Algorithm for Adjacent Vertex Sum Reducible Edge Coloring of Random Graphs

Jingwen Li, Shucheng Zhang, Rong Luo, Linyu Lan

Abstract—Let graph G(p,q) be a simple graph, if there is a positive integer $k(1 \le k \le |E|)$ and the mapping $f: E(G) \rightarrow \{1, 2, ..., k\}$, such that for any edge $uv \in E(G)$, when d(u) = d(v), S(u) = S(v), where $S(u) = \sum_{uv \in E(G)} f(uv)$. d(u) represents the degree of point u, then f is the coloring of Adjacent Vertex Sum Reducible Edge of G. The maximum value k is called the Adjacent Vertex Sum Reducible Edge chromatic number of graph G, denoted as $\chi'_{Avsr}(G)$. Based on the existing graph coloring concepts and reducible concepts, this paper proposes a new concept of adjacent vertex sum reducible edge coloring algorithm.

Index Terms—adjacent vertex sum reducible edge coloring, adjacent vertex sum reducible edge chromatic number, random graphs, algorithm

I. INTRODUCTION

s an important research topic in graph theory, graph A coloring has vital, theoretical and practical significance. Many problems in real life can be transformed into graph coloring problems, such as frequency allocation of communication segment, class scheduling, resource allocation, traffic scheduling, and so on. Graph coloring can be traced back to the famous "four-color conjecture". In 1965, M. Behzad proposed and studied the total coloring problem of graphs for the first time, and put forward the famous total coloring conjecture. In 1993, A. C. Burris and R. H. Schelp proposed the concept of Vertex Distinguishing Edge Coloring and related conjectures. Subsequently, Professor Zhang Zhongfu proposed the concept of Adjacent Vertex Distinguishing Edge Coloring of graphs based on Vertex Distinguishing Edge Coloring in 2002, and proposed the concept of Reducible Coloring series of graphs based on lemmas and conjectures in 2009. The new concept of Adjacent Vertex Sum Reducible Edge Coloring proposed is based on Adjacent Vertex Distinguishable Edge Coloring and Reducible Edge Coloring of graphs. To solve the Adjacent Vertex Sum Reducible Edge Coloring of random graphs, a

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new algorithm is proposed in this paper, whose design idea combines random search algorithm, such as mountain searching algorithm, bee swarm algorithm, and genetic algorithm. Then the coloring results obtained by the algorithm are analyzed and some theorems are summarized and proved for several graph classes.

II. PRELIMINARY KNOWLEDGE

This paper mainly discusses the Adjacent Vertex Sum Reducible Edge Coloring of special graphs, such as paths, circles, stars, fans and general linkage graphs.

Definition 1: Assume that G(V, E) is a simple diagram, if any positive integer $k(1 \le k \le |E|)$ and mapping $f: E(G) \rightarrow$ $\{1, 2, ..., k\}$, for any two points, $u, v \in V(G)$, when d(u) =d(v) and $uv \in E(G)$, there are S(u) = S(v), the S(u) = $\sum_{uv \in E(G)} f(uv), f$ is the Adjacent Vertex Sum Reducible Edge Coloring of G. And $\chi'_{avsr}(G) = \max\{k|k - AVSREC \ of \ G\}$ is Adjacent Vertex Sum Reducible Edge chromatic number. Obviously $\chi'_{avsr}(G)$ exists.

Definition 2: Suppose that the vertex set of graph G_n is $\{u_1, u_2, ..., u_n\}$, and the vertex set of H_m is $\{v_1, v_2, ..., v_m\}$. If the graphs G_n and H_m have central nodes, use u_1 or v_1 to denote. $G_n \uparrow H_m$ represents the graph obtained by connecting the central node v_1 of H_m to any vertex of G_n , $V(G_n \uparrow H_m) = V(G_n) \cup V(H_m)$, $E(G_n \uparrow H_m) = E(G_n) \cup E(H_m) \cup \{u_1v_1\}, E(G_n \uparrow H_m) = |E(G_n)| + |E(H_m)|$. An example is shown in figure 1.



Fig. 1. $C_n \uparrow S_5$

Definition 3: Let the Bunch graph be the concatenation graph of $m(m \ge 2)$ circle graphs of order $n (n \ge 4, n \equiv 0 \pmod{2})$, an example is shown in figure 2.



Definition 4^[6]: *Kite*: (n, t) - K is a graph consisting of a loop graph with vertex n and a road graph with length t.

Definition 5: If a tree graph has the same degree of other nodes except the leaf nodes, this type of graph is called $T_{(n)}$ graph. some samples are shown in figure 3.



Fig.3. Graph T_(n)

Lemma 1: For a simple graph G, when $\chi_{avsr}(G) = k$ exists but $\chi_{avsr}(G) = k + 1$ does not exist, the adjacent vertex sum reducible edge chromatic number of the graph G is k.

Proof: According to Definition 1, Lemma 1 obviously holds.

III. ALGORITHM

A. Preliminary Preparation

According to the definition of Adjacent Vertex Sum Reducible Edge Coloring, this paper divides all graphs into two types, one is graphs with adjacent same degree points, the other is graphs without adjacent same degree points. This article sets up a graph classification function Classify, a balance operator Balance that judges the balance in the process of increasing the color number, and an adjustment operator Surpass. The equilibrium state mentioned here refers to the state where the sum of the chromatic numbers of adjacent points of the same degree is equal and satisfies the continuity of the chromatic numbers.

B. The basic principle of the algorithm

The idea of the AVSREC algorithm is to transform the adjacency matrix of the graph into the initial coloring matrix that meets the requirements of AVSREC, and then increase the chromatic number by adjusting the operator, and make the coloring matrix meet AVSREC again by the balance operator. After many iterations, finally slowly tend to the optimal solution and complete the coloring process.

C. Fake Code

AVSREC Algorithm

Input: The adjacency matrix M of the graph G(p,q) **OutPut**: Coloring matrix that satisfies the AVSREC Begin

1 Calculate the SameDegreeList, maxColor, Balance. set a flag parameter count = 1. Initialize a matrix FinalAdjust.

2 while(count > 0)

3 count = 0

4 for $i \leftarrow 0$ to n

- 5 ei ++
- 6 count ++ 7 if (Surpass
- 7 if (Surpass) 8 ei ––
- 9 count --

10 endif *if*(*Balance*) 11 12 $FinalAdjust \leftarrow M$ 13 endif 14 if (maxColor == EdgeNum && Balance) 15 return FinalAdjust 16 endif 17 endfor 18 endwhile 19 return FinalAdjust 20 End

D. Figure 4 is an example of the coloring result of the graph $G_{(7,8)}$ obtained by the AVSREC algorithm



Fig.4. The coloring result example of the graph $G_{(7,8)}$

IV. CONCLUSION AND PROOF

Theorem 1: Suppose P_n is a path of $n(n \ge 2)$ vertices,

 $\chi_{avsr}(P_n) = 2$

Theorem 2: Suppose S_n is a star graph of n + 1 (n > 2) vertices, there is

 $\chi'_{avsr}(S_n) = n$ **Theorem 3:** Suppose C_n is a circle graph of n vertices, there is

$$\chi_{avsr}(C_n) = \begin{cases} 1, n \equiv 1 \pmod{2}; \\ 2, n \equiv 0 \pmod{2}; \end{cases}$$

According to the definition of Adjacent Vertex Sum Reducible Edge Coloring, adjacent points of the same degree must guarantee the same chromatic number, above three theorems are true.

Theorem 4: For friendship graphs $T_{(2,n)}$ $(n \ge 1)$, there is

 $\chi_{avsr}(T_{(2,n)}) = \Delta$ **Proof:** Suppose the friendship $T_{(2,n)}$ $(n \ge 1)$ represents a graph composed of n C_3 graphs, u_0 represents the center point, and the other two points of each triangle are represented by u_i and $v_i (1 \le i \le n, 1 \le j \le n)$, the sample is shown in the Figure 5.



Fig.5. Friendship graph $T_{(2,n)}$

According to the definition of AVSREC, the $T_{(2,n)}$ satisfies f coloring regulation

> $f(u_i v_i) = i, i = 1, 2, ..., n$ $f(u_0u_i) = f(u_0v_i) = 2n - i + 1(i = 1, 2, ..., n)$

The color sums of all adjacent 2 degree vertices in the figure are the same, and the chromatic number is equal to 2n.

Since the chromatic number of $f(u_iv_i)$ is mapped to $\{1,2,3,\ldots,n\}$, and $f(u_0u_i)$ and $f(u_0v_i)$ have the same chromatic number, the chromatic numbers of $f(u_0 u_i)$ and $f(u_0 v_i)$ are mapped to $\{n + 1, n + 2, n + 3, ..., 2n\}$. Therefore, the solution space of the maximum coloring number k of f is {1,2,3, ..., 2n}. when $f(u_1v_1)=1$, $f(u_0u_1)=1$ $f(u_0v_1) = 2n = \Delta$, so $\chi'_{avsr}(T_{(2,n)}) = \Delta (\Delta$ represents the maximum degree in the figure). The partial coloring result graphs of friendship graph are shown in Figure 6.



Fig.6. Partial coloring result graphs of the $T_{(2,n)}$ graph

Theorem 5: For the fan graph $F_n (n \ge 3)$, there is

 $\chi'_{avsr}(F_n) = 2n - 1$ **Proof:** Suppose the vertices set of the F_n graph is V = $\{u_0, u_1, u_2, ..., u_n\}, u_0$ represents the center point, u_1 and u_n represent the two 2 degree vertex of the fan graph. When $n \equiv 1 \pmod{2}$, the graph F_n satisfies AVSREC, f:

 $f(u_{0}u_{i}) = 2i - 1, i = 1, 2, ..., n$ $f(u_{i}u_{i+1}) = \begin{cases} 2n - 1 - i, i \equiv 1 \pmod{2}; \\ n + 1 - i, i \equiv 0 \pmod{2}; \\ i = 1, 2, ..., n - 1; \end{cases}$ When $n \equiv 0 \pmod{2}$, the graph F_n satisfies AVSREC, f: $f(u_0 u_i) = 2i, i = 1, 2, ..., n - 1$

$$f(u_0u_n) = 2n - 1$$

$$f(u_iu_{i+1}) = \begin{cases} n - i, i \equiv 1 \pmod{2}; \\ 2n - 1 - i, i \equiv 0 \pmod{2}; \\ i = 1, 2, ..., n - 1; \end{cases}$$

In both cases, the sum of the chromatic numbers of adjacent three-degree vertices is the same, which is equal to 3n, and the maximum chromatic numbers in the graph is dyed to the number of edges. At the same time, the coloring graphs still satisfy AVSREC. Therefore, for the graph F_n ($n \ge 3$), $\chi_{avsr}(F_n) = 2n - 1.$

Figure 7 shows the partial coloring result graphs of the F_n graph.



Theorem 6: For the joint graph $C_n \uparrow P_m \uparrow C_n (n \ge 3, m \ge 3)$ 3), there is

 $\chi'_{avsr}(C_n \uparrow P_m \uparrow C_n) = 6$ **Proof**: $C_n \uparrow P_m \uparrow C_n$ is shown in the Figure 8:



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When $n \equiv 1 \pmod{2}$, the graph $C_n \uparrow P_m \uparrow C_n$ satisfies f coloring regulation:

$$f(u_{i}u_{i+1}) = \begin{cases} 6, i \equiv 0 \pmod{2}; \\ 1, i \equiv 1 \pmod{2}; \\ i = 1, 2, ..., n - 1; \\ f(u_{i}u_{i+1}) = \begin{cases} 5, i \equiv 0 \pmod{2}; \\ 2, i \equiv 1 \pmod{2}; \\ i \equiv 1 \pmod{2}; \\ i \equiv 1, 2, ..., n - 1; \\ 3, i \equiv 1 \pmod{2}; \\ i \equiv 2, ..., m - 1; \\ f(u_{n}u_{1}) = 1, f(u_{n}u_{1}) = 2. \end{cases}$$

When $n \equiv 0 \pmod{2}$, the graph $C_n \uparrow P_m \uparrow C_n$ satisfies AVSREC, f:

$$f(u_{i}u_{i+1}) = \begin{cases} 6, i \equiv 0 \pmod{2}; \ i = 1, 2, ..., n-1; \\ 1, i \equiv 1 \pmod{2}. \end{cases}$$

$$f(u_{i}u_{i+1}) = \begin{cases} 5, i \equiv 0 \pmod{2}; \\ 2, i \equiv 1 \pmod{2}; \\ i \equiv 1 \pmod{2}. \end{cases}$$

$$f(v_{i}v_{i+1}) = \begin{cases} 4, i \equiv 0 \pmod{2}; \\ 3, i \equiv 1 \pmod{2}. \end{cases}$$

$$f(u_{n}u_{1}) = 6, \quad f(u_{n}u_{1}) = 5. \end{cases}$$

Therefore, for the joint graph $C_n \uparrow P_m \uparrow C_n (n \ge 3, m \ge 1)$, $\chi_{avsr}^{'}(C_n \uparrow P_m \uparrow C_n) = 6.$

Figure 9 shows the partial coloring result graphs of the $C_n \uparrow P_m \uparrow C_n$ graph.



Theorem 7: For the kite graph (n, t) - K, there is $\chi'_{avsr}((n, t) - K) = 4$

Proof: Suppose the vertices set of the (n, t) - K graph is $V = V(C_n) \cup V(P_m) = \{u_1, u_2, ..., u_n, v_1, v_2, ..., v_m\}$, where $u_1 = v_1$.

When $n \equiv 1 \pmod{2}$, the graph (n, t) – K satisfies f coloring regulation:

$$f(u_{i}u_{i+1}) = \begin{cases} 4, i \equiv 0 \pmod{2} \\ 1, i \equiv 1 \pmod{2} \\ i = 1, 2, ..., n-1; \\ f(u_{n}u_{1}) = 1; \\ f(v_{i}v_{i+1}) = \begin{cases} 3, i \equiv 1 \pmod{2} \\ 2, i \equiv 0 \pmod{2} \\ i = 1, 2, ..., m-1_{\circ} \end{cases}$$

When $n \equiv 1 \pmod{2}$, the graph (n, t) - K satisfies f coloring regulation:

$$f(u_{i}u_{i+1}) = \begin{cases} 4, i \equiv 0 \pmod{2}; \\ 1, i \equiv 1 \pmod{2}; \\ i = 1, 2, ..., n - 1; \\ f(u_{n}u_{1}) = 4; \\ f(v_{i}v_{i+1}) = \begin{cases} 2 i \equiv 1 \pmod{2} \\ 3 i \equiv 0 \pmod{2} \end{cases} i = 1, 2, ..., m - 1_{\circ} \end{cases}$$

Suppose that when $\chi'_{avsr}(C_n) = 5$, (n, t) - K also has at least one vertex whose color sum S(u) are different from the other degrees of the vertex of 2 degrees. Contradictions with assumptions. Therefore, according to Lemma 1, when $n \equiv$ $0 \pmod{2}$, $\chi'_{avsr}((n, t) - K) = 4$.

Obviously (n, t) - K graph satisfies the AVSREC, and the $\chi'_{avsr}((n, t) - K) = 4$

Figure 10 shows the partial coloring result graphs of the (n, t) - K graph.



Fig.10. Partial coloring result graphs of the (n, t) - K graph

Theorem 8: For the joint graph $C_n \uparrow P_2 \uparrow C_n (n \ge 3)$, there is (3 $n = 1 \pmod{2}$)

$$\chi_{avsr}(C_n \uparrow P_2 \uparrow C_n) = \begin{cases} 3 n \equiv 1 \pmod{2} \\ 5 n \equiv 0 \pmod{2} \end{cases}$$

Proof: Suppose the vertices set of C_n is $V = \{u_1, u_2, ..., u_n\}$, the vertices set of another C_n connected to P_2 is $V = \{u_1, u_2, ..., u_n\}$, the vertices set of P_2 is $V = \{v_1, v_2\}$, where $u_1 = v_1, u_1 = v_2$.

When $n \equiv 1 \pmod{2}$, the graph $C_n \uparrow P_2 \uparrow C_n$ satisfies AVSREC, f:

$$f(u_{i}u_{i+1}) = f(u_{i}u_{i+1}) = \begin{cases} 2, i \equiv 0 \pmod{2}; \\ 1, i \equiv 1 \pmod{2}; \\ i = 1, 2, ..., n - 1; \end{cases}$$

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$$f(u_1u'_1) = 3;$$

 $f(u_nu_1) = f(u'_nu'_1) = 1.$

Assuming $\chi_{avsr}(C_n \uparrow P_2 \uparrow C_n) = 4$, let $f(u_n u_1) = 4$ or $f(u_n u_1) = 4$. It is also known that the structure of $C_n \uparrow P_2 \uparrow C_n$ graph, when $f(u_n u_1) = 4$ or $f(u_n u_1) = 4$, there is at least one 2-degree vertex whose chromatic number sum are different from the rest vertices of the same degree. Then this would not meet the definition of AVSREC and contradict the assumption. Therefore, according to the Lemma 1, when $n \equiv 1 \pmod{2}, \chi_{avsr}(C_n \uparrow P_2 \uparrow C_n) = 3$.

When $n \equiv 0 \pmod{2}$, the graph $C_n \uparrow P_2 \uparrow C_n$ satisfies f coloring, f: (A i = 0 (mod 2))

$$f(u_{i}u_{i+1}) = \begin{cases} 4, i \equiv 0 \pmod{2}; i = 1, 2, ..., n - 1; \\ 1, i \equiv 1 \pmod{2}. \end{cases}$$

$$f(u_{i}u_{i+1}) = \begin{cases} 3, i \equiv 0 \pmod{2}; i = 1, 2, ..., n - 1; \\ 2, i \equiv 1 \pmod{2}. \end{cases}$$

$$f(u_{n}u_{1}) = 4, f(u_{n}u_{1}) = 3, f(u_{1}u_{1}) = 5. \end{cases}$$

Assuming $\chi'_{avsr}(C_n \uparrow P_2 \uparrow C_n) = 6$, let $f(u_n u_1) = 6$ or $f(u'_n u'_1) = 6$. According to the structure of the $C_n \uparrow P_2 \uparrow C_n$ graph, if $f(u_n u_1) = 6$ or $f(u'_n u'_1) = 6$, there is at least one 2-degree vertex whose chromatic number sum is different from the rest vertices of the same degree. Then this would not meet the definition of AVSREC and contradict the assumption. Therefore, according to the Lemma 1, when $n \equiv 0 \pmod{2}$, $\chi'_{avsr}(C_n \uparrow P_2 \uparrow C_n) = 5$.

Theorem 9: For the joint graph $C_n \uparrow P_2 \uparrow S_m$ $(n \ge 3, m \ge 2)$, there is:

$$\chi'_{avsr}(C_n \uparrow P_2 \uparrow S_m) = \begin{cases} 5, n \equiv 0 \pmod{2}, m = 2\\ 3, n \equiv 1 \pmod{2}, m = 2\\ \Delta + 2, m \ge 3 \end{cases}$$

Proof: When $n \equiv 0 \pmod{2}$, m = 2, the graph $C_n \uparrow P_2 \uparrow S_2$ meets Adjacent Vertices Sum Reducible Edge Coloring, the coloring results are shown in the Figure 11(a).

When $n \equiv 1 \pmod{2}$, m = 2, the graph $C_n \uparrow P_2 \uparrow S_2$ meets Adjacent Vertex Sum Reducible Edge Coloring, the coloring results are shown in the Figure 11(b).

Figure 11 shows the partial coloring result graphs of the $C_n \uparrow P_2 \uparrow S_2$ graph.



When $m \ge 3$, suppose the vertices set of C_n is $V = \{u_1, u_2, ..., u_n\}$, the vertices set of S_m is $V = \{v_0, v_1, ..., v_m\}$, the vertices set of P_2 is $V = \{v_1, v_2\}$, where $u_1 = v_1, v_0 = v_2$.

The graph
$$C_n \uparrow P_2 \uparrow S_m$$
 satisfies AVSREC, f:

$$f(u_i u_{i+1}) = \begin{cases} 2, i \equiv 0 \pmod{2}, i = 1, 2, ..., n - 1; \\ 1, i \equiv 1 \pmod{2}. \end{cases}$$

$$f(u_1 v_0) = 3, f(v_0 v_i) = i + 3, i = 1, 2, ..., m.$$

Prove $\chi_{avsr}(C_n \uparrow P_2 \uparrow S_m) \ge \Delta + 2$.

Assuming $\chi_{avsr}(C_n \uparrow P_2 \uparrow S_m) = \Delta + 3$, let $f(v_0v_i) = \Delta + 3$. According to the structure of $C_n \uparrow P_2 \uparrow S_m$, if $f(v_0v_i) = \Delta + 3$, the chromatic number $\Delta + 2$ is missing in the graph. According to the definition of AVSREC, the chromatic number is not continuous, so the hypothesis does not hold. According to the Lemma 1, $\chi_{avsr}(C_n \uparrow P_2 \uparrow S_m) = \Delta + 2$.

Theorem 10: For the joint graph $B_m^n (m \ge 2, n \ge 4, n \equiv 0 \pmod{2})$, there is

$$\chi_{avsr}(B_m^n) = 3m - 1$$

joint graphs B_6^4 , B_m^4 are shown in Figure 12(a), 12(b)



Fig.12. B_6^4 and B_m^4 graphs

Proof: The joint graph
$$B_m^n$$
 satisfies the AVSREC, f:
 $f(v_i^2 v_{i+1}^1) = 1 \ (i = 1, 2, ..., m - 1)$
 $f(v_i^1 v_i^2) = m + i + 1, (i = 1, 2, ..., m)$
 $f(v_i^n v_i^1) = m - i + 2, (i = 1, 2, ..., m)$
 $f(v_i^j v_i^{j+1}) = \begin{cases} m - i + 2, j \equiv 0 \ (j \mod 2); \\ 2m + i, j \equiv 1 \ (j \mod 2) \end{cases}$
 $(i = 2, 3, ..., m - 1; j = 2, 3, ..., n - 1);$
 $f(v_1^j v_1^{j+1}) \begin{cases} f(v_1^1 v_1^2) - 1, j \equiv 0 \ (j \mod 2); \\ f(v_1^1 v_1^2), j \equiv 1 \ (j \mod 2) \end{cases}$
 $(j = 2, 3, ..., n - 1)$
 $f(v_m^j v_m^{j+1}) = \begin{cases} 2, j \equiv 0 \ (j \mod 2); \\ f(v_m^1 v_m^2), j \equiv 1 \ (j \mod 2); \end{cases}$
 $f(v_m^m v_m^{j+1}) = \begin{cases} 2, j \equiv 0 \ (j \mod 2); \\ f(v_m^1 v_m^2), j \equiv 1 \ (j \mod 2) \end{cases}$
 $(i = 2, 3, ..., n - 1)$

Prove that $\chi_{avsr}(B_m^n) \ge 3m - 1$. Assuming that $\chi_{avsr}(B_m^n) = 3m$, that $f(v_{m-1}^j v_{m-1}^{j+1}) = 3m(j \equiv 1(j \mod 2))$ satisfies the chromatic number of adjacent vertices of the same degree is equal, but according to the definition of AVSREC, the chromatic number is not continuous, therefore the assumption is not true. So, the maximum chromatic number of the B_m^n graph is $\chi_{avsr}(B_m^n) = 3m - 1$.

Theorem 11: For the tree graph $T_{(n)}(3 \le n \le 19)$, there is $\chi'_{avsr}(T_{(n)}) = n - 1$

Table 1 shows the coloring results of the tree graph within n vertices $(3 \le n \le 19)$.

Table.1 Tree graph results within n vertices $(3 \le n \le 19)$									
G(p,q)	Number Chromatic Number K								
		K=2	K=3	K	=4	K=	5	K=6	K=7
(3,2)	1	1							
(4,3)	2	1	1						
(5,4)	3	1	0	2					
(6,5)	6	1	0	1		4			
(7,6)	11	1	0	1		2		7	
(8,7)	23	1	0	1		4		4	13
(9,8)	47	1	3	2		5		5	6
(10,9)	106	1	7	3		9	12		12
(11,10)	235	1	1	3		10)	16	31
(12,11)	551	1	2	4		15		29	55
(13,12)	1301	2	5	6		20)	43	101
(14,13)	3159	6	13	17		34		71	165
(15,14)	7741	14	33	27		59		94	315
(16,15)	19320	30	67	89		112	2	176	495
(17,16)	48629	99	157	178		262		371	874
(18,17)	123867	160	342	497		60	1	939	1616
(19,18)	317955	384	847	1238		1592		2081	3454
G(p,q)	Number	Chromatic Number K							
		K=8	K=9	K	=10	K=11		K=12	K=13
(9,8)	47	25							
(10,9)	106	11	51						
(11,10)	235	30	38	105					
(12,11)	551	82	84	94		18	5		
(13,12)	1301	150	181	195		19:	5	403	
(14,13)	3159	303	404	443		43	1	408	864
(15,14)	7741	545	773	1060		102	8	961	902
(16,15)	19320	1060	1619	2113		259	7	2483	2152
(17,16)	48629	1895	3308	40	550	552	4	6388	5649
(18,17)	123867	3392	6671	- 99	947	1278	88	14186	15502
(19,18)	317955	6317	13043	20	975	29121		33276	36395
G(p,q)	Number	Chromatic Number K							
		K=14	K=1	5	K=16		K=17		K=18
(15,14)	7741	1930							
(16,15)	19320	2008	4319						
(17,16)	48629	5036	4480		9768				
(18,17)	123867	13399	11374	4	10043		2	2410	
(19,18)	317955	36980	30 3124		26550		2	2733	51728

From the data in the table 1, we can get the proportion of the graphs in the tree graph where the number of stains is equal to the number of edges. As shown in Figure 13, we can see that the proportion of the number of tree graphs within 19 points equal to the number of edges under the corresponding number of points becomes smaller and smaller as the number of points increases.



According to the results of the algorithm, it can be included

that all graphs within 19 points satisfy this theorem. However, due to the limitation of machine computing power and algorithm efficiency, the experiment of larger point tree graph was not carried out. there is the following Conjecture:

Conjecture1: For the $T_{(n)}$ $(n \ge 20)$, $\chi_{avsr}(T_{(n)}) = n - 1$. As shown in Figure 14, the partial coloring result graphs of the $T_{(n)}$ graph are given.



Fig.14. Partial coloring results graphs of the $T_{(n)}$ graph

V. CONCLUSION

In this paper, the concepts of Adjacent Vertex Sum Reducible Edge Coloring are proposed based on the existing concepts of graph coloring. By referring to the existing intelligent algorithms, a new algorithm of Adjacent Vertex Sum Reducible Edge Coloring is designed, which calculates for all non-isomorphisms graphs within 20 points, about hundreds of millions of graphs. Some theorems and guesses are given after the analysis of the calculation results.

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