Exponential Weibull Distribution with Loss Data under Deductible Insurance Contract

Murati Somboon, Noocharin Tippayawannakorn, Piyachat Leelasilapasart, Kobkun Raweesawat

***Abstract*—This study estimates the pure premium under a deductible insurance contract by applying the Exponential Weibull (EW) distribution with loss data. We simulate loss models under two types of deductible insurance contracts: Franchise deductibles and fixed amount deductibles. The loss data is simulated with sample sizes of 100 and 500 based on the EW distribution for all combinations of three parameters: α = {1.5, 2}; λ= {0.9, 1}, and γ = {0.6, 0.7}. We assume that the deductible was determined at 5% of each loss. The quantile matching method is used to estimate the parameter. The results indicate that applying the EW distribution to loss data under the deductible to estimate the pure premium is suitable. Since the loss data is the right skewed distribution, it aligns with the nature of non-life insurance losses. Furthermore, it can be utilized as a planning tool to determine the pure premium for new insurance products without actual loss data. Also, it can enhance insurance premiums when the loss has already occurred.**

***Index Terms*— deductible insurance contract, exponential weibull distribution, pure premium, quantile matching**

I. Introduction

THE non-life insurance industry is experiencing consistent growth. The insurance business is a type of financial industry that provides financial protection to individuals and organizations against potential losses or damages. Non-life insurance, also known as property and casualty insurance, provides coverage for property, liability, and other risks. Non-life insurance benefits both the insured and society, as it helps to mitigate the financial impact of damages. The insurance company pays the claim amount to the insured or beneficiaries according to the terms of the insurance contract. The insured pays a premium in exchange for the coverage, which is considered the insurer's primary income. To manage various risks, insurers must determine the expected cost of losses that may occur in the future, relying on historical loss data.

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The operation of non-life insurance typically involves several components: assessing risks, the type and value of property, location, coverage limits, deductible amount, past claim history, etc. The reduction of operational costs in risk management is a crucial factor. One approach to reducing costs is by implementing a deductible insurance contract, in which the insured pays a deductible. The purpose of a deductible is to share the risk between the policyholder or insured and the insurance company. It will result in the insured reducing the damage claims or reducing the severity of the loss that may occur. This method reduces moral hazard, a condition that causes a higher risk due to the hope of taking advantage of insurance by a dishonest act of the insured. In Thailand, there are two popular types of deductible insurance contracts: franchise and fixed amount deductibles. Losses subject to deductible insurance contracts will be recorded when the damage occurs. When the actual loss is known, it increases the accuracy of insurance premium calculations. In non-life insurance, loss data is right-skewed, as losses with lower amounts tend to have a higher frequency of occurrence.

The influence of deductibles on pure risk premiums and the derivation of simple and practical formulae for premiums under various deductible methods in terms of the limited expected value function were investigated by [1]. They apply the results to typical loss distributions, for example, Lognormal, Pareto, Burr, Weibull, and Gamma. Furthermore, they analyzed the company's loss data, fitted distributions, and demonstrated how the distribution and deductible influence the premium.

The Exponential Weibull (EW) distribution, a three-parameter distribution consisting of a scale parameter and two shape parameters, was proposed by [2]. A study comparing the failure rates of the EW distribution with those of the two-parameter Weibull and Gamma distributions was conducted [3]. It was found that EW distribution has an excellent physical interpretation and can be applied in various fields, including human mortality in insurance, the likelihood of losing when taking risks, and engineering work. Gauss et al. [4] presented the EW distribution with three parameters. These parameters were estimated by the maximum likelihood method and applied to a real data set. The Exponential Inverted Weibull distribution and empirical application of aggregate loss were studied [5], and the pure reinsurance premium calculation was presented for the case of stop loss reinsurance. EW distribution and EIW distribution were compared in studies [6] to [7]. Suzanne et al. [8] conducted simulated experiments to investigate the performance of the self-determined probability-weighted moments compared to the methods of maximum likelihood and moments in estimating the parameters of the EW distribution. Additionally, [9] analyzed the problem of truncated normal distribution in sparse data estimation with a small sample size. [10] proposed a new class of the two parameters generalized exponential and discusses its properties.

This study focuses on the EW distribution, which exhibits a right-sided skewed pattern similar to the loss information of non-life insurance. The quantile matching method is used in parameter estimation to accurately predict the loss and insurance premium.

The paper is organized as follows: Section 2 briefly reviews the EW distribution. Section 3 summarizes the quantile matching method, while Section 4 presents the loss under the deductible insurance contracts. Section 5 describes the numerical setting and simulation procedure. Section 6 presents the numerical results of the study, while Section 7 concludes.

II. The Exponential Weibull Distribution

Mudholkar and Srivastava [2] proposed the Exponential Weibull (EW) distribution as an extension of the Weibull distribution by adding a second shape parameter. This study estimates the pure premium under a deductible insurance contract, with loss data assumed to follow the EW distribution. For $x > 0$, the probability density function and the cumulative distribution function of the EW distribution are given by

$$f(x;\alpha,\lambda,\gamma)=\alpha\gamma\lambda^{\gamma}x^{\gamma-1}(1-\exp\{-(\lambda x)^{\gamma}\})^{\alpha-1}\exp\{-(\lambda x)^{\gamma}$$

$$F(x;\alpha,\lambda,\gamma)=[1-\exp\{-(\lambda x)^{\gamma}\}]^{\alpha},$$

where α and γ are shape parameters and λ is a scale parameter; α, λ, $\gamma > 0$. When $\alpha = 1$, the EW distribution coincides with the Weibull distribution with a scale parameter λ. For $\gamma = 1$, it represents the Exponentiated Exponential distribution. Thus, the EW distribution is a generalization of the Exponentiated Exponential distribution and the Weibull distribution. In particular, the expectation of the EW distribution as:

$$E(X)=\begin{cases}\alpha\lambda^{-1}\Gamma\left(\dfrac{1}{\gamma}+1\right)\displaystyle\sum_{i=0}^{\alpha-1}\binom{\alpha-1}{i}(-1)^{i}(i+1)^{-\frac{1}{\gamma}-1}\ ;\ \text{if}\ \ \alpha\in N\\ \alpha\lambda^{-1}\Gamma\left(\dfrac{1}{\gamma}+1\right)\displaystyle\sum_{i=0}^{\infty}\frac{{}_{\alpha-1}P_i}{i!}(-1)^{i}(i+1)^{-\frac{1}{\gamma}-1};\ \text{if}\ \ \alpha\notin N,\end{cases}$$

$$E(X^2)=\begin{cases}\alpha\lambda^{-2}\Gamma\left(\dfrac{2}{\gamma}+1\right)\displaystyle\sum_{i=0}^{\alpha-1}\binom{\alpha-1}{i}(-1)^{i}(i+1)^{-\frac{2}{\gamma}-1}\ ;\ \text{if}\ \ \alpha\in N\\ \alpha\lambda^{-2}\Gamma\left(\dfrac{2}{\gamma}+1\right)\displaystyle\sum_{i=0}^{\infty}\frac{{}_{\alpha-1}P_i}{i!}(-1)^{i}(i+1)^{-\frac{2}{\gamma}-1};\ \text{if}\ \ \alpha\notin N,\end{cases}$$

where ${}_{\alpha}P_i=\alpha(\alpha-1)(\alpha-2)\ldots(\alpha-i+1)$ and N is the set of natural numbers. Also, the variance can be easily obtained from the above.

III. Quantile Matching Method

Quantile matching is a method of point estimation that selects one piece of data from a sample and assumes that its quantile corresponds to the same quantile of the population. The method of moments or maximum likelihood is not effective for distributions with fat tail characteristics or more than two parameters. In such cases, quantile matching can be used. This approach involves estimating parameters from the percentile or quantile function or the cumulative probability function. Therefore, quantile matching can be considered a reliable method for estimating parameters in distributions that do not conform to assumptions.

In this paper, the quantile matching method has been employed to estimate the parameters of the loss data following the EW distribution $\theta(\alpha,\lambda,\gamma)$. We set three quantile values to find the inverse function of the EW distribution as a function of the parameter to be estimated. The following are the underlying steps involved in this estimation process:

1) Assume that k represents the number of parameters. In this study, we use $k=3$ and quantile $\delta=\left(\frac{1}{4},\frac{2}{4},\frac{3}{4}\right)$, where

$$\delta_i=F(x_{\delta_i};\theta)=[1-\exp\{-(\lambda x)^{\gamma}]^{\alpha}\qquad ;0<\delta_i<1.$$

2) Construct the inverse function [9], which is a function depending on the parameters of the distribution at various quantile levels, referred to as the population quantile. The function is expressed as follows:

$$x_{\delta_i}=F^{-1}(\delta_i;\theta)=F^{-1}(\delta_i;\alpha,\lambda,\gamma)\qquad ;0<\delta_i<1.$$

3) Let $\hat{x}_{\delta_i}$ be the inverse function obtained from a random sample at the quantile level δ_i (A sample quantile).

4) Find the optimal solution of $\hat{\theta}(\hat{\alpha},\hat{\lambda},\hat{\gamma})$ by quantile matching using `qme` [10] in the R program. The Nelder -Mead Simplex method of random search technique is used [11] – [12] by matching the inverse function of the quantile of the distribution and the quantile of the random sample according to the following equation

$$x_{\delta_i}(\hat{\theta})=\hat{x}_{\delta_i},\qquad for\ i=1,2,\ldots,k.$$

IV. Loss under deductible insurance contract

In insurance, a deductible refers to the amount the policyholder pays toward an insured loss. The main idea of a deductible is to reduce claim handling costs by excluding coverage for the numerous small claims and to provide motivation to the insured to prevent claims through a limited degree of participation in claim costs [13]. This study assumes that the loss data will be governed by two types of deductible insurance contracts, which are:

Franchise deductible, which is the insurer pays all claims. If the loss exceeds the liability for the agreed deductible will be

$$Y=X\quad ;x>d, \tag{1}$$

and the cumulative distribution as

$$F_Y(Y)=\frac{F_X(Y)-F_X(d)}{1-F_X(d)}, \tag{2}$$

where Y represents the loss under the franchised deductible (Unit: Baht)

d is the loss per contract (unit: Baht).

Substitute the cumulative distribution function for the EW distribution as follows:

$$F_Y(Y)=\frac{[1-\exp\{(\lambda Y)^{\gamma}\}]^{\alpha}-[1-\exp\{(\lambda d)^{\gamma}\}]^{\alpha}}{1-[1-\exp\{(\lambda d)^{\gamma}\}]^{\alpha}}. \tag{3}$$

The parameter of estimation under the franchise deductible is given by

$$E(\hat{Y}) = \int_d^{\infty} y(\hat{f}_Y y)dy$$
$$= \int_d^{\infty} y(\hat{\alpha}\hat{\gamma}\hat{\lambda}^{\hat{\gamma}} y^{\hat{\gamma}-1}[1-\exp\{-(\hat{\lambda}y)^{\hat{\gamma}}\}]^{\hat{\alpha}-1}\exp\{-(\hat{\lambda}y)^{\hat{\gamma}}\})dy$$
$$= \frac{\int_0^{\infty} y\hat{f}_X(y) - \int_0^{d} y\hat{f}_X(y)}{1-\hat{F}_X(d)}. \quad (4)$$

For $\alpha \in N$

$$E(\hat{Y}) = \frac{\hat{\alpha}\hat{\lambda}^{-1}\Gamma\left(\frac{1}{\hat{\gamma}}+1\right)\sum_{i=0}^{\alpha-1}\binom{\hat{\alpha}-1}{i}(-1)^i(i+1)^{-\frac{1}{\hat{\gamma}}-1}}{1-[1-\exp\{-(\hat{\lambda}d)^{\hat{\gamma}}\}]^{\hat{\alpha}}} - \frac{\int_0^d y\hat{\alpha}\hat{\gamma}\hat{\lambda}^{\hat{\gamma}} y^{\hat{\gamma}-1}[1-\exp\{-(\hat{\lambda}y)^{\hat{\gamma}}\}]^{\hat{\alpha}-1}\exp\{-(\hat{\lambda}y)^{\hat{\gamma}}\}dy}{1-[1-\exp\{-(\hat{\lambda}d)^{\hat{\gamma}}\}]^{\hat{\alpha}}}, \quad (5)$$

and $\alpha \notin N$

$$E(\hat{Y}) = \frac{\hat{\alpha}\hat{\lambda}^{-1}\Gamma\left(\frac{1}{\hat{\gamma}}+1\right)\sum_{i=0}^{\alpha-1}\binom{{}_{\hat{\alpha}-1}P_i}{i}(-1)^i(i+1)^{-\frac{1}{\hat{\gamma}}-1}}{1-[1-\exp\{-(\lambda d)^{\gamma}\}]^{\alpha}} - \frac{\int_0^d y\alpha\gamma\lambda^{\gamma} x^{\gamma-1}[1-\exp\{-(\lambda x)^{\gamma}\}]^{\alpha-1}\exp\{-(\lambda x)^{\gamma}\}dy}{1-[1-\exp\{-(\lambda d)^{\gamma}\}]^{\alpha}}. \quad (6)$$

Fixed amount deductible is a method that the insurer pays compensation equal to the deductible or specified by the company if the loss exceeds the amount of first liability so

$$Y = \begin{cases} 0 & ; x \le d, \\ X & ; x > d \end{cases} \quad (7)$$

and the cumulative distribution as

$$P(Y \le y) = P(X - d \le y \mid X > d). \quad (8)$$

From (8), it can be rewritten as

$$P(Y \le y) = P(X \le y - d \mid X > d). \quad (9)$$

The parameter of estimation under the fixed amount deductible is given by

$$E(\hat{Y}) = \left[\int_d^{\infty} y(\hat{f}_Y y)dy\right] - d$$
$$= \left[\int_d^{\infty} y(\hat{\alpha}\hat{\gamma}\hat{\lambda}^{\hat{\gamma}} y^{\hat{\gamma}-1}[1-\exp\{-(\hat{\lambda}y)^{\hat{\gamma}}\}]^{\hat{\alpha}-1}\exp\{-(\hat{\lambda}y)^{\hat{\gamma}}\})dy\right] - d$$
$$= \frac{\int_0^{\infty} y\hat{f}_X(y) - \int_0^{d} y\hat{f}_X(y)}{1-\hat{F}_X(d)} - d. \quad (10)$$

For $\alpha \in N$

$$E(\hat{Y}) = \frac{\hat{\alpha}\hat{\lambda}^{-1}\Gamma\left(\frac{1}{\hat{\gamma}}+1\right)\sum_{i=0}^{\alpha-1}\binom{\hat{\alpha}-1}{i}(-1)^i(i+1)^{-\frac{1}{\hat{\gamma}}-1}}{1-[1-\exp\{-(\hat{\lambda}d)^{\hat{\gamma}}\}]^{\hat{\alpha}}} - \frac{\int_0^d y\hat{\alpha}\hat{\gamma}\hat{\lambda}^{\hat{\gamma}} y^{\hat{\gamma}-1}[1-\exp\{-(\hat{\lambda}y)^{\hat{\gamma}}\}]^{\hat{\alpha}-1}\exp\{-(\hat{\lambda}y)^{\hat{\gamma}}\}dy}{1-[1-\exp\{-(\hat{\lambda}d)^{\hat{\gamma}}\}]^{\hat{\alpha}}} - d, \quad (11)$$

and $\alpha \notin N$

$$E(\hat{Y}) = \frac{\hat{\alpha}\hat{\lambda}^{-1}\Gamma\left(\frac{1}{\hat{\gamma}}+1\right)\sum_{i=0}^{\alpha-1}\binom{{}_{\hat{\alpha}-1}P_i}{i}(-1)^i(i+1)^{-\frac{1}{\hat{\gamma}}-1}}{1-[1-\exp\{-(\lambda d)^{\gamma}\}]^{\alpha}} - \frac{\int_0^d y\alpha\gamma\lambda^{\gamma} x^{\gamma-1}[1-\exp\{-(\lambda x)^{\gamma}\}]^{\alpha-1}\exp\{-(\lambda x)^{\gamma}\}dy}{1-[1-\exp\{-(\lambda d)^{\gamma}\}]^{\alpha}}. \quad (12)$$

A franchise and a fixed amount deductible liability insurance contract are related in terms of format because their loss data yield identical parameter estimates. Therefore, the parameter estimates will present the results of the franchise deductible.

V. Numerical Illustration

We assume that the loss data (Y) to the contract follows the EW distribution with parameters $(\alpha, \lambda, \gamma)$ = (1.5, 0.9, 0.6), (2, 0.9, 0.6), (1.5, 0.9, 0.7), (2, 0.9, 0.7), (1.5, 1, 0.6), (2,1,0.6), (1.5, 1, 0.7), and (2, 1, 0.7). The deductible (d) is set at 5% of the initial loss. The simulation experiments with sample sizes of 100 and 500 are performed using the R program. For each situation of sample size and parameter values, the experiment will be repeated 1,000 times. Further, assume that the loss data will be under two deductible insurance contracts, which are:

1. Calculate deducible from loss distribution which is a 5% quantile of $EW(\alpha,\lambda,\gamma)$

$$x_{0.05} = F_X^{-1}(0.05;\alpha,\lambda,\gamma) = d_{0.05}. \quad (13)$$

2. Estimate parameters using the quantiles matching estimation method, and the qmedist function from the fitdest package in the R program is used. The Nelder-Mead simplex optimization method is employed to determine the initial parameter values using the moment estimator.
3. Use the estimated parameters to apply for the loss data with EW distribution to estimate the pure premium under an insurance contract that has a deductible as equations (5) and (12).

VI. Numerical Result

Numerical results from the simulation experiments, as explained in Section V, are summarized below.

TABLE I
THE ESTIMATES OF α, λ, γ AND THE PURE PREMIUM FOR THE LOSS DATA WITH THE EW DISTRIBUTION WHEN $n = 100$ WITH $\lambda = 0.9$ AND $\gamma = 0.6$.

α		1.5	2
$\hat{\alpha}$		1.0578	1.8997
$\hat{\lambda}$		0.0925	0.0937
$\hat{\gamma}$		0.2898	0.2782
Deductible (Baht)		727.50	638.00
Pure Premium (Baht)	Franchise	9,443.03	8,040.14
	Fixed amount	8,715.53	7,402.14

TABLE II

THE ESTIMATES OF α, λ, γ AND THE PURE PREMIUM FOR THE LOSS DATA WITH THE EW DISTRIBUTION WHEN $n = 100$ WITH $\lambda = 0.9$ AND $\gamma = 0.7$.

α		1.5	2
$\hat{\alpha}$		1.0155	1.2736
$\hat{\lambda}$		0.0693	0.0774
$\hat{\gamma}$		0.2764	0.3329
Deductible (Baht)		575.26	379.24
Pure Premium (Baht)	Franchise	7,641.16	8,698.99
	Fixed amount	7.065.90	8,319.75

From Tables 1 – 2, the loss data with the EW distribution when n = 100 with the parameters λ = 0.9, γ = 0.6 when $\alpha = 1.5$ and 2 shows that the deductible is equal to 727.50 baht and 638.00 baht, respectively. The estimated pure premium under the franchised deductible is 9,443.03 and 8,040.14 baht, respectively. The estimated pure premium under the fixed amount deductible is 8,715.53 and 7,402.14 baht, respectively. For the parameters λ = 0.9, γ = 0.7 when α = 1.5 and 2, the deductible is equal to 575.26 baht and 379.24 baht, respectively. The estimated pure premium under the franchised deductible is 7,641.14 and 8,698.99 baht, respectively. The estimated pure premium under the fixed amount deductible is 7,065.90 and 8,319.75 baht, respectively.

TABLE III

THE ESTIMATES OF α, λ, γ AND THE PURE PREMIUM FOR THE LOSS DATA WITH THE EW DISTRIBUTION WHEN $n = 100$ WITH $\lambda = 1$ AND $\gamma = 0.6$.

α		1.5	2
$\hat{\alpha}$		0.9923	1.3164
$\hat{\lambda}$		0.1024	0.1042
$\hat{\gamma}$		0.3015	0.3025
Deductible (Baht)		404.16	202.54
Pure Premium (Baht)	Franchise	7,742.58	6,374.42
	Fixed amount	7.338.42	6,171.88

TABLE IV

THE ESTIMATES OF α, λ, γ AND THE PURE PREMIUM FOR THE LOSS DATA WITH THE EW DISTRIBUTION WHEN $n = 100$ WITH $\lambda = 1$ AND $\gamma = 0.7$.

α		1.5	2
$\hat{\alpha}$		0.9460	2.1041
$\hat{\lambda}$		0.0810	0.0856
$\hat{\gamma}$		0.3345	0.3513
Deductible (Baht)		958.77	280.92
Pure Premium (Baht)	Franchise	9,818.25	9,340.31
	Fixed amount	8,859.48	9,059.39

From Tables 3 – 4, the loss data with the EW distribution when n = 100 with the parameters λ = 1, γ = 0.6 when α = 1.5 and 2 shows that the deductible is equal to 404.16 baht and 202.54 baht, respectively. The estimated pure premium under the franchised deductible is 7,742.58 and 6,374.42 baht, respectively. The estimated pure premium under the fixed amount deductible is 7,338.42 and 6,171.88 baht, respectively. For the parameters λ =1, γ = 0.7 when α = 1.5 and 2 shows that the deductible is equal to 958.77 baht and 280.92 baht, respectively. The estimated pure premium under the franchised deductible is 9,818.25 and 9,340.31 baht, respectively. The estimated pure premium under the fixed amount deductible is 8,859.48 and 9,059.39 baht, respectively.

TABLE V

THE ESTIMATES OF α, λ, γ AND THE PURE PREMIUM FOR THE LOSS DATA WITH THE EW DISTRIBUTION WHEN $n = 500$ WITH $\lambda = 0.9$ AND $\gamma = 0.6$.

α		1.5	2
$\hat{\alpha}$		0.9219	1.1472
$\hat{\lambda}$		0.2002	0.1996
$\hat{\gamma}$		0.4887	0.0401
Deductible (Baht)		727.50	911.42
Pure Premium (Baht)	Franchise	7,057.46	8,522.61
	Fixed amount	6,329.96	7,611.19

TABLE VI

THE ESTIMATES OF α, λ, γ AND THE PURE PREMIUM FOR THE LOSS DATA WITH THE EW DISTRIBUTION WHEN $n = 500$ WITH $\lambda = 0.9$ AND $\gamma = 0.7$.

α		1.5	2
$\hat{\alpha}$		0.1441	0.2301
$\hat{\lambda}$		1.5065	1.8131
$\hat{\gamma}$		0.0116	0.0147
Deductible (Baht)		575.26	379.24
Pure Premium (Baht)	Franchise	5,544.06	5,187.59
	Fixed amount	4,968.80	4,808.35

From Tables 5 – 6, the loss data with the EW distribution when n = 500 with the parameters λ = 0.9, γ = 0.6 when $\alpha = 1.5$ and 2, the deductible is equal to 727.50 baht and 911.42 baht, respectively. The estimated pure premium under the franchised deductible is 7,057.46 and 8,522.61 baht, respectively. The estimated pure premium under the fixed amount deductible is 6,329.96 and 7,611.19 baht, respectively. For the parameters λ =0.9, γ = 0.7 when α = 1.5 and 2 shows that the deductible is equal to 575.26 baht and 379.24 baht, respectively. The estimated pure premium under the franchised deductible is 5,544.06 and 5,187.59 baht, respectively. The estimated pure premium under the fixed amount deductible is 4,968.80 and 4,808.35 baht, respectively.

TABLE VII

THE ESTIMATES OF α, λ, γ AND THE PURE PREMIUM FOR THE LOSS DATA WITH THE EW DISTRIBUTION WHEN $n = 500$ WITH $\lambda = 1$ AND $\gamma = 0.6$.

α		1.5	2
$\hat{\alpha}$		0.9625	1.0933
$\hat{\lambda}$		0.2288	0.2218
$\hat{\gamma}$		0.4880	0.4877
Deductible (Baht)		404.16	202.54
Pure Premium (Baht)	Franchise	6,993.85	8,012.11
	Fixed amount	6,589.69	7,809.57

TABLE VIII

THE ESTIMATES OF α, λ, γ AND THE PURE PREMIUM FOR THE LOSS DATA WITH THE EW DISTRIBUTION WHEN $n = 500$ WITH $\lambda = 1$ AND $\gamma = 0.7$.

α		1.5	2
$\hat{\alpha}$		0.3848	0.7642
$\hat{\lambda}$		1.5709	0.1952
$\hat{\gamma}$		0.4466	0.5734
Deductible (Baht)		958.77	280.92
Pure Premium (Baht)	Franchise	9,256.35	9,849.81
	Fixed amount	8,297.58	9,568.89

From Tables 7 – 8, the loss data with the EW distribution when n = 500 with the parameters λ = 1, $\gamma = 0.6$ when $\alpha = 1.5$ and 2, the deductible is equal to 404.16 baht and 202.54 baht, respectively. The estimated pure premium under the franchised deductible is 6,993.85 and 8,012.11 baht, respectively. The estimated pure premium under the fixed amount deductible is 6,589.69 and 7,809.57 baht, respectively. For the parameters λ =1, $\gamma = 0.7$ when $\alpha = 1.5$ and 2 shows that the deductible is equal to 958.77 baht and 280.92 baht, respectively. The estimated pure premium under the franchised deductible is 9,256.35 and 9,849.81 baht, respectively. The estimated pure premium under the fixed amount deductible is 8,297.58 and 9,568.89 baht, respectively.

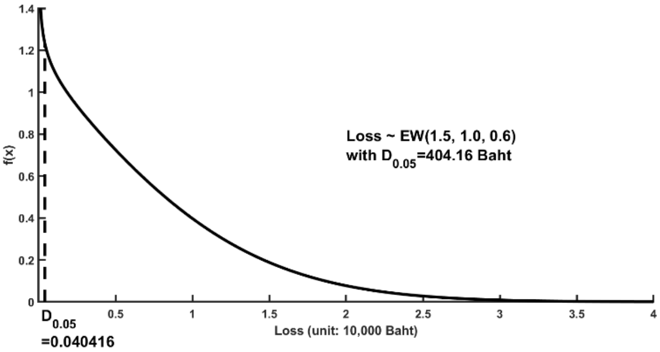

Fig. 1. Loss data with the EW distribution with parameters α = 1.5, λ= 1 and γ = 0.6.

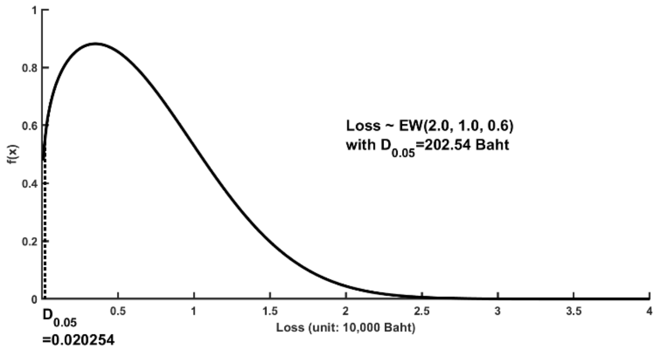

Fig. 2. Loss data with the EW distribution with parameters α = 2, λ= 1 and γ = 0.6.

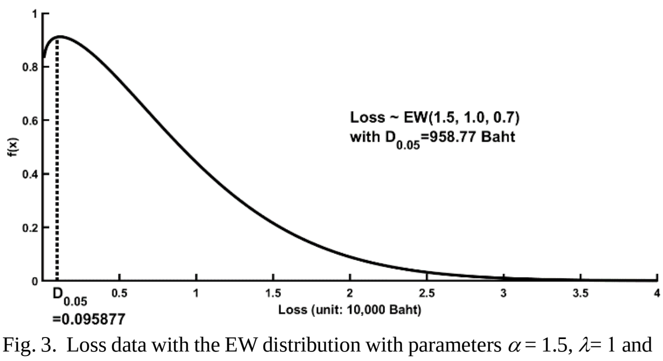

Fig. 3. Loss data with the EW distribution with parameters α = 1.5, λ= 1 and γ = 0.7.

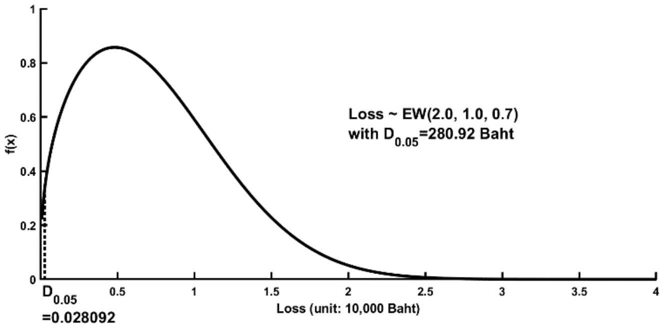

Fig. 4. Loss data with the EW distribution with parameters α = 2, λ = 1 and γ = 0.7

Figs. 1 – 4 display the simulation results for the EW distribution loss data when n = 500 with parameters $(\alpha, \lambda, \gamma) = (1.5, 1, 0.6)$, (2, 1, 0.6), (1.5, 1, 0.7), and (2, 1, 0.7), respectively, under the deductible 5% of the loss data, which is equal to 404.16 baht, 202.54 baht, 958.77 baht and 280.92 baht, respectively; the other case provides similar results.

According to Tables 1 – 8, almost all cases yield larger deductible values when α = 1.5 than when α = 2, with the exception of one case when (λ, γ) = (0.9, 0.6). This finding is in accordance with the loss distributions, as shown in Figs 1 – 4. The EW distribution when α = 1.5, as displayed in Figs 1 and 3, skews more positively than when α = 2, as shown in Figs 2 and 4. This means that the loss with α = 1.5 is higher than with α = 2.

VII. Conclusion

In this study, we utilize the EW distribution to simulate loss situations for two deductible insurance contracts: the Franchise deductible and the Fixed amount deductible. The EW parameters are estimated using the quantile matching method, known for its simplicity and efficiency in estimating pure premiums.

The results show that these estimated parameters can be employed to determine deductibles and pure premiums in non-life insurance, particularly in property insurance. Additionally, this study highlights the possibility of developing a fitting distribution model that accurately represents losses in the non-life insurance industry, leading to a more efficient determination of deductibles and pure premiums.

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