Research on Dynamically Corrective Hit Probability Model of Anti-air Missile Integrated in War Game System

Jun Huang, Pengfei Wu, Xiaobao Li

Abstract—**The hit probability model is an essential performance measure for anti-air missiles, aircraft, and guided targets in different combat situations and environments. A combination of analytical and numerical fitting methods is proposed to meet the requirements of war game systems including being real-time and accurate. In this approach, a dynamically corrected hit probability model is obtained for the anti-air missiles for which the corrections are made on the distance, speed, and maneuverability correction for the aircraft target. With this method, corrections are also made on the penetrating altitude of the aircraft and guided targets, countering azimuth angle for the guided targets, and terminal maneuverability and echo or infrared signal characteristics correction for the guided targets. After that, war game case analyses show that the proposed hit probability correction method are successfully operated in real-time with a model accuracy which is 6% higher than that of existing models.**

Index Terms—**anti-air missile, hit probability, correction model, war game**

I. Introduction

Hit probability of the anti-air missiles belongs to the operational effectiveness evaluation index system of the missile weapons. From launch to detonation, an anti-air missile passes through multiple phases including initial guidance, mid-course guidance, terminal guidance, and intercepting target. There are two basic types of hit probability modeling methods including mathematical analytic, and analog methods. Most of the existing research works are focused on estimating the killing zone, launching zone of the surface-to-air missile, or the attack zone of the air-to-air missiles with the same hit probability, where data fitting methods including numerical fitting and network fitting are often used [1].

As a common mathematical analytic method, the event probability analytic method divides a hitting process into several parts. The probability of each part is then assessed to obtain the hit probability. The analytic hit probability model is usually expressed as the integral of the guidance error probability distribution density in the dispersion circle. According to the incidence of systematic errors, the guidance error follows Rayleigh or Reis probability density functions. Distribution density parameters cover expectation and mean square error of the missile miss-distance, which are obtained using the Bayes method based on the predictive and posterior data [2]–[5].

Analytic hit probability model of the air-to-air missile includes acquisition probability in the mid-course guidance as to the integral of the guidance error probability density, pursuing probability in the terminal guidance obtained by the integral of intercepting angle probability distribution density, and the target detection distance of the seeker. For the aircraft targets, the hit probability is affected by the approach angle of the target. Its highest value is in head-on and tail-on countering cases. It also has higher values in forward and backward countering cases and lower values for the beam countering situation [6].

An analog method must be established and the differential equations of the missile trajectory should be solved including mass equation, kinetic equation, kinematical equation, guidance equation, and control equation [7]. Intercepting simulations and hardware in the loop simulations are then carried out to acquire the fire tests data. Finally, based on the miss distance, the theoretical hit probability is calculated and corrected [8]. As the theoretical investigation may be carried out by anyone, this method can technically be implemented only by the manufacturer or development organization of the missile weapons. Therefore, the absolutely solvable 6 DOF differential equations for the missile trajectory are not usually reported in the public domain. There exist several methods that are used for parameter estimation and test evaluation of the hit probability, they are however not suitable for anti-air war games. Therefore, it is important to develop a dynamically corrective hit probability model of the anti-air missiles integrated into the war game system.

II. Correction Method

Aircraft target and guided targets including the aerodynamic missile, reentry vehicle, hypersonic vehicle, can be intercepted by an anti-air missile. In this paper, the hit probability correction method of the anti-air missile in the war game system is proposed including 6 sides based on the referenced hit probability of the aircraft and guided targets provided by the war game system, P_{jf} and P_{jd} respectively.

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A. Distance and Speed Correction for Aircraft Target

According to the law of hit probability distribution over hit distance [9], [10], a specific calculation method of the distance corrected hit probability for aircraft target, P_{jx} is expressed as the following

$$P_{jx}=\begin{cases}P_{jf}\cdot(1+P_f\text{-}P_R) & P_R\geq P_f\\ P_{jf} & R_{fj}/R_{fy}\leq P_R<P_f\end{cases} \tag{1}$$

where, P_f = 0.5, $P_R = R/R_{fy}$, R refers to the hit distance between the aircraft target and anti-air missile, R_{fj} and R_{fy} refer to the near boundary, and far boundary of maximum hit zone of an anti-air missile respectively. In cases where $P_R\geq P_f$ the anti-air missile has no power hence the hit probability is approximately linearly reduced by the longer hit distances.

According to the law of hit probability distribution over speed, a specific calculation method of the speed corrected hit probability for aircraft target, P_{sx} is expressed as

$$P_{sx}=\begin{cases}P_{jf}-50\% & v\geq v_{\max}\\ P_{jf}-25\% & v\geq 0.8v_{\max}\\ P_{jf}-15\% & v\geq 0.7v_{\max}\\ P_{jf}-10\% & v\geq 0.6v_{\max}\\ P_{jf}-5\% & v\geq 0.5v_{\max}\end{cases} \tag{2}$$

where, v denotes the speed of the aircraft target, and $v_{\max}$ is the maxim speed of the aircraft target.

B. Maneuverability Correction for Aircraft Target

The impact of maneuvering types and radius [11] on the hit probability has been studied in the existing methods. However, the maneuvering position and situation factors have not been considered. Noting the law of hit probability distribution over the cruising altitude [9], and using the referenced maneuverability coefficient, A, a specific calculation method of the cruising altitude corrected maneuverability coefficient for the aircraft target, A_1, is obtained as

$$A_1=\begin{cases}\max(0.5A,A\cdot(1-0.5\cdot\dfrac{H_f-3000}{H_{E\max}-H_f})) & \textit{with SM}\\ \max(0.25A,A\cdot(1-0.75\cdot\dfrac{H_f-3000}{H_{E\max}-H_f})) & \textit{without SM}\end{cases} \tag{3}$$

where $H_{E\max}$ represents the maximum working altitude of the aircraft target, H_f is the cruising altitude of the aircraft target which plays a corrective role if it is larger than or equal to 3000 m. Super-maneuverability, abbreviated as SM, combines supersonic maneuver and poststall maneuver with the distinctive feature of the 4.5th and the 5th generation fighters such as Cobra, tail slide, falling leaf, Hammerhead, Herbst, and pendulum maneuvers.

Pre-set maneuverability coefficient corrected by the airman's training level for the aircraft target, A_2, is also expressed as

$$A_2=\begin{cases}0.3\cdot A_1 & \textit{new}\\ 0.5\cdot A_1 & \textit{internship}\\ 0.8\cdot A_1 & \textit{ordinary}\\ 1.0\cdot A_1 & \textit{senior}\\ 1.2\cdot A_1 & \textit{ace}\end{cases} \tag{4}$$

Using the load and maneuverability coefficients corrected by the airman's training level for the aircraft target, f_G and A_2, a specific calculation method of load corrected maneuverability coefficient for the aircraft target, A_3, is expressed as

$$A_3=[0.4+0.6(1-f_G)]A_2 \tag{5}$$

$$\begin{aligned}f_G&=\min\left(0.99,\frac{G_c-\left(G_{\text{Empty}}+0.6\text{G}_{\text{Fuel}}\right)}{G_{\text{valid}}}\right)\\ &=\min\left(0.99,\frac{G_{\text{Payload}}+0.4G_{\text{Fuel}}}{G_{\max}-\left(G_{\text{Empty}}+0.6\text{G}_{\text{Fuel}}\right)}\right)\end{aligned} \tag{6}$$

$$G_{\text{C}}=G_{\text{Empty}}+G_{\text{Payload}}+G_{\text{Fuel}} \tag{7}$$

$$G_{\text{valid}}=G_{\max}-(G_{\text{Empty}}+0.6\cdot G_{\text{Fuel}}) \tag{8}$$

where G_{C} denotes the gross weight, G_{Empty} is the empty weight, G_{Payload} denotes the payload weight, G_{Fuel} is the fuel weight, $G_{\max}$ denotes the maxim load weight, and G_{valid} is the valid load weight.

Using the damage scale coefficient and load corrected maneuverability coefficient for the aircraft target, f_D and A_3, a specific calculation method of the damage corrected maneuverability coefficient for the aircraft target, A_4, is also expressed as the following

$$A_4=(1-f_D)A_3 \tag{9}$$

Maneuverability correction by countering azimuth angle for the aircraft target is illustrated in Fig. 1, where T is the center of the circle representing the aircraft target, M denotes the anti-air missile, $\overrightarrow{TM}$ is the relative position vector from T to M, and $\vec{V}_T$ is the velocity vector of the aircraft target in the direction of 0 degree. In this illustration, the clockwise direction is considered positive rotation, and countering azimuth angle for aircraft target, q, denotes the intersection angle between $\vec{V}_T$ and $\overrightarrow{TM}$.

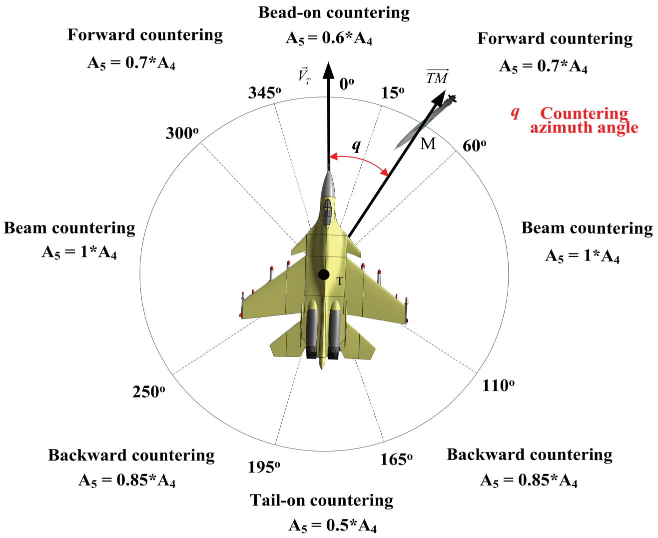


Fig. 1 Maneuverability correction by countering the azimuth angle of the aircraft target.

Noting the law of hit probability distribution over countering azimuth angle, using the damage corrected maneuverability coefficient for aircraft target, A_4, a specific calculation method of maneuverability coefficient corrected by countering azimuth angle for aircraft target, A_5, is also expressed as

$$A_5=\begin{cases}0.6\cdot A_4 & q\in[-15^\circ,15^\circ]\\ 0.7\cdot A_4 & q\in(15^\circ,60^\circ)\cup(300^\circ,345^\circ)\\ 1\cdot A_4 & q\in[60^\circ,110^\circ]\cup[250^\circ,300^\circ]\\ 0.85\cdot A_4 & q\in(110^\circ,165^\circ)\cup(195^\circ,250^\circ)\\ 0.5\cdot A_4 & q\in[165^\circ,195^\circ]\end{cases}\tag{10}$$

Similarly, using the maneuverability coefficient corrected by countering the azimuth angle of the aircraft target, i.e., A_5, the maneuverability corrected hit probability for aircraft target, P_{jfx}, is expressed as

$$P_{jfx}=P_{jf}-10\%\cdot A_5\tag{11}$$

C. Penetrating Altitude Correction for the Aircraft and Guided Targets

If the anti-air missile cannot intercept the skimming targets, targets at lower altitudes have a lower hit probability. According to the law of hit probability distribution over penetrating altitude, and using the penetrating altitude and referenced hit probability for aircraft target, H_{tf} and P_{jf}, a specific calculation method of the penetrating altitude corrected hit probability for aircraft target, P_{hfx}, is expressed as

$$P_{hfx}=\begin{cases}P_{jf} & H_{tf}\ge 91.44m\\ P_{jf}-5\% & 60.96m\le H_{tf}<91.44m\\ P_{jf}-15\% & 30.48m\le H_{tf}<60.96m\\ P_{jf}-30\% & 0\le H_{tf}<30.48m\end{cases}\tag{12}$$

Similarly, using the penetrating altitude and referenced hit probability for guided target, H_{td} and P_{jd}, a specific calculation method of the penetrating altitude corrected hit probability for guided target, P_{hdx}, is expressed as

$$P_{hdx}=\begin{cases}P_{jd} & H_{td}\ge 91.44m\\ P_{jd}-5\% & 60.96m\le H_{td}<91.44m\\ P_{jd}-15\% & 30.48m\le H_{td}<60.96m\\ P_{jd}-30\% & 0\le H_{td}<30.48m\end{cases}\tag{13}$$

D. Countering Azimuth Angle Correction for Guided Target

Fig. 2 illustrates the maneuverability correction by countering azimuth angle for a guided target, where, *T* is the center of the circle represents the guided target, *M* denotes the anti-air missile, $\overline{TM}$ represents the relative position vector from T to M, $\vec{V}_T$ represents the velocity vector of the guided target in the direction of 0 degree. In this illustration, the clockwise direction is considered as positive rotation, and the countering azimuth angle for guided target, *q*, denotes the intersection angle between $\vec{V}_T$ and $\overline{TM}$.

According to the law of hit probability distribution over the countering azimuth angle for the guided target, *q*, a specific calculation method of hit probability corrected by countering azimuth angle for guided target, P_{adx} is expressed as the following, and shown in Fig. 2

$$P_{adx}=\begin{cases}P_{jd}(1.0-0.5/90\cdot q) & 0^\circ\le q\le 90^\circ\\ P_{jd}[1.0+0.5/90\cdot(q-360)] & 270^\circ\le q<360^\circ\end{cases}\tag{14}$$

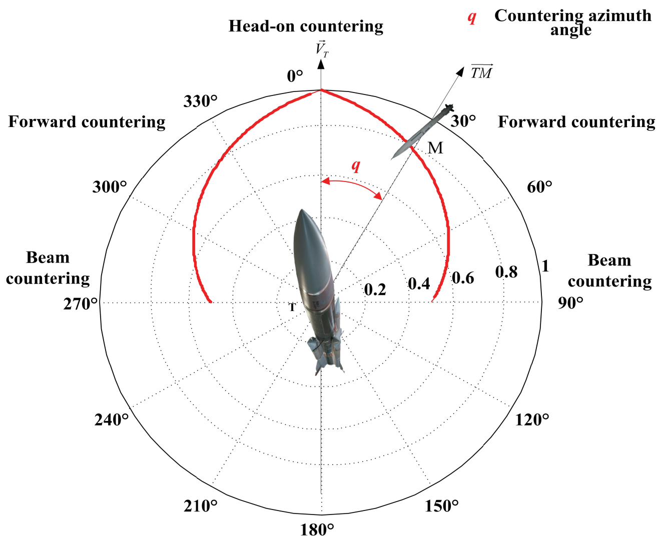


Fig. 2 Hit probability correction by countering the azimuth angle for the guided target.

E. Terminal Maneuverability Correction for the Guided Target

The guided target's zoom-and-dive maneuver can be considered as a sinusoidal maneuver with 1/2 or 1 cycle. The target's zoom-and-dive maneuver is shown in Fig. 3. In Fig. 3, *Oxyz* is earth-rectangular coordinate, *v* is the linear speed of target along the direction of the *x*-axis, H_0 represents the initial height before the zoom-and-dive maneuver and final height after the zoom-and-dive maneuver, Z_0 represents the yawing distance of the guided target, and R represents the amplitude of zoom-and-dive maneuver in the vertical plane. Along the x-axis direction between x_1 and x_2, the trajectory equation of the target's zoom-and-dive maneuver is

$$\begin{cases} x = x_1 + vt & 0 \le t \le \dfrac{x_2 - x_1}{v} \\ y = \mathrm{H}_0 - \mathrm{sign}(\sin(\xi_0))R & \\ \quad + R\sin(\xi_0 + k_y \pi \dfrac{vt}{x_2 - x_1}) & \\ z = z_0 \qquad \xi_0 = 0, k_y = 1; \xi_0 = -\pi/2, k_y = 2 & \end{cases} \tag{15}$$

$$\mathrm{sign}(x) = \begin{cases} 1 & x > 0 \\ 0 & x = 0 \\ -1 & x < 0 \end{cases}$$

where ξ_0 denotes the initial phase angle of the zoom-and-dive maneuver (0 or -π/2). In the above, $k_y = 1$ and $k_y = 2$ indicate that the periodic numbers of the zoom-and-dive maneuver parallel to xOy plane are 1/2 and 1, respectively.

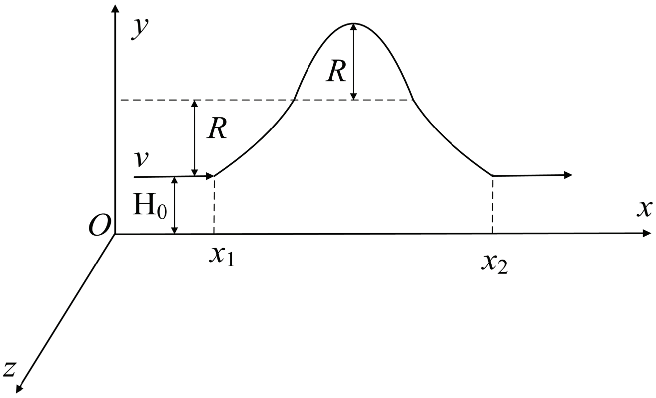


Fig.3 Target's zoom-and-dive maneuver trajectory.

The guided target's S maneuvers in the vertical and horizontal planes can be also considered as sinusoidal maneuvers with multiple cycles. The target's S maneuver in the vertical plane is shown in Fig. 4, where Oxyz is the earth-rectangular coordinate, v is the linear speed of the target along the direction of the x-axis. Similarly, here H_0 represents the initial height before S maneuver and the final height after S maneuver, z_0 represents the yawing distance of the guided target, and R is the amplitude of S maneuver in the vertical plane. Along the x-axis direction between x_1 and x_2, the trajectory equation of the target's S maneuver in the vertical plane is

$$\begin{cases} x = x_1 + vt & 0 \le t \le \dfrac{x_2 - x_1}{v} \\ y = \mathrm{H}_0 - sign(\sin(\xi_0))\left|\sin(\xi_0)\right| R & \\ \quad + R\sin(\xi_0 + k_y \pi \dfrac{vt}{x_2 - x_1}) & \\ z = z_0 & -\pi < \xi_0 \le \pi \end{cases} \tag{16}$$

where ξ_0 is the initial phase angle of S maneuver in the vertical plane, $k_y = 2k$ (k=1,2,…) sets the periodic number of sinusoidal motions parallel to the xOy plane as k .

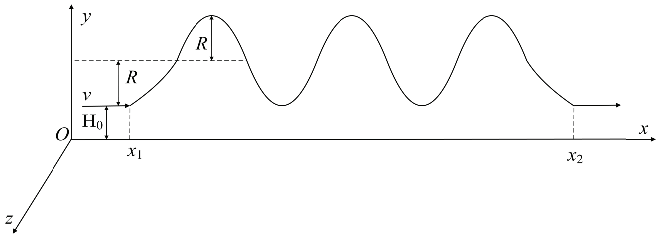


Fig.4 Target's S maneuver trajectory.

Along the direction of the x-axis between x_1 and x_2, the trajectory equation of the target's S maneuver in the horizontal plane is expressed as

$$\begin{cases} x = x_1 + vt & 0 \le t \le \dfrac{x_2 - x_1}{v} \\ y = \mathrm{H}_0 & -\pi < \xi_0 \le \pi \\ z = z_0 - sign(\cos(\xi_0))\left|\cos(\xi_0)\right| R & \\ \quad + R\cos(\xi_0 + k_z \pi \dfrac{vt}{x_2 - x_1}) & \end{cases} \tag{17}$$

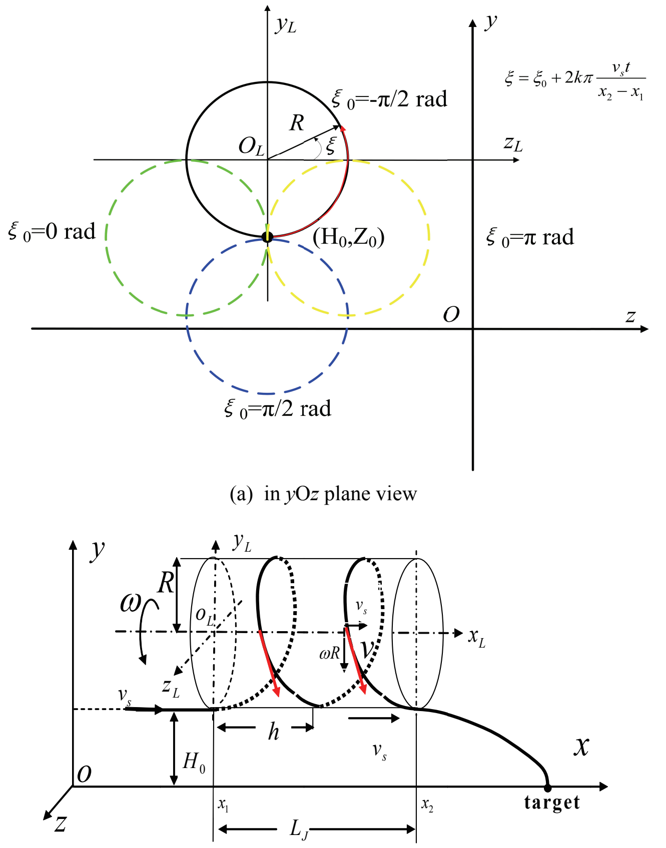
where ξ_0 is the initial phase angle of S maneuver in the horizontal plane, H_0 represents the flight height of the guided target, z_0 represents the initial yawing distance before S maneuver and final yawing distance after S maneuver, and R represents the amplitude of S maneuver in the horizontal plane. Here, $k_z = 2k$ (k=1,2,…) sets the periodic number of sinusoidal motion parallel to xOz plane as k .

The target's spiral maneuver trajectory is shown in Fig. 5 with yOz plane view and 3-dimensional view, where Oxyz is earth-rectangular coordinate, x_L represents the spiral axis line of target spiral maneuver, $\mathrm{O_L}$ is the starting point of target spiral maneuver on spiral axis line, z_L and y_L represent coordinate axes parallel to coordinate axes z and y with origin $\mathrm{O_L}$, respectively. Also v_s is the linear speed of target along the direction of the x_L axis, ω denotes the angular speed of target around the x_L axis, and v is the resultant velocity of target with constant value. In Fig. 5, L_J also denotes the displacement of the target along the direction of x_L axis between x_1 and x_2, h is the screw pitch, R represents the radius of the spiral, H_0 represents the initial height before the spiral maneuver and final height after spiral maneuver, and z_0 represents the initial yawing distance before spiral maneuver and final yawing distance after spiral maneuver. Here, ξ_0 is the initial phase angle of spiral maneuver. In Fig.(a), the upper solid circle represents the spiral maneuver trajectory from yOz plane view when ξ_0=-π/2, and the left, lower and right dotted circles refer to the spiral maneuver trajectory from yOz plane view when ξ_0=0, π/2 and π. $k_z = k_y = 2k$ (k=1,2,…), k_z is the periodic number of circular motion parallel to xOz plane as k , and k_y sets the periodic number of sinusoidal motion parallel to xOy plane as k . Along the x-axis direction between x_1 and x_2, the trajectory equation of target's spiral maneuver trajectory is

$$\begin{cases} x = x_1 + v_s t & 0 \le t \le \dfrac{2k\pi}{\omega} = \dfrac{x_2 - x_1}{v_s} \\ y = H_0 - sign(\sin(\xi_0))\left|\sin(\xi_0)\right| R & \\ \quad + R\sin(\xi_0 + 2k\pi \dfrac{v_s t}{x_2 - x_1}) & -\pi < \xi_0 \le \pi \\ z = Z_0 + sign(\cos(\xi_0))\left|\cos(\xi_0)\right| R & \\ \quad + R\cos(\xi_0 + 2k\pi \dfrac{v_s t}{x_2 - x_1}) & \end{cases} \tag{18}$$

According to the law of hit probability distribution over the capability for the guided target of the typical maneuver [9], [12] (e.g., spiral, S, and zoom-and-dive maneuvers) and combing with the horizontal distance from the predicted impact point to the launch position of the anti-air missile, R_y, a specific calculation method of the terminal maneuverability corrected hit probability for guided target, P_{jdx} is expressed as

$$P_{jdx} = \begin{cases} 0.5 \cdot P_{jd} & 9km \le R_y \le 21km, spiral\ maneuver \\ 0.66 \cdot P_{jd} & 9km \le R_y \le 21km, S\ maneuver \\ 0.75 \cdot P_{jd} & R_y \le 4.5km, zoom-and-dive\ maneuver \end{cases} \tag{19}$$



(a) in yOz plane view

(b) in 3-dimensional view

Fig. 5 Target's spiral maneuver trajectory.

F. Signal Characteristics Correction for the Guided Target

If an anti-air missile is terminally guided by radar, according to the law of hit probability distribution over RCS of the guided target [13], which decreases the detective range and hit probability distribution, a specific calculation method of echo signal characteristics corrected hit probability for guided target, P_{xdx} is

$$P_{xdx} = \begin{cases} P_{jd} - 20\% & RCS \le 0.01m^2 \\ P_{jd} - 15\% & 0.01m^2 < RCS \le 0.1m^2 \\ P_{jd} - 10\% & 0.1m^2 < RCS \le 1m^2 \\ P_{jd} & RCS > 1m^2 \end{cases} \tag{20}$$

If the anti-air missile is in infrared terminal guidance, based on the detective range of infrared seeker for guided target, R_D, a specific calculation method of infrared signal characteristics corrected hit probability for the guided target, P_{xdx} is expressed as

$$P_{xdx} = \begin{cases} P_{jd} - 20\% & R_D \le 0.25nm \\ P_{jd} - 15\% & 0.25nm < R_D \le 0.5nm \\ P_{jd} - 10\% & 0.5nm < R_D \le 1nm \\ P_{jd} & R_D > 1nm \end{cases} \tag{21}$$

III. War Game Case Analyses

The designed hit probability correction model of the anti-air missile can successfully be carried out in real-time [14] whose accuracy is improved by 6% taking the corresponding model as the referenced standard in the intelligent joint war-game system [15], [16] such as COMMAND Modern Operations based on the HLA architecture and EADSIM [17], [18].

A. A Case for the Aircraft Target

Here, P_{jx}, P_{sx} and P_{jfx} are calculated respectively using the A and B methods. Also, P_{hfx} is obtained using the method C. Then, comprehensively corrected hit probability is acquired as follows.

(1) Distance Correction

Given that the referenced hit probability of a certain type of anti-air missile is 90%, the far boundary of the maximum hit zone of the anti-air missile is 92.6 km, the launch position of the anti-air missile is at 9.581 degrees north latitude, and 112.809 degrees east longitude. The predicted hit position of the aircraft target is at 9.736 north latitude and 112.277 degrees east longitude, and the hit distance is 60.85 km and more than one second of the far boundary of the maximum hit zone. This means that the distance correction of the hit probability is feasible and using (1) P_{jx} = 75.85%.

(2) Speed Correction

The speed of the aircraft target is 1179.724 km/h, which is equal to or more than 60% of the maxim speed, i.e., 1713.1 km/h. Therefore, using (2) P_{sx} = 65.85%.

(3) Maneuverability Correction

1) Maneuverability Coefficient Corrected by the Cruising Altitude

The referenced maneuverability coefficient of the Su-27SK fighter is 4.5, with a cruising altitude of 24.384 m which is lower than 3000 m. This means that the maneuverability coefficient remains 4.5, hence altitude correction is not required.

2) Maneuverability Coefficient Corrected by the Airman's Training Level

Because of the airman's ordinary training level, using (4), the maneuverability coefficient is obtained as 3.6.

3) Load Corrected Maneuverability Coefficient

Given that the load coefficient is 0.32, using (5) the load corrected maneuverability coefficient for aircraft target is 2.92, i.e., $A_3 = 2.92$.

4) Damage Corrected Maneuverability Coefficient

Given that the damage scale coefficient is 0, using (9) the damage corrected maneuverability coefficient is 2.92, i.e., $A_4 = 2.92$.

5) Countering Azimuth Angle Corrected Maneuverability Coefficient

Given that the countering azimuth angle for aircraft target is 126 degrees, using (10) the maneuverability coefficient corrected by countering azimuth angle for aircraft target is 2.5, i.e., $A_5 = 2.5$.

6) Comprehensive Maneuverability Correction

Using (11), the maneuverability corrected hit probability for aircraft target is 41%, i.e., $P_{jfx} = 41\%$.

(4) Penetrating Altitude Correction

Because this type of anti-air missile can intercept the skimming targets, the corrective condition of (12) is not satisfied. Hence, the hit probability for the aircraft target remains 41%.

(5) Comprehensive Correction

In the process of countering the aircraft, the war game system outputs message is:

A certain type of anti-air missile with the referenced hit probability of 90%, is intercepting the Su-27SK fighter, the hit probability was corrected respectively by the distance and speed is equal to 66%, the comprehensive maneuverability correction is also -25%, so the final hit probability is 41%.

B. *A Case for the Guided Target*

Here, P_{hdx} is calculated using the method C. P_{adx} is also obtained by using the method D. Similarly, P_{jdx} and P_{xdx} are gained using the method E and F. Finally, the comprehensively corrected hit probability is acquired as follows.

(1) Penetrating Altitude Correction

Since this anti-air missile can intercept the skimming targets, the corrective condition of (13) is not met.

(2) Countering Azimuth Angle Correction

The referenced hit probability of this type of anti-air missile is 80% and the countering azimuth angle for aircraft target is 0 degree. Therefore, using (14) the hit probability corrected by countering azimuth angle for the guided target is 80%, i.e., $P_{adx} = 80\%$.

(3) Terminal Maneuverability Correction

Given that AS-18 anti-ship missile penetrates in zoom-and-dive maneuver, the horizontal distance from the predicted impact point to the launch position of the anti-air missile is 13.85264 nm, i.e., R_y = 13.85264 nm, which is longer than 4.5 km. Therefore the corrective condition of (19) is not satisfied.

(4) Signal Characteristics Correction

This type of anti-air missile is terminally guided by semi-active radar and RCS is 0.12 m^2. Therefore, using (20) the hit probability for guided target corrected by echo signal characteristics is 70%, i.e., $P_{xdx} = 70\%$.

(5) Comprehensive Correction

In this process of countering guided missiles, the war game system outputs message is:

A certain type of anti-air missile with the referenced hit probability is 80% is intercepting an AS-18 anti-ship missile. With no need for correction by the penetrating altitude, terminal maneuverability, and countering azimuth angle, the signal characteristics correction is -10%, hence the final hit probability is 70%.

IV. CONCLUSION

In this paper, combing analytical and numerical fitting methods, a dynamically corrective hit probability model of the anti-air missile for aircraft target and guided target based on the war game system is proposed. The proposed method corrects the distance, speed, and maneuverability of aircraft targets. Corrections are also made to the penetrating altitude correction for the aircraft and guided targets, countering azimuth angle, terminal maneuverability, and echo or infrared signal characteristics correction for the guided target. Using this method enables to meet the requirements of real-time and accuracy in war game systems and provides theoretical-practical values to the weapon system application engineering.

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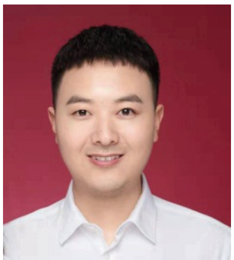

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