A Hybrid Firefly Algorithm with Butterfly Optimization Algorithm and its Application

Jinqian Zhang*, *Member, IAENG*, Xuefeng Xie, Min Wang, Mengjian Zhang*

***Abstract*—Butterfly optimization algorithm (BOA) is a new nature-inspired algorithm that imitates the food-searching and mating behavior of butterflies to solve the global optimization problem. In nature, butterflies not only determine the locate nectar or mates by smell, but also the visual function of butterflies cannot be ignored. We proposed a novel hybrid firefly algorithm (FA) with BOA, namely FA-BOA, in which we take the visual function of the similarity of fireflies and butterflies into consideration. To substantiate the optimization performance of the proposed algorithm, FA-BOA is tested on a set of eight benchmark functions. Besides, the proposed algorithm is used to solve two real-world engineering design problems (Three-bar truss design and Speed reducer design). Experimental results demonstrate that the proposed algorithm is effective and outpeforms other optimization algorithms in terms of convergence accuracy and stability.**

***Index Terms*—butterfly optimization algorithm, firefly algorithm, high dimension, speed reducer design, three-bar truss design**

I. Introduction

An innovative optimization system is proposed to emulate the food-searching strategy of butterflies. Based on the butterfly food forging approach, BOA is a nature-driven meta-heuristic algorithm [1]. Arora et al. [2] applied BOA to optimize the node localization problem in wireless sensor networks and obtained excellent solutions. Malisetti et al. [3] utilized a novel BOA based on quasi-objection for the cluster head selection problem in WSNs. Provas et al. [4] proposed a two-step BBO-BOA (hBBO-BOA) to solve the economic load distribution problem of an integrated power system composed of conventional thermal power generating units and renewable energy sources. Kun et al. [5] proposed a modified adaptive BOA to address the "early search blindness" and the relatively poor adaptability of the sensory modality. Zhang et al. [6] proposed a hybrid BOA with PSO algorithm to improve the global optimization capability of the basic BOA. Above the BOA research (improvement research or application research), there are very few papers on the BOA hybrid algorithm.

Firefly algorithm has the advantages of clear flow, few parameters, and easy implementation, and is widely used in computer networks, image processing, and engineering design [7-8]. Researchers have studied hybrid optimization on the Firefly Algorithm, which improved the ability of FA global optimization. Farahani SM et al. [9] mixed GA into FA to enhance the global search capability of firefly algorithm. The introduction of GA balances the exploration and mining capabilities of FA. Abdullah et al. [10] incorporated DE algorithm into the standard firefly algorithm. The hybrid algorithm increases the information sharing between fireflies, avoids convergence to local optimum, and improves the search efficiency of the algorithm. Guo et al. [11] combined the standard FA with Harmony Search (HS) algorithm , in order to integrate the exploration ability of HS with the mining ability of FA. Li et al. [12] proposed a fuzzy adaptive firefly algorithm to search for multi-level thresholding for color satellite images.

Hybrid algorithms mainly combine the advantages of two or more optimization algorithms to improve the optimization performance. However, the bionic algorithm is based on the imitation of biological characteristics, and the essential characteristics of the algorithm need to be considered in the improvement. In this article, the habits the photosensitive characteristics of butterflies in nature are taken into consideration. Therefore, we chose FA as a complementary optimization algorithm to make up for the limitations of the BOA, and proposed a new hybrid method.

Olfactory signals may be a key factor in the butterfly's foraging process, playing an attractive role. Visual signals may be an important factor for butterflies to find food sources, and then mainly rely on olfactory signals to stimulate foraging. Some species of butterflies use visual signals as the dominant factor to locate nectar sources, other species may rely mainly on olfactory information, and some species make comprehensive use of these two sensory channels [13-15]. The details are as follows:

1) Ômura and Honda [14] found that the newly emerged Vanessa indica Herbst mainly relies on vision, and secondly relies on smell to visit flowers.

2) Some species of butterflies (Cethosia bibles Drury, Tirumala limniace, and Idea leuconoe) have a certain response to visual signals when foraging, but they rely more heavily on smell.

3) Heliconius Melpomene L. relies on the combination of

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Jinqian Zhang is a postgraduate student in the School of Electrical Engineering, Guizhou University, Guiyang, 550025 PR CHINA (e-mail: 928683496@qq.com).

Xuefeng Xie is a senior engineer of Guizhou Aerospace Kaishan Petroleum Instruments Co., Ltd, Guiyang 550025 PR CHINA (e-mail: 2550649632@qq.com).

Min Wang is a postgraduate student in the College of Electrical Engineering, Guizhou University, Guiyang, 550025 PR CHINA (e-mail: w_min0715@163.com).

Mengjian Zhang is a postgraduate student in the School of Electrical Engineering, Guizhou University, Guiyang, 550025 PR CHINA (e-mail: mjz960106@163.com).

visual and olfactory signals to locate nectar information, but its newly emerging adults mainly rely on smell to select flowers [15].

Besides, the combination of visual and olfactory signals can increase the number of flower visitors visiting flowers and the degree of foraging activities [13]. Different flower-visiting insects can assign different weights when using the visual and olfactory information of nectar sources [16]. Therefore, the use of multi-channel signals is beneficial for butterflies to distinguish unrewarded flowers in the ever-changing distribution of food resources and improve the efficiency of flower visits.

Since the BOA only relies on olfactory foraging or ignores its visual signal, we use the characteristics of the firefly in the FA algorithm to find the global optimal value through the visual signal. In addition, the proposed hybrid algorithm FA-BOA of the FA and the BOA improves the ability to find the global optimum, which is more in line with the habit of butterflies in nature.

The remainder of the paper is organized as follows. Section II introduces the principle of the FA, the BOA, and the FA-BOA. Section III illustrates the experimental results for 8 representative test functions. Section IV presents the simulation results of two classic engineering application problems. Finally, the conclusion and future studies are summarized in Section V.

II. Firefly Algorithm and Butterfly Algorithm

A. Principle of Firefly algorithm

The FA is a intelligence algorithm with simple structure yet superior performance which is utilized to solve complex optimization problems in continuous search space. The flashing and attraction behavior exhibited by fireflies is crucial to their evolution.

The position of each firefly represents a feasible solution to the problem to be solved, and the brightness of the firefly represents the suitability and superiority of the firefly position. Then, the position update formula of firefly i attracted by the brighter firefly j is defined as:

$$x_{id}(t+1) = x_{id}(t) + \beta \cdot (x_{jd}(t) - x_{id}(t)) + \alpha(t)\varepsilon_i \tag{1}$$

where x_{id} and x_{jd} are the D-dimensional positions of the fireflies i and j. Furthermore, β is the attractiveness, α represents the step factor, and t indicates the iteration number. Finally, ε is uniformly distributed in the range of [−0.5, 0.5].

The parameter α of the firefly algorithm is calculated as follows:

$$\alpha(t) = \alpha_0 \theta^t \tag{2}$$

where α_0 is the initial step size factor of the FA, which is taken as 1; the value of θ range is [0.95, 0.99].

The relative fluorescence and attraction of two fireflies can be approximated as:

$$\begin{cases} I = I_0 \exp(-\gamma r_{ij}^2) \\ \beta = \beta_0 \exp(-\gamma r_{ij}^2) \end{cases} \tag{3}$$

where I_0 represents the highest fluorescence brightness obtained when $\gamma = 0$; β_0 denotes the maximum attraction, i.e. the attraction at $r = 0$, and r_{ij} represents the distance between firefly i and j, calculated as follows.

$$r_{ij} = \left\| x_i - x_j \right\| = \sqrt{\sum_{d=1}^{D} (x_{id} - x_{jd})^2} \tag{4}$$

The basic steps of the Firefly Algorithm are summarized as the pseudo-code shown in Algorithm 1.

Algorithm 1: Firefly Algorithm

```
Objective function f(x), x = (x1, x2, ⋯, xdim)
Initialize a population of fireflies xi (i = 1, 2, ⋯, n)
Define light absorption coefficient γ
While (t < MaxGeneration)
  For i = 1: n all n fireflies
    For j = 1: i all n fireflies
      Light intensity Ii at xi is determined by f(xi)
      If (Ij > Ii)
        Move firefly i towards j in all dimensions
      End if
      Attractiveness varies with distance r via exp[-γr^2]
      Evaluate new solutions and update light intensity
    End for j
  End for i
  Rank the fireflies and find the current best
End while
Output the best solution found
```

B. Principle of Butterfly Optimization Algorithm

The BOA [1] is a intelligent optimization algorithm derived from simulating the food search and mating behavior of butterflies (butterflies use scent to locate nectar or mating objects). In BOA, each butterfly produces a scent of a certain intensity, the magnitude of which is related to the physical intensity of the stimulus. The specific formula for fragrance size is:

$$f_i = cI^a \tag{5}$$

where f_i is the scent size, that is, the scent intensity that other butterflies can perceive. c represents the sensory modality , taking values at [0, 1]. And I is the stimulus intensity, a takes vaule in [0, 1]. Theoretically, the value of any sensory morphological coefficient c can be in the range of [0, ∞]. The parameter c in the optimal search phase of the BOA can be formulated as follows:

$$c_{t+1} = c_t + \left(\frac{0.025}{c_t \cdot T_{\max}}\right) \tag{6}$$

where $T_{\max}$ is the maximum number of iterations of BOA, and the initial starting value of parameter c_t is 0.01.

Also, each algorithm consists of two key steps. In the global search stage, butterflies moves towards the optimal butterfly (solution g_{best}), and the global position update is expressed by Eq. (7).

$$x_i^{t+1} = x_i^t + (r^2 \cdot g_{best} - x_i^t) \cdot f_i \tag{7}$$

where x^t_i is the solution vector for the ith butterfly in tth iteration; and r taks any value in [0, 1]. Here, g_{best} represents the optimal position among all solutions in the current iteration. In addition, the fragrance emitted by the ith butterfly is denoted by f_i. The position update for the local search stage can be formulated as follows:

$$x_j^{t+1} = x_i^t + (r^2 \cdot x_j^t - x_k^t) \cdot f_i \tag{8}$$

where x^t_j and x^t_k denote the solution vectors of the jth and kth individuals randomly selected from within the population in t iterations. If x^t_j and x^t_k belong to the same population, and r is a random number in [0, 1], denoting a local random wander.

In nature, both global and local searches can occur as butterflies look for food and mating partners. Therefore, switching the normal global search and the dense local search requires a switching parameter p to control. Each iteration generates a random number in [0, 1], which is compared with p to decide whether to perform global search or local search.

The above theoretical principles constitute the complete algorithm of BOA, and its pseudo-code is demonstrated in Algorithm 2.

Algorithm 2: Butterfly Optimization Algorithm

1. Objective function $f(x)$, $X = (X_1, X_2, \cdots, X_{\dim})$, Dim is No. of dimensions
2. Generate initial population of n Butterflies X_i (i = 1, 2, ⋯, n)
3. Stimulus Intensity I_i at X_i is determined by $f(X_i)$
4. Define sensor modality c, power exponent a, and switch probability p
5. **While** stopping criteria not met do
6. **For** each buttery bf in the population do
7. Calculate fragrance for bf using Eq. (5)
8. **End for**
9. Find the best *bf*
10. **For** each butterfly *bf* in the population do
11. Generate a random number r from [0, 1]
12. If r < P then
13. Move towards the best solution using Eq. (7)
14. Else
15. Move randomly using Eq. (8)
16. End if
17. E**nd for**
18. Update the value of a
19. **End while**
20. Output the best solution found.

The flowchart of BOA is shown in Fig. 1.

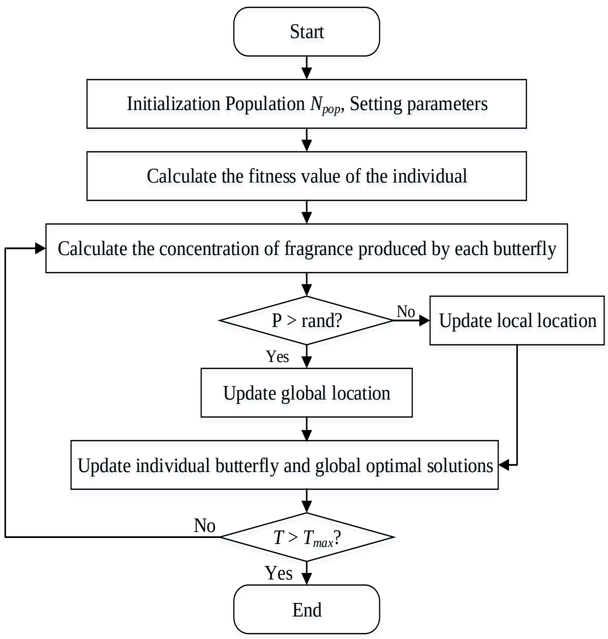


Fig. 1 Flowchart of BOA.

C. The Proposed Algorithm

In this subsection, we propose a novel hybrid FA-BOA, which is a combination of independent FA and BOA. In the local search process of BOA, due to the small moving range of the individual butterfly, it is easy to fall into the local optimum. However, FA has strong global detection ability and local development ability. Therefore, integrating FA in the local search stage of BOA can make full use of a small number of butterfly individuals in the local search process, and guide the butterfly individuals to move to the target position, enhancing the algorithm. local development capabilities.

The mathematical model of the global search stage of FA-BOA can be calculated as follows:

$$X_i^{t+1} = X_i^t + (r^2 \cdot g_{best} - X_i^t) \cdot f_i \tag{9}$$

In the local search stage of FA-BOA, the location update of the optimization process can be expressed as:

$$X_i^{t+1} = X_i^t + \beta \cdot (X_i^k - X_j^k) + \alpha \cdot \varepsilon \tag{10}$$

where the parameter ε obeys a uniform distribution in [−0.5, 0.5], and the parameter β can be calculated by:

$$\beta = \beta_0 \exp(-\gamma r_{ij}^2) \tag{11}$$

The parameter α of FA-BOA can be determined by:

$$\alpha(t) = \alpha_0 \cdot \theta^t \tag{12}$$

where α_0 is the starting value of the algorithm step size and is set to 1; the value range of θ is [0.95, 0.99].

And the computational flowchart of FA-BOA is shown in Fig. 2.

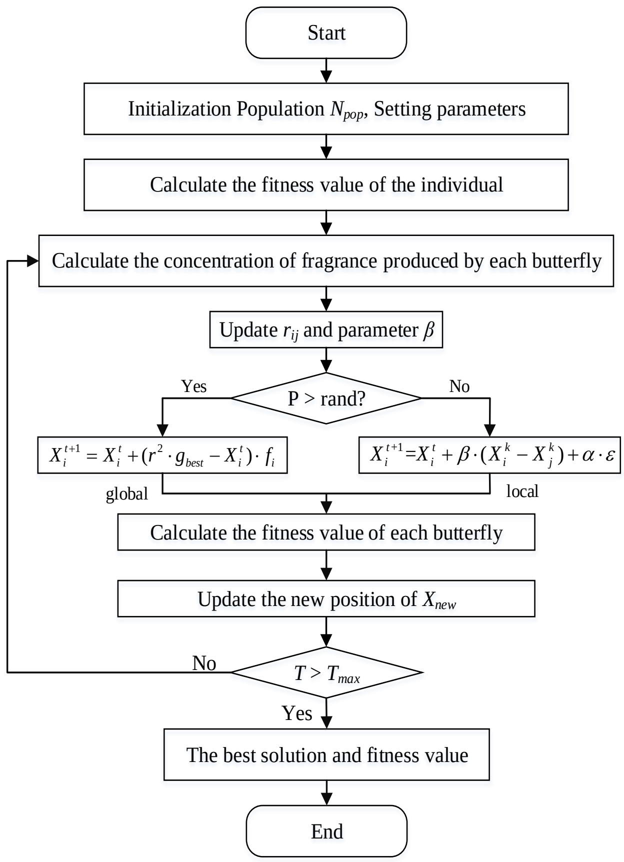


Fig. 2 Flowchart of FA-BOA.

Based on the above explanation, the pseudo-code of hybrid

FA-BOA is given in Algorithm 3.

Algorithm 3: Pseudo-code of hybrid FA with BOA

1. Generate the initialize population of the butterflies X_i (i = 1, 2, ⋯, n) randomly
2. Initialize the parameter r_1, r_2, β, γ, ε
3. Define sensor modality c, power exponent a, and switch probability p
4. **For** i = 1: n
5.   Calculate the fitness value of each butterfly
6. **End for**
7. **While** t < T_{max}
8.   **For** i =1: n
9.   Update the fragrance of the current search agent by Eq. (5)
10.   **For** j =1: n
11.   Update r_{ij} and β by Eq. (4) and Eq. (11)
12.   If rand < P
13.   Update the position using Eq. (9)
14.   Else
15.   Update the position using Eq. (10)
16.   End if
17.   Calculate the fitness value of each butterfly
18.   Find the best f
19.   **End for** j
20.   **End for** i
21.   Update the parameter c using Eq. (6)
22. $t = t + 1$
23. **End while**
24. **End** FA-BOA

III. Algorithm Simulation and Result Analysis

A Benchmark set and compared algorithm

In the simulation experiments, eight benchmark functions were selected from Ref. [17] and Ref. [18]. A set of benchmark functions of different types are used to evaluate the optimization capability of FA-BOA.

The benchmark functions include two distinct types, namely unimodal (UM) and multi-modal (MM). The UM (F1-F4) benchmark functions, with only one global minimum, can strengthen the capabilities of algorithm. If the MM (F5–F8) benchmark functions are used, it shows the diversification capabilities of the optimization algorithms.

The mathematical formulas, value ranges and theoretical optimal values of the four UM and four MM test functions are shown in Tables I.

The simulation results and optimization performance of FA-BOA compared with other types of optimization algorithms FA, BOA, GOA [19], GWO algorithm [20], LBOA [2], PSO algorithm [21], and WOA [22].

The performance evaluation indicators include the best scheme (Best), the worst scheme (Worst), the standard deviation (Std) and the average result (Avg). The selected GOA, GWO algorithms, LBOA, and WOA are all powerful and novel optimization algorithms, while FA, BOA, and PSO algorithms are selected as algorithms that are heavily adopted in optimization contexts.

B parameter settings

The proposed FA-BOA has been tested using the Matlab R2018b running Windows 10 with an AMD Ryzen7-4800H 2.90 GHz processor and 16.00 GB RAM, executed to check the performance of the FA-BOA. All tests were performed using 30 populations in a maximum of 500 iterations. All stored simulation results are the average of 30 independent runs, and the obtained results are used for comparison.

Meanwhile, algorithms of FA, BOA, GOA, GWO algorithm, LBOA, PSO algorithm, and WOA all use the settings parameters presented from the original work. The basic parameter settings of each algorithms are shown in Tab. II.

C Quantitative results of FA-BOA

In this section, we select eight comparison algorithms to test each benchmark function together to evaluate the performance of the proposed FA-BOA. The simulation results of each algorithm on the test function are shown in Tab. III, where N/A means that the algorithm is not suitable for solving this function. Notably, the optimal solution obtained is highlighted in bold.

According to the results in Tab. III, the proposed FA-BOA outperforms the other algorithms in the average (Avg) vaule and standard deviation (Std) values of solving the functions F1, F2, F3, F4, F5, F6, and F8. Additionally, the FA only gets the optimal value on the function F7, while FA-BOA gets the optimal value on other functions.

As can be seen from the simulation results for the functions (F1-F4), FA-BOA is very competitive in optimizing the unimodal function, compared with FA, BOA, GOA, GWO algorithms, LBOA, PSO algorithms, and WOA. These experimental results demonstrate the excellent optimization accuracy of FA-BOA with one-global minimal functions.

At the same time, we can also see that FA-BOA shows excellent optimization performance for optimizing the multimodal functions. FA-BOA can find superior average results for the test functions F5, F6, and F8.

Obviously, these results imply that the improved FA-BOA has a good ability to avoid falling into the trap of local optima and seek the global optima.

It can be seen from Tab. III that the variances of FA-BOA on eight test functions are all minimum, indicating that FA-BOA optimization algorithm has good robustness.

According to the simulation results, to analyze the robustness of the proposed FA-BOA and other algorithms, the convergence curves for the eight test functions (Dim = 30) are demonstrated in Fig. 3.

The convergence curves in Fig. 3 can verify that the proposed FA-BOA has faster convergence speed than other algorithms. The simulation results verify that the improved algorithm FA-BOA can effectively improve the convergence trend of the basic BOA.

The unimodal functions only provide a global optimum, so they are applied to investigate the development phase. The obtained results proved that FA-BOA exhibited a very excellent development ability compared with that of the other competitive algorithms.

The MM functions were applied as they have many local optima compared to the unimodal functions. Usually, the complexity of variables increases with the size or dimension of the problem. In order to optimize each stage to obtain better solutions and get rid of the trap of local optimization, these evaluations make it easy for us to understand that fa-boa has high comprehensive ability in the exploration and utilization stage.

TABLE I. SIMULATION TEST FUNCTIONS

No.	Function	Formula	Dim	Range	Optima
F1	Sphere	$F_1(x)=\sum_{i=1}^{n}x_i^2$	30	[-100,100]	0
F2	Schwefel 2.22	$F_2(x)=\sum_{i=1}^{n}\lvert x_i\rvert+\prod_{i=1}^{n}\lvert x_i\rvert$	30	[-10,10]	0
F3	Sum Square	$F_3(x)=\sum_{i=1}^{n}nx_i^2$	30	[-10,10]	0
F4	Zakharov	$F_4(x)=\sum_{i=1}^{n}x_i^2+(\sum_{i=1}^{n}0.5ix_i)^2+(\sum_{i=1}^{n}0.5ix_i)^4$	30	[-5,10]	0
F5	Cigar	$F_5(x)=x_1^2+10^6\cdot\sum_{i=2}^{n}x_i^2$	30	[-100,100]	0
F6	Alpine	$F_6(x)=\sum_{i=1}^{n}\lvert x_i\sin(x_i)+0.1x_i\rvert$	30	[-5,5]	0
F7	Penalized1	$F_7(x)=\frac{\pi}{n}\{10\sin^2(\pi y_i)+\sum_{i=1}^{n-1}(y_i-1)^2[1+10\sin^2(\pi y_{i+1})]+(y_n-1)^2\}$ $+\sum_{i=1}^{n}u(x_i,10,100,4), y_i=1+\frac{1}{4}(x_i+1)$	30	[-100,100]	0
F8	Schwefel 2.26	$F_8(x)=-\sum_{i=1}^{n}\lvert x_i\sin(\sqrt{\lvert x_i\rvert})\rvert$	30	[-10,10]	0

TABLE II. PARAMETER SETTINGS

No.	Algorithms	Population Size	Parameter Settings
1	Butterfly Optimization Algorithm (BOA)	30	$a=0.1, c(0)=0.01, p=0.6$
2	Firefly Algorithm (FA)	30	$\beta_0=1, \gamma=1$
3	Grasshopper Optimization Algorithm (GOA)	30	$c_{\max}=1, c_{\min}=0.00004, \tau=1.5$
4	Grey Wolf Optimizer (GWO)	30	$a_{frist}=2, a_{final}=0$
5	Butterfly Optimization Algorithm with Lévy flights (LBOA)	30	$a=0.1, c(0)=0.01, p=0.6, \lambda=1$
6	Particle Swarm Optimization (PSO)	30	$c_1=c_2=2, V_{\max}=1, V_{\min}=1, \omega_{\max}=0.9, \omega_{\min}=0.2$
7	Whale Optimization Algorithm (WOA)	30	$a_1=[2\ \ 0], a_2=[-2\ \ -1], b=1$
8	FA-BOA	30	$a=0.1, c(0)=0.01, p=0.6, \beta_0=0.2, \alpha_0=0.2$

TABLE III. SIMULATION RESULTS OF EIGHT COMPARISON ALGORITHMS

Functions		BOA	FA	GOA	GWO	LBOA	PSO	WOA	FA-BOA
F1	Best	1.0832E-11	8.0585E-08	1.1830E+01	3.4318E-29	3.9633E-15	3.8737E-07	2.5066E-88	**0**
	Worst	1.4304E-11	1.3532E-07	1.1219E+02	4.6471E-26	3.4806E-11	1.4108E-04	1.0263E-71	**1.5823E-205**
	Avg	1.2572E-11	1.1132E-07	3.7080E+01	3.0913E-27	4.7553E-12	1.0397E-05	5.2314E-73	**5.4560E-207**
	Std	8.2541E-13	1.3178E-08	2.4387E+01	8.9724E-27	6.3009E-12	2.5157E-05	2.0772E-72	**0**
F2	Best	1.3568E-09	9.0934E-06	1.7757E+01	8.1813E-17	1.2481E-09	3.5642E-03	2.0942E-51	**1.2402E-128**
	Worst	5.8275E-09	1.5498E-04	8.0473E+01	3.7793E-16	5.3183E-09	1.5909E-02	9.3275E-51	**5.8079E-128**
	Avg	4.6481E-09	1.3875E-04	1.4826E+01	1.0042E-16	1.3017E-09	3.8583E-03	6.8064E-52	**3.8296E-129**
	Std	1.3568E-09	9.0934E-06	1.7757E+01	8.1813E-17	1.2481E-09	3.5642E-03	2.0942E-51	**1.2402E-128**
F3	Best	9.6696E-12	1.1249E-08	1.7678E+00	2.2652E-30	1.5064E-13	7.6080E-06	3.3296E-82	**1.3568E-304**
	Worst	1.3489E-11	2.0808E-08	9.0017E+01	9.1990E-28	9.5905E-12	5.6376E-04	4.3402E-70	**3.6041E-260**
	Avg	1.2040E-11	1.5004E-08	2.6763E+01	2.0962E-28	2.7997E-12	8.0024E-05	1.5064E-71	**1.2897E-261**
	Std	8.4649E-13	2.2728E-09	2.3256E+01	2.5850E-28	2.3330E-12	1.2592E-04	7.9194E-71	**0**
F4	Best	1.0951E-11	6.8495E-09	1.0056E-01	2.0318E-29	1.2908E-13	3.4770E-06	8.6514E-85	**0**
	Worst	1.3865E-11	1.2414E-08	6.0027E+02	2.5490E-27	1.5820E-11	3.2815E-04	4.9501E-70	**0**
	Avg	1.2515E-11	9.5769E-09	5.9637E+01	3.8551E-28	3.7920E-12	7.2819E-05	1.6569E-71	**0**
	Std	7.9152E-13	1.2563E-09	1.1646E+02	5.5358E-28	3.9472E-12	8.0401E-05	9.0364E-71	**0**
F5	Best	1.8916E-17	1.7655E-11	1.3150E-03	4.6813E-247	1.7319E-22	4.3816E-67	5.1021E-131	**0**
	Worst	1.2033E-11	5.5092E-09	1.6605E+03	1.0030E-200	2.0166E-17	3.3251E-60	3.5758E-99	**0**
	Avg	5.4713E-12	1.6806E-09	2.5190E+02	3.3436E-202	3.8248E-18	1.8044E-61	1.1919E-100	**0**
	Std	4.0994E-12	1.5644E-09	4.9994E+02	0.0000E+00	5.3847E-18	6.3373E-61	6.5284E-100	**0**
F6	Best	2.3642E-10	7.0607E-06	1.0134E-01	5.2452E-17	1.3845E-21	2.3744E-04	1.6368E-58	**4.3851E-292**
	Worst	8.3800E-10	7.7259E-05	7.0200E+00	2.0992E-03	2.9702E-13	2.8310E-03	2.1097E-50	**1.3152E-82**
	Avg	4.0765E-10	2.5784E-05	2.2003E+00	5.7573E-04	2.3671E-14	8.0691E-04	8.6689E-52	**1.1140E-83**
	Std	1.3879E-10	1.6866E-05	1.4606E+00	6.3003E-04	5.5900E-14	6.5512E-04	3.8416E-51	**3.2240E-83**

F7	Best	1.8199E-01	**9.8098E-10**	7.5530E+00	1.3053E-02	1.9983E-01	1.0369E+00	9.6675E-03	3.3960E-09
	Worst	9.9164E-01	7.2643E-01	1.9290E+02	8.6832E-02	4.8926E-01	1.0950E+01	1.3889E-01	**3.8095E-08**
	Avg	6.5520E-01	3.4581E-02	2.8408E+01	4.0546E-02	3.2379E-01	4.6114E+00	3.3954E-02	**1.3782E-08**
	Std	1.8025E-01	1.3443E-01	3.4255E+01	1.7448E-02	8.5837E-02	2.5105E+00	2.6628E-02	**8.5531E-09**
F8	Best	6.3003E-65	6.1162E-05	5.2213E-01	9.7568E-02	7.4001E-30	1.3576E-05	3.1889E-72	**8.6082E-136**
	Worst	2.6196E+01	1.3883E-04	8.1281E+00	6.3312E+00	9.6865E-10	3.9299E+00	7.0607E-01	**4.8783E-78**
	Avg	4.3649E+00	9.6079E-05	1.9503E+00	7.6681E-01	3.2316E-11	3.3792E-01	1.5271E-01	**1.6261E-79**
	Std	8.9379E+00	1.9545E-05	1.7168E+00	1.2359E+00	1.7685E-10	7.7290E-01	2.0720E-01	**8.9066E-79**

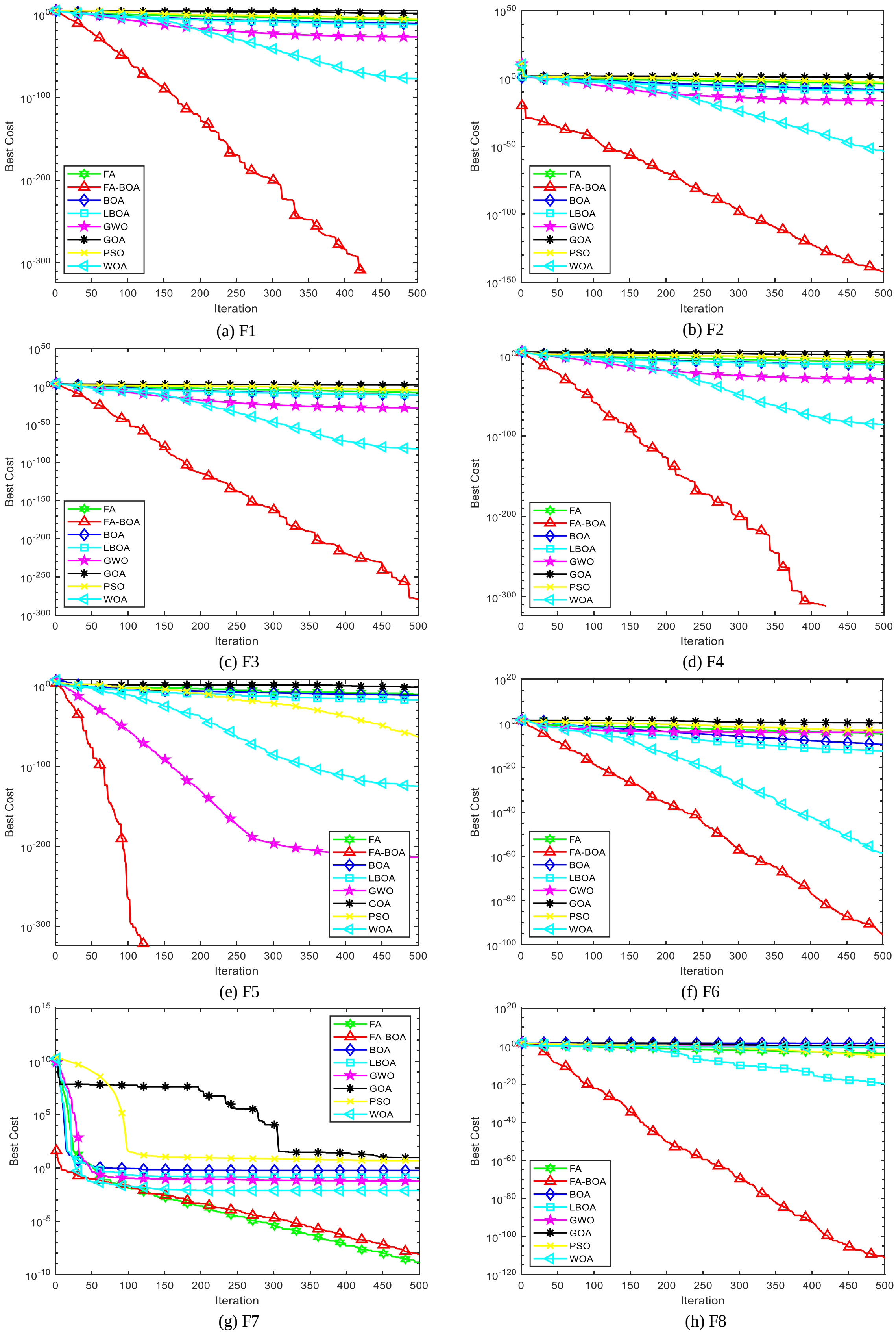


Fig. 3 Convergence curve for eight algorithms with Dim= 30.

It is not convincing to compare the advantages of different algorithms only according to the average value, standard deviation value and convergence analysis. We should utilize more statistical tests to assess the optimization performance of the proposed FA-BOA. Friedman test [23] and Wilcoxon rank-sum (WRS) test [24] are required to verify whether there is a substantial improvement compared with existing algorithms on a specific problem.

We usually use the WRS test and the Friedman rank test to verify the statistical significance of the proposed FA-BOA. We set the alpha in the WRS test to 0.05. The null hypothesis reflects the significant difference between the proposed algorithm and other algorithms. If this statistic (*H*) is greater than 0.05, null is accepted; otherwise, an alternative is accepted. In Tab. IV, the *p*-values calculated by FA-BOA and other algorithms in the Wilcoxon rank-sum tests for each test functions are given. For instance, if the best algorithm is FA-BOA, then do pairwise comparisons between FA-BOA and FA, FA-BOA, and BOA, and so on. According to the results in Tab. IV, the *p*-values of FA-BOA are all less than 0.05. It proves that the performance of our proposed FA-BOA is statistically significantly superior to almost all comparison algorithms. In conclusion, FA-BOA has higher convergence accuracy and faster convergence speed than other algorithms.

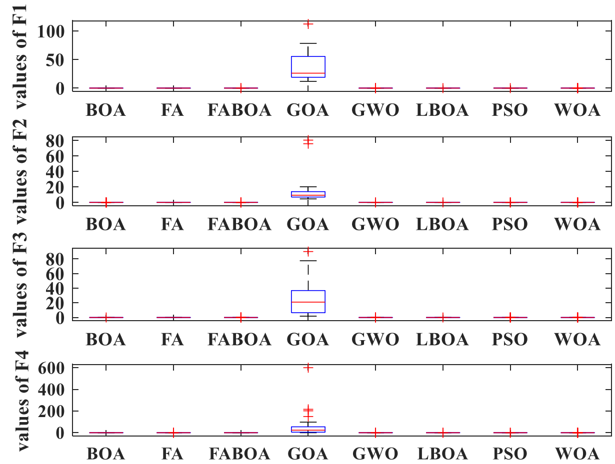
TABLE IV. THE *P*-VALUE AND HYPOTHESIS (*H*) OF WRS TEST

No.		FA	BOA	GOA	GWO	LBOA	PSO	WOA
F1	*p*	2.56E-06	2.56E-06	2.56E-06	2.56E-06	2.56E-06	2.56E-06	2.56E-06
	H	1	1	1	1	1	1	1
F2	*p*	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06
	H	1	1	1	1	1	1	1
F3	*p*	3.79E-06	3.79E-06	3.79E-06	3.79E-06	3.79E-06	3.79E-06	3.79E-06
	H	1	1	1	1	1	1	1
F4	*p*	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06
	H	1	1	1	1	1	1	1
F5	*p*	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06
	H	1	1	1	1	1	1	1
F6	*p*	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06
	H	1	1	1	1	1	1	1
F7	*p*	1.73E-06	1.48E-02	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06
	H	1	1	1	1	1	1	1
F8	*p*	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06
	H	1	1	1	1	1	1	1

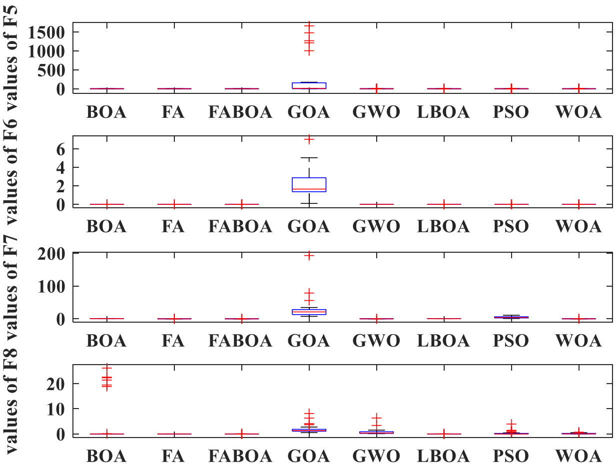
TABLE V. RANK SUMMARY OF STATISTICAL ASSESSMENT RESULTS

No.	BOA	FA	FA-BOA	GOA	GWO	LBOA	PSO	WOA
F1	5	6	1	8	3	4	7	2
F2	5	6	1	8	3	4	7	2
F3	5	6	1	8	3	4	7	2
F4	5	6	1	8	3	4	7	2
F5	6	7	1	8	2	5	4	3
F6	4	5	1	8	6	3	7	2
F7	6	1	2	8	4	5	7	3
F8	3	5	1	8	7	2	6	4
Avg	4.875	5.25	1.125	8	3.875	3.875	6.5	2.5
Final	5	6	1	8	3	3	7	2

We utilized Friedman test on the mean of the eight test functions computed by each algorithm to make the statistical results more convincing. The results of the Friedman rank test of each algorithm are shown in Tab. IV. To summarize, we can count the final order of the rank means, the eight algorithms order is FA-BOA > WOA > GWO > LBOA > FA > BOA > PSO > GOA. From these tables, it can be seen that there are significant differences between the proposed FA-BOA and the other algorithms of the eight test functions with Dim = 30.



(a) F1-F4



(b) F5-F8

Fig. 4 Boxplot graphs of the comparison algorithms.

The Fig. 4 shows the box plots of the simulation results of eight fixed-dimensional test functions by the eight algorithms. For the benchmark functions, the GOA results are relatively poor in the eight algorithms. For benchmark function F8, the WOA is also underperforming. By contrast, FA-BOA achieved the best results for all test functions.

According to all the experiments carried out, the FA-BOA with excellent solution results has shown that it can well balance the exploration and exploitation phases. As FA-BOA can achieve good results in various test functions, it needs to have a good balance in both exploration and exploitation phases.

IV. FA-BOA FOR CLASSICAL ENGINEERING PROBLEMS

This section discusses two engineering design problems, three-bar truss design, and speed reducer design which are employed to analyze and evaluate the ability of FA-BOA. Since the fitness function may directly affects the location update of search agents, constraint processing is a challenging job for the algorithm. Most classical engineering problems contain optimization objectives and multiple constraints, which evaluate the ability of FA-BOA from the perspective of solving the optimization objective with constraints.

A. Three-bar Truss Design

This case is designed for a three-bar planar truss [25] structure shown in Fig. 5. The goal of the three bar truss design problem is to minimize the volume of the truss under static pressure and meet the stress (σ) on each truss member constraints. This problem can be transformed into the problem of optimal cross-sectional area (A_1, A_2).

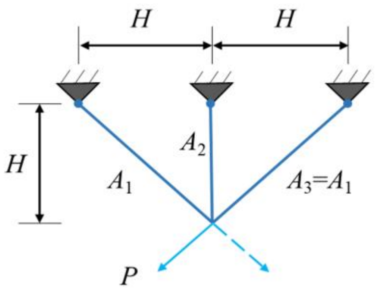


Fig. 5 Three-bar truss

The mathematical formulation of this problem is given below:

$$Minimize: f\left(A_1, A_2\right) = \left(2\sqrt{2A_1} + A_2\right) \times l$$

$$\text{s.t.}\begin{cases} g_1 = \dfrac{\sqrt{2}A_1 + A_2}{\sqrt{2}A_1^2 + 2A_1A_2} P - \sigma \\ g_2 = \dfrac{A_2}{\sqrt{2}A_1^2 + 2A_1A_2} P - \sigma \\ g_3 = \dfrac{1}{A_1 + \sqrt{2}A_2} P - \sigma \end{cases}$$

Variable range:

$$A_1, A_2 \in [0,1],\ l = 100cm,\ P = 2kN/(cm)^2,\ \sigma = 2kN/(cm)^2$$

TABLE VI. STATISTICAL RESULTS OF THE BEST THREE-BAR TRUSS MODEL OF FA-BOA

No.	No. iterations	Times	Average	S.D.
60	1000	30	**263.8958434**	5.977E-13

The statistical values of the best solution obtained by the FA-BOA are shown in Tab. VI. The solution by FA-BOA is (A_1, A_2) = (0.788675, 0.408248) with the objective value equal to 263.8958.

TABLE VII. COMPARISON OF RESULTS FOR THE THREE-BAR TRUSS DESIGN PROBLEM

Item	PSO	CS [26]	MBA [27]	FA-BOA
A_1	7.881 E-01	7.886 E-01	7.886 E-01	7.887 E-01
A_2	4.099 E-01	4.09 E-01	4.086 E-01	4.082 E-01
Best	2.639 E+02	2.64 E+02	2.639 E+02	2.639 E+02
Worst	2.639 E+02	N/A	2.650 E+02	2.639 E+02
Avg	2.641 E+02	2.639 E+02	2.639 E+02	2.639 E+02
Std	9.000 E-05	1.658 E-01	3.930 E-03	5.977 E-13

A comparison results between statistical performance and the best solutions obtained by FA-BOA and other considered algorithms is presented in Tab. 7. The algorithms used for comparison are PSO algorithm, CS algorithm, and MBA. With regard to the best solution, PSO algorithm, MBA, and FA-BOA all obtain the same solution, f(x) = 263.8958. From the quality of actual results, FA-BOA outperformed the methods considered in other literature by producing the optimal worst and standard deviation value.

B. Speed Reducer Design

In the comparative study of algorithms, reducer design problem is applied to analyze the accuracy performance (see Fig. 6). The optimization objective is to minimize the overall weight of the reducer. Constraints mainly include: bending stress of gear teeth, surface stress, force transmitted by lateral deflection of axis 1 and 2, stress in axis 1 and 2, etc.

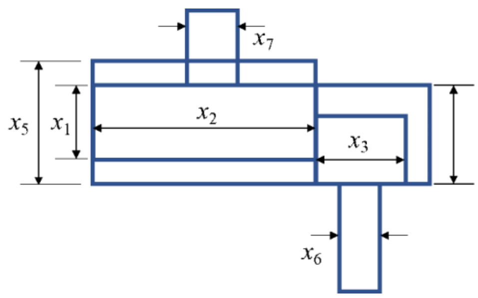


Fig. 6 Speed reducer

The objective function and constraints of this problem are expressed as follows:

$$Minimize: f(x_1, x_2, x_3, x_4, x_5, x_6, x_7)$$
$$= 0.7854x_1x_2^2(3.3333x_3^2 + 14.9334x_3 - 43.0934)$$
$$-1.508x_1(x_6^2 + x_7^2) + 7.4777(x_6^3 + x_7^3) + 0.7854(x_4x_6^2 + x_5x_7^2)$$

$$\text{s.t.}\begin{cases} g_1 = \dfrac{27}{x_1x_2^2x_3} P - 1 \le 0,\ g_2 = \dfrac{397.5}{x_1x_2^2x_3^2} - 1 \le 0 \\ g_3 = \dfrac{1.93x_4^3}{x_2x_3x_6^4} - 1 \le 0,\ g_4 = \dfrac{1.93x_5^3}{x_2x_3x_7^4} - 1 \le 0 \\ g_5 = \dfrac{\sqrt{\left(\dfrac{745x_4}{x_2x_3}\right)^2 + 1.69\times10^6}}{110x_6^3} - 1 \le 0 \\ g_6 = \dfrac{\sqrt{\left(\dfrac{745x_5}{x_2x_3}\right)^2 + 157.6\times10^6}}{85x_7^3} - 1 \le 0 \\ g_7 = \dfrac{x_2x_3}{40} - 1 \le 0,\ g_8 = \dfrac{5x_2}{x_1} - 1 \le 0,\ g_9 = \dfrac{x_1}{12x_2} - 1 \le 0 \\ g_{10} = \dfrac{1.5x_6 + 1.9}{x_4} - 1 \le 0,\ g_{11} = \dfrac{1.1x_7 + 1.9}{x_5} - 1 \le 0 \end{cases}$$

Variable range:

$$2.6 \le x_1 \le 3.6,\ 0.7 \le x_2 \le 0.8,\ 17 \le x_3 \le 28,\ 7.3 \le x_4 \le 8.3,$$
$$7.8 \le x_5 \le 8.3,\ 2.9 \le x_6 \le 3.9,\ and\ 5 \le x_7 \le 5.5$$

The corresponding statistical values of the Best FA-BOA model and the simple bounds of the speed reducer problem are presented in Tab. VIII. A comparison between the statistical performance and the best solutions obtained by the FA-BOA and the other comparison algorithms is given in Tab.

IX. The algorithms used for comparison are PSO algorithm, CS algorithm, and MBA.

Table VIII. STATISTICAL RESULTS OF THE SPEED REDUCER DESIGN PROBLEM OF FA-BOA

Item	Bound	Value
x_1	(2.6-3.6)	3.5
x_2	(0.7-0.8)	0.7
x_3	(17-28)	17
x_4	(7.3-8.3)	7.3
x_5	(7.8-8.3)	7.8
x_6	(2.9-3.9)	3.45836
x_7	(5.0-5.5)	5.245858
objective function value	/	**2999.0372**
No.	/	60
No. iterations	/	1000

Table IX. COMPARISON RESULTS FOR THE SPEED REDUCER DESIGN PROBLEM OF DIFFERENT ALGORITHMS

Item	PSO	CS [26]	MBA [27]	FA-BOA
x_1	3.508 E+00	3.501 E+00	3.500 E+00	3.500 E+00
x_2	0.700 E+00	0.700 E+00	0.700 E+00	0.700 E+00
x_3	1.700 E+01	1.700 E+01	1.700 E+01	1.700 E+01
x_4	7.300 E+00	7.605 E+00	7.300 E+00	7.300 E+00
x_5	8.300 E+00	7.818 E+00	7.716 E+00	7.800 E+00
x_6	3.365 E+00	3.352 E+00	3.350 E+00	3.458 E+00
x_7	5.290 E+00	5.287 E+00	5.287 E+00	5.246 E+00
Best	3.012 E+03	3.001 E+03	2.994 E+03	2.999 E+03
Worst	3.150 E+03	3.009 E+03	3.000 E+03	2.999 E+03
Mean	3.051 E+03	3.007 E+03	2.997 E+00	2.999 E+03
S.D.	1.807 E+01	4.968 E+00	1.560 E+00	2.850 E-10

Tab. IX presents the comparison results obtained by FA-BOA and other algorithms. Obviously, the solution of FA-BOA is better than PSO algorithm, CS algorithm in the literature. Although the optimal target values derived from the MBA is better than that of FA-BOA, the reported values are not feasible due to the violation of the fifth constraint (x_5) in the results of MBA. For the solution results, FA-BOA outperformed the considered algorithms by producing the optimal worst, mean, and standard deviation values.

V. CONCLUSIONS

This paper proposes a hybrid algorithm that incorporates two heuristic optimization algorithms, FA and BOA. The proposed algorithm combines the advantages of both FA and BOA, where a set of roaming random butterflies in the search space is initialized. During this roaming, the evolution of these butterflies is performed by integrating FA and BOA, where FA acts as a local search to optimise where the butterflies are found. Meanwhile, the randomization parameter is reduced so that it gradually decreases as it approaches the optimal value, and the performance of FA is improved. From the comparisons of simulation results, it can be seen that there is a certain research space for the hybrid intelligence algorithms to solve difficult continuous optimization problems, and the hybrid FA-BOA is a parctical and valuable method for solving the unconstrained nonlinear optimization problems. The optimization algorithm proposed in this paper has the following advantages.

1) It can efficiently make up for the defect that BOA is easy to fall into local optimum.

2) Compared with other existing algorithms, it has better optimization performance and competitiveness.

3) It can efficiently and stably find the global minimum for the optimization problems.

4) Due to the BOA algorithm only relies on smell to forage or ignores its visual signal, we use the characteristics of the firefly in the FA to search for the global optimum through the visual signal. Besides, the proposed hybrid algorithm FA-BOA of the FA and the BOA improves the ability to find the global optimum, which is more in line with the behavior of butterflies in nature.

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Jinqian Zhang (M'21) is received his B. E. degree from Chengdu University of Information Technology in 2017. He is currently a master student in School of Electronical Engineering, Guizhou University, China. His main research interests include mathematical modeling of complex system and intelligent optimization algorithms.

Xuefeng Xie is a senior engineer of Guizhou Aerospace Kaishan Petroleum Instruments Co., Ltd, His main research interests include swarm intelligent optimization algorithms and petroleum instruments.

Mengjian Zhang (M'21) born in 1996, CCF Student Member, is currently a master student in School of Electronical Engineering, Guizhou University, China. His main research interests include swarm intelligent optimization algorithms and wireless sensor network.

Min Wang, born in 1993, M. S. candidate. Her research interests include Swarm intelligent optimization algorithm and Load decomposition and identification.