

# Event-Triggered Sliding Model Control Without A Priori Bounded Uncertainty

Xiangyu Kong, Zhen Wang

**Abstract**—This article presents a disturbance estimation-based event-triggered sliding mode control design for the linear time-invariant system without continuous monitoring. Comparing with traditional design methods, the proposed method here considers the influence of control input on the sliding model in the process of sliding model estimation. As a result, sliding mode control is only triggered at triggering instances, which reduces the amount of communication and lowers the frequency of controller updates in practice. With the obtained control law, the stability of the output trajectory is proved. Furthermore, continuous monitoring can be avoided in triggering condition evaluating. Zeno behavior is proved to be excluded for the proposed event-triggered control. Moreover, numerical examples show the effectiveness of the proposed strategies, which executed on both linear dynamic simulation and practical spring-mass-damper system.

**Index Terms**—Event-Triggered Control, Sliding Model Control, Disturbance Estimation, Spring-Mass-Damper System, Intermittent Monitoring.

## I. INTRODUCTION

**E**VENT-TRIGGERED control (ETC) technique receives a lot of attention due to its ability to enhance control efficiency and reduce the burden of communication or actuation for control implementation [1], [2]. There is clear distinction to update controller from traditional sampling systems [3], [4]. Meanwhile, it is important to exclude Zeno phenomenon for an ETC. Nevertheless, because of the difficulty of uncertain processing, most of literature investigate systems without uncertainty [5], [6].

In general, sliding mode control (SMC) technology has been employed to deal with the variety of uncertainty problems, such as time-delay systems [7], fuzzy systems [8], to name just a few. In practical applications, a lot of research results have been investigated on discrete-time sliding mode control [9]–[12]. Meanwhile, we know that the trajectory of a system driven by sliding-mode control could chatter and will not stay on the sliding surface, but will remain bounded within a bounded region [9], and the magnitude of which depends on the bounds of disturbance. In fact, in all these strategies, the measured data of the system will be transmitted to the controller all the time, which will cause the increasing of the control computation cost and application cost. Additionally, much of the work mentioned above employs time-triggered technique. Furthermore, event-triggered SMC has been studied in [13]–[16]. Under the discrete-time

control scheme, the event-triggered system cannot be an ideal sliding model, which makes the trajectory just close to the sliding surface. Behera and Bandyopadhyay [13] studied the linear time-invariant (LTI) system by defining the practical sliding mode and designing a global event-triggering SMC. In [14], the robust self-triggered SMC was presented to stabilize an LTI system. A kind of periodic event-triggering SMC was proposed for LTI systems [15]. The authors in [16] investigated quantized feedback control for event-triggered SMC linear system. In the above works, controller is updated under continuous monitoring, which results in much communication consumption. However, in practical applications, energy environment is limited, even a large number of problems have minimum energy requirements.

Based on the analysis of the previous researches, this paper aims to save resources and implement intermittent monitoring in a framework of limited energy environment. Although there have been a large number of classical results concerning sliding mode control, but intermittent monitoring techniques have not been studied sufficiently. This paper focuses on a new design technique based on co-design of both event-triggered technique and SMC for LTI with intermittent monitoring. The contribution highlights are summarized as follows:

- Event-triggered SMC design with both continuous monitoring and intermittent monitoring are explored.
- By fusing the techniques of event-triggered method and SMC, a novel control design under both disturbance estimation and monitoring mechanism is proposed to ensure the output the stability of the system. Furthermore, computing overhead as well as communication frequency can be well reduced.
- Compared with the traditional sliding model design [13]–[16], this method considers the influence of control input and monitoring mechanism which not only can make the estimation of sliding mode more accurate, but also reduces energy consumption. Meanwhile, the design excludes Zeno behavior.

The rest of this paper is structured as follows. In Section II, We present the model formulation and preliminaries. Subsequently, in Section III, we address the development of event-triggered sliding mode control (SMC) with continuous monitoring. Section IV focuses on the topic of event-triggered SMC with intermittent monitoring. To validate the efficacy of the proposed event-triggered SMC, numerical simulations are provided in Section V. Finally, Section VI presents the conclusions and outlines potential avenues for future research.

## A. Notation

The symbols  $\mathbb{R}$  and  $\mathbb{R}^n$  respectively represent the set of real numbers and the  $n$ -dimensional Euclidean space. Denote

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$|\cdot|$  and  $\|\cdot\|$  as the absolute value of a scalar and Euclidean norm of a vector, respectively.

## II. MODEL FORMULATION AND PRELIMINARIES

In this section, the system is described, and the control objective is also given. Then, some basic assumptions are introduced.

### A. System Description and Control Objective

The LTI system can be described as follows:

$$\begin{cases} \dot{x}(t) = Ax + B(u + d), \\ y = Cx(t), \quad x_0 = x(t_0), \end{cases} \quad (1)$$

where  $x \in \mathbb{R}^n$  denotes the state,  $u \in \mathbb{R}$  represents the control input, and  $y \in \mathbb{R}$  is the system output. The matrices  $A \in \mathbb{R}^{n \times n}$ ,  $B \in \mathbb{R}^{n \times 1}$  and  $C \in \mathbb{R}^{1 \times n}$  are the system matrices. Here,  $d \in \mathbb{R}$  is the matched external disturbance.

The purpose of this paper is to establish a stable sliding surface with output estimation information, which ensures system output stability. To achieve this task, the event-triggered SMC with continuous or intermittent monitoring will be designed for improving the performance of system output. Furthermore, the robustness of the system output will be discussed, which againsts matched disturbances.

### B. Some Assumptions

*Assumption 1:* The pair  $(A, B)$  is controllable.

*Assumption 2:* The disturbance  $d$  is bounded. And there exists  $d_1 > 0$  such that  $\sup_{t \geq 0} \|d(t)\| \leq d_0$  and  $\sup_{t \geq 0} \|\dot{d}(t)\| \leq d_1$ .

## III. EVENT-TRIGGERED SMC WITH CONTINUOUS MONITORING

This section first develops the estimation sliding surface design for systems output. Then, an output estimation-based event-triggered SMC design of LTI system with continuous monitoring is presented.

### A. Sliding Model Design

To design sliding mode control to ensure the system output stability, one defines the following sliding model

$$s = \dot{y} + \lambda y \quad (2)$$

where the constant  $\lambda > 0$ . Based on (1), we have

$$s = (CA + \lambda C)x + CB(u + d). \quad (3)$$

By taking the time derivative of  $s$ , one obtains

$$\begin{aligned} \dot{s} = & (CA^2 + \lambda CA)x + (CAB + \lambda CB)u \\ & + CB\dot{u} + (CAB + \lambda CB)d + CB\dot{d}. \end{aligned} \quad (4)$$

Because of the existence of the unknown disturbance  $d$ , it is impossible to evaluate the sliding variable  $s$  directly.

To overcome this difficulty, we resort to the estimation technique on the uncertain disturbance  $d$  to realize the estimate of the sliding variable  $s$  defined in (3).

Note that system output  $y$  can make the following differential equation holds,

$$\dot{y} = CAx + CB(u + d). \quad (5)$$

In the sequel, the estimation error of output  $y$  can be defined as

$$e \triangleq y - \hat{y} \quad (6)$$

where  $\hat{y}$  is an estimation of  $y$ ,

$$\dot{\hat{y}} = CAx + CB(u + \nu) + \beta e, \quad \nu = \rho \frac{e}{|e| + \varepsilon}. \quad (7)$$

Furthermore, we design the following the estimator for matched disturbance

$$\dot{\hat{d}} = \frac{\beta}{CB}e + \nu = \frac{\beta e}{CB} + \frac{\rho e}{|e| + \varepsilon} \quad (8)$$

where the constant  $\beta > 0, \rho > d_0$ , and  $\varepsilon > 0$ . In virtue of (3), (7) and (8), we define

$$\hat{s} = CAx + CB(u + \hat{d}) + \lambda Cx. \quad (9)$$

The effectiveness of (7) and (8) can be proven by the following theorem.

*Theorem 1:* If  $\rho$  in (7) is sufficiently large, the disturbance estimation error  $d - \hat{d}$ , will become arbitrarily small, where  $\rho$  represents the estimator gain and  $\hat{d}$  is given by (8).

*Proof:* From (5) and (7), we obtain

$$\begin{aligned} \dot{e} &= -\beta e - CB(v - d) \\ &= -\beta e - CB\left(\frac{\rho e}{|e| + \varepsilon} - d\right). \end{aligned} \quad (10)$$

It is clear that, if  $\rho$  is sufficiently large,  $e$  will be a arbitrarily small number, as well as  $\dot{e}$ . From (8) and (10), we can obtain  $\dot{e} = CB(d - \hat{d})$ . Thus, smaller  $\dot{e}$  means smaller  $d - \hat{d}$ . Therefore, the disturbance estimation is successful.

Define the Lyapunov function  $V_1 = \frac{1}{2}e^2$ , and take the derivative with respect to time

$$\begin{aligned} \dot{V}_1 &= e\dot{e} = e(-\beta e - CB(v - d)) \\ &\leq -\beta e^2 - |CB e| \left(\frac{\rho |e|}{|e| + \varepsilon} - d_0\right) \\ &\leq -\beta n_1^2, \quad \text{for all } e \notin \{e : |e| < n_1 = (\varepsilon d_0)/(\rho - d_0)\} \end{aligned}$$

Therefore, we can obtain that for some finite time  $T_1$  there is  $|e(t)| < n_1$  for all  $t > T_1$  (see [17]). As  $\rho$  becomes sufficiently large,  $n_1 = (\varepsilon d_0)/(\rho - d_0)$  will become arbitrarily small. Then we can conclude that within a finite time if  $\rho$  is sufficiently large,  $e$  will become a arbitrarily small number.

For  $\dot{e}$ , let  $V_2 = \frac{1}{2}\dot{e}^2$ , and take its time derivative

$$\begin{aligned} \dot{V}_2 &= \dot{e}\ddot{e} = \dot{e}\left(-\beta\dot{e} - CB\left(\frac{\rho\varepsilon\dot{e}}{(|e|+\varepsilon)^2} - \dot{d}\right)\right) \\ &\leq -\beta\dot{e}^2 - \frac{\rho\varepsilon|CB\dot{e}|^2}{(n_1+\varepsilon)^2} + |CB\dot{e}|d_1, \quad t \geq T_1 \\ &\leq -\beta\dot{e}^2 - \frac{\rho\varepsilon|CB\dot{e}|}{n_1+\varepsilon} \left(|\dot{e}| - \frac{d_1(n_1+\varepsilon)^2}{\rho\varepsilon}\right), \quad t \geq T_1, \\ &\leq \beta n_2^2, \quad \text{for all } e \notin \{e : |\dot{e}| < n_2 = d_1(n_1 + \varepsilon)^2/(\rho\varepsilon)\} \end{aligned}$$

Then, we can get that for some finite time  $T_2$  there will have  $|\dot{e}| < n_2$  for all  $t > T_2$  [17]. As  $\rho$  gets sufficiently large,  $n_2 = d_1(n_1 + \varepsilon)^2/(\rho\varepsilon)$  will become arbitrarily small. So we can obtain that within a finite time if  $\rho$  is sufficiently large,  $\dot{e} = CB(d - \hat{d})$  will become a arbitrarily small number. ■

According to the above Theorem, we know  $d - \hat{d}$  is bounded. Hence, the variable  $\hat{s}$  can be taken as an estimation of  $s$ .

### B. Event-Triggered SMC

To facilitate the future discussion, we define the  $\hat{y}(t_k)$  as

$$\hat{y}_k = \hat{y}(t_k), \quad k \in \mathbb{N} \quad (11)$$

where denoting  $t_k$  the  $k$ th triggering instant to be determined, For future proof, one defines sampled error based on (6) and (11)

$$e(t) \triangleq y(t) - \hat{y}(t_k), \quad t \in [t_k, t_{k+1}). \quad (12)$$

To stabilize the output signal  $y$ , we propose the following event-triggered control protocol:

$$\begin{aligned} CB\dot{u} = & -(CA^2 + \lambda CA)x \\ & - (CAB + \lambda CB)u(t_k) - K \operatorname{sgn}(\hat{s}(t_k)) \end{aligned} \quad (13)$$

where  $\operatorname{sgn}(\cdot)$  represents the sign function, and  $K \geq |\Delta p| = |(CAB + \lambda CB)d + CB\dot{d}|$ . The main result is as follows.

**Theorem 2:** Given  $\gamma > 0$  and  $\eta > 0$ , if  $K$  satisfies  $K > \sup_{t \geq 0} |\Delta p| + \gamma + \eta$  and  $|\Delta p| \leq d_1 + d_0(|C||A||B| + |\lambda||C||B|)$ . Then, the system output of (1) under event-triggered control (13) will reach the sliding surface  $s = 0$  with the triggering instant  $t_k$  defined as  $t_k = \min\{t : t \geq t_{k-1}, f(t) > 0\}$ , of which

$$f(t) = (|A|^2 + |\lambda||A|)|e(t)| - \gamma \quad (14)$$

is the triggering function.

*Proof:* Design the Lyapunov function  $V(s) = 1/2s^2$ .

$$\begin{aligned} \dot{V} &= s\dot{s} \\ &= s[(CA^2 + \lambda CA)x + (CAB + \lambda CB)u(t_k) \\ &\quad + CB\dot{u} + (CAB + \lambda CB)d + CB\dot{d}] \\ &= s[-(A^2 + \lambda A)e - K \operatorname{sgn}(s(\hat{t}_k)) + \Delta p] \\ &= -s(A^2 + \lambda A)e + s[-K \operatorname{sgn}(s(\hat{t}_k)) + \Delta p]. \end{aligned}$$

When  $\operatorname{sign}(\hat{s}(t_k)) = \operatorname{sign}(s(t))$ ,  $\forall t \in [t_k, t_{k+1})$  is satisfied, one obtains that

$$\begin{aligned} \dot{V} &\leq |s||A^2 + \lambda A||e| - |s|K + s\Delta p \\ &\leq |s|(|A^2 + \lambda A||e| - K + \Delta p) \\ &\leq -|s|(K - \gamma - d_0 - d_1(|C||A||B| + |\lambda||C||B|)) \\ &\leq -\eta|s| \end{aligned}$$

where  $\eta > 0$ . This can make sure the sliding trajectory converges to the sliding surface. However, if the trajectory hits the manifold  $s = 0$ , the sign of  $s$  will change in  $[t_k, t_{k+1})$ , which cannot guarantee  $\dot{V} < 0$ . It is possible that the trajectory crosses the manifold and eventually moves away, but, it will stay in the bounded region of the  $s = 0$  since  $\dot{V} < 0$  outside this bounded region. Therefore, the ultimate band is the region where  $\operatorname{sign}(\hat{s}(t_k)) \neq \operatorname{sign}(s(t))$ . In virtue of event-triggered condition (14), we have  $|s(t_k) - s(t)| = |(CA + \lambda C)e^*| < \frac{\gamma}{|A|}$ .

It is clear that there is a bounded region  $\gamma/|A|$  with  $\|s\| < \gamma/|A|$ . The detailed proof reference [15]. ■

**Remark 1:** Compare with [19], our work improves the control efficiency and reduces the burden of communication or actuation.

The following result indicates that the proposed control design is Zeno free.

**Theorem 3:** The system (1) with the event-triggered control (13) subject to the triggering condition (14) does not exhibit Zeno behavior.

*Proof:* Differentiating error  $\|e(t)\|$  for all  $t \in [t_k, t_{k+1})$

$$\begin{aligned} \frac{d}{dt}\|e(t)\| &\leq \left\| \frac{d}{dt}y(t) \right\| = \|C\| \|Ax + B(u + d)\| \\ &= \|C\| \|Ae(t) - Ax(t_k) + B(u + d)\| \\ &= \|C\| (\|Ae(t) - Ax(t_k) - s(t_k) - \lambda Cx(t_k)\|) \\ &\leq \|C\| (\|Ae(t)\| + \|A - \lambda Cx(t_k)\| + \|s(t_k)\|) \\ &= \|C\| (\|A\| \|e(t)\| + \rho_1(\|x(t_k)\|) + \beta_1). \end{aligned}$$

Thus, we have

$$\|e(t)\| \leq \frac{\rho_1(\|x(t_k)\|) + \beta_1}{\|A\|} (e^{\|A\|(t-t_k)} - 1), \quad \|e(t_k) = 0\|$$

for all times  $t \in [t_k, t_{k+1})$ . According to the triggering condition, we have

$$\begin{aligned} \frac{\gamma}{\|A\|^2 + |\lambda||A|} &= \|e(t_{k+1})\| \\ &\leq \frac{\rho_1(\|x(t_k)\|) + \frac{\gamma}{\|A\|}}{\|A\|} (e^{\|A\|T_k} - 1) \end{aligned} \quad (15)$$

where  $T_k := t_{k+1} - t_k$ .

According to (15), one has

$$T_k \geq \frac{1}{\|A\|} \ln \left( 1 + \frac{\gamma}{(\|A\| + \lambda)(\rho_1(\|x(t_k)\|) + \beta_1)} \right) \quad (16)$$

where  $\rho_1(\|x(t_k)\|) = (\|A - \lambda C\|)\|x(t_k)\|$  and  $\beta_1 = \frac{\gamma}{\|A\|}$ . Thus, the lower bound of  $T_k$  ensures no Zeno phenomenon. ■

## IV. EVENT-TRIGGERED SMC WITH EVENT INTERMITTENT MONITORING

In the previous section, one may observe that the proposed event-triggered control relies on continuous output information in the triggering condition which requires uninterrupted communication between sensor (for output information) and actuator. In this section, an improved event-triggered SMC is developed under the triggering condition (17) with intermittent monitoring.

**Theorem 4:** Given  $\gamma > 0$  and  $\eta > 0$ , the system output of (1) under event-triggered control (13) will reach the sliding surface (3) with the triggering instant  $t_k$  defined as  $t_k = \min\{t : t \geq t_{k-1}, F(\cdot) \geq 0\}$ . Here, the triggering function is defined as

$$F(t) = \bar{e} - Y(t_k) \quad (17)$$

where

$$\bar{e} = \int_{t_k}^t \left( \frac{\gamma\|C\|}{\|A\| + |\lambda|} + \rho_1(\|y(t_k)\|) + \beta_1 \right) d\tau \quad (18)$$

is the measurement error of system output, and

$$Y(t_k) = \theta \hat{y}(t_k), \quad \theta \in (0, 1). \quad (19)$$

*Proof:*

From the proof of Theorem 3, it is easy to know

$$|e(t)| < |\bar{e}(t_k)|, \quad t \in [t_k, t_{k+1}). \quad (20)$$

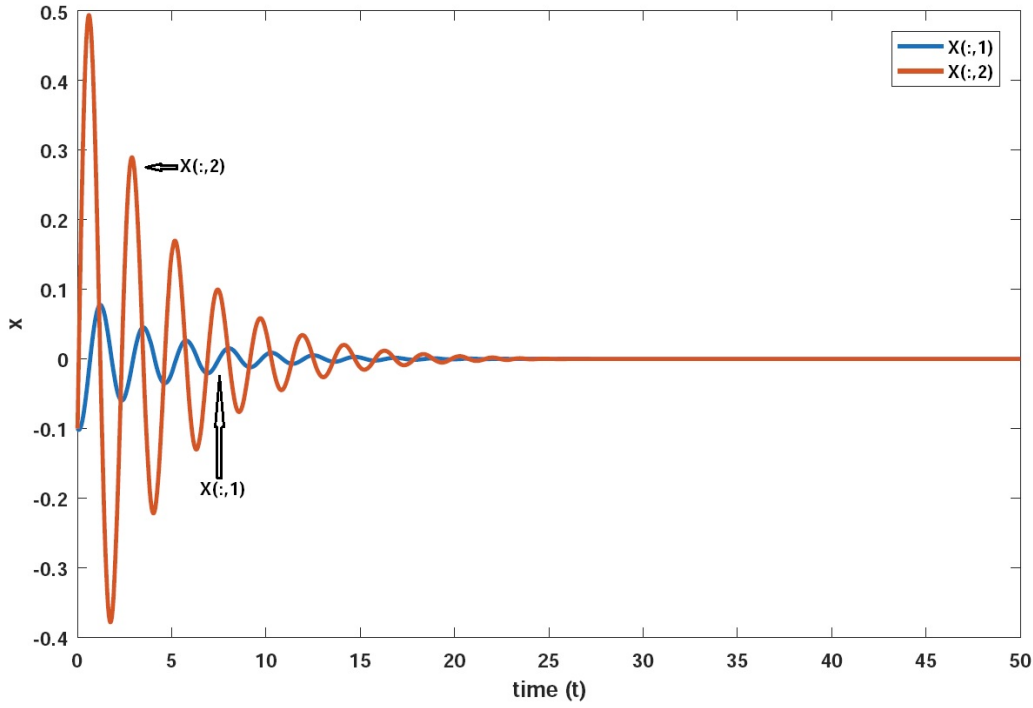


Fig. 1. States trajectories of the system with sliding mode dynamic.

According to (17) and (18), one can obtain

$$\int_{t_k}^t \left( \frac{\gamma \|C\|}{\|A\| + |\lambda|} + \rho_1 (\|y(t_k)\|) + \beta_1 \right) d\tau < Y(t_k). \quad (21)$$

Substituting (17) and (20) into (21), we have

$$|e(t)| < Y(t_k), \quad t \in [t_k, t_{k+1}). \quad (22)$$

By invoking (12), one obtains

$$y(t_k) < \frac{1}{1-\theta} y(t).$$

Thus, a further conclusion comes out

$$|e(t)| < Y(t_k) < \frac{\theta}{1-\theta} y(t). \quad (23)$$

Here, the output of system (1) with Intermittent monitoring under the input control (13) and event-triggered condition (17) will reach the sliding model surface  $s = 0$ . This ends the proof. ■

*Remark 2:* The triggering function (17) indicates that  $t_{k+1}$  can be determined by both  $y(t_k)$  and  $\hat{y}(t_k)$ , which implies that continuous monitoring is avoided.

*Remark 3:* In [13]–[16], the controller update depends on continuous monitoring of output information. On the contrary, continuous monitoring is not required for the novel controller designed here.

*Remark 4:* It's worth noting that though intermittent monitoring reduces energy consumption, however, system performance could get worse. Therefore, (17) has more triggering times than (14), which can be found from Fig. 8 and 12 as well as Table 1.

Suppose all assumptions hold in this article, the procedures of disturbance estimation-based event-triggered adaptive sliding model control policy for the system (1) is shown in Algorithm 1.

TABLE I  
EVENT-TRIGGERED FREQUENCY COMPARISON

output sampling with continuous monitoring :	1084
output estimation sampling with continuous monitoring:	647
output estimation sampling with intermittent monitoring:	2390

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#### Algorithm 1: Disturbance Estimation-Based ETSMCP

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Input: initial states of the SMC system

step 1: Disturbance estimation based on the scalar variable  $y$  defined in (2)

step 2: Estimation of Sliding variable  $s$  via (3)

step 3: With Estimation of Sliding variable  $s$ , we get

$$s = CAx + CB(u + d) + \lambda Cx$$

Step 4: With satisfying the triggering condition (17), the event-triggered control policy for system (1) with disturbances as

$$\begin{aligned} \omega &= CB\dot{u} \\ &= -(CA^2 + \lambda CA)x(t_k) - (CAB + \lambda CB)u \\ &\quad - K \operatorname{sgn}(\hat{s}(t_k)) \end{aligned}$$

where  $t \in [t_k, t_{k+1})$ .

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## V. ILLUSTRATIVE EXAMPLE

To verify event-triggered SMC, a typical linear dynamics simulation and a practical application will be presented.

### A. Linear Dynamics Simulation

Consider the continuous-time line system system (1) as

$$\dot{x} = \begin{bmatrix} 0 & 0.5 \\ -15 & -0.5 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0.02 \end{bmatrix} (u + d). \quad (24)$$

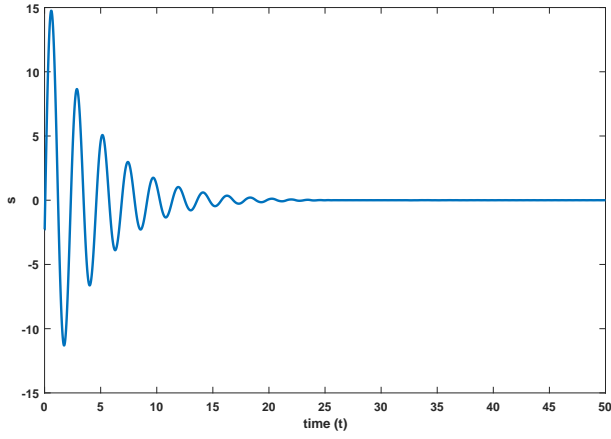


Fig. 2. Evolution of the sliding mode function.

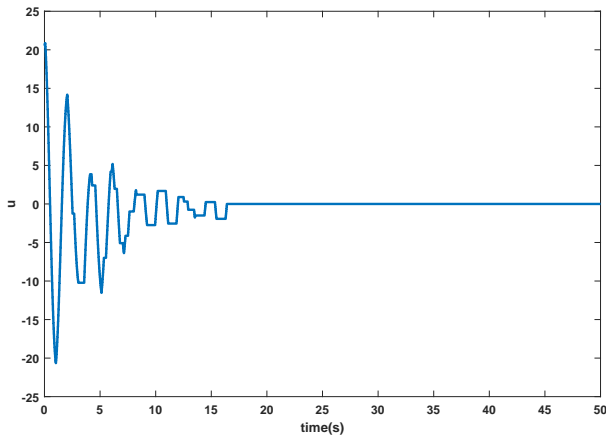


Fig. 3. Evolution of the SMC.

The sliding surface is

$$s = \dot{z} + \lambda z, \quad z = [0.8, 1.5]x. \quad (25)$$

Designing the event-triggered SMC as

$$\begin{aligned} \omega &= CB\dot{u} \\ &= -(CA^2 + \lambda CA)x(t_k) - (CAB + \lambda CB)u \\ &\quad - K \operatorname{sgn}(\hat{s}(t_k)). \end{aligned} \quad (26)$$

To verify the validity of the theoretical analysis in this paper, some parameters are chosen as  $d = 0.5 \sin(10t)$ ,  $\gamma = 1$ ,  $\lambda = 20$ ,  $\beta = 0.1$ ,  $\rho = 50$ ,  $\varepsilon = 0.5$  and  $K = 50$ . The control system (24) under SMC (25) can achieve practical sliding mode. According to the ETSMC algorithm, the simulation results are shown in Figs. 1, 2, 3 and 4. From Fig. 1, we can see that the system states are all convergent. As is shown in Fig. 2, the dynamics of the dynamic surface function are quickly convergent before 25 s. Fig. 3 depicts the dynamics of sliding mode controller asymptotically converge to zero with time going on. Fig. 4 displays the event-inter time for (24), from which one can find that the Zeno behavior does not happen. It can be observed that by using the dynamic sliding mode controller, the closed loop system is stable.

### B. Application to a Spring-Mass-Damper System

Consider the following spring-mass-damper system [18], which is depicted in Fig. 5 as follows

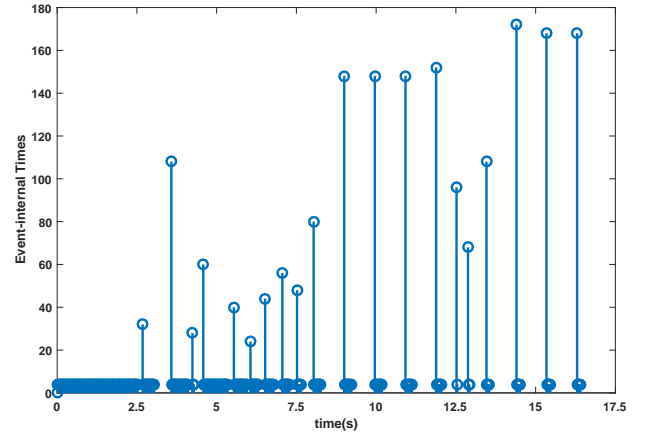


Fig. 4. Event interval time of the SMC.

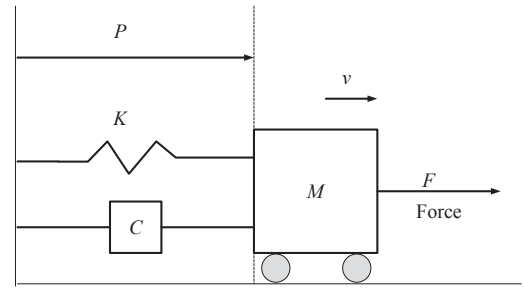


Fig. 5. Simple diagram of the mass-spring-damper system.

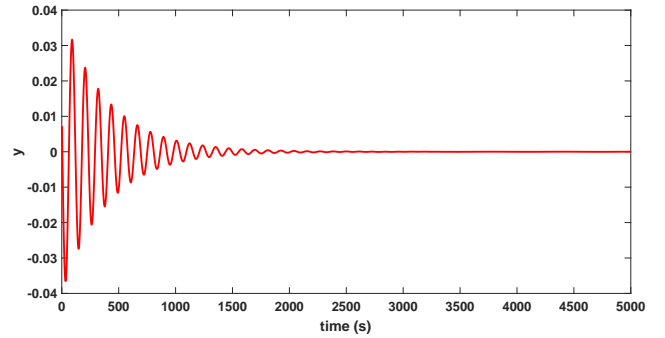


Fig. 6. Output trajectories with continuous monitoring.

$$\dot{P} = V, \quad (27a)$$

$$M\dot{V} = -K_1P - C_1V + F. \quad (27b)$$

Let  $x_1 = P$ ,  $x_2 = V$ ,  $u = F$ ,  $\xi = K_1/M$ ,  $\delta = C_1/M$  and  $\varphi = 1/M$ . And assuming the system (27) has an uncertain term and output term. Then, the (27) can be turned into

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{bmatrix} 0 & 1 \\ -\xi & -\delta \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \varphi \end{bmatrix} (u + d). \quad (28)$$

$$y = C_2 \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}. \quad (29)$$

To test the feasibility of theoretical analysis, some parameters are chosen as  $d = 0.8 \sin(t)$ ,  $\xi = 30$ ,  $\delta = 0.65$ ,

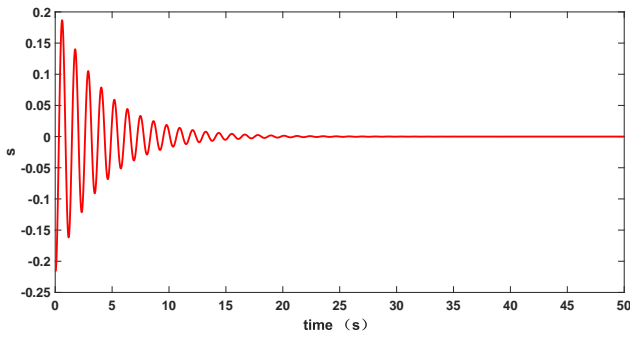


Fig. 7. Evolution of the sliding mode function with continuous monitoring.

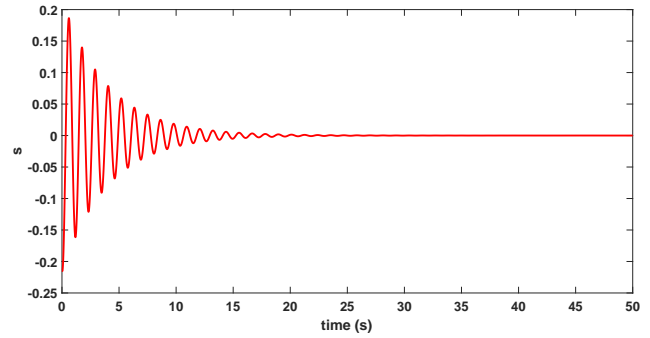


Fig. 11. Evolution of the sliding mode function with intermittent monitoring.

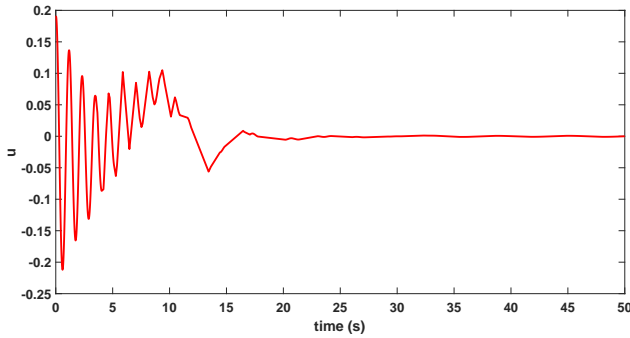


Fig. 8. Evolution of the SMC with continuous monitoring.

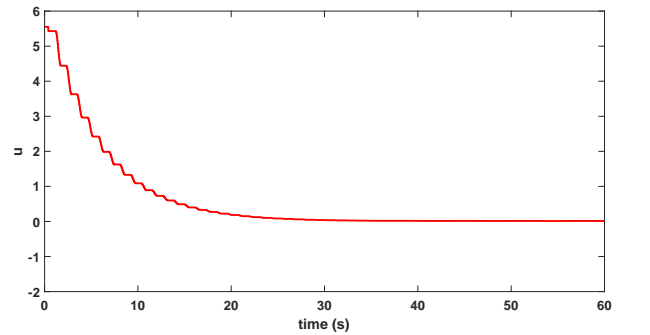


Fig. 12. Evolution of the SMC with intermittent monitoring.

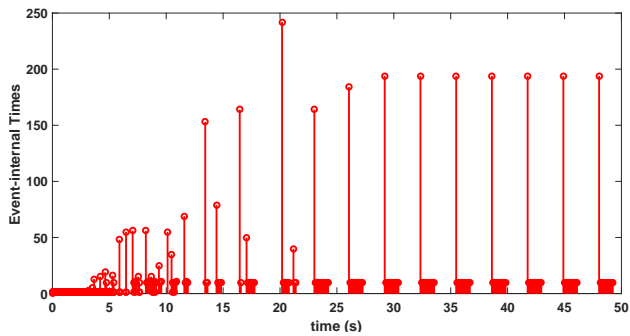


Fig. 9. Event interval time of the SMC with continuous monitoring.

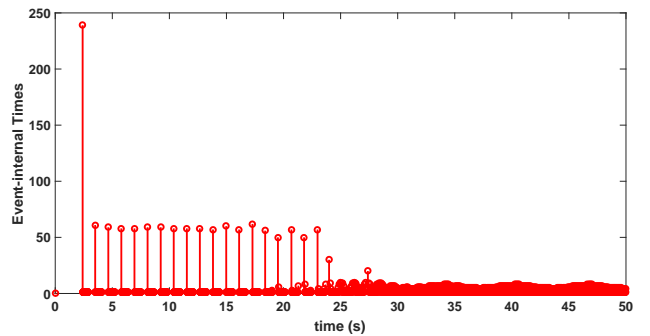


Fig. 13. Event interval time of the SMC with intermittent monitoring.

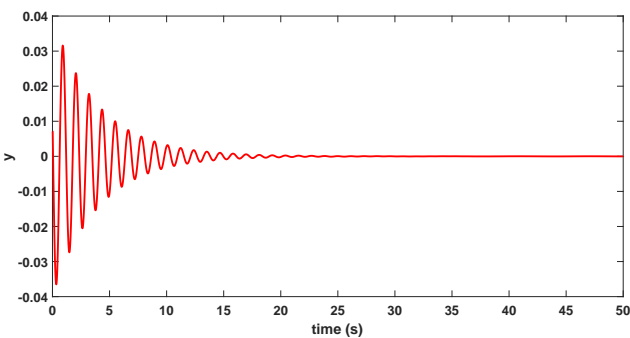


Fig. 10. Output trajectories with intermittent monitoring.

$\varphi = 0.001$ ,  $\gamma = 0.1$ ,  $\lambda = 0.05$ ,  $\beta = 2$ ,  $\rho = 5$ ,  $\varepsilon = 0.00001$ .  $K = 0.001$  and  $C_2 = [1, 8.5]$ . Dynamic trajectory of (29) is shown in Figs. 5 and 10. In addition, Figs. 7 and 11 depict the dynamic behaviors of event-triggered sliding variable. Figs. 8 and 12 show the input. As implied in Fig. 9 and 13, though intermittent monitoring reduces energy consumption,

however, systems conservatives will get worse. Therefore, (17) has more triggering times than (14), which can be found from Figs. 9 and 13 as well as Table 1.

## VI. CONCLUSION

This paper has presented the co-design technical process of event-triggered strategy and SMC based on disturbance estimation for continuous-time LTI system with matched disturbance. A new event-triggered SMC without continuous monitoring has been employed, which lowers the energy consumption of communications. New control design proposed effectively guarantees stability of system output. Moreover, Zeno behavior has been excluded under the designed event-triggered rule. In the future work, we will concentrate on the optimal control for the completely unknown nonlinear system with unmatched disturbance based on event-triggered SMC via adaptive dynamic programming approach.

## REFERENCES

- [1] L. Xing, C. Wen, Z. Liu, H. Su and J. Cai, "Event-triggered adaptive control for a class of uncertain nonlinear systems", *IEEE Transactions on Automatic Control*, vol. 62, no. 4, pp. 2071-2076, 2017.
- [2] P. Tabuada, "Event-triggered real-time scheduling of stabilizing control tasks", *IEEE Transactions on Automatic Control*, vol. 52, no. 9, pp. 1680-1685, 2007.
- [3] X. Xie, D. Yue and S. Hu, "Fault estimation observer design of discrete-time nonlinear systems via a joint real-time scheduling law", *IEEE Transactions on Systems, Man, and Cybernetics*, vol. 47, no. 7, pp. 1451-1463, 2017.
- [4] Y. Wu, Y. Li, S. He and Y. Guan, "Sampled-data synchronization of network systems in industrial manufacture", *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 50, no. 9, pp. 3210-3219, 2020.
- [5] T. Liu and Z. Jiang, "Event-triggered control of nonlinear systems with state quantization", *IEEE Transactions on Automatic Control*, vol. 64, no. 2, pp. 797-803, 2019.
- [6] X. Yi, K. Liu, D.V. Dimarogonas and K.H. Johansson, "Dynamic event-triggered and self-triggered control for multi-agent systems", *IEEE Transactions on Automatic Control*, vol. 64, no. 8, pp. 3300-3307, 2019.
- [7] L. Wu and W. Zheng, "Passivity-based sliding mode control of uncertain singular time-delay systems", *Automatica*, vol. 45, no. 9, pp. 2120-2127, 2009.
- [8] D. Fulwani, B. Bandyopadhyay and L. Fridman, "Nonlinear sliding surface: towards high performance robust control", *IET Control Theory & Applications*, vol. 6, no. 2, pp. 235-242, 2012.
- [9] W. Gao, Y. Wang and A. Homaiifa, "Discrete-time variable structure control systems", *IEEE Transactions on Industrial Electronics*, vol. 42, no. 2, pp. 117-122, 1995.
- [10] S. Chakrabarty and B. Bandyopadhyay, "A generalized reaching law for discrete time sliding mode control", *Automatica*, vol. 52, pp. 83-86, 2015.
- [11] S. Janardhanan and B. Bandyopadhyay, "Output feedback sliding-mode control for uncertain systems using fast output sampling technique", *IEEE Transactions on Industrial Electronics*, vol. 53, no. 5, pp. 1677-1682, 2006.
- [12] Z. Galias and X. Yu, "Euler's discretization of single-input sliding mode control system", *IEEE Transactions on Automatic Control*, vol. 52, no. 9, pp. 1726-1730, 2007.
- [13] A.K. Behera and B. Bandyopadhyay, "Robust sliding mode control: an event-triggering approach", *IEEE Transactions on Circuits and Systems II: Express Briefs*, vol. 64, no. 2, pp. 146-150, 2017.
- [14] A.K. Behera and B. Bandyopadhyay, "Self-triggering-based sliding-mode control for linear systems", *IET Control Theory & Applications*, vol. 9, no. 17, pp. 2541-2547, 2015.
- [15] A.K. Behera, B. Bandyopadhyay and X. Yu, "Periodic event-triggered sliding mode control", *Automatica*, vol. 96, no. 96, pp. 61-72, 2018.
- [16] B. Zheng, X. Yu and Y. Xue, "Quantized feedback sliding-mode control: An event-triggered approach", *Automatica*, vol. 91, no. 91, pp. 126-135, 2018.
- [17] M. Chen and M. Tomizuka, "Disturbance estimator and its application in estimation of system output derivatives", *IEEE Conference on Decision and Control*, IEEE, 1989.
- [18] H. Modares and F.L. Lewis, "Optimal tracking control of nonlinear partially-unknown constrained-input systems using integral reinforcement learning", *Automatica*, vol. 50, no. 7, pp. 1780-1792, 2014.
- [19] M. Tseng and M. Chen, "Chattering reduction of sliding mode control by low-pass filtering the control signal", *Asian Journal of Control*, vol. 12, no. 3, pp. 392-398, 2010.