Mathematical Model for Two-Submerged Breakwaters on Preventing Wave Resonance

Ikha Magdalena, Gabriel Jonathan, L.H. Wiryanto

***Abstract*—This study aims to explore the impact of wave resonance in basins, which can cause damage to harbors and shorelines. To mitigate this risk, researchers propose the use of submerged breakwaters. The study's objective is to develop a mathematical model that examines wave resonance in a basin with two submerged breakwaters. The model will determine the basin's natural resonant frequency and generate a resonant wave. A numerical model based on the Linearized Shallow Water Equations (LSWE) will also be solved using a staggered finite volume method to simulate the phenomenon. The study's findings suggest that constructing two rectangular submerged breakwaters in the basin is sufficient to prevent wave resonance.**

***Index Terms*—shallow water equations, wave resonance, semi closed basin, finite volume, resonant period, submerged breakwater, oscillations**

I. Introduction

THE presence of wave resonance in basins may result in damages to shoreline or harbors. Building a submerged breakwater on the basin is one way to reduce the risk posed by this event. The objective of this study is to develop a mathematical model to investigate the presence of wave resonance in a basin containing two submerged breakwaters. A model will be developed to derive the natural resonant frequency for basin, which will be used to induce an incoming resonant wave into the basin. A numerical model will also be proposed to simulate the phenomenon numerically. Both models will use the Linearized Shallow Water Equations as their foundation (LSWE). The numerical results proposed in this study suggested that the construction of two rectangular submerged breakwater on basin is sufficient to prevent the wave resonance.

Numerous research have also determined the wave resonant period for semi-closed basins with constant width [1], [2], [3], [4], [5]. Additionally, researchers have investigated phenomena associated with wave resonance, such as sloshing inside a basin [6], [7], [8], [9].

The construction of a submerged breakwater is one approach to preventing the occurrence of the resonance phenomenon in a basin. However, despite its significance, relatively few studies and publications examine the influence of submerged breakwater construction on resonance phenomena of basins. Recent studies in this topics only observed only one rectangular submerged breakwater block in a basin with zero friction [10], which was found to be ineffective at preventing the wave resonance phenomenon. It also has been discovered that the addition of sufficient friction to the submerged breakwater surface was able to prevent the water wave from resonating. In this observation, we will identify the effects of two submerged rectangular breakwater blocks on the wave resonance phenomena in basins with smooth surface. The primary model for this study is the Linearized Shallow Water Equations, or LSWE. The SWE is known to be performing really well to perform simulation even in complex topographic areas [11]. To acquire the natural period of a basin, we converted the PDE system LSWE into an ordinary differential equation. Following that, we will solve the ODE to obtain the basin's natural period. Using the obtained basin's natural period, we will perform a number of numerical simulations to observe the presence of resonance phenomena in basins. The numerical approach is based on the method of staggered finite volume discussed in [12], [13]. Other phenomena have also been simulated using the similar method, such as wave amplitude attenuation because of porous media [14], [15], [16], wave shoaling and refraction [17], wave run-up [18], dam break problems [19], [20], [21], [22], and heat transfer phenomena [23], [24].

The following outline will be used to organize the remaining of the paper. The base mathematical model will be discussed in the Section II. After that, we will derive the final equation linked to the analytical solution in the Section III and numerical solution for the governing equation in the Section IV. The numerical simulation results with its analysis will be presented in the Section V. Lastly, the closing conclusion will be presented in the Section VI of the paper.

II. Mathematical Model

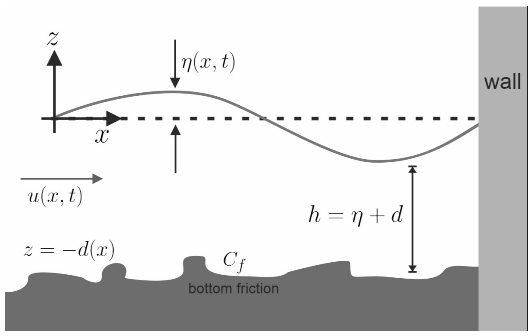

Fig. 1. An illustration of the fluid flow in a semi-closed basin.

This section will examine the mathematical model utilized to simulate the phenomenon of resonance in a basin with

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Ikha Magdalena is a Lecturer and researcher at the Faculty of Mathematics and Natural Sciences, Bandung Institute of Technology, Indonesia (corresponding author; email address: ikha.magdalena@itb.ac.id).

Gabriel Jonathan is a research assistant at the Faculty of Mathematics and Natural Sciences, Bandung Institute of Technology, Indonesia (e-mail: gabejonathan29@gmail.com).

L.H. Wiryanto is a Professor at the Faculty of Mathematics and Natural Sciences, Bandung Institute of Technology, Indonesia (e-mail: leo@math.itb.ac.id).

submerged breakwaters. Linearized Shallow Water Equations (LSWEs) form the basis of the mathematical model below:

$$\eta_t + (hu)_x = 0, \tag{1}$$

$$u_t + g\eta_x = 0 \tag{2}$$

where η represents the wave height, h represents the water thickness, and u denotes the horizontal water wave velocity. We assume that the total water thickness $h(x,t)$ can be approximated by $h(x,t) \approx d(x)$ given the fact that the value of water wave height is insignificant to water depth $d(x)$. Therefore, we are reffering the water thickness as $d(x)$.

III. Analytical Solutions

In this section, we will solve the analytical solution derived from Eqs. (1)–(2) to obtain the analytical natural period T_1 of the basin. First, we reduce the partial differential equations system (1) and (2) into an ordinary differential equation. We define the following ansatz:

$$\eta(x,t) = F(x)e^{-i\omega t}, \tag{3}$$

$$u(x,t) = G(x)e^{-i\omega t}, \tag{4}$$

based on the oscillating motion of a monochromatic wave with an angular frequency of ω. Through substituting Eqs. (3)–(4) into Eqs. (1)–(2), we obtained the two equations coupling F and G:

$$-i\omega F(x) + h_x G(x) + hG_x(x) = 0, \tag{5}$$

$$G_x(x) = -\frac{ig}{\omega}F_{xx}(x). \tag{6}$$

Substituting Eq. (6) into the Eq. (5) yields a second-order ODE:

$$\omega^2 F(x) + gF_x(x)h_x + hgF_{xx}(x) = 0. \tag{7}$$

We will use Eq. (7) to determine the analytical natural period T_1 of the basin. Here is a basin with two block

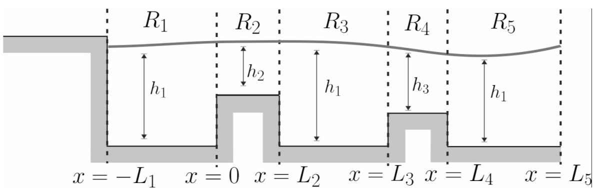

Fig. 2. Basin containing a pair of submerged rectangular breakwaters.

of rectangular submerged breakwater, with smooth bottom surface all over the R_1, R_2, R_3, R_4, R_5 areas. The basin's depth is then described as:

$$h(x) = \begin{cases} h_1, & x \in R_1 \cup R_3 \cup R_5 \\ h_2, & x \in R_2 \\ h_3, & x \in R_4, \end{cases}$$

with

$$\begin{aligned} R_1 &= \{x \in \mathbb{R} \mid -L_1 \le x < 0\} \\ R_2 &= \{x \in \mathbb{R} \mid 0 \le x < L_2\} \\ R_3 &= \{x \in \mathbb{R} \mid L_2 \le x < L_3\} \\ R_4 &= \{x \in \mathbb{R} \mid L_3 \le x < L_4\} \\ R_5 &= \{x \in \mathbb{R} \mid L_4 \le x \le L_5\} \end{aligned}$$

with $h_1 = mh_2$, $h_1 = nh_3$ for some constants m, n. The analytical natural period T_1 of this basin will be determined. To reach the desired result, we will apply Eq. (7) to each basin area. We can obtain consecutively the following analytical solutions for regions R_1, R_2, R_3, R_4, R_5:

$$F_1(x) = A\sin\left(\frac{\omega x}{\sqrt{mgh_2}}\right) + B\cos\left(\frac{\omega x}{\sqrt{mgh_2}}\right), \tag{8}$$

$$F_2(x) = C\sin\left(\frac{\omega x}{\sqrt{gh_2}}\right) + D\cos\left(\frac{\omega x}{\sqrt{gh_2}}\right), \tag{9}$$

$$F_3(x) = E\sin\left(\frac{\omega x}{\sqrt{mgh_2}}\right) + F\cos\left(\frac{\omega x}{\sqrt{mgh_2}}\right), \tag{10}$$

$$F_4(x) = G\sin\left(\sqrt{\frac{n}{m}}\frac{\omega x}{\sqrt{gh_2}}\right) + H\cos\left(\sqrt{\frac{n}{m}}\frac{\omega x}{\sqrt{gh_2}}\right), \tag{11}$$

$$F_5(x) = I\sin\left(\frac{\omega x}{\sqrt{mgh_2}}\right) + J\cos\left(\frac{\omega x}{\sqrt{mgh_2}}\right), \tag{12}$$

with variables $A, B, C, D, E, F, G, H, I, J$ that are unknown. To create a connection between Eqs. (8)–(12), we can utilize the water flow continuity conditions in conjunction with the basin's boundary conditions. Continuity condition of flow in a basin necessitates that the water level on both sides of a step be identical, resulting in the following conditions:

$$\eta(0^-, t) = \eta(0^+, t), \tag{13}$$

$$\eta(L_2^-, t) = \eta(L_2^+, t), \tag{14}$$

$$\eta(L_3^-, t) = \eta(L_3^+, t), \tag{15}$$

$$\eta(L_4^-, t) = \eta(L_4^+, t), \tag{16}$$

Flow continuity also stipulates that the mass flow rate of water entering one region from another must be identical, thereby imposing the following additional conditions:

$$\left.\frac{dM_1}{dt}\right|_{x=0} = \left.\frac{dM_2}{dt}\right|_{x=0}, \tag{17}$$

$$\left.\frac{dM_2}{dt}\right|_{x=L_2} = \left.\frac{dM_3}{dt}\right|_{x=L_2}, \tag{18}$$

$$\left.\frac{dM_3}{dt}\right|_{x=L_3} = \left.\frac{dM_4}{dt}\right|_{x=L_3}, \tag{19}$$

$$\left.\frac{dM_4}{dt}\right|_{x=L_4} = \left.\frac{dM_5}{dt}\right|_{x=L_4}. \tag{20}$$

Now we will construct a boundary condition for the basin. The water flow encounters a solid wall at $x = -L_1$. Consequently, at $x = -L_1$ the horizontal flow velocity must be zero. According to [2], the water level η at $x = L_5$ must remain at a minimum of $\eta = 0$. The following boundary conditions were obtained:

$$u(-L_1) = 0, \tag{21}$$

$$\eta(L_5, t) = 0. \tag{22}$$

By substituting the Eqs. (13)–(20) into the Eqs. (8)–(12) and through several extensive mathematical operations, we finally arrived at the final analytical solution:

$$\cos\left(\frac{X}{\sqrt{m}}\right)\left[\sqrt{n}\left(a_2\sin\left(\frac{X}{\sqrt{m}}\right)+b_2\cos\left(\frac{X}{\sqrt{m}}\right)\right)\cos\left(\frac{\beta X}{\sqrt{m}}\right)+\varepsilon\sin\left(\sqrt{\frac{n}{m}}\beta X\right)-Z\varepsilon\cos\left(\sqrt{\frac{n}{m}}\beta X\right)\right]$$
$$-\sqrt{n}\sin\left(\sqrt{\frac{n}{m}}\beta X\right)\left[Z\varepsilon\sin\left(\sqrt{\frac{n}{m}}\beta X\right)+\varepsilon\cos\left(\sqrt{\frac{n}{m}}\beta X\right)-\left(a_2\sin\left(\frac{X}{\sqrt{m}}\right)+b_2\cos\left(\frac{X}{\sqrt{m}}\right)\right)\sin\left(\frac{\beta X}{\sqrt{m}}\right)\right]=0, \tag{23}$$

with

$$a_1=\sin\left(\frac{\gamma X}{\sqrt{m}}\right)-\tan\left(\frac{\delta X}{\sqrt{m}}\right)\cos\left(\frac{\gamma X}{\sqrt{m}}\right), \tag{24}$$

$$b_1=\sqrt{n}\left[\cos\left(\frac{\gamma X}{\sqrt{m}}\right)+\tan\left(\frac{\delta X}{\sqrt{m}}\sin\left(\frac{\gamma X}{\sqrt{m}}\right)\right)\right], \tag{25}$$

$$Z=\frac{-a_1\sin\left(\sqrt{\frac{n}{m}}\gamma X\right)-b_1\cos\left(\sqrt{\frac{n}{m}}\gamma X\right)}{-a_1\cos\left(\sqrt{\frac{n}{m}}\gamma X\right)+b_1\sin\left(\sqrt{\frac{n}{m}}\gamma X\right)}, \tag{26}$$

$$a_2=-\sqrt{m}\tan\left(\frac{\delta X}{\sqrt{m}}\right)\sin(X)+\cos(X), \tag{27}$$

$$b_2=-\tan\left(\frac{\delta X}{\sqrt{m}}\right)\cos(X)-\frac{1}{\sqrt{m}}\sin(X), \tag{28}$$

$$\varepsilon=\frac{a_2\sin\left(\frac{X}{\sqrt{m}}\right)+b_2\cos\left(\frac{X}{\sqrt{m}}\right)}{a_3\sin\left(\frac{\beta X}{\sqrt{m}}\right)+b_3\cos\left(\frac{\beta X}{\sqrt{m}}\right)}. \tag{29}$$

$$a_3=Z\sin\left(\sqrt{\frac{n}{m}}\beta X\right)+\cos\left(\sqrt{\frac{n}{m}}\beta X\right), \tag{30}$$

$$b_3=\frac{1}{\sqrt{n}}\left[Z\cos\left(\sqrt{\frac{n}{m}}\beta X\right)-\sin\left(\sqrt{\frac{n}{m}}\beta X\right)\right]. \tag{31}$$

$X=\frac{\omega L_2}{\sqrt{gh_2}}$, $h_1=mh_2$, $h_1=nh_3$, $L_1=\alpha L_2, L_3=\beta L_2, L_4=\gamma L_2$ and $L_5=\delta L_2$ for some constants $m,n,\alpha,\beta,\gamma,\delta$. Since $\omega=\frac{2\pi}{T_1}$, the basin's analytical natural period T_1 can be found by locating the root X of Eq. (23).

Next, we will determine if the phenomenon of resonance occurs for different basin topographies when two blocks of submerged breakwaters are present. For the simulations, we will adjust the parameters $\alpha,\beta,\gamma,\delta,m,n$ to modify the brekwater's shape. The Table I provides the T_1 values of basins with various values of parameters:

IV. Numerical Method

In this section, we provide the numerical scheme to simulate the water wave motion in a basin to confirm the existence of resonance phenomenon. By using the staggered method, we discretize the basin's domain into $0=x_{1/2},x_1,x_{3/2},x_2,..,x_{N_x},x_{N_x+1/2}=L$. As shown in Fig. 3, the mass conservation equation (1) is stored in the cells with dashes, whereas the other cells are used for computation

TABLE I
The analytical natural period T_1 for different basin topographies.

No	m	n	α	β	γ	δ	Analytical T_1 (s)
(a)	2	2	1	2	3	4	$3.035732\frac{L_2}{\sqrt{gh_2}}$
(b)	1.5	2	1	2	3	4	$3.0366492\frac{L_2}{\sqrt{gh_2}}$
(c)	2	1.5	1	2	3	4	$3.0359882\frac{L_2}{\sqrt{gh_2}}$
(d)	2	2	2	3	4	6	$2.416610\frac{L_2}{\sqrt{gh_2}}$
(e)	1.5	2	2	3	4	6	$2.385925\frac{L_2}{\sqrt{gh_2}}$
(f)	2	1.5	2	3	4	6	$2.396915\frac{L_2}{\sqrt{gh_2}}$
(g)	2	2	3	4	5	7	$2.595538\frac{L_2}{\sqrt{gh_2}}$
(h)	1.5	2	3	4	5	7	$2.619326\frac{L_2}{\sqrt{gh_2}}$
(i)	2	1.5	3	4	5	7	$2.618074\frac{L_2}{\sqrt{gh_2}}$

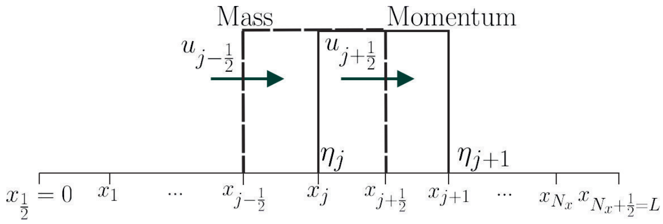

Fig. 3. Illustration of finite volume method on a staggered grid.

of the momentum balance equation (2). A staggered grid discretization was constructed as follows: the half-grid points $x_{j+1/2}$, with $j=0,1,2,..,N_x$ will contain the information of $u(x,t)$, while the full-grid points x_j with $j=1,2,..,N_x$ will stores the values of wave elevation $\eta(x,t)$ and $h(x)$.

The approximation of Eqs. (1)-(2) obtained by using the Finite Volume Method is expressed as:

$$\frac{\eta_i^{n+1}-\eta_i^n}{\Delta t}+\frac{({}^*hu)^n_{i+\frac{1}{2}}-({}^*hu)^n_{i-\frac{1}{2}}}{\Delta x}=0, \tag{32}$$

$$\frac{u^{n+1}_{i+\frac{1}{2}}-u^n_{i+\frac{1}{2}}}{\Delta t}+g\frac{\eta^{n+1}_{i+1}-\eta^{n+1}_i}{\Delta x}=0. \tag{33}$$

Because the basin's water depth h is defined only on the full grid, the $({}^*hu)_{i+\frac{1}{2}}$ value is unknown. To estimate h's value on the half-grids using the First Order Upwind Method, we may write

$${}^*h^n_{j+\frac{1}{2}}=\begin{cases}h^n_j, & \text{for } u_{j+\frac{1}{2}}>0\\ h^n_{j+1}, & \text{for } u_{j+\frac{1}{2}}\le 0\end{cases} \tag{34}$$

$\sqrt{gh_{max}}\Delta t/\Delta x\ \le\ 1$ is the stability condition for the numerical scheme proposed above, with h_{max} denotes the highest water depth $h(x)$ of basin [12].

V. Simulation Results and Discussion

Using the numerical scheme presented in the previous section, we will conduct multiple numerical simulations in this section. Using the MATLAB program, all of the results presented in this section were obtained through numerical simulations. We will validate the presence of the resonance phenomenon based on the results of these simulations. The simulations have the following parameters: : $h_2=1.25$ m for basins with $(\alpha, \beta, \gamma\, \delta) = (1, 2, 3, 4)$ and $h_2 = 2.5$ m for basins with $(\alpha, \beta, \gamma\, \delta) = (2, 3, 4, 6)$ and $(\alpha, \beta, \gamma\, \delta) = (3, 4, 5, 7)$, $L_2 = 5$ m, $T = 400$ s, $\Delta x = 0.1$, $\Delta t = \frac{\Delta x}{\sqrt{gh_{max}}}$, and $g = 9.81$ m/s^2. We imposed the following initial and boundary conditions: $u(0, t) = 0$, $\eta(x, 0) = 0$ and $u(x, 0) = 0$. We also used $L = 25$ m for basins with $(\alpha, \beta, \gamma\, \delta) = (1, 2, 3, 4)$ and $L = 40$ m for basins with $(\alpha, \beta, \gamma\, \delta) = (2, 3, 4, 6)$ and $(\alpha, \beta, \gamma\, \delta) = (3, 4, 5, 7)$. On each iteration of the simulation, consider a wave with 10 cm amplitude and period of T_1 s enters the basin from the right. The left end of the basin is bounded by a solid wall.

For each parameter listed in Table I, we will measure the water's elevation at the left boundary of the basin ($x = -L_1$). Figure 4 provides a summary of the outcomes.

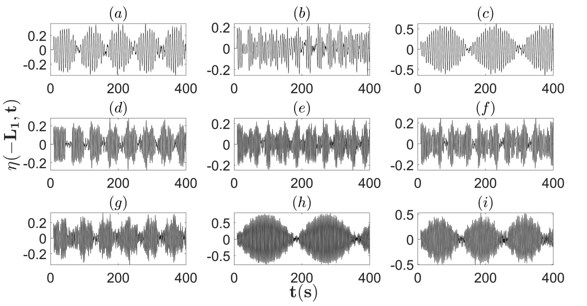

Fig. 4. Water level at basin boundary ($x = -L_1$) for a variety of parameters

As illustrated in the graph presented in Figure 4, there is no further amplification of the elevation of water waves over time since the water elevation continues to fluctuate up and down. Hence, numerical simulations indicated that no resonance phenomenon occurred for any combination of the parameters provided in the Table I.

Additionally, we will conduct additional experiments to verify the effectiveness of two rectangular submerged breakwater blocks in preventing the wave resonance phenomenon. By [10], the presence of one submerged rectangular breakwater was found to be ineffective at preventing the resonance phenomenon. We set the m and n parameters in Eq. (23) to $m = 2$ and $n = 1$ to describe a basin containing a single rectangular submerged breakwater. The Table II provides the T_1 values for basins with various parameters and $(m, n) = (2, 1)$. We conduct another set of numerical

TABLE II
ANALYTICAL NATURAL PERIOD T_1 FOR DIFFERENT BASIN TOPOGRAPHIES WITH $(m, n) = (2, 1)$.

α	β	γ	δ	Analytical T_1 (s)
1	2	3	4	$2.8571\frac{L_2}{\sqrt{gh_2}}$
2	3	4	6	$3.3128\frac{L_2}{\sqrt{gh_2}}$
3	4	5	7	$2.6400\frac{L_2}{\sqrt{gh_2}}$

simulations, using same parameter values as the previous simulations except the $T = 250$ s. For each numerical simulation iteration, we suppose that a wave with amplitude of 0.1 m with period of T_1 s listed in the Table II enters the basin from the right side. Similar to previous simulations, we will observe the water level at the left boundary of the basin ($\eta(-L_1, t)$). The simulation results are provided in the Figure 5.

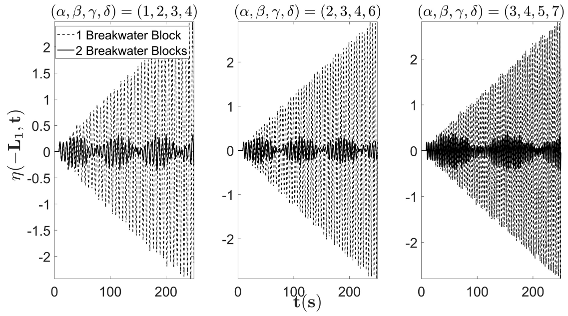

Fig. 5. Water level at ($x = -L_1$) for a number of parameters with $(m, n) = (2, 1)$.

As shown in the Figure 5, the water elevation $\eta(-L_1, t)$ on the basin containing a single submerged rectangular breakwater block is amplified over time, indicating the presence of the water wave's resonance. By constructing another rectangular submerged breakwater, we can demonstrate that the elevation of the water wave has been prevented.

VI. Conclusion

The shallow water equations simulated using the finite volume method on staggered grids works well to simulate the effect of submerged breakwater on basin's wave resonance phenomenon. Moreover, the proposed model solves the model numerically with no generation of damping error. In a semi-closed basin, the existence of two rectangular breakwater blocks submerged prevented the occurrence of any resonance phenomenon.

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