}-x_{q\Sigma}}{x'_{d\Sigma}x_{q\Sigma}}sin2x_1\Big]+d_2(t)-D^\alpha\tau_1 \end{aligned} \tag{16}$$

selecting the Lyapunov function as

$$V_2(t)=V_1(t)+\frac{1}{2}\xi_2^2+\frac{1}{2\eta}\widetilde{\rho}_2^2 \tag{17}$$

taking the α-th derivative of $V_2(t)$, one obtains

$$\begin{aligned} D^\alpha V_2(t) &\le D^\alpha V_1(t)+\xi_2 D^\alpha\xi_2+\frac{1}{\eta}\widetilde{\rho}_2 D^\alpha\hat{\rho}_2 \\ &= \omega_0\xi_1\xi_2-\sqrt{2}m_1\Big(\frac{\xi_1^2}{2}\Big)^{1/2}-||\widetilde{\rho}||\frac{\xi_1^2}{||\Xi||^2}+\xi_2\times \\ &\quad \Big\{\frac{1}{T_{ab}}\Big[(\xi_3+\tau_2)-Fx_2-\frac{E_q{}'V_s}{x'_{d\Sigma}}sinx_1-\frac{V_s^2}{2}\times \\ &\quad \frac{x'_{d\Sigma}-x_{q\Sigma}}{x'_{d\Sigma}x_{q\Sigma}}sin2x_1\Big]+d_2(t)-D^\alpha\tau_1\Big\} \\ &\quad +\widetilde{\rho}_2|\xi_2| \end{aligned} \tag{18}$$

substituting τ_2 from the second equation of (9) into (18), one has

$$\begin{aligned} D^\alpha V_2(t) &\le -\sqrt{2}m_1\Big(\frac{\xi_1^2}{2}\Big)^{1/2}+\frac{1}{T_{ab}}\xi_2\xi_3-m_2|\xi_2|+\rho_2|\xi_2| \\ &\quad -\hat{\rho}_2|\xi_2|-||\widetilde{\rho}||\frac{\xi_1^2+\xi_2^2}{||\Xi||^2}+\widetilde{\rho}_2|\xi_2| \\ &\le \frac{1}{T_{ab}}\xi_2\xi_3-\sqrt{2}m_1\Big(\frac{\xi_1^2}{2}\Big)^{1/2}-\sqrt{2}m_2\Big(\frac{\xi_2^2}{2}\Big)^{1/2} \\ &\quad -||\widetilde{\rho}||\frac{\xi_1^2+\xi_2^2}{||\Xi||^2} \end{aligned} \tag{19}$$

similar to step 1, if $\xi_3=0$, then $D^\alpha V_2(t)<0$, that is ξ_2, $\widetilde{\rho}_2$ will converge to zero.

Step 3: We continue to investigate the third new subsystem with transformation variable ξ_3, that is

$$\begin{aligned} D^\alpha\xi_3 &= D^\alpha x_3-D^\alpha\tau_2 \\ &= \frac{1}{e_{qh}T_w}\Big[-x_3+e_y(\xi_4+\tau_3)+\frac{ee_yT_w}{T_y}(\xi_4+\tau_3)\Big] \\ &\quad +d_3(t)-D^\alpha\tau_2 \end{aligned} \tag{20}$$

This step is to verify the stability of system (20) with the following Lyapunov function

$$V_3(t)=V_2(t)+\frac{1}{2}\xi_3^2+\frac{1}{2\eta}\widetilde{\rho}_3^2 \tag{21}$$

taking the α-th derivative of $V_3(t)$, it yields

$$\begin{aligned} D^\alpha V_3(t) &\le D^\alpha V_2(t)+\xi_3 D^\alpha\xi_3+\frac{1}{\eta}\widetilde{\rho}_3 D^\alpha\hat{\rho}_3 \\ &= \frac{1}{T_{ab}}\xi_2\xi_3-\sqrt{2}m_1\Big(\frac{\xi_1^2}{2}\Big)^{1/2}-\sqrt{2}m_2\Big(\frac{\xi_2^2}{2}\Big)^{1/2} \\ &\quad -||\widetilde{\rho}||\frac{\xi_1^2+\xi_2^2}{||\Xi||^2}+\xi_3 D^\alpha\xi_3+\widetilde{\rho}_3|\xi_3| \\ &= \frac{1}{T_{ab}}\xi_2\xi_3-\sqrt{2}m_1\Big(\frac{\xi_1^2}{2}\Big)^{1/2}-\sqrt{2}m_2\Big(\frac{\xi_2^2}{2}\Big)^{1/2} \\ &\quad -||\widetilde{\rho}||\frac{\xi_1^2+\xi_2^2}{||\Xi||^2}+\xi_3\Big\{\frac{1}{e_{qh}T_w}\times \\ &\quad \Big[-x_3+(e_y+\frac{ee_yT_w}{T_y})(\xi_4+\tau_3)\Big]+d_3(t) \\ &\quad -D^\alpha\tau_2\Big\}+\widetilde{\rho}_3|\xi_3| \end{aligned} \tag{22}$$

substituting τ_3 from the third equation of (9) into (2), we have

$$\begin{aligned} D^\alpha V_3(t) \quad \le \quad & \frac{e_yT_y+ee_yT_w}{e_{qh}T_wT_y}\xi_3\xi_4-\sqrt{2}m_1\Big(\frac{\xi_1^2}{2}\Big)^{1/2} \\ & -\sqrt{2}m_2\Big(\frac{\xi_2^2}{2}\Big)^{1/2}-\sqrt{2}m_3\Big(\frac{\xi_3^2}{2}\Big)^{1/2} \\ & -||\widetilde{\rho}||\frac{\xi_1^2+\xi_2^2+\xi_3^2}{||\Xi||^2} \end{aligned} \tag{23}$$

obviously, when $\xi_4=0$, then ξ_3 and $\widetilde{\rho}_3=0$ will converge to zero.

Step 4: In the last step, the actual controller is designed. Similar to the above steps, the last subsystem with transformation variable ξ_4 is determined as

$$\begin{aligned} D^\alpha\xi_4 &= D^\alpha x_4-D^\alpha\tau_3 \\ &= \frac{1}{T_y}\Big(u(t)-x_4\Big)+d_4(t)-D^\alpha\tau_3 \\ &= \frac{1}{T_y}\Big(u(t)-x_4\Big)+d_4(t)-D^\alpha\tau_3 \end{aligned} \tag{24}$$

the overall Lyapunov function is constructed as

$$V_4(t)=V_3(t)+\frac{1}{2}\xi_4^2+\frac{1}{2\eta}\widetilde{\rho}_4^2 \tag{25}$$

according to the previous inequality are induced in the step

1 to 3, the α-th derivative of eq. (25) is

$$\begin{aligned}
D^{\alpha}V_4(t) &\leq D^{\alpha}V_3(t)+\xi_4 D^{\alpha}\xi_4+\frac{1}{\eta}\widetilde{\rho}_4 D^{\alpha}\hat{\rho}_4\\
&= \frac{e_yT_y+ee_yT_w}{e_{qh}T_wT_y}\xi_3\xi_4-\sqrt{2}m_1\Big(\frac{\xi_1^2}{2}\Big)^{1/2}\\
&\quad -\sqrt{2}m_2\Big(\frac{\xi_2^2}{2}\Big)^{1/2}-\sqrt{2}m_3\Big(\frac{\xi_3^2}{2}\Big)^{1/2}\\
&\quad -||\widetilde{\rho}||\frac{\xi_1^2+\xi_2^2+\xi_3^2}{||\Xi||^2}+\xi_4(D^{\alpha}x_4-D^{\alpha}\tau_3)\\
&\quad +\widetilde{\rho}_4|\xi_4|\\
&= \frac{e_yT_y+ee_yT_w}{e_{qh}T_wT_y}\xi_3\xi_4-\sqrt{2}m_1\Big(\frac{\xi_1^2}{2}\Big)^{1/2}\\
&\quad -\sqrt{2}m_2\Big(\frac{\xi_2^2}{2}\Big)^{1/2}-\sqrt{2}m_3\Big(\frac{\xi_3^2}{2}\Big)^{1/2}\\
&\quad -||\widetilde{\rho}||\frac{\xi_1^2+\xi_2^2+\xi_3^2}{||\Xi||^2}+\widetilde{\rho}_4|\xi_4|\\
&\quad +\xi_4\left[\frac{1}{T_y}(u(t)-x_4)+d_4(t)-D^{\alpha}\tau_3\right] \qquad (26)
\end{aligned}$$

according to Assumption 1, we have

$$\begin{aligned}
D^{\alpha}V_4(t) \leq\ & \frac{e_yT_y+ee_yT_w}{e_{qh}T_wT_y}\xi_3\xi_4-\sqrt{2}m_1\Big(\frac{\xi_1^2}{2}\Big)^{1/2}\\
& -\sqrt{2}m_2\Big(\frac{\xi_2^2}{2}\Big)^{1/2}-\sqrt{2}m_3\Big(\frac{\xi_3^2}{2}\Big)^{1/2}\\
& -||\widetilde{\rho}||\frac{\xi_1^2+\xi_2^2+\xi_3^2}{||\Xi||^2}+\frac{1}{T_y}\xi_4 u(t)\\
& -\frac{1}{T_y}\xi_4x_4-\xi_4D^{\alpha}\tau_3+\hat{\rho}_4|\xi_4| \qquad (27)
\end{aligned}$$

substituting $u(t)$ from (11) into (27), one has

$$\begin{aligned}
D^{\alpha}V_4(t) \leq\ & -\sqrt{2}m_1\Big(\frac{\xi_1^2}{2}\Big)^{1/2}-\sqrt{2}m_2\Big(\frac{\xi_2^2}{2}\Big)^{1/2}-\sqrt{2}m_3\Big(\frac{\xi_3^2}{2}\Big)^{1/2}\\
& -\sqrt{2}m_4\Big(\frac{\xi_4^2}{2}\Big)^{1/2}-||\widetilde{\rho}||\frac{\xi_1^2+\xi_2^2+\xi_3^2+\xi_4^2}{||\Xi||^2}\\
=\ & -m\left[\Big(\frac{\xi_1^2}{2}\Big)^{1/2}+\Big(\frac{\xi_2^2}{2}\Big)^{1/2}+\Big(\frac{\xi_3^2}{2}\Big)^{1/2}+\Big(\frac{\xi_4^2}{2}\Big)^{1/2}\right]\\
& -\sqrt{2\eta}\Big(\frac{||\widetilde{\rho}||^2}{2\eta}\Big)^{1/2} \qquad (28)
\end{aligned}$$

furthermore, according to lemma 2, it yields

$$\begin{aligned}
D^{\alpha}V_4(t) \leq\ & -\bar{m}\left[\frac{1}{2}\Big(\xi_1^2+\xi_2^2+\xi_3^2+\xi_4^2\Big)+\frac{\widetilde{\rho}_1^2}{2\eta}+\frac{\widetilde{\rho}_2^2}{2\eta}\right.\\
& \left.+\frac{\widetilde{\rho}_3^2}{2\eta}+\frac{\widetilde{\rho}_4^2}{2\eta}\right]^{1/2}=-\bar{m}V_4^{1/2}(t) \qquad (29)
\end{aligned}$$

where $m=min\{\sqrt{2}m_1,\sqrt{2}m_2,\sqrt{2}m_3,\sqrt{2}m_4\}>0$, $\bar{m}=min\{m,\sqrt{2\eta}\}>0$. According to the theory results of ref.[16], $V(t)=0$ for all $t\geq T$, here T can be estimated as

$$T=t_0+\left(\frac{\Gamma(\alpha+1)\Gamma(1/2)V^{\alpha-1/2}(t_0)}{\bar{m}\Gamma(\alpha+1/2)}\right)^{1/\alpha} \qquad (30)$$

which implies that $\xi_i=0$ and $\widetilde{\rho}_i=0$ for all $t\geq T$. Hence, the finite-time stabilization of system (6) with saturated nonlinear input and external disturbances is achieved. This completes the proof.

IV. Simulation Results

In this section, some simulation results are presented to demonstrate the effectiveness and feasibility of the proposed control strategy. In the system $\alpha=0.98$, the initial conditions are selected as $x(0)=(0.1,0.1,0.1,0.1)^T$, $\rho=(0,0,0,0)^T$ considering external disturbance $d_1(t)=0.01sin(t)$, $d_2(t)=0.03sin(t)$, $d_3(t)=0.05sin(t)$, $d_4(t)=0.07sin(t)$, the phase trajectories maps of system (6) without control are shown in Fig.1.

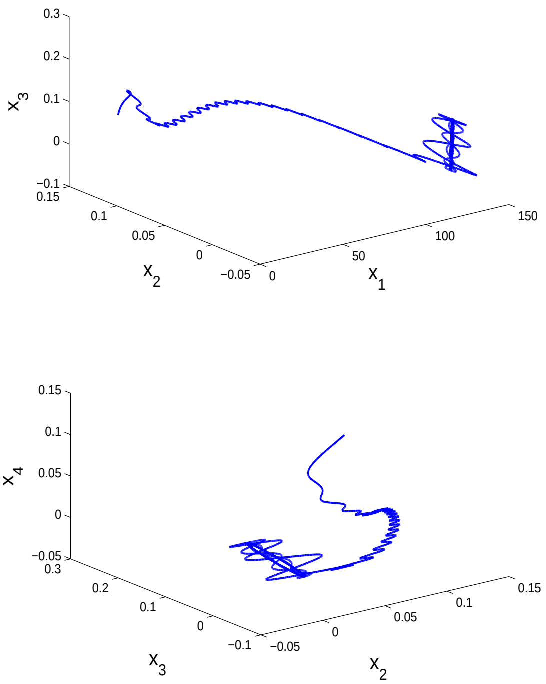
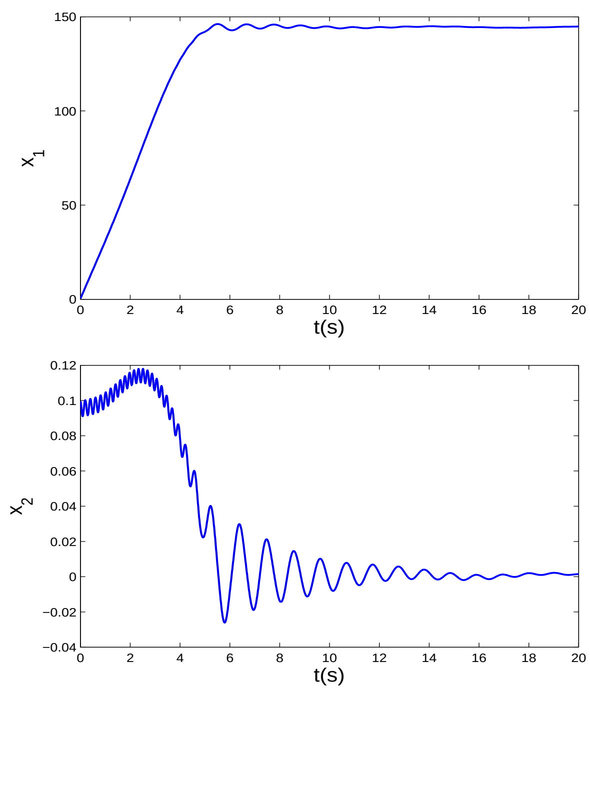


Fig. 1: Phase trajectories map of system (6) without control

In order to verify the control effect of the proposed controller, the state trajectories of system (6) without controller firstly presented in Fig.2.



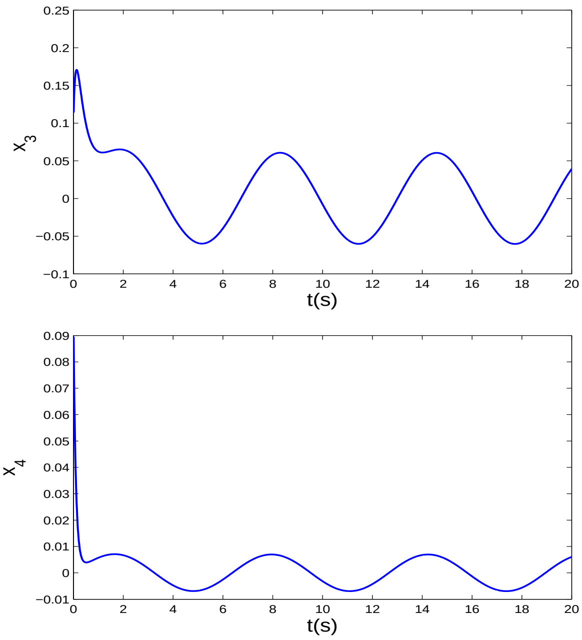
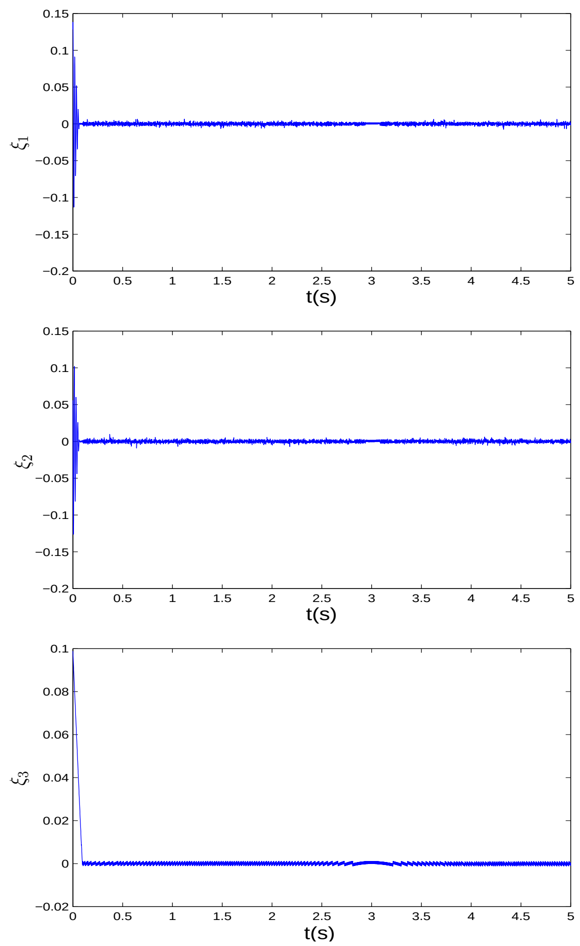


Fig. 2: State trajectories of system (6) without control

Letting $m_1 = m_2 = m_3 = m_4 = 1$, $\eta = 1$, when the actual controller $u(t)$ is activated, the time responses of the transformation variables are shown in Fig.3. It is clearly that all variables trajectories are converge to zero in given time, which implies that under the control of the proposed control strategy, the finite-time stabilization of the controlled system with external disturbance is realized.



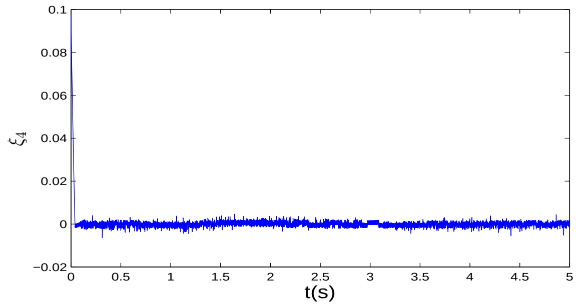


Fig. 3: Time responses of transformation variables with controller activated

V. Conclusions

This paper researched the problem of finite-time stabilizing fractional-order hydro-turbine governing system with backstepping method. The system is perturbed by unknown external disturbances, the bounds of external disturbances are assumed to be unknown in advance. In order to deal with these unknown parameters, some appropriate fractional-order version of adaptive rules are proposed. Through design virtual controllers step by step, then a comprehensive actual controller can be designed finally. Fractional-order finite-time stability theory is applied to demonstrate the finite-time stability of the closed-loop system and simulation results are presented to verified the feasibility and effectiveness of the proposed control strategy.

References

[1] W. J. Yang, N. L. Saarinen, J.D. Yang, etc, "Wear and tear on hydro power turbines-Influence from primary frequency control," *Renewable Energy*, vol. 87, pp. 88-95, 2016.

[2] W. C. Guo, J. D. Yang, "Modeling and dynamic response control for primary frequency regulation of hydro-turbine governing system with surge tank," *Renewable Energy*, vol. 121, pp. 173-187, 2018.

[3] R. L. Bagley, R. A. Calico, "Fractional order state equations for the control of viscoelastically damped structure," *Journal of Guidance Control and Dynamics*, vol. 14, pp. 304311, 1991.

[4] A. Dumlu, K. Erenturl, "Trajectory tracking control for a 3-DOF parallel manipulator using fractional-order $\mathrm{PI}^{\lambda}\mathrm{D}^{\mu}$ control," *IEEE Transactions on Industrial Electronics*, vol. 61, pp. 3417-3426, 2014.

[5] J. Ni, L. Liu, C. Liu, X. Hu, "Fractional order fixed-time nonsingular terminal sliding mode synchronization and control of fractional order chaotic systems," *Nonlinear Dynamics*, vol. 89, pp. 2065-2083, 2017.

[6] H. Liu, Y. Pan, Y. Chen, "Adaptive fuzzy backstepping control of fractional-order nonlinear systems," *IEEE Trans. Syst. Man Cybern. Syst*, vol. 47, pp. 22092217, 2017.

[7] M. P. Aghababa, S. Khanmohammadi, G. Alizadeh, "Finite-time synchronization of two different chaotic systems with unknown parameters via sliding mode technique," *Applied Mathematical Modelling*, vol. 35, pp. 3080-3091, 2011.

[8] N. Bigdeli, H. A. Ziazi, "Finite-time fractional-order adaptive intelligent backsstepping sliding mode control of uncertain fractional-order chaotic systems," *Journal of the Franklin Institute*, vol. 354, pp. 160-183, 2017.

[9] R. Z. Luo, M. C. Huang, H.P. Su, "Robust control and synchronization of 3-D uncertain fractional-order chaotic systems with external disturbances cia adding one power integrator control," *Complexity*, 8417536, 2019.

[10] M. K. Shukla, B. B. Sharma, "Backstepping based stabilizaiton and synchronizaiton of a class of fractional order chaotic systems," *Chaos Solitons and Fractals*, vol. 102, pp. 274-284, 2017.

[11] M. K. Shukla, B. B. Sharma, "Stabilizaiton of a class of fractional order chaotic systems via backstepping approach," *Chaos Solitons and Fractals*, vol. 98, pp. 56-62, 2017.

[12] K. Y. Shao, H. X. Guo, F. Han, "Finite-time projective synchronization of fractional-order chaotic systems via soft variable structure control," *Journal of Mechancial Science and Technology*, vol. 34, no. 1, pp. 369-376, 2020.

[13] M. P. Aghababa, S. Khanmohammadi, G. Alizadeh, "Finite-time synchronization of two different chaotic systems with unknown parameters via sliding mode technique," *Applied Mathematical Modelling*, vol. 35, no. 6, pp. 3080-3091, 2011.

[14] M. K. Shukla, B. B. Sharma, "Control and synchronizaiton of a class of uncertain fractional order chaotic systems via adaptive backstepping control," *Asian Journal of Control*, vol. 20, pp. 707-720, 2018.

[15] S. H. Huang, B. Zhou, etc, "Robust fixed-time sliding mode control for fractional-order nonlinear hydro-turbine governing system," *Renewable Energy*, vol. 139, pp. 447-458, 2019.

[16] Y. Xu, W. X. Li, "Finite-time synchronization of fractional-order complex-valued couled systems," *Physica A*, vol. 549, 123903, 2020.

[17] V. D. Lecce, A. Amato, A. Quarto, and M. Minoia, "Bigraph theory for distributed and autonomous cyber-physical system design," *IAENG International Journal of Computer Science*, vol. 47, no. 1, pp. 37-46, 2020.

[18] O. I. Kharchenko, "Modeling nonlinear stochastic filter by volterra transfer functions," *Engineering Letters*, vol. 28, no. 2, pp. 262-267, 2020.

[19] A. Sambas, S. Vaidyanathan, et al, "Investigation of chaos behavior in a new two-scroll chaotic system with four unstable equilibrium points, its synchronization via four control methods and circuit simulation," *IAENG International Journal of Applied Mathematics*, vol. 50, no. 1, pp. 12-21, 2020.

[20] M. Li, Y. Q. Wang, "Existence and Iteration of Monotone Positive Solutions for Fractional Boundary Value Problems with Riesz-Caputo Derivative," *Engineering Letters*, vol. 29, no. 2, pp. 327-331, 2021.

[21] C. F. Wang, Y. P. Deng, and P. P. Shen, "A Global Optimization Algorithm for Solving Indefinite Quadratic Programming," *Engineering Letters*, vol. 28, no. 4, pp. 1058-1062, 2020.

[22] F. R. Shi, N. N. Zhao, et al, "Adaptive Fuzzy Funnel Control for Pure-Feedback Nonlinear System with Input Constraint," *IAENG International Journal of Computer Science*, vol. 48, no. 2, pp. 334-342, 2021.

[23] W. H. Zhang, Y. L. Shang, et al, "Finite-Time Stabilization of General Stochastic Nonlinear Systems with Application to a Liquid-Level System," *IAENG International Journal of Applied Mathematics*, vol. 51, no. 2, pp. 295-299, 2021.