Assessment of Water Quality in Titicaca Lake, Peru Using the Grey Clustering Method

Alexi Delgado, Andy Cotrina, Jose Cueto, Rosmely Figueroa, Jhony Huarcaya, Hugo Lopez and Laberiano Andrade-Arenas

Abstract—**Due to the effect of pollution in water bodies such as lakes, lagoons, rivers, among others, the application of methodologies, to assess water quality, lead the analysis of the various components and parameters from altered environmental. In this way, the grey clustering method is a good alternative to evaluate the quality of water, considering that it simplifies a complex system; it groups the same types of factors in their individual categories without losing information. This study evaluated the water quality of the aquatic ecosystem in the Titicaca Lake watershed (Peruvian sector), by the results of the monitoring of water quality in Titicaca Lake, carried out by the National Water Authority (ANA) from Peru. According to the monitoring points, they were divided into four study areas and the parameters studied were chlorophyll content (Chla), total nitrogen (NT), total phosphorus (PT) and chemical oxygen demand (COD). The results of the evaluation showed that 45% of the monitoring points of the Titicaca Lake watershed present an upper mesotrophic level, classified as an acceptable water quality and 55% present a eutrophic level, classified as a poor water quality. This means that the degree of water pollution in the Titicaca Lake watershed is high because there are discharge of sewage and solid waste without any type of treatment. Finally, the results obtained could be of great help to environmental NGOs, local and regional authorities, to improve the environmental management programs in the Titicaca Lake watershed.**

Index Terms—**eutrophication-levels, grey-clustering-method, Titicaca-Lake-watershed, water-quality**

I. Introduction

HUMAN activity modifies the characteristics and quality of water, according to the use assigned to the water, generating sewage. Wastewater is frequently discharged into sewers or natural water bodies [1]. Generally, developed countries treat around 70% of the municipal and industrial wastewater they generate. This percentage falls to 38% in upper-middle-income countries and 28% in lower-middle-income countries [2]. Peru is the ninth country with the largest amount of fresh water [3] formed by 1134 courses of water between rivers and streams [4]. Wastewater contains a high load of nutrients. Nutrient loads remain one of the most common forms of water pollution, and most nutrient emissions originate from agriculture [5].

The theory of the grey system focuses on the analysis of cases with incomplete information [6]. This advantage determines its wide range of applicability [7]. This theory was proposed by Deng Julong in 1982 [8]. Also, it is a branch of science that can be used to manage uncertainty [9][10]. This methodology solves multiple problems in decision making [11]. The grey clustering method, which is based on grey systems theory, can be used by the grey whitenization weight functions [12]. In this work we apply the method of the central point of the triangular whitening weight functions (CTWF) [13], because normally for people the central points of grey classes tend to be safer than other points in the range of the grey class, as evidenced by studies on water management [14]. Therefore, the results could be more reliable [12].

The case study will be carried out taking into account the monitoring of water quality in Titicaca Lake (Peruvian sector), the monitoring points were carried out in a network made up of forty (40) points, of which eleven (11) belong to the Inland Puno Bay (BIP), nine (09) to the Puno Bay, sixteen (16) to the greater lake in the Peruvian sector and four (04) to the Wiñaymarca lake in the Peruvian sector [15]. The Titicaca Lake is a natural body of water that receives all the pollutant load contained in domestic, municipal, and industrial wastewater, as well as those generated by illegal and informal mining activities in the area. The generating source of domestic and municipal wastewater is represented by the 63 main urban centers, whose population ranges between 1,128 and 268,018 inhabitants, in this case corresponding to the urban centers of Cojata and Juliaca, respectively [14].

The objective of the research work is to apply the grey clustering method to the 40 water quality monitoring points located in the Titicaca Lake watershed - Peruvian sector, which were distributed in 4 sectors, including: Bahía Interior de Puno, Bahía of Puno, Major Lake - Peruvian Sector and Huiñaymarca Lake - Peruvian Sector [15], using the criteria, according to the Zhalong wetland eutrophication classification standards [16].

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In the present work, Section II presents antecedents of similar studies. In Section III, details of the grey clustering method are described. In Section IV the description of the case study is presented, followed by the results and the discussion in Section V. Finally, the conclusions are presented in Section VI.

II. Literature Review

According to the report carried out by the National Water Authority (ANA) from Peru, in 2014, which a monitoring was developed, being the criteria taken into account for the evaluation of water quality, in the case of the Peruvian sector they have been the values of the physical, chemical and microbiological parameters established in Category 4: Conservation of the Aquatic Environment, subcategory: lagoons and lakes, of the National Environmental Quality Standards for Water, established in Supreme Decree No. 002-2008- MINAM, Peru. The parameters analyzed were Thermotolerant Coliforms, BOD5, Total Suspended Solids, Total Nitrogen, Chemical Oxygen Demand (COD), Ammonia Nitrogen, Nitrates, Chlorophyll, Phosphates, Total Metals Run (Al, As, Ba, B, Be, Bi, Ca, Cd, Cr, Cu, Co, Fe, K, Li, Mg, Mn, Mo, Na, Ni, P, Pb, Sb, Se, Sn, Sr, Zn, etc.) and Mercury. In this work, the following analysis parameters such as Chlorophyll, Total Phosphorus, Total Nitrogen and COD were considered [15].

Beltrán, Palomino, Moreno, Peralta, in 2015, the aquatic quality of the interior bay of Puno was evaluated, for this reason 12 random sampling stations were established, recording data monthly between December 2010 and April 2011. Parameters such as Temperature, dissolved oxygen, pH, phosphates, nitrates, and nitrites of the water indicate that the area near the exit of the stabilization lagoon of the city (Espinar Island), which is a critical zone of contamination in the interior bay of Puno. In addition, the main activities carried out in the interior bay are fishing, extraction of reeds, transit of boats for tourism, commerce, and livestock [17].

Belizario, Capacoila, Huaquisto, Cornejo and Chui, in 2019, measured the content of phosphorus, arsenic, aluminum, iron and manganese in the surface waters of the Coata River, a tributary of Titicaca Lake, the measurements were made in the dry season and avenue, taking five strategic points of the Coata river, for the determination of the elements they used inductive coupling plasma (ICP) that consists of an ionization source, and an optical emission spectrophotometer (OES). Performed by atomic emission spectrometry. Obtaining as a result maximum concentrations of aluminum 1,043 mg / L, iron 0.856 mg / L, manganese 0.460 mg / L, arsenic 0.029 mg / L and phosphorus 10.287 mg / L, which exceed the permissible limits established in the Environmental Quality Standards of the Ministry of the Environment of Peru. They concluded that the contamination in the tributary of Titicaca Lake is due to discharges of sewage and solid waste discharges without any type of treatment from the city of Juliaca, Puno, Peru [18].

In addition, according to study from S. Liu, in 2011 they explain that the theory of grey systems was born in a scientific background and that the amazing progress of this theory of grey systems has achieved in the world of learning a wide reach across the spectrum of science. The characteristics of the undetermined systems include incomplete information and inaccuracies in the data. The authors also compare grey systems with other types of uncertainty models, such as stochastic probability, approximate set theory, and fuzzy mathematics. The fundamental characteristic of uncertain systems is the incompleteness and inadequacy of their information due to the dynamics of the evolutions of the system, the biological limitations of human sensory organs, and the limitations of the relevant economic conditions and technological availability. Accurate models suffer from inaccuracies when the information available is incomplete and the data collected inaccurate, any quest to pursue accurate models is generally meaningless [12].

Finally, the studies developed in the Titicaca Lake watershed, and the methodologies used to evaluate the quality of the water, they provide results of the level of contamination caused by net presence of substances, but they do not specify or analyze the synergistic effect of the contaminants on the ecosystems; in fact, they do not provide a specific grouping of effects or alterations in a systemic evaluation, as requires an aquatic ecosystem. In this way, the present research work incorporates the grey clustering methodology, which is an innovative, objective and synergy approach that simplifies a complicated evaluation system. Therefore, it was possible to evaluate the degree of contamination of the water of the aquatic ecosystem in the Titicaca Lake watershed.

III. Methodology

The analysis was performed using the grey clustering method, which is a method that divides a series of objects according to the grey incidence matrix or the whitenization functions corresponding to predetermined classes [19]. The flow chart of the steps of the grey clustering method, applied to assess water quality, are presented in Fig. 1.

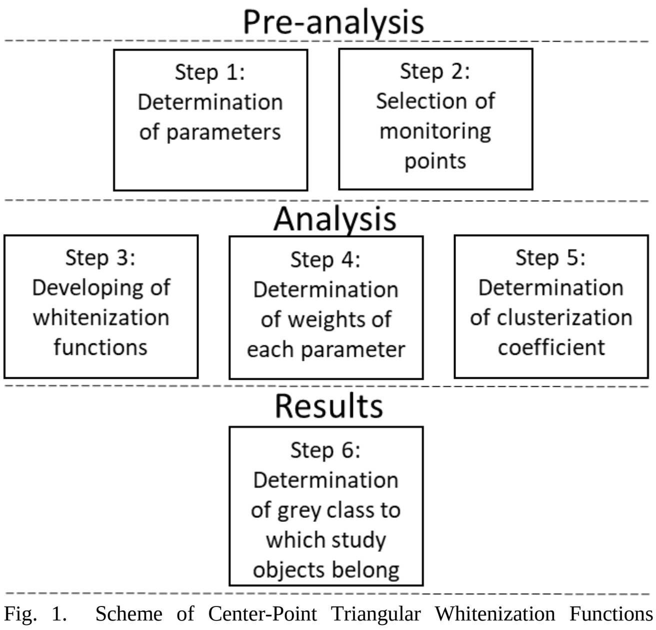

Fig. 1. Scheme of Center-Point Triangular Whitenization Functions (CTWF) model to assess water quality.

The steps of the grey clustering method are presented as follows [19]:

Step 1: Determine the parameters and their value for the analysis (C1, C2…., Cn), these parameters and values are based on the eutrophication classification standards according to Linfei Zhou and Shiguo Xu [16].

Step 2: The points to be analyzed were taken from 40 water quality monitoring points located in the Titicaca Lake watershed - Peruvian sector, which were distributed in 4 sectors, among them: Bahía Interior de Puno, Bahía de Puno, Major Lake – Peruvian sector and Huiñaymarca Lake - Peruvian Sector [15].

Step 3: The intervals of the criteria, according to the eutrophication classification standards according to Linfei Zhou and Shiguo Xu are divided into 6 grey classes and their central points (λ1, λ2, λ3, λ4, λ5 and λ6), identified in Fig. 2:

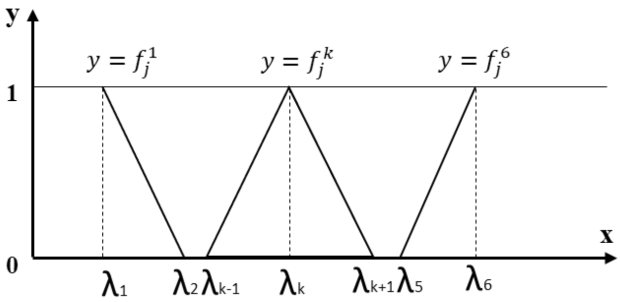

Fig. 2. Whitenization functions based on five main points.

Whitening functions are determined by f_j^k, where the monitoring points are represented and k the grey classes. They are obtained according to Equations: (1) – (3).

$$f_j^1(x_{ij}) = \begin{cases} 1, \ x \ni [0, \lambda_j^1] \\ \dfrac{\lambda_j^2 - x}{\lambda_j^2 - \lambda_j^1}, \ x \in \langle \lambda_j^1, \lambda_j^2 \rangle \\ 0, x \in [\lambda_j^2, +\infty\rangle \end{cases} \tag{1}$$

$$f_j^k(x_{ij}) = \begin{cases} \dfrac{x - \lambda_j^{k-1}}{\lambda_j^k - \lambda_j^{k-1}}, \ x \ni \langle \lambda_j^{k-1}, \lambda_j^k] \\ \dfrac{\lambda_j^{k+1} - x}{\lambda_j^{k+1} - \lambda_j^k}, \ x \in \langle \lambda_j^k, \lambda_j^{k+1} \rangle \\ 0, x \in [0, \lambda_j^{k-1}] U [\lambda_j^2, +\infty\rangle \end{cases} \tag{2}$$

$$f_j^6(x_{ij}) = \begin{cases} \dfrac{x - \lambda_j^4}{\lambda_j^5 - \lambda_j^4}, \ x \in \langle \lambda_j^4, \lambda_j^5 \rangle \\ 1, \ x \in [\lambda_j^5, +\infty\rangle \\ 0, x \in [0, \lambda_j^4] \end{cases} \tag{3}$$

Where k represents the eutrophication classification standards according to Linfei Zhou and Shiguo Xu.

Step 4: Determine the weights for each parameter, λ_j^k , where λ is the mean value taking as reference the table of the guidelines of the eutrophication classification standards according to Linfei Zhou and Shiguo Xu for each parameter, k (1,2,3,4,5,6) is the number of the whitnization function and j is the monitored point. Thus, Equation (4).

$$n_j^k = \frac{\frac{1}{\lambda_j^k}}{\sum_{j=1}^{m} \frac{1}{\lambda_j^k}} \tag{4}$$

Step 5: Calculate the clusterization coefficient for each monitoring point through Equation (5).

$$\sigma_i^k = \sum_{j=1}^{m} f_j^k (X_{ij}) * n_j \tag{5}$$

Step 6: According to the grey clustering method, the highest value of σ_i^k represents the type of classification that the analyzed point belongs it.

IV. Case Study

A. *Context Description*

The analysis of the level of eutrophication was carried out in the Titicaca Lake from Peruvian sector located in the extreme southeast of Peru, it has an area of 4,996 km2 which is showed in Fig. 3.

Fig. 3. Map of the study area - Titicaca Lake Peruvian sector

B. *Definition of object of study*

For the analysis of the level of eutrophication in the Titicaca Lake (Peruvian sector), the information was collected from 40 monitoring points obtained from the "monitoring report of the evaluation of the water quality of Titicaca Lake" carried out from March 10 to March 22. March 2014 by the National Water Authority (ANA) from Peru. This monitoring network made up of 40 points comprises the following sectors: Bahía Interior Puno, Bahía Puno, Major Lake, Peruvian sector and Minor Lake or Wiñaymarca Peruvian sector which contain 11, 09, 16 and 04 monitoring points respectively, which are represented in Fig. 4.

C. *Definition of evaluation criteria*

In the present study, the evaluation criteria to determine the water quality in the Titicaca Lake watershed ware established by the water quality parameters that cause eutrophication, that is, those that indicate the considerable presence of nutrients, in Table I, the evaluation criteria of interest are presented.

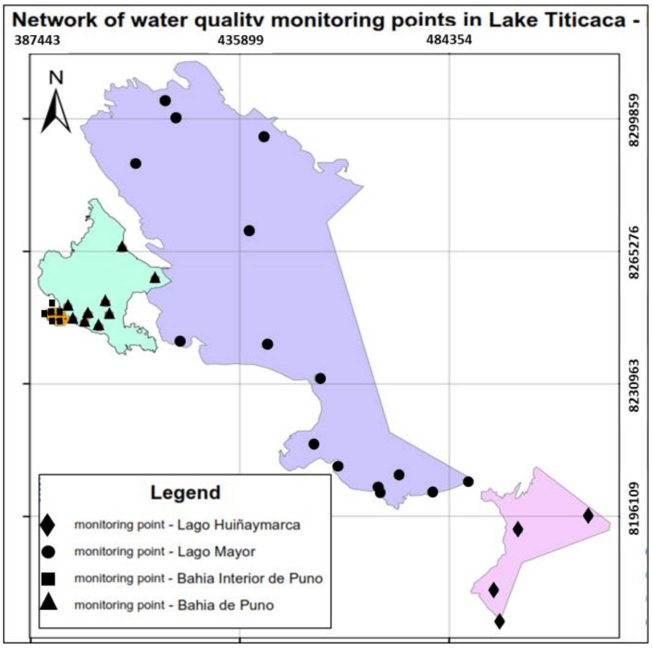

Fig. 4. Map of the water quality monitoring network according to the monitoring sectors.

TABLE I
CRITERIA FOR EVALUATING WATER QUALITY

Evaluation criteria	Unit of measurement	Notation
Chlorophyll A (Chl a)	mg/m^3	C_1
Total Phosphorus (TP)	mg/m^3	C_2
Total Nitrogen (TN)	mg/m^3	C_3
Chemical Oxygen Demand (COD_5)	mg/L	C_4

D. Definition of grey classes

The grey classes were defined based on the indicators of water quality levels, according to the levels of eutrophication in a body of water, that is, they indicate the presence of nutrients in the water; these are based according to the eutrophication classification standards of the Zhalong wetland [2], which are presented in Table II and Table III.

TABLE II
EUTROPHICATION LEVELS, FOR THE EVALUATION OF WATER QUALITY

Level	Description	Notation
Oligotrophic	Level with **low** nutrient content in aquatic systems as a consequence of minimal plant production	λ_1
Lower-Mesotrophic	**Acceptable** level of nutrient content in aquatic systems	λ_2
Mesotrophic	**Ideal** level of nutrient content in aquatic systems, between Oligotrophic and Eutrophic	λ_3
Upper-Mesotrophic	**Acceptable** level of nutrient content in aquatic systems	λ_4
Eutrophic	Level with **high** nutrient content in aquatic systems, as a consequence high plant production	λ_5
Hyper eutrophic	**Super excessive** level of nutrient content in aquatic systems, as a consequence of excessive plant production	λ_6

E. Calculations using the CTWF method

Calculations for the case study are presented below:

Step 1*:* The parameters are based according to the classification standards of the eutrophication of the Zhalong wetland as shown in Table III.

TABLE III
STANDARD DATA FOR THE EVALUATION OF WATER QUALITY

Classification standards for eutrophication from Zhalong wetland						
Parameters	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6
C_1	1,0	2,0	4,0	10	65	160
C_2	2,5	5,0	25	50	200	600
C_3	30	50	300	500	2000	6000
C_4	0,3	0,4	2,0	4,0	10	25

Step 2: The non-dimensioned standard values for each criterion, according to the eutrophication classification standard, were determined through arithmetic average, the results are presented in Table IV.

TABLE IV
NON-DIMENSIONED STANDARD VALUES OF THE CRITERIA FOR THE EVALUATION

Non-dimension standard values in the case study						
Parameters	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6
C_1	0.025	0.050	0.099	0.248	1.612	3.967
C_2	0.017	0.034	0.170	0.340	1.360	4.079
C_3	0.020	0.034	0.203	0.338	1.351	4.054
C_4	0.043	0.058	0.288	0.576	1.439	3.597

Likewise, the same procedure of Table IV is applied to the results of the monitoring report of March 2014, for the evaluation of the water quality of Titicaca Lake (Peruvian sector), prepared by the National Water Authority (ANA). As an example, the results from 10 points are presented in Table V.

TABLE V
NON-DIMENSIONED DATA FROM THE MONITORING OF THE WATER QUALITY OF TITICACA LAKE

M. Point	**BInt01**	**BInt02**	**BInt03**	**BInt04**	**BInt05**
C1	1.000	1.000	1.000	1.000	1.000
C2	0.116	1.157	1.088	1.099	1.054
C3	0.733	0.733	0.733	1.687	0.733
C4	1.099	0.502	1.150	1.175	1.250

Through them, the same procedure was carried out for the other monitoring points.

Step 3*:* The values of Table V are replaced. Into Equations (1) – (3), the center point triangular whitenization functions of the six grey classes are obtained for each criterion. then, as an example, the one corresponding to the first criterion Chlorophyll A (Chla; j = 1) is shown in Equations (6) – (11) and Fig. 5.

$$f_j^1(x_{i1}) = \begin{bmatrix} 1, \ x \in [0,0.025] \\ \dfrac{0.05-x}{0.025}, \ x \in \langle 0.025,0.050\rangle \\ 0, x \in [0.050,+\infty\rangle \end{bmatrix} \quad (6)$$

$$f_j^2(x_{i1}) = \begin{cases} \frac{x-0.025}{0.025}, & x \in \langle 0.025, 0.050] \\ \frac{0.099-x}{0.050}, & x \in \langle 0.050, 0.099\rangle \\ 0, x \in [0, 0.025]\, U\, [0.099, +\infty\rangle \end{cases} \quad (7)$$

$$f_j^3(x_{i1}) = \begin{cases} \frac{x-0.050}{0.050}, & x \in \langle 0.050;\ 0.099\rangle \\ \frac{0.248-x}{0.149}, & x \in [0.099;\ 0.248\rangle \\ 0, x \in [0;\ 0.050]\, U\, [0.248;\ +\infty > \end{cases} \quad (8)$$

$$f_j^4(x_{i1}) = \begin{cases} \frac{x-0.099}{0.050}, & x \in \langle 0.099;\ 0.248\rangle \\ \frac{1.612-x}{1.364}, & x \in [0.248;\ 1.612\rangle \\ 0, x \in [0;\ 0.099]\, U\, [1.612;\ +\infty > \end{cases} \quad (9)$$

$$f_j^5(x_{i1}) = \begin{cases} \frac{x-0.248}{1.364}, & x \in \langle 0.248;\ 1.612] \\ \frac{3.967-x}{2.355}, & x \in \langle 1.612;\ 3.967\rangle \\ 0, x \in [0;\ 0.248]\, U\, [3.967;\ +\infty\rangle \end{cases} \quad (10)$$

$$f_j^6(x_{ij}) = \begin{cases} \frac{x-1.612}{2.355}, & x \in \langle 1.612;\ 3.967\rangle \\ 1, & x \in [3.967;\ +\infty\rangle \\ 0, & x \in [0;\ 1.612] \end{cases} \quad (11)$$

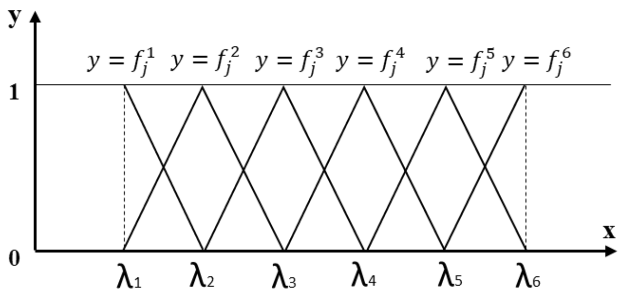

Fig. 5. CTWF for chlorophyll.

The values of Table V are replaced, the results obtained are presented in Table VI.

TABLE VI
VALUES OF THE TRIANGULAR WHITENIZATION FUNCTIONS (CTWF) FOR THE TITICACA LAKE

	C1	C2	C3	C4
	0	0	0	0
	0	0.370	0	0
BInt01	0	0.630	0	0
	0.449	0	0.610	0.390
	0.551	0	0.390	0.610
	0	0	0	0
	C1	C2	C3	C4
	0	0	0	0
	0	0	0	0
BInt02	0	0	0	0.260
	0.449	0.200	0.610	0.740
	0.551	0.800	0.390	0
	0	0	0	0
	C1	C2	C3	C4
	0	0	0	0
	0	0	0	0
BInt03	0	0	0	0
	0.449	0.250	0.610	0.390
	0.551	0.750	0.390	0.610
	0	0	0	0

Step 4: The weights of the parameters for each criterion were determined by Equation (4). These values are shown in Table VII.

TABLE VII
ZHALONG WETLAND CRITERIA WEIGHT VALUES

Parameters	ʎ1	ʎ2	ʎ3	ʎ4	ʎ5	ʎ6
C1	0.235	0.209	0.415	0.346	0.222	0.247
C2	0.343	0.305	0.241	0.252	0.264	0.239
C3	0.287	0.307	0.202	0.254	0.265	0.241
C4	0.135	0.179	0.143	0.149	0.249	0.272
	1	1	1	1	1	1

Step 5: The clustering coefficient values were calculated using Equation (5). The results of the three monitoring points are shown in Table VIII.

TABLE VIII
CLUSTERING COEFFICIENT VALUES FOR THE 11 WATER QUALITY MONITORING POINTS OF TITICACA LAKE

	C1	C2	C3	C4	
	0	0	0	0	0
	0	0.37	0	0	0.112721153
BInt01	0	0.63	0	0	0.152061538
	0.449	0	0.61	0.39	0.367889627
	0.551	0	0.39	0.61	0.377814119
	0	0	0	0	0
	C1	C2	C3	C4	
	0	0	0	0	0
	0	0	0	0	0
BInt02	0	0	0	0.26	0.037066667
	0.449	0.2	0.61	0.74	0.470391947
	0.551	0.8	0.39	0	0.436713254
	0	0	0	0	0
	C1	C2	C3	C4	
	0	0	0	0	0
	0	0	0	0	0
BInt03	0	0	0	0	0
	0.449	0.25	0.61	0.39	0.430893839
	0.551	0.75	0.39	0.61	0.575440932
	0	0	0	0	0

Step 6: Finally, the classification of each monitoring point was determined according to the location of its max clustering coefficient in the intervals of the parameters, as can be seen in Table IX.

V. RESULTS AND DISCUSSION

A. About the Case Study

The results of the water quality analysis using the grey clustering method are given in Table IX, in which we can appreciate as follows:

Inner Bay of Puno

The sector of the interior bay of Puno is at the eutrophic level. These differ from that shown by Beltran, which would indicate an elevation in the eutrophication process of the interior bay of Puno. These results could be explained by the growth of Lemma spp., which covered approximately 10km2 of the interior bay and were several centimeters thick. The abundance of duckweed in the interior bay reflects a state of concentration of nutrients and the low flow of water in this area; due to this, the interior bay of Puno is shallow and of little flow, characteristics that combined with the high inputs of nutrients from the drains of the city of Puno, which gives a severely polluted to body of water.

Puno Bay

The Puno Bay sector presents a eutrophic level of eutrophication in its 9 monitoring points, these results obtained can be contrasted with the report presented by ANA (2014), which presents a eutrophic trend according to its trophic classification.

TABLE IX
CLASSIFICATION OF THE MONITORING POINTS FOR THE TITICACA LAKE

Inner Bay of Puno			
M. Point	Code	σ	Level
1	BInt04	0.730102668	Eutrophic (ʎ5)
2	BInt11	0.696350471	Eutrophic (ʎ5)
3	BInt07	0.640166979	Eutrophic (ʎ5)
4	BInt06	0.631748338	Eutrophic (ʎ5)
5	BInt05	0.617774498	Eutrophic (ʎ5)
6	BInt08	0.581597781	Eutrophic (ʎ5)
7	BInt10	0.580613939	Eutrophic (ʎ5)
8	BInt03	0.575440932	Eutrophic (ʎ5)
9	BInt09	0.472912115	Upper-Mesotrophic (ʎ4)
10	BInt02	0.470391947	Upper-Mesotrophic (ʎ4)
11	BInt01	0.377814119	Eutrophic (ʎ5)
Lake Maggiore			
M. Point	Code	σ	Level
1	LTiti01	0.541439066	Upper-Mesotrophic (ʎ4)
2	LTiti02	0.541439066	Upper-Mesotrophic (ʎ4)
3	LTiti03	0.541439066	Upper-Mesotrophic (ʎ4)
4	DRam01	0.541439066	Upper-Mesotrophic (ʎ4)
5	LTiti04	0.541439066	Upper-Mesotrophic (ʎ4)
6	LTiti05	0.541439066	Upper-Mesotrophic (ʎ4)
7	LTiti07	0.541439066	Upper-Mesotrophic (ʎ4)
8	LTiti06	0.541439066	Upper-Mesotrophic (ʎ4)
9	LTiti08	0.541439066	Upper-Mesotrophic (ʎ4)
10	LTiti09	0.541439066	Upper-Mesotrophic (ʎ4)
11	JChoc01	0.541439066	Upper-Mesotrophic (ʎ4)
12	LTiti10	0.541141358	Upper-Mesotrophic (ʎ4)
13	Bpoma	0.541141358	Upper-Mesotrophic (ʎ4)
14	LTiti11	0.541439066	Upper-Mesotrophic (ʎ4)
15	BCutu01	0.541439066	Upper-Mesotrophic (ʎ4)
16	LTiti12	0.418178807	Eutrophic (ʎ5)
Lake Huayñamarca			
M. Point	Code	σ	Level
1	RDesa01	0.470879291	Eutrophic (ʎ5)
2	LTiti14	0.548495538	Upper-Mesotrophic (ʎ4)
3	LTiti13	0.418178807	Eutrophic (ʎ5)
4	LTiti13.1	0.418178807	Eutrophic (ʎ5)
Puno Bay			
M. Point	Code	σ	Level
1	RWilly	0.576	Eutrophic (ʎ5)
2	BPuno01	0.41	Eutrophic (ʎ5)
3	BPuno02	0.41	Eutrophic (ʎ5)
4	BPuno03	0.41	Eutrophic (ʎ5)
5	BPuno04	0.41	Eutrophic (ʎ5)
6	BPuno05	0.41	Eutrophic (ʎ5)
7	BPuno06	0.41	Eutrophic (ʎ5)
8	BPuno07	0.41	Eutrophic (ʎ5)
9	BPuno08	0.41	Eutrophic (ʎ5)

Major lake

The quality of the water in the greater lake results in 93.75% with a higher mesotrophic level, classified as acceptable water quality, but not ideal, and in 6.25% with a eutrophic level, classified as low water quality, that is, it is contaminated. This is mainly due to sources of contamination such as domestic solid waste dumps, discharge of wastewater without any type of treatment that are tributaries to the greater lake sector, such as the Coata River, which presents values in phosphorus content that exceed excessively environmental quality standards [18] [20]. Confirming that the aquatic system in this sector presents an excessive plant production, due to the high content of nutrients.

Minor lake

It was determined that 75% of the points analyzed (3) of Huayñamarca Lake have a eutrophic level, and 25% (1) have an upper mesotrophic level. Point LTiti14 is the only point that presents an upper mesotrophic level, this due to the little discharges of wastewater and with a low nitrogen and phosphorus load, from small, populated centers, such as Zepita and Tinicachi. Regarding the points of eutrophic level. Firs, RDesa01 taken at the Rio Desaguadero Bridge, presents a higher level (0.4708) due to its narrow channel and being the only emissary that evacuates the waters of Titicaca Lake, which is sustained by the low transparency it presents; second, point LTiti13 responds to wastewater from the Sucauma; and third, point LTiti13.1 is affected mainly by the effluent from the Katari River that discharges wastewater from Cohana Bay, which contains the waters from livestock activities with high organic load and residues large-scale El Alto.

B. About the Methodology

The theory of grey systems incorporates uncertainty in its analysis method, it also allows working with small samples and information that is difficult to handle for probability. Similarly, one of the benefits is that it allows us to explore the realistic laws of evolution and movement of events and materials into of a research problem. It also allows the construction of models with small amounts of data; however, it has some advantages and disadvantages compared to other theories such as probability and statistics models, the approximate set theory, and fuzzy logic models [12].

VI. CONCLUSION

From the 40 monitoring points studied in the Titicaca Lake of the Peruvian sector: Inner Bay of Puno (11), 81.8% were identified at the eutrophic level and 18.2% at the upper mesotrophic level; Bahía de Puno (09), were identified 100% at the eutrophic level; major lake (16) 93.75% were identified at the upper mesotrophic level and 6.25% at a eutrophic level; and minor lake (04) were identified 75% at an eutrophic level, and 25% present an upper mesotrophic level.

In addition, the grey clustering method generated reliable results, which allow a truthful analysis of the level of eutrophication of the Titicaca Lake in the Peruvian sector. This makes possible quantitatively to classify the bodies of water with the aim of proposing a better intake in the management of water resources.

Finally, in futures research work, it is expected that the reader will have the initiative to add new parameters when applying the grey clustering methodology, to obtain more precise data in the analysis for the determination of eutrophication levels of the bodies of water.

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