Active Event-Triggered Fault-Tolerant Control Design for Switched Pure-Feedback Nonlinear Systems

Zhongfeng Li, Yingxin Wei, Lidong Wang

Abstract—**The proposed research paper introduces an active fault-tolerant control (FTC) strategy for pure-feedback switched nonlinear systems using event-triggered input. The proposed methodology employed fuzzy logic systems to directly approximate unknown functions, thereby enabling estimation of the unavailable states through a designed observer. Under the framework of backstepping technology, it is demonstrated that all of the signals in the closed-loop system remain bounded under fault-free conditions. Additionally, the fault detection function only depend on the available output. The combination of backstepping technology and an event-triggered mechanism is incorporated into designing the active FTC scheme, ensuring the boundedness of every closed-loop signal under a specific class of switching signals with a designated average dwell time in the proposed control project. A simulation is then offered to cinch the efficiency of the suggested controller.**

Index Terms—**active fault-tolerant control, event-triggered control, pure-feedback, switched nonlinear systems.**

I. Introduction

SWITCHED nonlinear systems have multi-mode modeling characteristics and are prevalent in various engineering applications such as power, traffic, aerospace, and chemical control systems [1]-[2]. The systems are composed of multiple subsystems that operate under a switching rule, making their multi-mode modeling characteristics more consistent with real-world control systems [3]. Ongoing research in this field investigates the effective utilization of active fault-tolerant control (FTC) in specific classes of switched nonlinear systems [4]-[6]. Some studies seek active FTC for switched nonlinear systems which were constructed as strict-feedback form [4], some explore inadequacy in actuator faults for uncertain switched systems [5], while other research aims to propose a finite-time FTC technique to handle the challenges of actuator faults, alongside the biases that arise in a class of lower triangular switched nonlinear systems [6]. The development of virtual controllers and control signals for pure-feedback systems using backstepping technology presents a significant challenge. Researchers have addressed this issue using various techniques, such as the mean value theorem and implicit function, as described in [7]-[8]. The non-affine input dilemma in pure-feedback systems is addressed in this work through the use of filtered signals, a concept previously explored in [9].

With the increasing demands for control technology and equipment, failures are inevitable in actual work processes. However, the development of FTC technology provides an effective solution for ensuring the security of complex systems [10]. Active FTC methods primarily include model following reconfiguration control and control law reconfiguration. The basic principle of model following reconfiguration control is to use the method of model reference adaptive control, which cinches that the system output always goes after the signal of the reference model, with or without any faults, as reported in [11]. The control law reconstruction method usually requires a fault detection and diagnosis module to redesign the control law, including online selection and online design, in order to achieve the desired control effect by using obtained fault information [12]. In addition, [13] proposed an active FTC scheme for a continuous stirred tank reactor system.

The use of event-triggered systems in research has garnered interest due to its potential to save communication resources and reduce communication stress associated with large data transfer. Previous studies (e.g. [14]-[18]) have explored the use of event-triggered systems to control uncertain nonlinear systems, pure-feedback systems, nonlinear systems with pure-feedback, nonlinear time-delay systems with non-strict feedback shape, as well as arbitrary switched nonlinear system with pure-feedback shape. This paper, unlike previous works, investigates switching signals that are predicated upon the average dwell time.

Building on previous analyses, this study introduces an event-triggered active FTC technique for pure-feedback switched nonlinear systems featuring unknown functions. The primary dedications of this paper are discussed below.

1. Firstly, this study presents an active event-triggered FTC technique for switched nonlinear systems with pure-feedback shape. It is challenging to design such a scheme due to the simultaneous presence of event-triggered strategy and switching signals. A pure-feedback system with non-affine inputs is more commonly used in modeling than a strict-feedback system with affine inputs. In addition, the proposed event-triggered programme is anticipated to assist in reducing the unnecessary usage of communication resources.

2. By introducing filter signals during the controller design process, it can not only release the assumption conditions in [7]-[8], but avoid the algebraic loop produced by mean value theorem and implicit function.

Manuscript received December 06, 2022; revised May 14, 2023. This work was supported in part by Regional joint fund project of Liaoning Provincial Department of science and technology (Grant No.2020-YKLH-32), in part by 2021 scientific research fund project (key project) of Liaoning Provincial Department of Education (Grant No. LJKZ1199).

Zhongfeng Li is a senior engineer and associate professor of School of Electrical Engineering, Yingkou Institute of Technology, Yingkou, Liaoning, CO 115014, China (e-mail: afeng0601@163.com).

Yingxin Wei is a postgraduate of School of Electronic and Information Engineering, University of Science and Technology Liaoning, Anshan, Liaoning, CO 114051, China (e-mail: weiyx0104@163.com).

Lidong Wang is a professor of School of Electronic and Information Engineering, University of Science and Technology Liaoning, Anshan, Liaoning, CO 114051, China (Corresponding author: e-mail: wangld5600@163.com).

This paper's structure is as follows. Section II lays out the problem formulations and preliminary steps are discussed, while the stability analysis without faults and fault detection is covered in Section III. Section IV contains the design for an event-triggered active FTC method. A simulation example is offered in Section V, while Section VI is where this study comes to an end.

II. Preparatory knowledge

A. System Description

Considering the following pure-feedback switched system

$$\begin{cases} \dot{x}_i = f_{i,\sigma(t)}\left(\bar{x}_i, x_{i+1}\right), i = 1, 2, \cdots, n-1 \\ \dot{x}_n = f_{n,\sigma(t)}\left(\bar{x}_n, u\right) + \hbar(t-t_\hbar) d_{\sigma(t)}(\bar{x}_n, u) \\ y = x_1 \end{cases}, \quad (1)$$

where $\bar{x}_i = [x_1, x_2, \cdots, x_i]^T$, $1 \le i \le n$, denote system states, $u \in \mathbb{R}$ and $y \in \mathbb{R}$ stand for the control input and system output, separately. A switching signal is represented as $\sigma(t) : [0, +\infty) \to S = \{1, 2, \cdots, s\}$, and $\sigma(t) = p,\ p \in S$, denotes the activation of the pth subsystem. Unknown smooth nonlinear functions are represented by $f_{i,p}(\cdot,\cdot)$. The output y is assumed to be the only accessible variable for measurements in this study. Moreover, the fault functions are represented by $d_p\left(\bar{x}_n, u\right)$.

$$\hbar(t-t_\hbar) = \begin{cases} 0, & t < t_\hbar \\ 1, & t \ge t_\hbar, \end{cases}$$

where $t_\hbar$ is the unknown fault occurring time during the system operation.

In the light of [19], the idea of average dwell time (ADT) can be provided. If a switching signal σ is supposed to have an ADT of τ_a, then, there are positive values N_0 and τ_a satisfying

$$N_\sigma(t_1, t_2) \le N_0 + \frac{t_2 - t_1}{\tau_a}\ , \forall t_2 \ge t_1 \ge 0 \quad (2)$$

where $N_\sigma(t_1, t_2)$ means the switching times in the interval $[t_1, t_2)$.

Assumption 1 [20]: The following inequality can be satisfied by positive constants $\epsilon_i, 1 \le i \le n$,

$$|f_i(Z_1) - f_i(Z_2)| \le \epsilon_i \left\| Z_1 - Z_2 \right\|. \quad (3)$$

Assumption 2 [23]: There are unknown constants κ_i^*, δ_i^*, κ_d^*, δ_d^* and g_i, $1 \le i \le n$, such that $|\kappa_{i,p}| \le \kappa_i^*$, $|\delta_{i,p}| \le \delta_i^*$, $|\kappa_{d,p}| \le \kappa_d^*$, $|\delta_{d,p}| \le \delta_d^*$, and $|\hat{x}_{i+1} - \hat{x}_{i+1,f}| \le g_i$.

Lemma 1 [21]: Let $f(x)$ defined on a compact set Θ be a continuous function, for each positive constant δ, then there is a fuzzy logic system (FLS) satisfying

$$\sup_{x\in\Theta} \left| f(x) - \theta^T \varphi(x) \right| \le \delta. \quad (4)$$

Lemma 2 [9]: For any $a > 0$, the inequality $|x| - x\tanh(\frac{x}{a}) \le 0.2785a = a_0$ exists.

B. Triggering Signal

In the light of [22], a fixed threshold event-triggered control project is considered in this paper. Define an intermediate control signal as

$$v(t) = \alpha_n - \bar{\varsigma}\tanh(\frac{\chi_n \bar{\varsigma}}{l}), \quad (5)$$

where l and $\bar{\varsigma}$ are positive design parameters. α_n stands for virtual control signal, χ_n is virtual error surface, both of them are going to be defined later.

The triggering event is defined as

$$u(t) = v(t_k), \forall t \in [t_k, t_{k+1}), \quad (6)$$
$$t_{k+1} = \inf\{t \in \mathbb{R}\, ||\Gamma(t)| \ge \varsigma\}, t_1 = 0, \quad (7)$$

where $\Gamma(t) = v(t) - u(t)$ means the estimation error, $\varsigma(\bar{\varsigma} > \varsigma > 0)$ is a positive design factor. The controller update time t_k, $k \in z^+$, means that the time will be updated to t_{k+1} anytime (7) is triggered, and $u(t_{k+1})$ will be delivered to the actuator. For $t \in [t_k, t_{k+1})$, the control signal keeps constant, meaning $v(t)$.

III. Controller design

In this part, we complete the controller design process for fault-free situations. To design the control scheme completely, an observer is first constructed. For this purpose, we define the following equations

$$F_{i,p}\left(\bar{x}_i, x_{i+1}\right) = f_{i,p}\left(\bar{x}_i, x_{i+1}\right) - x_{i+1} \quad (8)$$
$$F_{n,p}\left(\bar{x}_n, u\right) = f_{n,p}\left(\bar{x}_n, u\right) - u, \quad (9)$$

according to which the system (1) is described as

$$\begin{cases} \dot{x}_i = x_{i+1} + F_{i,p}\left(\hat{\bar{x}}_i, \hat{x}_{i+1,f}\right) + \Delta F_{i,p} \\ \dot{x}_n = u + F_{n,p}\left(\hat{\bar{x}}_n, u_f\right) + \Delta F_{n,p} \\ y = x_1, \end{cases} \quad (10)$$

where $\hat{\bar{x}}_i$ and $\hat{\bar{x}}_n$ are the respective measured values of $\bar{x}_i$ and $\bar{x}_n$. $\Delta F_{i,p} = F_{i,p}\left(\bar{x}_i, x_{i+1}\right) - F_{i,p}\left(\hat{\bar{x}}_i, \hat{x}_{i+1,f}\right)$, $\Delta F_{n,p} = F_{n,p}\left(\bar{x}_n, u\right) - F_{n,p}\left(\hat{\bar{x}}_n, u_f\right)$. Additionally, $\hat{x}_{i,f} = E_L(s)\hat{x}_i \approx \hat{x}_i, u_f = E_L(s)u \approx u$, and $E_L(s)$ is a Butterworth low-pass filter (LPF), whose parameters are given in [9].

Let $\hat{x}_{n+1,f} = u_f$, we have

$$\dot{\hat{\bar{x}}}_n = C_p\hat{\bar{x}}_n + H_p y + \sum_{i=1}^{n} Z_i[F_{i,p}\left(\hat{\bar{x}}_i, \hat{x}_{i+1,f}\right) + \Delta F_{i,p}] + Z_n u \quad (11)$$

where $C_p = \begin{bmatrix} -h_{1,p} & & \\ \vdots & I_{n-1} & \\ -h_{n,p} & \cdots & 0 \end{bmatrix}$, $H_p = \begin{bmatrix} h_{1,p} \\ h_{2,p} \\ \vdots \\ h_{n,p} \end{bmatrix}$, $Z_n = \begin{bmatrix} 0 \\ \vdots \\ 1 \end{bmatrix}$, $Z_i = \begin{bmatrix} \underbrace{0 \ \cdots \ 1}_{i} & \cdots & 0 \end{bmatrix}^T$. $h_{i,p}$, $1 \le i \le n$, $p \in S$, stand for design parameters. C_p represent strict Hurwitz matrixes. For any given matrix $Q_p > 0$, which is a define symmetric matrix, there is a positive matrix P_p to make

$$C_p^T P_p + P_p C_p \le -Q_p. \quad (12)$$

In line with Lemma 1, the unknown functions $F_{i,p}(\cdot,\cdot)$ are approximated by FLSs

$$\hat{F}_{i,p}\left(\hat{\bar{x}}_i, \hat{x}_{i+1,f} | \theta_{i,p}\right) = \theta_{i,p}^T \varphi_i(\hat{\bar{x}}_i, \hat{x}_{i+1,f})\ , 1 \le i \le n. \quad (13)$$

Define the following optimal parameter vector

$$\theta_{i,p}^* = \arg\min_{\theta_{i,p}\in\Xi_i} [\sup_{(\hat{\bar{x}}_i, \hat{x}_{i+1,f})\in\Phi_i} |F_{i,p}\left(\hat{\bar{x}}_i, \hat{x}_{i+1,f}\right) - \hat{F}_{i,p}\left(\hat{\bar{x}}_i, \hat{x}_{i+1,f} | \theta_{i,p}\right)|], \quad (14)$$

where Ξ_i is the compact sets for $\theta_{i,p}$, Φ_i is the compact sets for $(\hat{\bar{x}}_i, \hat{x}_{i+1,f})$.

After that, the estimated error can be obtained

$$\kappa_{i,p} = F_{i,p}\left(\hat{\bar{x}}_i, \hat{x}_{i+1,f}\right) - \hat{F}_{i,p}\left(\hat{\bar{x}}_i, \hat{x}_{i+1,f}|\theta_{i,p}\right), \quad (15)$$

$$\delta_{i,p} = F_{i,p}\left(\hat{\bar{x}}_i, \hat{x}_{i+1,f}\right) - \hat{F}_{i,p}\left(\hat{\bar{x}}_i, \hat{x}_{i+1,f}|\theta^*_{i,p}\right). \quad (16)$$

Let $w_{i,p} = \delta_{i,p} - \kappa_{i,p}$, $2 \le i \le n$, $w_{d,p} = \delta_{d,p} - \kappa_{d,p}$, with the help of Assumption 2, there exist unknown constants $w_i^* > 0$, $w_d^* > 0$ to full fill $|w_{i,p}| \le w_i^* = \delta_i^* + \kappa_i^*$, $|w_{d,p}| \le w_d^* = \delta_d^* + \kappa_d^*$.

An observer in the following form can be constructed

$$\begin{cases} \dot{\hat{x}}_i = \hat{x}_{i+1} + \hat{F}_{i,p}\left(\hat{\bar{x}}_i, \hat{x}_{i+1,f}|\theta_{i,p}\right) + h_{i,p}(y - \hat{x}_1) \\ \dot{\hat{x}}_n = u + \hat{F}_{n,p}\left(\hat{\bar{x}}_n, u_f|\theta_{n,p}\right) + h_{n,p}(y - \hat{x}_1) \\ \hat{y} = \hat{x}_1. \end{cases} \quad (17)$$

The observer error is defined as $e = [e_1, e_2, \cdots e_n]^T$, where $e_i = x_i - \hat{x}_i$, $1 \le i \le n$. The derivative of e can be obtained through (11) and (17)

$$\begin{aligned} \dot{e} &= C_p e + \sum_{i=1}^{n} Z_i[F_{i,p}\left(\hat{\bar{x}}_i, \hat{x}_{i+1,f}\right) \\ &\quad - \hat{F}_{i,p}\left(\hat{\bar{x}}_i, \hat{x}_{i+1,f}|\theta_{i,p}\right) + \Delta F_{i,p}] \\ &= C_p e + \kappa_p + \Delta F_p, \end{aligned}$$

where $\Delta F_p = [\Delta F_{1,p}, \Delta F_{2,p}, \cdots, \Delta F_{n,p}]^T$, $\kappa_p = [\kappa_{1,p}, \kappa_{2,p}, \cdots \kappa_{n,p}]^T$.

Define the Lyapunov function $V_{0,p} = e^T P_p e$.

Then, we can deduce the time derivation of $V_{0,p}$

$$\begin{aligned} \dot{V}_{0,p} &= 2e^T P_p (C_p e + \kappa_p + \Delta F_p) \\ &\le -Q_p \|e\|^2 + 2e^T P_p \Delta F_p + 2e^T P_p \kappa_p. \end{aligned} \quad (18)$$

According to Assumption 1, we have

$$\begin{aligned} \Delta F_{i,p} &= [F_{i,p}\left(\bar{x}_i, x_{i+1}\right) - F_{i,p}\left(\hat{\bar{x}}_i, \hat{x}_{i+1,f}\right)] \\ &\le [\epsilon_i(\left|\bar{x}_i - \hat{\bar{x}}_i\right| + |x_{i+1} - \hat{x}_{i+1,f}|)] \\ &\le \epsilon_i(\|e\| + g_i). \end{aligned}$$

Using Youngs inequality, we get

$$2e^T P_p \Delta F_p \le \|e\|^2 + \|P_p\|^2 \sum_{i=1}^{n} \epsilon_i^2 (\|e\|^2 + g_i^2) \quad (19)$$

$$2e^T P_p \kappa_p \le \|e\|^2 + \|P_p\|^2 \left\|\kappa_p^*\right\|^2 \quad (20)$$

where $\kappa_i^* = [\kappa_1^*, \kappa_2^*, \cdots \kappa_n^*]^T$.

Combining (18), (19) and (20), one has

$$\dot{V}_{0,p} \le -\beta \|e\|^2 + M, \quad (21)$$

where $\beta = \lambda_{\min}(Q_p) - 2 - \|P_p\|^2 \sum_{i=1}^{n} \epsilon_i^2$, $M = \|P_p\|^2 [\sum_{i=1}^{n} \epsilon_i^2 g_i^2 + \|\kappa_i^*\|^2]$.

Next, in the light of observer (17), the controller will be designed under the framework of backstepping recipe.

The coordinate transformation is given as:

$$\begin{cases} \chi_1 = x_1 \\ \chi_i = \hat{x}_i - \alpha_{i-1}, 2 \le i \le n, \end{cases} \quad (22)$$

where α_{i-1} stand for the virtual control signals.

At the beginning of the design process, to facilitate writing, let θ_i^* represent $\theta_{i,p}^*$, θ_i represent $\theta_{i,p}$, $\tilde{\theta}_i$ represent $\tilde{\theta}_{i,p}$, and $\tilde{\theta}_i = \theta_i^* - \theta_i$.

Step 1:

According to (10), (16) and (22), we have

$$\begin{aligned} \dot{\chi}_1 &= \chi_2 + \alpha_1 + e_2 + \delta_{1,j} + \tilde{\theta}_1^T \varphi_1(\hat{\bar{x}}_1, \hat{x}_{2,f}) \\ &\quad + \theta_1^T \varphi_1(\hat{\bar{x}}_1, \hat{x}_{2,f}) + \Delta F_{1,p}. \end{aligned} \quad (23)$$

Define the Lyapunov function as

$$V_{1,p} = V_{0,p} + \frac{1}{2}\chi_1^2 + \frac{1}{2\eta_1}\tilde{\theta}_1^T \tilde{\theta}_1 + \frac{1}{2\bar{\eta}_1}\tilde{\delta}_1^2,$$

where $\eta_i > 0$, $\bar{\eta}_i > 0$, $1 \le i \le n$, are designed constants. $\hat{\delta}_1$ is the evaluation of δ_1^*, and $\tilde{\delta}_1 = \delta_1^* - \hat{\delta}_1$.

Then, we have

$$\begin{aligned} \dot{V}_{1,p} &= \dot{V}_{0,p} + \chi_1(\chi_2 + \alpha_1 + e_2 + \delta_{1,p} \\ &\quad + \tilde{\theta}_1^T \varphi_1(\hat{\bar{x}}_1, \hat{x}_{2,f}) + \theta_1^T \varphi_1(\hat{\bar{x}}_1, \hat{x}_{2,f}) \\ &\quad + \Delta F_{1,p}) + \frac{1}{\eta_1}\tilde{\theta}_1^T \dot{\tilde{\theta}}_1 + \frac{1}{\bar{\eta}_1}\tilde{\delta}_1 \dot{\tilde{\delta}}_1. \end{aligned}$$

Along with Young's inequality and Assumption 1, it holds that

$$\chi_1 e_2 \le \frac{1}{2}\chi_1^2 + \frac{1}{2}\|e_2\|^2 \quad (24)$$

$$\chi_1 \Delta F_{1,p} \le \frac{1}{2}\chi_1^2 + \frac{1}{2}\epsilon_1^2(\|e_1\|^2 + g_1^2). \quad (25)$$

According to (21), (24) and (25), we have

$$\begin{aligned} \dot{V}_{1,p} &\le -(\beta - \frac{1}{2} - \frac{1}{2}\epsilon_1^2)\|e\|^2 + M + \chi_1(\chi_2 \\ &\quad + \alpha_1 + \theta_1^T \varphi_1(\hat{\bar{x}}_1, \hat{x}_{2,f}) + \chi_1) + |\chi_1| \delta_1^* \\ &\quad + \frac{1}{\eta_1}\tilde{\theta}_1^T(\eta_1 \varphi_1(\hat{\bar{x}}_1, \hat{x}_{2,f})\chi_1 - \dot{\theta}_1) \\ &\quad + \frac{1}{\bar{\eta}_1}\tilde{\delta}_1 \dot{\tilde{\delta}}_1 + \frac{1}{2}\epsilon_1^2 g_1^2. \end{aligned} \quad (26)$$

In the light of (26), Assumption 2 and Lemma 2, one has

$$\begin{aligned} \dot{V}_{1,p} &\le -(\beta - \frac{1}{2} - \frac{1}{2}\epsilon_1^2)\|e\|^2 + M \\ &\quad + \chi_1[\chi_2 + \alpha_1 + \theta_1^T \varphi_1(\hat{\bar{x}}_1, \hat{x}_{2,f}) + \chi_1 \\ &\quad + \hat{\delta}_1 \tanh(\frac{\chi_1}{a_1})] + \delta_1^* a_{10} + \frac{1}{\eta_1}\tilde{\theta}_1^T(\eta_1 \varphi_1(\hat{\bar{x}}_1, \hat{x}_{2,f})\chi_1 \\ &\quad - \dot{\theta}_1) + \frac{1}{\bar{\eta}_1}\tilde{\delta}_1(\bar{\eta}_1 \chi_1 \tanh(\frac{\chi_1}{a_1}) - \dot{\hat{\delta}}_1) + \frac{1}{2}\epsilon_1^2 g_1^2. \end{aligned} \quad (27)$$

Design the following signals

$$\begin{aligned} \alpha_1 &= -b_1\chi_1 - \chi_1 - \theta_1^T \varphi_1(\hat{\bar{x}}_1, \hat{x}_{2,f}) \\ &\quad - \hat{\delta}_1 \tanh(\frac{\chi_1}{a_1}) \end{aligned} \quad (28)$$

$$\dot{\theta}_1 = \eta_1 \varphi_1(\hat{\bar{x}}_1, \hat{x}_{2,f})\chi_1 - \gamma_1 \theta_1 \quad (29)$$

$$\dot{\hat{\delta}}_1 = \bar{\eta}_1 \chi_1 \tanh(\frac{\chi_1}{a_1}) - \bar{\gamma}_1 \hat{\delta}_1 \quad (30)$$

where $b_i > 0$, $\gamma_i > 0$, $a_i > 0$, $\bar{\gamma}_i > 0, 1 \le i \le n$, are design parameters.

Then, one has

$$\begin{aligned} \dot{V}_{1,p} &\le -\beta_1 \|e\|^2 - b_1\chi_1^2 + \chi_1\chi_2 + \frac{\gamma_1}{\eta_1}\tilde{\theta}_1^T \theta_1 \\ &\quad + \frac{\bar{\gamma}_1}{\bar{\eta}_1}\tilde{\delta}_1 \hat{\delta}_1 + M_1, \end{aligned} \quad (31)$$

where $\beta_1 = \beta - 1/2 - \epsilon_1^2/2$, $M_1 = M + \frac{1}{2}\epsilon_1^2 g_1^2 + \delta_1^* a_{10}$.

Step 2:

According to (10), (15), (22) and (28), we can obtain that

$$\begin{aligned}\dot{\alpha}_1 &= \frac{\partial\alpha_1}{\partial x_1}(\hat{x}_2+e_2+\theta_1^T\varphi_1(\hat{\bar{x}}_1,\hat{x}_{2,f})+\kappa_{2,p}\\&\quad+\Delta F_{1,p})+\frac{\partial\alpha_1}{\partial\hat{x}_1}\dot{\hat{x}}_1+\frac{\partial\alpha_1}{\partial\theta_1}\dot{\theta}_1+\frac{\partial\alpha_1}{\partial\hat{\delta}_1}\dot{\hat{\delta}}_1. \quad (32)\end{aligned}$$

From (22) and (32), we can deduce that

$$\begin{aligned}\dot{\chi}_2 &= \hat{x}_3+\hat{F}_{2,p}\left(\hat{\bar{x}}_2,\hat{x}_{3,f}|\theta_2\right)+h_{2,p}e_1-\dot{\alpha}_1\\&= \chi_3+\alpha_2+\tilde{\theta}_2^T\varphi_2(\hat{\bar{x}}_2,\hat{x}_{3,f})-\frac{\partial\alpha_1}{\partial x_1}(e_2\\&\quad+\kappa_{1,p}+\Delta F_{1,p})+B_2+w_{2,p} \quad (33)\end{aligned}$$

where $B_2 = \theta_2^T\varphi_2(\hat{\bar{x}}_2,\hat{x}_{3,f})+h_{2,p}e_1-\frac{\partial\alpha_1}{\partial x_1}(\hat{x}_2+\theta_1^T\varphi_1(\hat{\bar{x}}_1,\hat{x}_{2,f}))-\frac{\partial\alpha_1}{\partial\hat{x}_1}\dot{\hat{x}}_1-\frac{\partial\alpha_1}{\partial\theta_1}\dot{\theta}_1-\frac{\partial\alpha_1}{\partial\hat{\delta}_1}\dot{\hat{\delta}}_1$.

Define the following Lyapunov function

$$V_{2,p}=V_{1,j}+\frac{1}{2}\chi_2^2+\frac{1}{2\eta_2}\tilde{\theta}_2^T\tilde{\theta}_2+\frac{1}{2\bar{\eta}_2}\tilde{w}_2^2 \quad (34)$$

where $\hat{w}_i$ denote the evaluations of w_i^*, in addition, $\tilde{w}_i = w_i^*-\hat{w}_i$.

From (33) and (34), one has

$$\begin{aligned}\dot{V}_{2,p} &= \dot{V}_{1,p}+\chi_2\dot{\chi}_2+\frac{1}{\eta_2}\tilde{\theta}_2^T\dot{\tilde{\theta}}_2+\frac{1}{\bar{\eta}_2}\tilde{w}_2\dot{\tilde{w}}_2\\&\le \dot{V}_{1,p}+\chi_2(\chi_3+\alpha_2+\tilde{\theta}_2^T\varphi_2(\hat{\bar{x}}_2,\hat{x}_{3,f})\\&\quad-\frac{\partial\alpha_1}{\partial x_1}(e_2+\kappa_{1,p}+\Delta F_{1,p})+B_2)\\&\quad+|\chi_2|\,w_2^*+\frac{1}{\eta_2}\tilde{\theta}_2^T\dot{\tilde{\theta}}_2+\frac{1}{\bar{\eta}_2}\tilde{w}_2\dot{\tilde{w}}_2. \quad (35)\end{aligned}$$

Building upon Young's inequality, one has

$$\begin{aligned}-\chi_2\frac{\partial\alpha_1}{\partial x_1}(e_2+\kappa_{1,p}+\Delta F_{1,p}) &\le \frac{3}{2}\chi_2^2(\frac{\partial\alpha_1}{\partial x_1})^2+\frac{1}{2}\|e\|^2\\+\frac{1}{2}\kappa_1^{*2}+\frac{1}{2}\epsilon_1^2(\|e\|^2+g_1^2). \quad (36)\end{aligned}$$

With the help of Lemma 2, (31) and (36), we have

$$\begin{aligned}\dot{V}_{2,p} &\le -(\beta_1-\frac{1}{2}-\frac{1}{2}\pi_1^2)\|e\|^2-b_1\chi_1^2+\chi_1\chi_2\\&\quad+\frac{\gamma_1}{\eta_1}\tilde{\theta}_1^T\theta_1+\frac{\bar{\gamma}_1}{\bar{\eta}_1}\tilde{\delta}_1\hat{\delta}_1+M_1+\chi_2[\chi_3+\alpha_2\\&\quad+B_2+\frac{3}{2}\chi_2(\frac{\partial\alpha_1}{\partial x_1})^2+\hat{w}_2\tanh(\frac{\chi_2}{a_2})]\\&\quad+w_2^*a_{20}+\frac{1}{\eta_2}\tilde{\theta}_2^T(\eta_2\chi_2\varphi_2(\hat{\bar{x}}_2,\hat{x}_{3,f})-\dot{\theta}_2)\\&\quad+\frac{1}{\bar{\eta}_2}\tilde{w}_2(\bar{\eta}_2\chi_2\tanh(\frac{\chi_2}{a_2})-\dot{\hat{w}}_2)+\frac{1}{2}\epsilon_1^2g_1^2\\&\quad+\frac{1}{2}\kappa_1^{*2}. \quad (37)\end{aligned}$$

Design the following signals

$$\begin{aligned}\alpha_2 &= -b_2\chi_2-\chi_1-B_2-\frac{3}{2}\chi_2(\frac{\partial\alpha_1}{\partial x_1})^2\\&\quad-\hat{w}_2\tanh(\frac{\chi_2}{a_2}) \quad (38)\\\dot{\theta}_2 &= \eta_2\chi_2\varphi_2(\hat{\bar{x}}_2,\hat{x}_{3,f})-\gamma_2\theta_2 \quad (39)\\\dot{\hat{w}}_2 &= \bar{\eta}_2\chi_2\tanh(\frac{\chi_2}{a_2})-\bar{\gamma}_2\hat{w}_2. \quad (40)\end{aligned}$$

(37) can be further summarized as

$$\begin{aligned}\dot{V}_{2,p} &\le -\beta_2\|e\|^2+M_2-\sum_{i=1}^{2}b_i\chi_i^2+\chi_2\chi_3\\&\quad+\sum_{i=1}^{2}\frac{\gamma_i}{\eta_i}\tilde{\theta}_i^T\theta_i+\frac{\bar{\gamma}_1}{\bar{\eta}_1}\tilde{\delta}_1\hat{\delta}_1+\frac{\bar{\gamma}_2}{\bar{\eta}_2}\tilde{w}_2\hat{w}_2, \quad (41)\end{aligned}$$

where $\beta_2=\beta_1-1/2-\epsilon_1^2/2$, $M_2=M_1+w_2^*a_{20}+\frac{1}{2}\epsilon_1^2g_1^2+\frac{1}{2}\kappa_1^{*2}$.

Step n:

Guided by previous design steps, we have

$$\begin{aligned}\dot{\alpha}_{n-1} &= \frac{\partial\alpha_{n-1}}{\partial x_1}(\hat{x}_2+e_2+\theta_1^T\varphi_1(\hat{\bar{x}}_1,\hat{x}_{2,f})\\&\quad+\kappa_{2,p}+\Delta F_{1,p})+\sum_{i=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial\hat{x}_i}\dot{\hat{x}}_i\\&\quad+\sum_{i=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial\theta_i}\dot{\theta}_i+\frac{\partial\alpha_{n-1}}{\partial\hat{\delta}_1}\dot{\hat{\delta}}_1\\&\quad+\sum_{i=2}^{n-1}\frac{\partial\alpha_{n-1}}{\partial\hat{w}_i}\dot{\hat{w}}_i. \quad (42)\end{aligned}$$

From (7) we have $|v(t)-u(t)|\le\varsigma$ in the interval $[t_k,t_{k+1})$. Therefore, there is a time-varying argument $\varpi(t)$, with $\varpi(t_k)=0,\varpi(t_{k+1})=\pm1$ and $|\varpi(t)|\le1,\forall t\in[t_k,t_{k+1})$, so that $v(t)=u(t)+\varpi(t)\varsigma$.

From (22), (42) and above analysis, we have

$$\begin{aligned}\dot{\chi}_n &= v(t)-\varpi(t)\varsigma+\tilde{\theta}_n^T\varphi_n(\hat{\bar{x}}_n,u_f)-\frac{\partial\alpha_{n-1}}{\partial x_1}(e_2\\&\quad+\kappa_{1,p}+\Delta F_{1,p})+B_n+w_{n,p} \quad (43)\end{aligned}$$

where $B_n=h_{n,p}e_1+\theta_n^T\varphi_n(\hat{\bar{x}}_n,u_f)-\sum_{i=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial\hat{x}_i}\dot{\hat{x}}_i-\sum_{i=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial\theta_i}\dot{\theta}_i-\frac{\partial\alpha_{n-1}}{\partial\hat{\delta}_1}\dot{\hat{\delta}}_1-\sum_{i=2}^{n-1}\frac{\partial\alpha_{n-1}}{\partial\hat{w}_i}\dot{\hat{w}}_i$ -$\frac{\partial\alpha_{n-1}}{\partial x_1}(\hat{x}_2+\theta_1^T\varphi_1(\hat{\bar{x}}_1,\hat{x}_{2,f}))$.

State the Lyapunov function $V_{n,p}=V_{n-1,p}+\frac{1}{2}\chi_n^2+\frac{1}{2\eta_n}\tilde{\theta}_n^T\tilde{\theta}_n+\frac{1}{2\bar{\eta}_n}\tilde{w}_n^2$.

We can obtain $\dot{V}_{n,p}$ on the basis of (43)

$$\begin{aligned}\dot{V}_{n,p} &\le \dot{V}_{n-1,p}+\chi_n[v(t)-\varpi(t)\varsigma+B_n\\&\quad+\tilde{\theta}_n^T\varphi_n(\hat{\bar{x}}_n,u_f)-\frac{\partial\alpha_{n-1}}{\partial x_1}(e_2+\kappa_{1,p}\\&\quad+\Delta F_{1,p})]+|\chi_n|\,w_n^*+\frac{1}{\eta_n}\tilde{\theta}_n^T\dot{\tilde{\theta}}_n\\&\quad+\frac{1}{\bar{\eta}_n}\tilde{w}_n\dot{\tilde{w}}_n. \quad (44)\end{aligned}$$

Along with Young's inequality, we possess

$$\begin{aligned}-\chi_n\frac{\partial\alpha_{n-1}}{\partial x_1}(e_2+\kappa_{1,p}+\Delta F_{1,p}) &\le \frac{3}{2}\chi_n^2(\frac{\partial\alpha_{n-1}}{\partial x_1})^2+\\\frac{1}{2}\|e\|^2+\frac{1}{2}\kappa_1^{*2}+\frac{1}{2}\epsilon_1^2(\|e\|^2+g_1^2). \quad (45)\end{aligned}$$

Then, we can deduce that

$$\begin{aligned}\dot{V}_{n,p} \leq & -(\beta_1-(n-1)(\frac{1}{2}+\frac{1}{2}\epsilon_1^2))\|e\|^2 \\ & -\sum_{i=1}^{n-1} b_i\chi_i^2+\chi_{n-1}\chi_n+\sum_{i=1}^{n-1}\frac{\gamma_i}{\eta_i}\tilde{\theta}_i^T\theta_i \\ & +\frac{\bar{\gamma}_1}{\bar{\eta}_1}\tilde{\delta}_1\hat{\delta}_1+\sum_{i=2}^{n-1}\frac{\bar{\gamma}_i}{\bar{\eta}_i}\tilde{w}_i\hat{w}_i+M_{n-1} \\ & +\chi_n[\alpha_n-\bar{\varsigma}\tanh(\frac{\chi_n\bar{\varsigma}}{l})-\varpi(t)\varsigma \\ & +B_n+\frac{3}{2}\chi_n(\frac{\partial\alpha_{n-1}}{\partial x_1})^2+\hat{w}_n\tanh(\frac{\chi_n}{a_n})] \\ & +w_n^*a_{n0}+\frac{1}{\eta_n}\tilde{\theta}_n^T(\eta_n\chi_n\varphi_n(\hat{\bar{x}}_n,u_f)-\dot{\theta}_n) \\ & +\frac{1}{\bar{\eta}_n}\tilde{w}_n(\bar{\eta}_n\chi_n\tanh(\frac{\chi_n}{a_n})-\dot{\hat{w}}_n)+\frac{1}{2}\epsilon_1^2g_1^2 \\ & +\frac{1}{2}\kappa_1^{*2}. \end{aligned} \quad (46)$$

To proceed, it is necessary to opt for the adaptive laws of θ_n, virtual control signal α_n, and estimated value of $\hat{w}_n$

$$\alpha_n = -b_n\chi_n-\chi_{n-1}-B_n-\frac{3}{2}\chi_n(\frac{\partial\alpha_{n-1}}{\partial x_1})^2 -\hat{w}_n\tanh(\frac{\chi_n}{a_n}) \quad (47)$$

$$\dot{\theta}_n = \eta_n\chi_n\varphi_n(\hat{\bar{x}}_n,u_f)-\gamma_n\theta_n \quad (48)$$

$$\dot{\hat{w}}_n = \bar{\eta}_n\chi_n\tanh(\frac{\chi_n}{a_n})-\bar{\gamma}_n\hat{w}_n. \quad (49)$$

Applying (47)-(49) and Lemma 2, we possess

$$\begin{aligned}\dot{V}_{n,p} \leq & -\beta_n\|e\|^2+M_n-\sum_{i=1}^{n}b_i\chi_i^2+\sum_{i=1}^{n}\frac{\gamma_i}{\eta_i}\tilde{\theta}_i^T\theta_i \\ & +\frac{\bar{\gamma}_1}{\bar{\eta}_1}\tilde{\delta}_1\hat{\delta}_1+\sum_{i=2}^{n}\frac{\bar{\gamma}_i}{\bar{\eta}_i}\tilde{w}_i\hat{w}_i+l_0, \end{aligned} \quad (50)$$

where $\beta_n=\beta_1-(n-1)(1/2+\epsilon_1^2/2)$, $M_n=M_1+(n-1)(\frac{1}{2}\epsilon_1^2g_1^2+\frac{1}{2}\kappa_1^{*2})+\sum_{i=2}^{n}w_i^*a_{i0}$.

Based on Young's inequality, we can deduce that

$$\frac{\gamma_i}{\eta_i}\tilde{\theta}_i^T\theta_i \leq -\frac{1}{2}\frac{\gamma_i}{\eta_i}\tilde{\theta}_i^T\tilde{\theta}_i+\frac{1}{2}\frac{\gamma_i}{\eta_i}|\theta_i^*|^2 \quad (51)$$

$$\frac{\bar{\gamma}_1}{\bar{\eta}_1}\tilde{\delta}_1\hat{\delta}_1 \leq -\frac{1}{2}\frac{\bar{\gamma}_1}{\bar{\eta}_1}\tilde{\delta}_1^2+\frac{1}{2}\frac{\bar{\gamma}_1}{\bar{\eta}_1}\delta_1^{*2} \quad (52)$$

$$\frac{\bar{\gamma}_i}{\bar{\eta}_i}\tilde{w}_i\hat{w}_i \leq -\frac{1}{2}\frac{\bar{\gamma}_i}{\bar{\eta}_i}\tilde{w}_i^2+\frac{1}{2}\frac{\bar{\gamma}_i}{\bar{\eta}_i}w_i^{*2}. \quad (53)$$

From (51)-(53), (50) can be changed into

$$\begin{aligned}\dot{V}_{n,p} \leq & -\beta_n\|e\|^2+M_n+l_0-\sum_{i=1}^{n}b_i\chi_i^2 \\ & -\sum_{i=1}^{n}\frac{\gamma_i}{2\eta_i}\tilde{\theta}_i^T\tilde{\theta}_i-\frac{\bar{\gamma}_1}{2\bar{\eta}_1}\tilde{\delta}_1^2-\sum_{i=2}^{n}\frac{\bar{\gamma}_i}{2\bar{\eta}_i}\tilde{w}_i^2 \\ & +\sum_{i=1}^{n}\frac{\gamma_i}{2\eta_i}|\theta_i^*|^2+\frac{\bar{\gamma}_1}{2\bar{\eta}_1}\delta_1^{*2} \\ & +\sum_{i=2}^{n}\frac{\bar{\gamma}_i}{2\bar{\eta}_i}w_i^{*2}, \end{aligned} \quad (54)$$

We finally have

$$\dot{V}_{n,p} \leq -\varepsilon V_{n,p}+\phi, \quad (55)$$

where $\varepsilon = \min\left\{\frac{2\beta_n}{\lambda_{\max}(P_p)}, 2b_i, \gamma_i, \bar{\gamma}_i, 1\leq i\leq n\right\}$, $\phi = M_n+l_0+\sum_{i=1}^{n}\frac{\gamma_i}{2\eta_i}|\theta_i^*|^2+\frac{\bar{\gamma}_1}{2\bar{\eta}_1}\delta_1^{*2}+\sum_{i=2}^{n}\frac{\bar{\gamma}_i}{2\bar{\eta}_i}w_i^{*2}$.

Theorem 1: If the fault-free system (10) and the observer (17) satisfy Assumption 1 and 2. According to the bounded initial circumstances, the designed virtual controllers (28), (47), the adaptive laws (30), (48), (49), and the actual controller (6) can make all of closed-loop system signals bounded within a category of switching signals with an ADT

$$\tau_a > \frac{\log\mu}{\varepsilon} \quad (56)$$

and

$$\mu = \max\left\{\frac{\lambda_{\max}(P_p)}{\lambda_{\min}(P_k)}, p,k\in S\right\}. \quad (57)$$

Moreover, the event trigger intervals $\{t_{k+1}-t_k\}$ has a lower bound $t^*>0$, $\forall k\in z^+$.

Proof:

Choosing the nether Lyapunov function

$$\begin{aligned}V_p(\chi) = & e^TP_pe+\frac{1}{2}\sum_{i=1}^{n}\chi_i^2+\sum_{i=1}^{n}\frac{1}{2\eta_i}\tilde{\theta}_i^T\tilde{\theta}_i \\ & +\frac{1}{2\bar{\eta}_1}\tilde{\delta}_1^2+\sum_{i=2}^{n}\frac{1}{2\bar{\eta}_i}\tilde{w}_i^2 \end{aligned} \quad (58)$$

where $\chi=[e^T,\chi_1,\cdots,\chi_n,\tilde{\theta}_1,\cdots,\tilde{\theta}_n,\tilde{\delta}_1,\tilde{w}_2,\cdots,\tilde{w}_n]$, according to (55) one has

$$\dot{V}_p \leq -\varepsilon V_p+\phi.$$

To explain the stability of the system (10) through ADT condition. Define the following function

$$F(t) = e^{\varepsilon t}V_{\sigma(t)}(t). \quad (59)$$

On each interval $[t_k,t_{k+1})$, it can be determined the time derivative of $F(t)$

$$\begin{aligned}\dot{F}(t) = & \varepsilon e^{\varepsilon t}V_{\sigma(t)}(t)+e^{\varepsilon t}\dot{V}_{\sigma(t)}(t) \\ \leq & \phi e^{\varepsilon t}. \end{aligned}$$

From (57), we can get $V_p(t)\leq\mu V_k(t)$, based on which, one has

$$\begin{aligned}F(t_{k+1}) = & e^{\varepsilon t_{k+1}}V_{\sigma(t_{k+1})}(t_{k+1}) \\ \leq & \mu e^{\varepsilon t_{k+1}}V_{\sigma(t_k)}(t_{k+1}) = \mu F(t_{k+1}^-) \\ \leq & \mu[F(t_k)+\int_{t_k}^{t_{k+1}}\phi e^{\varepsilon t}dt]. \end{aligned}$$

Similar to the procedure in [19], for any $\varepsilon_a\in[0,\varepsilon-(\log\mu/\tau_a)]$, since $\tau_a>(\log\mu/\varepsilon)$, one has

$$F(T^-) \leq \mu^{N_\sigma(T,0)}F(0)+\mu^{1+N_0}e^{(\varepsilon-\varepsilon_a)T}\int_0^T\phi e^{\varepsilon_a t}dt,$$

on the basis of (59), it can be deduced as

$$\begin{aligned}e^{\varepsilon T}V_{\sigma(T^-)}(T^-) \leq & \mu^{N_\sigma(T,0)}V_{\sigma(0)}(0) \\ & +\mu^{1+N_0}e^{(\varepsilon-\varepsilon_a)T}\int_0^T\phi e^{\varepsilon_a t}dt. \end{aligned}$$

It is not stiff to known that $\underline{\omega}(\|\chi\|) \leq V(\chi) \leq \bar{\omega}(\|\chi\|)$, and $\underline{\omega}$, $\bar{\omega}$ are k_∞ functions. Therefore, we finally have

$$\begin{aligned} \underline{\omega}(T) &\leq V_{\sigma(T^-)}(T^-) \\ &\leq e^{-\varepsilon T}\mu^{N_\sigma(T,0)}\bar{\omega}(0) + \mu^{1+N_0}e^{-\varepsilon_a T}\int_0^T \phi e^{\varepsilon_a t}dt \\ &\leq e^{\log \mu}e^{(\log \mu/\tau_a - \varepsilon)T}\bar{\omega}(0) + \frac{\phi}{\varepsilon_a}\mu^{1+N_0}. \quad (60) \end{aligned}$$

With the help of (58) and (60), we establish the boundedness of e, χ_i, $\tilde{\theta}_i$, $\tilde{\delta}_1$ and $\tilde{w}_i$, by means of a the switching signal $\sigma(t)$, which is designed using $\tau_a > \log \mu/\varepsilon$. Because $\theta_i^*, \delta_1^*, w_i^*$ are constants, building upon $\tilde{\theta}_i = \theta_i^* - \theta_i$, $\tilde{\delta}_1 = \delta_1^* - \hat{\delta}_1$ and $\tilde{w}_i = w_i^* - \hat{w}_i$, their estimations θ_i, $\hat{\delta}_1$, $\hat{w}_i$ are bounded. After that, the physical controller u, along with the virtual control signals α_i, remains bounded due to these circumstances. $\hat{x}_i$ are bounded since (22), so that x_i are bounded. As a result, in this situation, all the signals in this closed-loop system are bounded for bounded initial values.

There is a time $t^* > 0$ such that for all $\forall k \in z^+, \{t_{k+1} - t_k\} \geq t^*$. To prove this, we review $\Gamma(t) = v(t) - u(t), \forall t \in [t_k, t_{k+1})$, and have

$$\frac{d|\Gamma|}{dt} = \frac{d}{dt}(\Gamma * \Gamma)^{\frac{1}{2}} = sign(\Gamma)\dot{\Gamma} \leq |\dot{v}|. \quad (61)$$

From (5), we can easily attain that $\dot{v}$ is a bounded function, so a constant ι exists with $|\dot{v}| < \iota$. By pointing out that $\Gamma(t_k) = 0$ and $\lim_{t\to t_{k+1}} \Gamma(t) = \varsigma$, we known that the lower bound t^* of event trigger intervals satisfies $t^* \geq \varsigma/\iota > 0$. Therefore, there is no Zeno behavior.

This finished the proof of Theorem 1.

Similar to [4], select the following evaluation function

$$L = \sqrt{\int_{t_0}^{t_0+t} e_1^T(\tau)e_1(\tau)d\tau}.$$

Design the following threshold

$$L_{th} = \sup_{d=0} L.$$

The following decision logic is used to detect the fault

$$\begin{cases} L \leq L_{th}, & Fault - free \\ L > L_{th}, & Faulty. \end{cases}$$

In this article, we assume that the states are not measurable, which means that the evaluation function L and the threshold L_{th} are determined solely based on the available output error $e_1(t)$.

IV. Event-triggered active FTC control design

In this slice, we will combine backstepping technology with fixed-threshold event-triggered strategy to devise an active fault-tolerant controller.

The system (1) in faulty case can be described as

$$\begin{cases} \dot{x}_i = f_{i,\sigma(t)}\left(\bar{x}_i, x_{i+1}\right), 1 \leq i \leq n-1 \\ \dot{x}_n = f_{n,\sigma(t)}\left(\bar{x}_n, u\right) + d_{\sigma(t)}(\bar{x}_n, u) \\ y = x_1. \end{cases} \quad (62)$$

In the light of the previous design process, system (62) changes into

$$\begin{cases} \dot{x}_i = x_{i+1} + F_{i,p}\left(\hat{\bar{x}}_i, \hat{x}_{i+1,f}\right) + \Delta F_{i,p} \\ \dot{x}_n = u + F_{n,p}\left(\hat{\bar{x}}_n, u_f\right) + d_p(\hat{\bar{x}}_n, u_f) + \Delta F_{n,p} + \Delta d_p \\ y = x_1, \end{cases} \quad (63)$$

where $\Delta d_p = d_p(\bar{x}_n, u) - d_p(\hat{\bar{x}}_n, u_f)$.

Then, let $\hat{x}_{n+1,f} = u_f$, one has

$$\begin{aligned} \dot{\bar{x}}_n &= C_p'\bar{x}_n + H_p'y + \sum_{i=1}^n Z_i[F_{i,p}\left(\hat{\bar{x}}_i, \hat{x}_{i+1,f}\right) \\ &\quad + \Delta F_{i,p}] + Z_n[u + d_p(\hat{\bar{x}}_n, u_f) + \Delta d_p] \quad (64) \end{aligned}$$

where $C_p' = \begin{bmatrix} -h_{1,p}' & & \\ \vdots & I_{n-1} & \\ -h_{n,p}' & \cdots & 0 \end{bmatrix}$, $H_p' = \begin{bmatrix} h_{1,p}' \\ h_{2,p}' \\ \vdots \\ h_{n,p}' \end{bmatrix}$.

Design the parameters $h_{i,p}'$, $1 \leq i \leq n$, $p \in S$, to make matrix C_p' a strict Hurwitz matrix. Then, for any given matrix $Q_p' > 0$, which is a define symmetric matrix, there is a positive matrix P_p' to make

$$C_p'^T P_p' + P_p' C_p' \leq -Q_p'. \quad (65)$$

A sate observer is modeled

$$\begin{cases} \dot{\hat{x}}_i' = \hat{x}_{i+1}' + \hat{F}_{i,p}'\left(\hat{\bar{x}}_i', \hat{x}_{i+1,f}' \middle| \theta_{i,p}'\right) + h_{i,p}'(y - \hat{x}_1') \\ \dot{\hat{x}}_n' = u' + \hat{F}_{n,p}'\left(\hat{\bar{x}}_n', u_f' \middle| \theta_{n,p}'\right) + \hat{d}_p(\hat{\bar{x}}_n', u_f' \left|\theta_{d,p}\right.) \\ +d_{n,p}'(y - \hat{x}_1') \\ \hat{y}' = \hat{x}_1' \end{cases} \quad (66)$$

where $\hat{x}_i'$, $\hat{F}_{i,p}'\left(\hat{\bar{x}}_i', \hat{x}_{i+1,f}' \middle| \theta_{i,,p}'\right)$, $\hat{d}_p(\hat{\bar{x}}_n', u_f' \left|\theta_{d,p}'\right.)$, $1 \leq i \leq n$, $p \in S$, are the estimations of x_i, $F_{i,p}(\bar{x}_i, x_{i+1,f})$, $d_p(\bar{x}_n, u_f)$, respectively.

The unknown functions $d_p(\bar{x}_n, u_f)$, $\hat{d}_p(\hat{\bar{x}}_n', u_f')$ are both approximated by FLSs

$$\hat{d}_p(\hat{\bar{x}}_n, u_f \left|\theta_{d,p}\right.) = \theta_{d,p}^T\varphi_d(\hat{\bar{x}}_n, u_f) \quad (67)$$

$$\hat{d}_p(\hat{\bar{x}}_n', u_f' \left|\theta_{d,p}\right.) = \theta_{d,p}^T\varphi_d(\hat{\bar{x}}_n', u_f'). \quad (68)$$

Define the following optimal parameter vector

$$\theta_{d,p}^* = \arg\min_{\theta_{d,p}\in\Xi_d}[\sup_{(\hat{\bar{x}}_n, u_f)\in\Phi_d} |d_p(\hat{\bar{x}}_n, u_f) - \hat{d}_p(\hat{\bar{x}}_n', u_f' \left|\theta_{d,p}\right.)|] \quad (69)$$

where Ξ_d, Φ_d are compact sets for $\theta_{d,p}$, $(\hat{\bar{x}}_n, u_f)$. Besides, define the following evaluated errors

$$\begin{aligned} \kappa_{d,p} &= d_p(\hat{\bar{x}}_n, u_f) - \hat{d}_p(\hat{\bar{x}}_n', u_f' \left|\theta_{d,p}\right.) \quad (70) \\ \delta_{d,p} &= d_p(\hat{\bar{x}}_n, u_f) - \hat{d}_p(\hat{\bar{x}}_n', u_f' \left|\theta_{d,p}^*\right.). \quad (71) \end{aligned}$$

Define the observation error as $e' = [e_1', e_2', \cdots e_n']^T$, where $e_i' = x_i - \hat{x}_i'$, $1 \leq i \leq n$. So we have

$$\dot{e}' = C_p'e' + \Delta F_p' + Z_n\Delta d_p + \kappa_p' + Z_n\kappa_{d,p}.$$

Select the Lyapunov function $V_{0,p}' = e'^T P_p' e'$, and $\dot{V}_{0,p}'$ is going to be obtained as

$$\begin{aligned} \dot{V}_{0,p}' &\leq -Q_p'\left\|e'\right\|^2 + 2e'^T P_p'\Delta F_p' + 2e'^T P_p' Z_n\Delta d_p \\ &\quad + 2e'^T P_p'\kappa_p' + 2e'^T P_p' Z_n\kappa_{d,p}. \quad (72) \end{aligned}$$

Based on Young's inequality, Assumption 1 and Assumption 2, we have

$$2e^{'T}P_p^{'}\Delta F_p^{'} \leq \left\|e^{'}\right\|^2 + \left\|P_p^{'}\right\|^2 \sum_{i=1}^{n} \epsilon_i^2(\left\|e^{'}\right\|^2 + g_i^2) \quad (73)$$

$$2e^{'T}P_p^{'}B_n\Delta d_p \leq \left\|e^{'}\right\|^2 + \left\|P_p^{'}\right\|^2 \left\|\Delta d_p\right\|^2 \quad (74)$$

$$2e^{'T}P_p^{'}\kappa_p^{'} \leq \left\|e^{'}\right\|^2 + \left\|P_p^{'}\right\|^2 \left\|\kappa_p^{'*}\right\|^2 \quad (75)$$

$$2e^{'T}P_p^{'}Z_n\kappa_{d,p} \leq \left\|e^{'}\right\|^2 + \left\|P_p^{'}\right\|^2 \left\|\kappa_d^{*}\right\|^2. \quad (76)$$

Substituting (73)-(76) into (72) yields

$$\dot{V}_{0,p}^{'} \leq -\beta^{'}\left\|e^{'}\right\|^2 + M^{'}, \quad (77)$$

where $\beta^{'} = \lambda_{\min}(Q_p^{'}) - 4 - \left\|P_p^{'}\right\|^2 \sum_{i=1}^{n}\epsilon_i^2$, $M^{'} = \left\|P_p^{'}\right\|^2 [\sum_{i=1}^{n}\epsilon_i^2 g_i^2 + \|\Delta d_p\|^2 + \left\|\kappa_p^{'*}\right\|^2 + \|\kappa_d^*\|^2]$.

According to the design procedure in fault-free case, we can easily get the results of the final step of the backstepping method. We first let θ_d^* represent $\theta_{d,p}^*$.

Design the following virtual controllers and intermediate control function with event-triggering mechanism (6)-(7)

$$\begin{aligned}\alpha_1^{'} &= -b_1^{'}\chi_1^{'} - \chi_1^{'} - \theta_1^{'T}\varphi_1^{'}(\hat{\bar{x}}_1^{'}, \hat{x}_{2,f}^{'}) \\ &\quad -\hat{\delta}_1^{'}\tanh(\frac{\chi_1^{'}}{a_1^{'}}),\end{aligned} \quad (78)$$

$$\begin{aligned}\alpha_i^{'} &= -b_i^{'}\chi_i^{'} - \chi_{i-1}^{'} - B_i^{'} - \frac{3}{2}\chi_i^{'}(\frac{\partial\alpha_{i-1}^{'}}{\partial x_1^{'}})^2 \\ &\quad -\hat{w}_i^{'}\tanh(\frac{\chi_i^{'}}{a_i^{'}}),\end{aligned} \quad (79)$$

$$\begin{aligned}\alpha_n^{'} &= -b_n^{'}\chi_n^{'} - \chi_{n-1}^{'} - B_n^{'} - \frac{3}{2}\chi_n^{'}(\frac{\partial\alpha_{n-1}^{'}}{\partial x_1^{'}})^2 \\ &\quad -\hat{w}_n^{'}\tanh(\frac{\chi_n^{'}}{a_n^{'}}) - \hat{w}_d\tanh(\frac{\chi_n^{'}}{a_d}),\end{aligned} \quad (80)$$

$$v(t)' = \alpha_n' - \varsigma'\tanh(\frac{\chi_n^{'}\bar{\varsigma}'}{l'}),$$

where $B_n^{'} = h_{n,p}^{'}e_1^{'} + \theta_n^{'T}\varphi_n^{'}(\hat{\bar{x}}_n^{'}, u_f^{'}) + \theta_d^{'T}\varphi_d^{'}(\hat{\bar{x}}_n^{'}, u_f^{'}) - \sum_{i=1}^{n-1}\frac{\partial\alpha_{n-1}^{'}}{\partial\hat{x}_i^{'}}\dot{\hat{x}}_i^{'} - \sum_{i=1}^{n-1}\frac{\partial\alpha_{n-1}^{'}}{\partial\theta_i^{'}}\dot{\theta}_i^{'} - \frac{\partial\alpha_{n-1}^{'}}{\partial\hat{\delta}_1^{'}}\dot{\hat{\delta}}_1^{'} - \sum_{i=2}^{n-1}\frac{\partial\alpha_{n-1}^{'}}{\partial\hat{w}_i^{'}}\dot{\hat{w}}_i^{'}$ - $\frac{\partial\alpha_{n-1}^{'}}{\partial x_1^{'}}(\hat{x}_2^{'} + \theta_1^{'T}\varphi_1^{'}(\hat{\bar{x}}_1^{'}, \hat{x}_{2,f}^{'}))$.

Design the adaptive laws below

$$\begin{aligned}\dot{\theta}_i^{'} &= \eta_i\chi_i^{'}\varphi_i^{'}(\hat{\bar{x}}_i^{'}, u_f^{'}) - \gamma_i^{'}\theta_i^{'}, i = 1, \cdots, n, &(81)\\ \dot{\theta}_d &= \eta_d\chi_n^{'}\varphi_d(\hat{\bar{x}}_n^{'}, u_f^{'}) - \gamma_d\theta_d, &(82)\\ \dot{\hat{\delta}}_1^{'} &= \bar{\eta}_1^{'}\chi_1^{'}\tanh(\frac{\chi_1^{'}}{a_1^{'}}) - \bar{\gamma}_1^{'}\hat{\delta}_1^{'}, &(83)\\ \dot{\hat{w}}_i^{'} &= \bar{\eta}_i^{'}\chi_i^{'}\tanh(\frac{\chi_i^{'}}{a_i^{'}}) - \bar{\gamma}_i^{'}\hat{w}_i^{'}, i = 2, \cdots, n, &(84)\\ \dot{\hat{w}}_d &= \bar{\eta}_d\chi_n^{'}\tanh(\frac{\chi_n^{'}}{a_d}) - \bar{\gamma}_d\hat{w}_d. &(85)\end{aligned}$$

Theorem 2: Under the Assumption 1 and 2, consider the faulty system (63) and observer (66). Given bounded initial circumstances, the adaptive laws (81)-(85), the virtual control signals (78)-(80), and the actual control input (6) ensure that every signals within the closed-loop system remain bounded with a family of switching signals constructed by the ADT

$$\tau_a^{'} = \frac{\log\mu'}{\varepsilon^{'}} \quad (86)$$

and

$$\mu' = \max\{\frac{\lambda_{\max}(P_p')}{\lambda_{\min}(P_k^{'})}, p, k \in S\}. \quad (87)$$

Moreover, the inter-execution intervals $\{t_{k+1} - t_k\}$ has a lower bounded $t^* > 0$, $\forall k \in z^+$.

Proof:

Define the following Lyapunov function

$$\begin{aligned}V_p^{'}(\chi^{'}) &= e^{'T}P_p^{'}e^{'} + \frac{1}{2}\sum_{i=1}^{n}\chi_i^{'2} + \sum_{i=1}^{n}\frac{1}{2\eta_i^{'}}\tilde{\theta}_i^{'T}\tilde{\theta}_i^{'} \\ &\quad + \frac{1}{2\eta_d}\tilde{\theta}_d^T\tilde{\theta}_d + \frac{1}{2\bar{\eta}_1^{'}}\tilde{\delta}_1^{'2} + \sum_{i=2}^{n}\frac{1}{2\bar{\eta}_i^{'}}\tilde{w}_i^{'2} \\ &\quad + \frac{1}{2\bar{\eta}_d}\tilde{w}_d^2,\end{aligned} \quad (88)$$

where $\chi^{'} = [e^{'T}, \chi_1^{'}, \cdots, \chi_n^{'}, \tilde{\theta}_1^{'}, \cdots, \tilde{\theta}_n^{'}, \tilde{\theta}_d, \tilde{\delta}_1^{'}, \tilde{w}_2^{'}, \cdots, \tilde{w}_n^{'}, \tilde{w}_d]$, $\tilde{\theta}_d = \theta_d^* - \theta_d$ stands for the error vector, $\hat{w}_d$ represents the evaluation of w_d^*, further more, $\tilde{w}_d = w_d^* - \hat{w}_d$.

Then, one has

$$V_p^{'} = -\varepsilon^{'}V_n^{'} + \phi^{'},$$

where $\varepsilon^{'} = \min\left\{\frac{2\beta_n^{'}}{\lambda_{\max}(P_p^{'})}, 2b_i^{'}, \gamma_i^{'}, \gamma_d, \bar{\gamma}_i^{'}, \bar{\gamma}_d, 1 \leq i \leq n\right\}$, $\phi^{'} = M_n^{'} + \sum_{i=1}^{n}\frac{\gamma_i^{'}}{2\eta_i^{'}}\left|\theta_i^{'*}\right|^2 + \frac{\bar{\gamma}_1^{'}}{2\bar{\eta}_1^{'}}\delta_1^{'*2} + \sum_{i=2}^{n}\frac{\bar{\gamma}_i^{'}}{2\eta_i^{'}}w_i^{'*2} + \frac{\gamma d}{2\eta_d}\left|\theta_d^*\right|^2 + \frac{\bar{\gamma}_d}{2\bar{\eta}_d}w_d^{*2} + l_0^{'}$, $M_n^{'} = M_1^{'} + (n-1)(\frac{1}{2}\epsilon_1^2 g_1^2 + \frac{1}{2}\kappa_1^{'*2}) + \sum_{i=2}^{n}w_i^{'*}a_{i0} + w_d^* a_{d0}$.

Then, design a function: $F^{'}(t) = e^{\varepsilon^{'}t}V_{\sigma(t)}'(t)$.

We can ultimately derive the nether inequality

$$\begin{aligned}\underline{\omega}^{'}(T) &\leq V_{\sigma(T^-)}^{'}(T^-) \\ &\leq e^{\log\mu^{'}}e^{(\log\mu^{'}/\tau_a^{'} - \varepsilon^{'})T}\bar{\omega}^{'}(0) \\ &\quad + \frac{\phi^{'}}{\varepsilon_a^{'}}\mu^{'1+N_0}.\end{aligned} \quad (89)$$

where $\underline{\omega}^{'}$ and $\bar{\omega}^{'}$ are k_∞ functions, so that $\underline{\omega}^{'}(\left\|\chi^{'}\right\|) \leq V^{'}(\chi^{'}) \leq \bar{\omega}^{'}(\left\|\chi^{'}\right\|)$.

According to (88) and (89), $e^{'}$, $\chi_i^{'}$, $\tilde{\theta}_i^{'}$, $\tilde{\theta}_d$, $\tilde{\delta}_1^{'}$, $\tilde{w}_i^{'}$ and $\tilde{w}_d$ are bounded with the switching signal $\sigma(t)$ designed by $\tau_a^{'} > \log\mu^{'}/\varepsilon^{'}$. Due to $\theta_i^{'*}$, θ_d^*, $\delta_1^{'*}$, $w_i^{'*}$, w_d^* are constants, their estimations $\theta_i^{'}$, θ_d, $\hat{\delta}_1^{'}$, $\hat{w}_i^{'}$, $\hat{w}_d$ are bounded. Consequently, the virtual controllers $\alpha_i^{'}$ as well as the event-triggered input u' are bounded. Additionally, $\hat{x}_i^{'}$ and x_i are bounded. Thus, even if there is a fault, all signals within the closed-loop system remain bounded for bounded initial values.

The proof of avoiding Zeno behavior is the same as before.

Theorem 2 has completed its proof.

V. Simulation

An illustrative numerical simulation is raised in this part to showcase the effectiveness of the built control approach. A second-order switched system is considered

$$\begin{cases} \dot{x}_1 = f_{1,\sigma(t)}(\bar{x}_1, x_2) \\ \dot{x}_2 = f_{2,\sigma(t)}(\bar{x}_2, u) + \hbar(t - t_\hbar) d_{\sigma(t)}(\bar{x}_2, u) \\ y = x_1 \end{cases} \tag{90}$$

where $\sigma(t) \in \{1, 2\}$, $f_{1,1} = x_1 + x_2 + x_2^3/7$, $f_{2,1} = x_1 x_2 + u^3/5$, $f_{1,2} = x_1 + \sin(x_2)$, $f_{2,2} = x_1 x_2 + u + \sin(u)$, $d_1 = 0.5 + x_2^2 + \sin(u)$, $d_2 = 1 + x_1^2 + \cos(u)$, The fault occurring time $t_\hbar = 32$.

Choose the following fault-free related parameters $\bar{\varsigma} = 20$, $l = 1$, $\varsigma = 1$, $b_1 = 0.15$, $b_2 = 0.25$, $a_1 = 10$, $a_2 = 0.2$, $\gamma_1 = \bar{\gamma}_1 = 1.5$, $\gamma_2 = 0.12$, $\bar{\gamma}_2 = 1.4$, $\eta_1 = 1$, $\bar{\eta}_1 = 4$, $\eta_2 = 3$, $\bar{\eta}_2 = 5$, $h_{1,1} = 12$, $h_{1,2} = 10$, $h_{2,1} = 1.6$, $h_{2,2} = 12$. Moreover, $x_1(0) = 0.5$, $x_2(0) = 0$, $\hat{x}_1(0) = 0.5$, $\hat{x}_2(0) = 0$. $\hat{\delta}_1(0) = 0.01$, $\hat{w}_2(0) = 0$, $\theta_1^T(0) = [0, 0, 0, 0, 0]^T$, $\theta_2^T(0) = [0, 0, 0, 0, 0.5]^T$. Additionally, $P_1 = \begin{bmatrix} 0.22 & -1 \\ -1 & 7.64 \end{bmatrix}$, $P_2 = \begin{bmatrix} 2.6 & -2 \\ -2 & 1.88 \end{bmatrix}$. Then, choose the average dwell time $\tau_a = 3$.

In faulty case, choose the following parameters $\bar{\varsigma}' = 9$, $l' = 0.1$, $\varsigma' = 0.1$, $b_1^{'} = 0.15$, $b_2^{'} = 0.25$, $a_1^{'} = 1$, $a_2^{'} = 0.2$, $a_d = 0.2$ $\gamma_1^{'} = \bar{\gamma}_1^{'} = 1.5$, $\gamma_2^{'} = 1.2$, $\bar{\gamma}_2^{'} = 1.4$, $\gamma_d = 1.5$, $\bar{\gamma}_d = 1.5$, $\eta_1^{'} = 1.2$, $\bar{\eta}_1^{'} = 3$, $\eta_2^{'} = 1.5$, $\bar{\eta}_2^{'} = 3.5$, $\eta_d = 2$, $\bar{\eta}_d = 3.5$, $h_{1,1}^{'} = 10$, $h_{1,2}^{'} = 10$, $h_{2,1}^{'} = 1$, $h_{2,2}^{'} = 15$. Meanwhile, $x_1^{'}(0) = 0.4$, $x_2^{'}(0) = 0$, $\hat{x}_1^{'}(0) = 0.4$, $\hat{x}_2^{'}(0) = 0$. $\hat{\delta}_1^{'}(0) = 0.01$, $\hat{w}_2^{'}(0) = 0$, $\hat{w}_d(0) = 0$, $\theta_1^{'T}(0) = [0, 0, 0, 0, 0]^T$, $\theta_2^{'T}(0) = [0, 0, 0, 0, 0.5]^T$, $\theta_d^T(0) = [0, 0, 0, 0, 0.5]^T$. The matrices $P_1^{'} = \begin{bmatrix} 0.2 & -1 \\ -1 & 10.2 \end{bmatrix}$, $P_2^{'} = \begin{bmatrix} 3.2 & -2 \\ -2 & 1.55 \end{bmatrix}$. Then, choose the average dwell time $\tau_a^{'} = 1.5$.

Figs. 1-10 display the simulation results, where Fig. 1 and Fig. 2 show the profiles of the values of states x_1, x_2 and their estimates $\hat{x}_1$, $\hat{x}_2$ in fault-free case. We can easily see that they all bounded and the observer performed well. Fig. 3 represents the trajectory of event-triggered input u and the in-process control signal v in fault-free case. The intervals between triggering events are displayed in Fig. 4 and Fig. 9, which mean that the control signals u and u' are intermittently conveyed. Additionally, the number of triggering events are 40 and 54, respectively. Fig. 5 and Fig. 10 describe the switching signal in fault-free case and faulty case, respectively. The faults occur at 32s. Figs. 6-7 show the system states x_1, x_2 and their evaluations $\hat{x}_1'$, $\hat{x}_2'$, obviously, the system states can be approximated even though there exist faults. Fig. 8 reveals the trajectory of event-triggered input u' and the intermediate control signal v' in FTC. Every signal within the system remain bounded while faults exist, as can be observed.

VI. Conclusion

The current study proposes an active event-triggered FTC technique for a category of switched nonlinear systems. Through Butterworth LPFs, this switched system is transformed from its pure-feedback form into an affine form. In order to estimate unknown nonlinear functions and errors, FLSs are employed. A backstepping technology-based event-triggered method is used to create an event-triggered FTC technique. The whole closed-loop system signals are guaranteed to stay bounded by the switching signals featuring an ADT under the suggested controller. The simulation findings attest to the viability of the proposed technique.

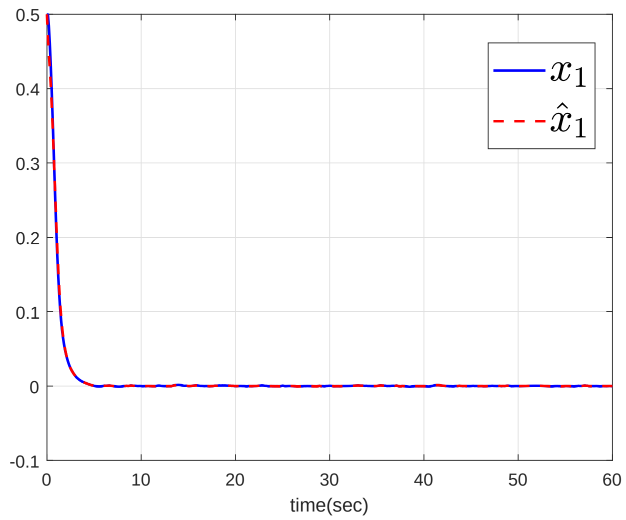

Fig. 1. The states of x_1 and $\hat{x}_1$ in fault-free situation

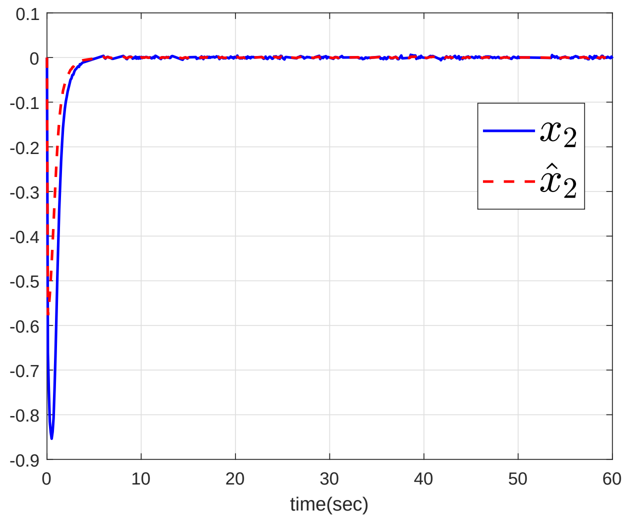

Fig. 2. The states of x_2 and $\hat{x}_2$ in fault-free situation

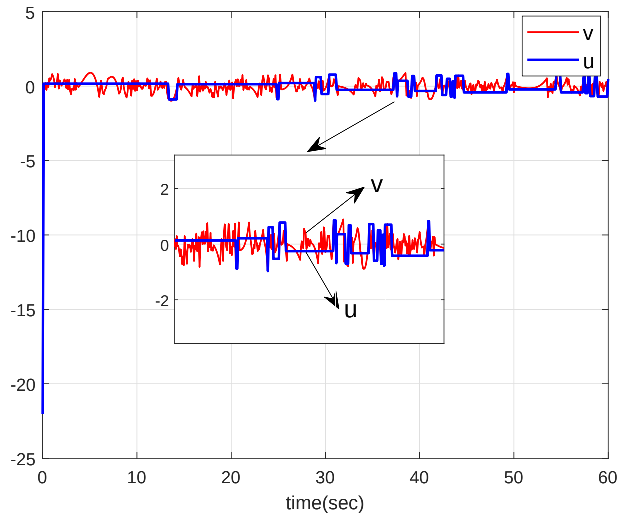

Fig. 3. The control input u and v in fault-free case

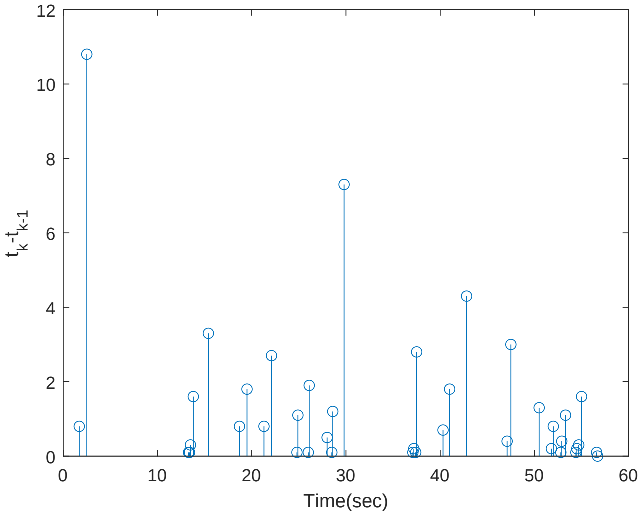

Fig. 4. Time intervals in fault-free case

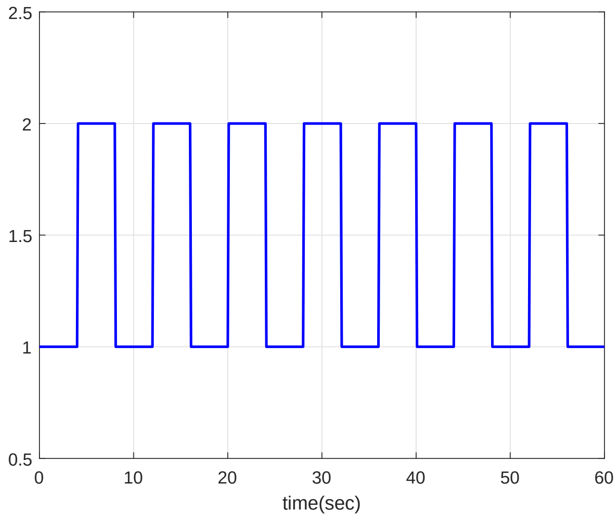

Fig. 5. Switching signal in fault-free case

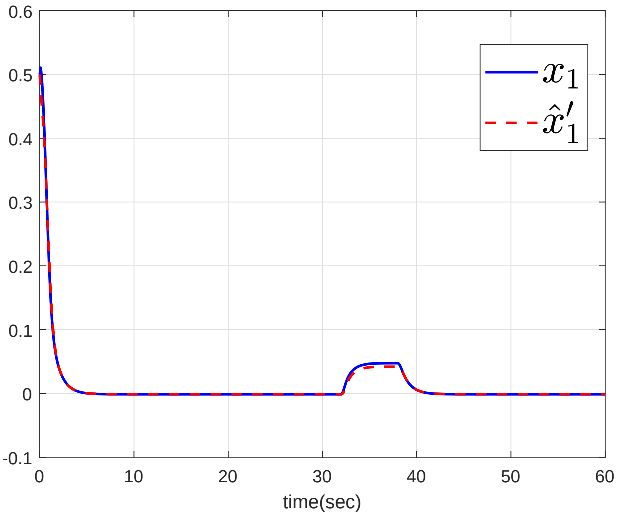

Fig. 6. The states of x_1 and $\hat{x}_1'$ in FTC

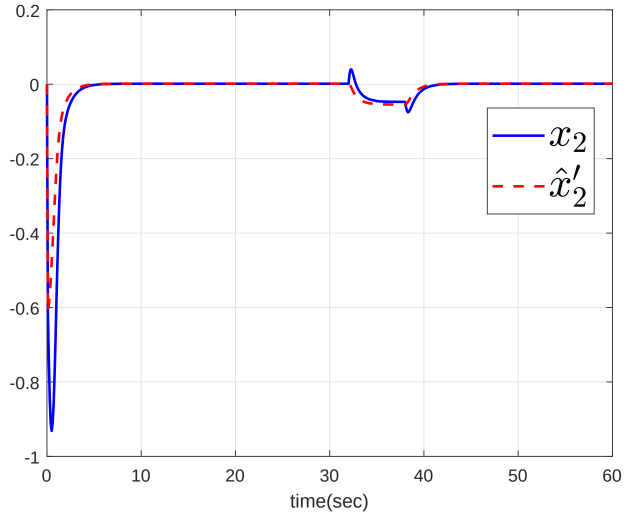

Fig. 7. The states of x_2 and $\hat{x}_2'$ in FTC

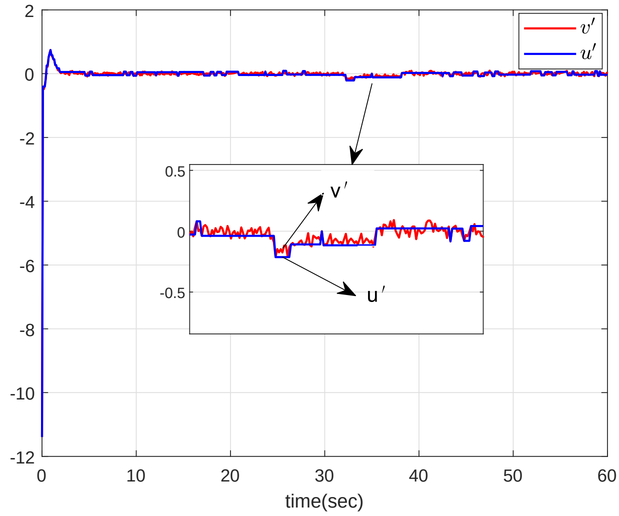

Fig. 8. The controller u' and v' in FTC

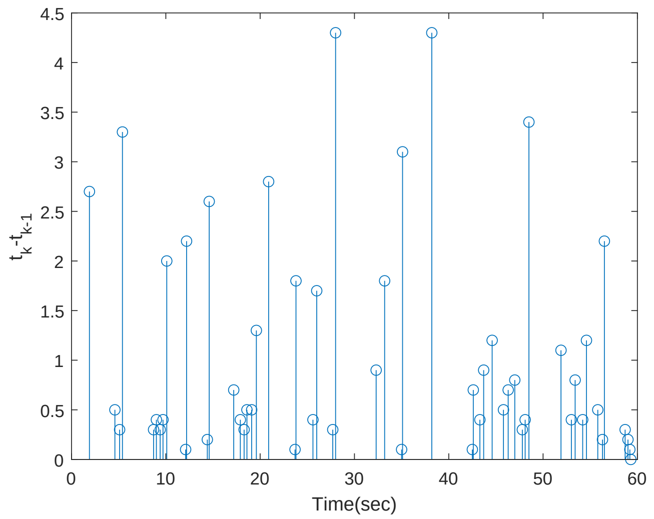

Fig. 9. Time intervals in FTC

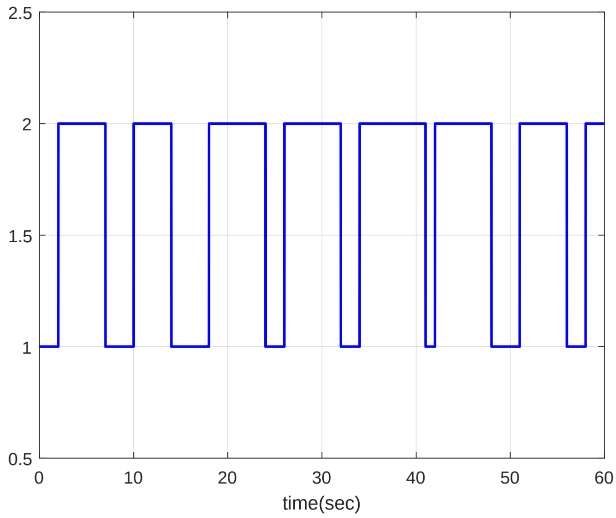

Fig. 10. Switching signal in FTC

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