Stability Analysis of Power Bogie System and Forward Design of Its Yaw Damper Damping Parameter

Yuewei Yu, Jingye Wang, Yunpeng Song and Leilei Zhao

***Abstract*—In this paper, a power bogie hunting motion model is proposed, in which the traction motor is suspended on the bogie frame, and the effectiveness of the model is validated by ADAMS/Rail software. The proposed model can effectively describe the hunting motion of power bogie under self-excitation. The study starts with the hunting motion model. Firstly, based on Hurwitz stability criterion, through the stability analysis of hunting motion, a mathematical calculation model of the yaw damper critical damping coefficient for the power bogie system is established. Then, by using the analytical calculation model, the variation law of the yaw damper critical damping coefficient of the power bogie with the traction motor suspension parameters is explored, which laid a foundation for the design of the yaw damper damping coefficient for power bogie systems. Finally, a forward design method of the yaw damper damping coefficient is given in terms of the analytical calculation method. The derived analytical calculation model and established design method can be used either during preliminary design or for other special purposes, especially in the case where many vehicle parameters are unknown.**

***Index Terms*—power bogie, hunting motion, stability analysis, yaw damper, forward design**

I. INTRODUCTION

As one of the important components of the bogie system for railway vehicles, yaw damper has a very important effect on the bogie hunting motion’s stability and the ability of its curving performance [1,2]. How to scientifically select the yaw damper damping coefficient is an urgent problem to be solved in the design of bogie systems. Also, it is the foundation and key of the yaw damper design [3]. In recent years, many researchers have conducted extensive research on yaw dampers, and achieved many valuable innovative results.

The first concern for researchers is the hunting stability analysis for trailer bogie systems affected by the yaw damper [4–10], and there are mainly two commonly used analysis methods at present [11], i.e. the linear stability analysis method, and the nonlinear stability analysis method. Many research results have pointed out that, both linear analysis and nonlinear analysis can obtain a relatively accurate analysis result, although the results of nonlinear analysis are slightly lower than those of linear analysis [12,13]. In other words, when conducting preliminary analysis and rough design of railway vehicles stability, in order to simplify the solution process, the linear stability analysis method can be preferred. In addition, many researchers have studied the curving performance of the trailer bogie affected by the yaw damper [14–17], which pointed out that when train passes through a curve track, the excessive wheel rail lateral force will aggravate the wear of wheels and tracks, and even cause train derailment. Therefore, when designing the yaw damper, it is necessary to consider both the design requirements for the stability of the bogie's hunting motion and its curve negotiation performance. For example, based on the analysis of hunting stability and curve performance, we have built a forward design method of the trailer bogie system’s yaw damper damping coefficient previously [18].

However, as the train operation speed increases, many studies have shown that, there are significant differences between the trailer bogie and the power bogie not only in terms of hunting stability analysis, but also in their curving performance analysis results, that is, traction motor have a very important effect on the bogie’s running performance [19]. Therefore, researchers have began to concentrate on the assessment of the power bogie systems [20,21], yet these studies were mainly conducted with the damping coefficient of the yaw damper known. It can be seen that, there has been no reliable forward design method for the yaw damper of power bogie systems.

To facilitate designers to design the power bogie yaw damper’s damping coefficient, in this paper, based on analytic method, a yaw damper damping coefficient forward design method of the power bogie system is established. The study starts with a hunting motion model of the suspension type traction motor power bogie, as shown in section II and section III. In section IV and section V, based on Hurwitz stability criterion, through the stability analysis of hunting motion, a mathematical calculation model of the yaw damper critical damping coefficient for the power bogie system is established, and the variation law of the yaw damper critical damping coefficient of the power bogie with the traction motor suspension parameters is explored, which are

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beneficial for the design of the yaw damper in power bogie systems, based on this, a simple analytical formula for the design of the yaw damper damping coefficient is derived.

II. Hunting motion model of power bogie system

A. Physical model

It is known that the hunting stability of the bogie system is a lateral dynamic problem, which has little relation with the vertical degree of freedom, therefore, when establishing a bogie hunting motion model, usually only the yaw and lateral displacements are considered [22]. On the other hand, in order to improve the computational efficiency of numerical analysis, a single bogie system model is often used to study the hunting motion mechanism of the bogie system instead of a vehicle model [23,24]. Based on this, a lateral stability dynamic model of the power bogie system is proposed, as shown in Figure 1. Here, the model is composed of one bogie, two wheel-sets, and two traction motors, with each component having degrees of freedom in both lateral and yaw directions. The reference coordinates are located on each subsystem mass center, and the positive x direction is the direction of train travel. Furthermore, the effects of the joint stiffness of the secondary lateral damper, yaw damper, and motor lateral damper are considered.

In Figure 1, M_b is the bogie frame mass, and its yaw moment of inertia is J_b; M_w is the mass of each wheel-set, and its yaw moment of inertia is J_w; M_m is the mass of each motor, in which the yaw moment of inertia of each motor is J_m; W is the axle load; K_{1x} and K_{2x} are the longitudinal stiffness of each wheel-set and each bogie; K_{1y} and K_{2y} are the lateral stiffness of each wheel-set and each bogie; K_m is the lateral stiffness of the motor suspension; $K_{m\varphi}$ is the yaw stiffness of the motor suspension; K_{ds}, K_{dt}, and K_{dm} are the equivalent stiffness of the yaw damper, secondary lateral damper, and motor lateral damper; C_s is the damping coefficient of a pair yaw damper; C_t is the damping coefficient of a pair secondary lateral damper; C_m and $C_{m\varphi}$ are the motor suspension's damping coefficients in lateral and yaw; r is the wheel rolling radius; a is half of the wheelbase; a_0 is the longitudinal length from the motor mass center to the bogie frame mass center; b is the half lateral distance between the contact point of the wheel and the rail; b_1, b_2, and b_3 are the half lateral installation distance of the wheel axle positioning spring, the central spring, and the yaw damper; b_0 is the half longitudinal installation distance of the secondary lateral damper; v is the operating speed of the train; O is the mass center of the bogie frame; O_1 and O_2 are the mass center of the front wheel-set and rear wheel-set; O_3 and O_4 are the mass center of the front motor system and rear wheel-set motor system; y_{w1} and φ_{w1} are the displacements of the front wheel-set in the lateral and yaw directions; y_{w2} and φ_{w2} are the displacements of the rear wheel-set in the lateral and yaw directions; y_b and φ_b are the the displacements of the bogie frame in the lateral and yaw directions; y_{m1} and φ_{m1} are the displacements of the front motor in the lateral and yaw directions; y_{m2} and φ_{m2} are the displacements of the rear motor in the lateral and yaw directions; x_{s1}~x_{s4} are the longitudinal displacements of the yaw damper piston rod; y_{d1}~y_{d4} are the lateral displacements of the secondary lateral damper piston rod; y_{e1}~y_{e4} are the lateral displacements of the motor damper piston rod.

It should be noted that, in order to improve the efficiency of the solution, certain assumptions are made on the premise of meeting the requirements of the stability analysis for the power bogie system's hunting motion, as follows:

1) The train travels at a constant speed on the straight rail.

2) The bogie is weakly coupled with the car body, only vertical loads are transmitted between the bogie and car body. When the bogie undergoes a hunting motion, the car body is almost in a static state.

3) The height of the bogie frame gravity center is consistent with the axle center line, ignoring the influence of the bogie frame roll vibration.

4) The elastic sliding contact between the wheel and rail is considered linear, and its sliding coefficients in all directions are equal. Neglecting the impact of the rail on wheel flange.

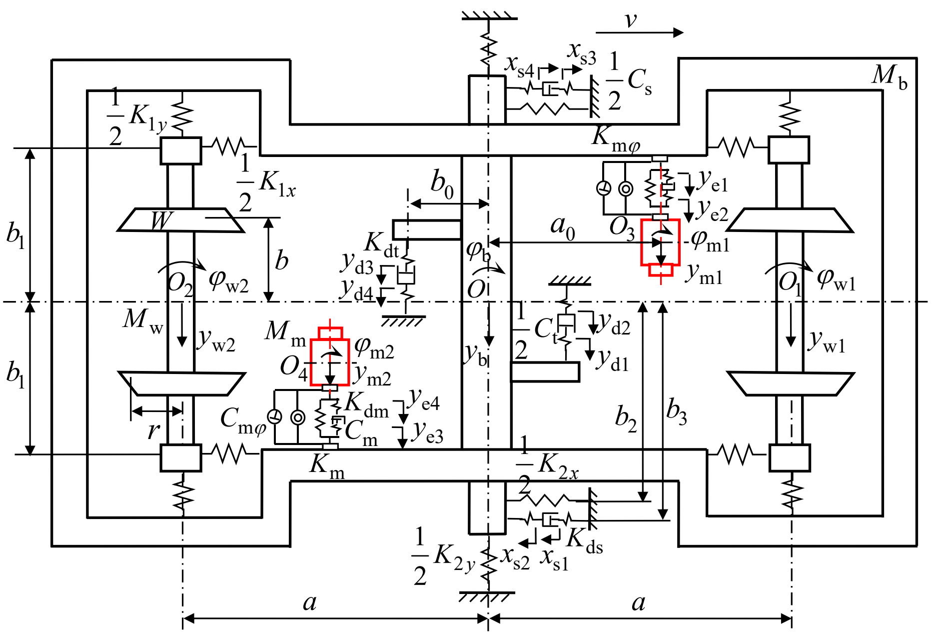

Fig. 1. Lateral stability dynamic model of the power bogie system

5) Neglecting the effect of small factors, e.g. the rotating moment of wheel axle around y direction, and the spin creep force between the wheel and rail. Besides, considering the wheel rail contact geometric relationship as linear.

6) The effect of the friction between subsystems is not considered, and the suspension force works in linear region.

B. Mathematical model

(1) Differential equations of the bogie frame:

$$\begin{cases} M_b\ddot{y}_b + K_{dt}(y_b + b_0\varphi_b - y_{d1}) + K_{dt}(y_b - b_0\varphi_b - y_{d3}) \\ \quad + K_{2y}y_b + K_{1y}(y_b + a\varphi_b - y_{w1}) + K_{1y}(y_b - a\varphi_b - y_{w2}) \\ \quad + K_{dm}(y_b + a_0\varphi_b - y_{e1}) + K_{dm}(y_b - a_0\varphi_b - y_{e3}) \\ \quad + K_m(y_b + a_0\varphi_b - y_{m1}) + K_m(y_b - a_0\varphi_b - y_{m2}) = 0 \\ J_b\ddot{\varphi}_b + K_{ds}b_3(b_3\varphi_b - x_{s4}) + K_{ds}b_3(b_3\varphi_b - x_{s2}) + K_{2x}b_2^2\varphi_b \\ \quad + K_{1x}b_1^2(\varphi_b - \varphi_{w1}) + K_{1x}b_1^2(\varphi_b - \varphi_{w2}) \\ \quad - K_{dm}a_0(y_b - a_0\varphi_b - y_{e3}) - K_m a_0(y_b - a_0\varphi_b - y_{m2}) \\ \quad + K_{dt}b_0\left[(y_b + b_0\varphi_b - y_{d1}) - (y_b - b_0\varphi_b - y_{d3})\right] \\ \quad + K_{dm}a_0\left[(y_b + a_0\varphi_b - y_{e1}) + (y_b + a_0\varphi_b - y_{m1})\right] \\ \quad + K_{1y}a\left[(y_b + a\varphi_b - y_{w1}) - (y_b - a\varphi_b - y_{w2})\right] = 0 \end{cases} \tag{1}$$

(2) Differential equations of the wheel-set:

$$\begin{cases} M_w\ddot{y}_{w1} - K_{1y}(y_b + a\varphi_b - y_{w1}) + \dfrac{W\lambda}{b}y_{w1} \\ \quad + 2f_{11}\left(\dfrac{\dot{y}_{w1}}{v} - \varphi_{w1}\right) = 0 \\ J_w\ddot{\varphi}_{w1} - K_{1x}b_1^2(\varphi_b - \varphi_{w1}) - Wb\lambda\varphi_{w1} \\ \quad + 2f_{22}\left(\dfrac{b\lambda}{r}y_{w1} + \dfrac{b^2\dot{\varphi}_{w1}}{v}\right) = 0 \\ M_w\ddot{y}_{w2} - K_{1y}(y_b - a\varphi_b - y_{w2}) + \dfrac{W\lambda}{b}y_{w2} \\ \quad + 2f_{11}\left(\dfrac{\dot{y}_{w2}}{v} - \varphi_{w2}\right) = 0 \\ J_w\ddot{\varphi}_{w2} - K_{1x}b_1^2(\varphi_b - \varphi_{w2}) - Wb\lambda\varphi_{w2} \\ \quad + 2f_{22}\left(\dfrac{b\lambda}{r}y_{w2} + \dfrac{b^2\dot{\varphi}_{w2}}{v}\right) = 0 \end{cases} \tag{2}$$

(3) Differential equations of the motor:

$$\begin{cases} M_m\ddot{y}_{m1} + K_{dm}(y_{m1} - y_{e2}) - K_m(y_b + a_0\varphi_b - y_{m1}) = 0 \\ J_m\ddot{\varphi}_{m1} + K_{m\varphi}(\varphi_{m1} - \varphi_b) + C_{m\varphi}(\dot{\varphi}_{m1} - \dot{\varphi}_b) = 0 \\ M_m\ddot{y}_{m2} + K_{dm}(y_{m2} - y_{e4}) - K_m(y_b - a_0\varphi_b - y_{m2}) = 0 \\ J_m\ddot{\varphi}_{m2} + K_{m\varphi}(\varphi_{m2} - \varphi_b) + C_{m\varphi}(\dot{\varphi}_{m2} - \dot{\varphi}_b) = 0 \end{cases} \tag{3}$$

(4) Differential equations of the yaw damper:

$$\begin{cases} K_{ds}x_{s1} + \dfrac{C_s}{2}(\dot{x}_{s1} - \dot{x}_{s2}) = 0 \\ K_{ds}x_{s3} + \dfrac{C_s}{2}(\dot{x}_{s3} - \dot{x}_{s4}) = 0 \\ \dfrac{C_s}{2}(\dot{x}_{s2} - \dot{x}_{s1}) + K_{ds}(x_{s2} - b_3\varphi_b) = 0 \\ \dfrac{C_s}{2}(\dot{x}_{s4} - \dot{x}_{s3}) + K_{ds}(x_{s4} - b_3\varphi_b) = 0 \end{cases} \tag{4}$$

(5) Differential equations of the motor damper:

$$\begin{cases} K_{dm}(y_{e1} - y_b - a_0\varphi_b) + C_m(\dot{y}_{e1} - \dot{y}_{e2}) = 0 \\ K_{dm}(y_{e3} - y_b + a_0\varphi_b) + C_m(\dot{y}_{e3} - \dot{y}_{e4}) = 0 \\ C_m(\dot{y}_{e2} - \dot{y}_{e1}) + K_{dm}(y_{e2} - y_{m1}) = 0 \\ C_m(\dot{y}_{e4} - \dot{y}_{e3}) + K_{dm}(y_{e4} - y_{m2}) = 0 \end{cases} \tag{5}$$

(6) Differential equations of the secondary lateral damper:

$$\begin{cases} K_{dt}y_{d2} + \dfrac{C_t}{2}(\dot{y}_{d2} - \dot{y}_{d1}) = 0 \\ \dfrac{C_t}{2}(\dot{y}_{d1} - \dot{y}_{d2}) + K_{dt}(y_{d1} - y_b - b_0\varphi_b) = 0 \\ K_{dt}y_{d4} + \dfrac{C_t}{2}(\dot{y}_{d4} - \dot{y}_{d3}) = 0 \\ \dfrac{C_t}{2}(\dot{y}_{d3} - \dot{y}_{d4}) + K_{dt}(y_{d3} - y_b + b_0\varphi_b) = 0 \end{cases} \tag{6}$$

Here, λ is the wheel-set tread equivalent taper; f_{11} and f_{22} are the lateral and longitudinal creep coefficients of each wheel, respectively.

III. HUNTING MOTION MODEL VERIFICATION

To verify the reliability of the hunting motion model described in Section II, taking a train as an example, a comparison with the data obtained by using ADAMS/Rail software is performed. Here, the structural values of the power bogie system are shown in Table I.

TABLE I
STRUCTURAL PARAMETER VALUES

Parameter	Unit	Value
M_b	kg	7 186
J_b	kg·m²	9 751
M_w	kg	3 085
J_w	kg·m²	2 024
M_m	kg	1 765
J_m	kg·m²	1 019
W	N	200 000
K_{1x}	MN/m	40
K_{1y}	MN/m	6
K_{2x}	MN/m	0.8
K_{2y}	MN/m	0.8
K_m	kN/m	626.48
$K_{m\varphi}$	kN·m/rad	361.69
K_{ds}	MN/m	40
K_{dt}	MN/m	40
K_{dm}	MN/m	40
C_t	kN·s/m	60
C_s	kN·s/m	1 000
$C_{m\varphi}$	kN·s·m/rad	11.52
C_m	kN·s/m	19.95
a	m	1.25
b	m	0.746 5
a_0	m	0.75
b_0	m	0.3
b_1	m	1.1
b_2	m	1.1
b_3	m	1.2
r	m	0.625
λ	-	0.15
f_{11}	kN	11 511
f_{22}	kN	11 511

Note that, when validated, the initial conditions imposed by the two models are the same, and the model in this paper is solved using Matlab/Simulink [25]. For example, when the front wheel-set is affected by the lateral disturbance y=sin6t, and the train operating speed v=250 km/h, the time-varying curve of the bogie acceleration under the two models obtained by simulation is shown in Figure 2.

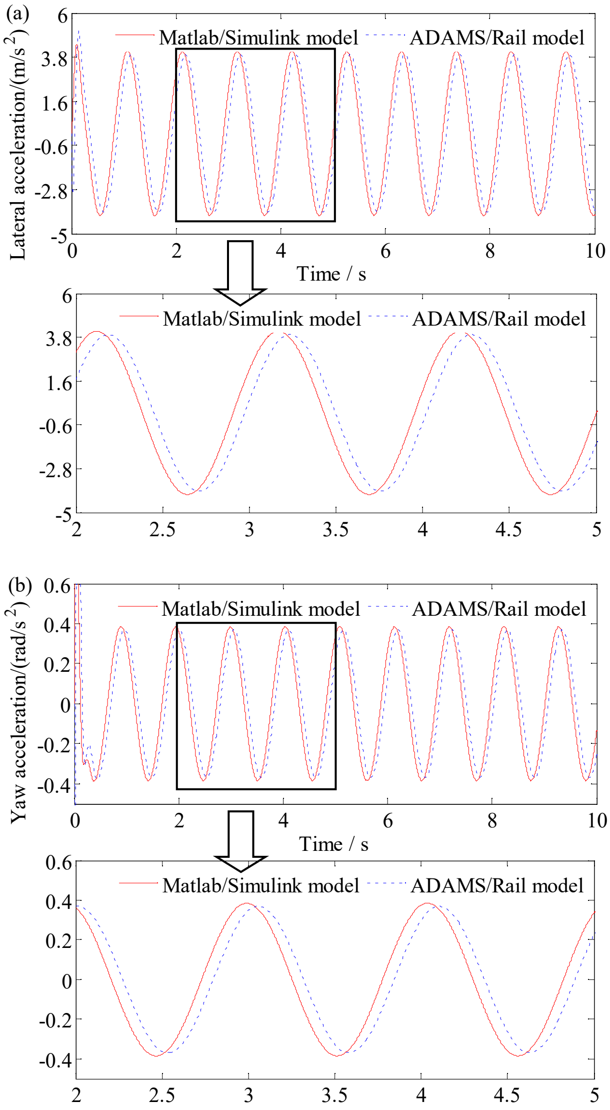

Fig. 2. Vibration response comparison results of the bogie frame: (a) lateral acceleration; (b) yaw acceleration

Table II gives the comparing results of the root mean square values of the lateral and yaw accelerations of the bogie frame.

TABLE II
COMPARING RESULTS OF THE ROOT MEAN SQUARE VALUES

Type	Lateral acceleration (m/s^2)	Yaw acceleration (rad/s^2)
Proposed model	2.895	0.282
ADAMS/Rail model	2.788	0.279
Relative deviation/%	3.838	1.075

From Figure 2 and Table II, it can be seen that, the lateral and yaw accelerations of the bogie frame have a good coincidence under the two models, and the vibration amplitudes of the acceleration are almost the same even though there is a little difference in phase. In addition, the deviation range of the root mean square values of the lateral and yaw accelerations is small, and the maximum deviation is only 3.838%. This implies that the hunting motion model of the power bogie system established is correct.

IV. STABILITY ANALYSIS

Considering that the linear stability method, which has the characteristics of fast calculation speed, and easy consideration of the variation of the wheel-rail parameters, it is very suitable for the preliminary research of regularity [21], and useful for preliminary parameter selection in railway vehicles engineering design [7], in this paper, the most commonly used linear system stability criterion, namely Hurwitz stability criterion, is used to study the hunting stability of the power bogie system.

On the basis of Laplace transformation, equations (1)~(6) can be rewritten in the matrix form, as follows

$$\begin{pmatrix}
\kappa_0 & 0 & \kappa_1 & 0 & \kappa_1 & 0 & \kappa_2 & 0 & \kappa_2 & 0 & 0 & 0 & 0 & 0 & \kappa_3 & 0 & \kappa_3 & 0 & \kappa_4 & 0 & \kappa_4 & 0 \\
0 & \kappa_5 & \kappa_6 & \kappa_7 & \kappa_8 & \kappa_7 & \kappa_9 & 0 & \kappa_{10} & 0 & 0 & \kappa_{11} & 0 & \kappa_{11} & \kappa_{12} & 0 & \kappa_{13} & 0 & \kappa_{14} & 0 & \kappa_{15} & 0 \\
\kappa_1 & \kappa_6 & \kappa_{16} & \kappa_{17} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \kappa_7 & \kappa_{18} & \kappa_{19} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\kappa_1 & \kappa_8 & 0 & 0 & \kappa_{16} & \kappa_{17} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \kappa_7 & 0 & 0 & \kappa_{18} & \kappa_{19} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\kappa_2 & \kappa_9 & 0 & 0 & 0 & 0 & \kappa_{20} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \kappa_3 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \kappa_{21} & 0 & 0 & 0 & 0 & 0 & \kappa_{22} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\kappa_2 & \kappa_{10} & 0 & 0 & 0 & 0 & 0 & 0 & \kappa_{20} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \kappa_3 & 0 & 0 & 0 & 0 \\
0 & \kappa_{21} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \kappa_{22} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \kappa_{23} & \kappa_{24} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \kappa_{11} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \kappa_{24} & \kappa_{23} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \kappa_{23} & \kappa_{24} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \kappa_{11} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \kappa_{24} & \kappa_{23} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\kappa_3 & \kappa_{12} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \kappa_{25} & \kappa_{26} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \kappa_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \kappa_{26} & \kappa_{25} & 0 & 0 & 0 & 0 & 0 & 0 \\
\kappa_3 & \kappa_{13} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \kappa_{25} & \kappa_{26} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \kappa_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \kappa_{26} & \kappa_{25} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \kappa_{27} & \kappa_{28} & 0 & 0 \\
\kappa_4 & \kappa_{14} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \kappa_{28} & \kappa_{27} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \kappa_{27} & \kappa_{28} \\
\kappa_4 & \kappa_{15} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \kappa_{28} & \kappa_{27}
\end{pmatrix}
\begin{pmatrix} y_{\mathrm{b}} \\ \varphi_{\mathrm{b}} \\ y_{\mathrm{w1}} \\ \varphi_{\mathrm{w1}} \\ y_{\mathrm{w2}} \\ \varphi_{\mathrm{w2}} \\ y_{\mathrm{m1}} \\ \varphi_{\mathrm{m1}} \\ y_{\mathrm{m2}} \\ \varphi_{\mathrm{m2}} \\ x_{\mathrm{s1}} \\ x_{\mathrm{s2}} \\ x_{\mathrm{s3}} \\ x_{\mathrm{s4}} \\ y_{\mathrm{e1}} \\ y_{\mathrm{e2}} \\ y_{\mathrm{e3}} \\ y_{\mathrm{e4}} \\ y_{\mathrm{d1}} \\ y_{\mathrm{d2}} \\ y_{\mathrm{d3}} \\ y_{\mathrm{d4}} \end{pmatrix}
=
\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{7}$$

where, $\kappa_0 = M_{\mathrm{b}}s^2 + 2K_{1y} + K_{2y} + 2K_{\mathrm{dm}} + 2K_{\mathrm{dt}} + 2K_{\mathrm{m}}$, $\kappa_1 = -K_{1y}$, $\kappa_2 = -K_{\mathrm{m}}$, $\kappa_3 = -K_{\mathrm{dm}}$, $\kappa_4 = -K_{\mathrm{dt}}$,

$\kappa_5=2K_{1y}a^2+2K_{dm}a_0^2+2K_{m}a_0^2+2K_{1x}b_1^2+K_{2x}b_2^2+2K_{ds}b_3^2+2K_{dt}b_0^2+J_bs^2$, $\kappa_6=\kappa_1a$, $\kappa_7=-K_{1x}b_1^2$, $\kappa_8=K_{1y}a$, $\kappa_9=\kappa_2a_0$, $\kappa_{10}=K_ma_0$, $\kappa_{11}=-K_{ds}b_3$, $\kappa_{12}=\kappa_3a_0$, $\kappa_{13}=K_{dm}a_0$, $\kappa_{14}=\kappa_4b_0$, $\kappa_{15}=K_{dt}b_0$, $\kappa_{16}=M_ws^2+2f_{11}s/v+K_{1y}+W\lambda/b$, $\kappa_{17}=-2f_{11}$, $\kappa_{18}=2bf_{22}\lambda/r$, $\kappa_{19}=2f_{22}b^2s/v-W\lambda b+K_{1x}b_1^2+J_ws^2$, $\kappa_{20}=M_ms^2+K_{dm}+K_m$, $\kappa_{21}=-K_{my}-C_{my}s$, $\kappa_{22}=J_ms^2+C_{my}s+K_{my}$, $\kappa_{23}=K_{ds}+C_ss/2$, $\kappa_{24}=-C_ss/2$, $\kappa_{25}=K_{dm}+C_ms$, $\kappa_{26}=-C_ms$, $\kappa_{27}=-C_ts/2$, $\kappa_{28}=K_{dt}+C_ts/2$.

According to equation (7), the Herwitz characteristic determinant of the hunting motion of the power bogie shown in Figure 1 can be solved, as follows

$$D(s)=\begin{vmatrix}
\kappa_0 & 0 & \kappa_1 & 0 & \kappa_1 & 0 & \kappa_2 & 0 & \kappa_2 & 0 & 0 & 0 & 0 & 0 & \kappa_3 & 0 & \kappa_3 & 0 & \kappa_4 & 0 & \kappa_4 & 0 \\
0 & \kappa_5 & \kappa_6 & \kappa_7 & \kappa_8 & \kappa_7 & \kappa_9 & 0 & \kappa_{10} & 0 & 0 & \kappa_{11} & 0 & \kappa_{11} & \kappa_{12} & 0 & \kappa_{13} & 0 & \kappa_{14} & 0 & \kappa_{15} & 0 \\
\kappa_1 & \kappa_6 & \kappa_{16} & \kappa_{17} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \kappa_7 & \kappa_{18} & \kappa_{19} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\kappa_1 & \kappa_8 & 0 & 0 & \kappa_{16} & \kappa_{17} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \kappa_7 & 0 & 0 & \kappa_{18} & \kappa_{19} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\kappa_2 & \kappa_9 & 0 & 0 & 0 & 0 & \kappa_{20} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \kappa_3 & 0 & 0 & 0 & 0 & 0 \\
0 & \kappa_{21} & 0 & 0 & 0 & 0 & 0 & \kappa_{22} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\kappa_2 & \kappa_{10} & 0 & 0 & 0 & 0 & 0 & 0 & \kappa_{20} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \kappa_3 & 0 & 0 & 0 & 0 \\
0 & \kappa_{21} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \kappa_{22} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \kappa_{23} & \kappa_{24} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \kappa_{11} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \kappa_{24} & \kappa_{23} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \kappa_{23} & \kappa_{24} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \kappa_{11} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \kappa_{24} & \kappa_{23} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\kappa_3 & \kappa_{12} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \kappa_{25} & \kappa_{26} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \kappa_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \kappa_{26} & \kappa_{25} & 0 & 0 & 0 & 0 & 0 & 0 \\
\kappa_3 & \kappa_{13} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \kappa_{25} & \kappa_{26} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \kappa_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \kappa_{26} & \kappa_{25} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \kappa_{27} & \kappa_{28} & 0 & 0 \\
\kappa_4 & \kappa_{14} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \kappa_{28} & \kappa_{27} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \kappa_{27} & \kappa_{28} \\
\kappa_4 & \kappa_{15} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \kappa_{28} & \kappa_{27}
\end{vmatrix} \tag{8}$$

According to equation (8), the characteristic equation of the Hurwitz stability criterion can be obtained by

$$\begin{aligned}&\chi_0s^{26}+\chi_1s^{25}+\chi_2s^{24}+\chi_3s^{23}+\chi_4s^{22}+\chi_5s^{21}+\chi_6s^{20}+\chi_7s^{19}\\&+\chi_8s^{18}+\chi_9s^{17}+\chi_{10}s^{16}+\chi_{11}s^{15}+\chi_{12}s^{14}+\chi_{13}s^{13}+\chi_{14}s^{12}\\&+\chi_{15}s^{11}+\chi_{16}s^{10}+\chi_{17}s^{9}+\chi_{18}s^{8}+\chi_{19}s^{7}+\chi_{20}s^{6}+\chi_{21}s^{5}\\&++\chi_{22}s^{4}+\chi_{23}s^{3}\chi_{24}s^{2}+\chi_{25}s+\chi_{26}=0\end{aligned} \tag{9}$$

where, χ_0~χ_{26} are the coefficients of the characteristic equation, which is represented by vehicle parameters and vehicle operating speed. Here, $\chi_0>0$, $\chi_1>0$, …, $\chi_{26}>0$.

According to Hurwitz stability criterion, it can be seen that, the stability of the system can be judged based on the following mathematical relationship [26]:

1) Assuming that all the real parts of the roots in equation (9) are all negative real numbers, at this time, the system is asymptotically stable;

2) If at least one real part of the root of equation (9) is positive, it indicates that the system is unstable;

3) It is assumed that the roots of equation (9) have no real part, but if there is a single root with zero real part, then the system is critically stable; if there are multiple roots with zero real part, at this time, the critical stability or instability of the system is determined by the relationship between the multiplicity of the root and the number of independent solutions.

V. Forward Design of the Yaw Damper Damping Coefficient for Power Bogie System

A. Analytical design formulae

It is known that, in railway vehicles, the yaw damper is mainly used to suppress bogie system's hunting motion, so that the train can have a higher running speed. Therefore, when the design speed range of the train is given, in order to ensure the stable operation of the power bogie system, the critical damping coefficient of the yaw damper can be obtained from the above theoretical analysis. The following is an introduction to the design of the yaw damper damping coefficient for power bogie systems.

As stated earlier, the coefficients in characteristic equation (9) can be taken as Hurwitz determinants, as follows

$$\Delta_n=\begin{vmatrix}\chi_1 & \chi_3 & \chi_5 & \cdots\cdots & \cdots & \cdots \\ \chi_0 & \chi_2 & \chi_4 & \cdots\cdots & \cdots & \cdots \\ 0 & \chi_1 & \chi_3 & \cdots\cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots\cdots & \cdots & \cdots \\ 0 & \cdots & \cdots & \cdots\cdots & \chi_{n-1} & \chi_n\end{vmatrix} \tag{10}$$

It can be concluded that, in order to make the system stable, the following conditions need to be met: the Hurwitz determinant and all its subdeterminants should be all positive numbers, i.e. $\Delta_1>0$, $\Delta_2>0$, $\Delta_3>0$, …, $\Delta_{n-1}>0$, $\Delta_n>0$ (n=1, 2, …, 26). Here, $\Delta_1=|\chi_1|$, $\Delta_2=\begin{vmatrix}\chi_1 & \chi_3\\ \chi_0 & \chi_2\end{vmatrix}$,

$$\Delta_3=\begin{vmatrix}\chi_1 & \chi_3 & \chi_5\\ \chi_0 & \chi_2 & \chi_4\\ 0 & \chi_1 & \chi_3\end{vmatrix}, \cdots, \Delta_n=\begin{vmatrix}\chi_1 & \chi_3 & \chi_5 & \cdots & \cdots & \cdots \\ \chi_0 & \chi_2 & \chi_4 & \cdots & \cdots & \cdots \\ 0 & \chi_1 & \chi_3 & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & \cdots & \cdots & \cdots & \chi_{n-1} & \chi_n\end{vmatrix}.$$

According to Hurwitz stability criterion, if $\chi_0>0$, $\chi_1>0$, ..., $\chi_{26}>0$, the system is stable as long as $\Delta_n>0$ is satisfied.

Therefore, based on the critical condition of stability, i.e. $\Delta_{26}=0$, an analytical model for calculating the yaw damper critical damping coefficient for power bogie's hunting motion can be established, that is

$$E_0C_s^{49}+E_1C_s^{48}+\sum_{i=2}^{49}E_iC_s^{49-i}+E_{48}C_s+E_{49}=0 \qquad (11)$$

where, E_0, E_1, …, E_{49} are the constant coefficients of equation (11), which is represented by vehicle parameters and vehicle operating speed.

According to equation (11), taking C_s as an unknown variable, and then solving the minimum positive real root of equation (11) regarding C_s, the required yaw damper critical damping coefficient for the power bogie hunting motion (i.e., C_{sl}) can be achieved.

Figure 3 gives the variation of the yaw damper critical damping coefficient C_{sl} for the power bogie hunting motion (parameter values can be found in section III) varying with the motor suspension parameters, and the vehicle operating speed is 300 km/h. Here, in order to draw a general conclusion, when exploring the variation law of the yaw damper critical damping coefficient of the power bogie with the traction motor suspension parameters, traction motor's suspension stiffness and damping is converted into the natural frequency and damping ratio, the conversion formula is as follows

$$\begin{cases} f_m=\dfrac{1}{2\pi}\sqrt{\dfrac{K_m}{M_m}} \\ \xi_m=\dfrac{C_m}{2\sqrt{K_mM_m}} \end{cases} \qquad (12)$$

where, f_m and ξ_m are the traction motor suspension's lateral natural frequency and damping ratio.

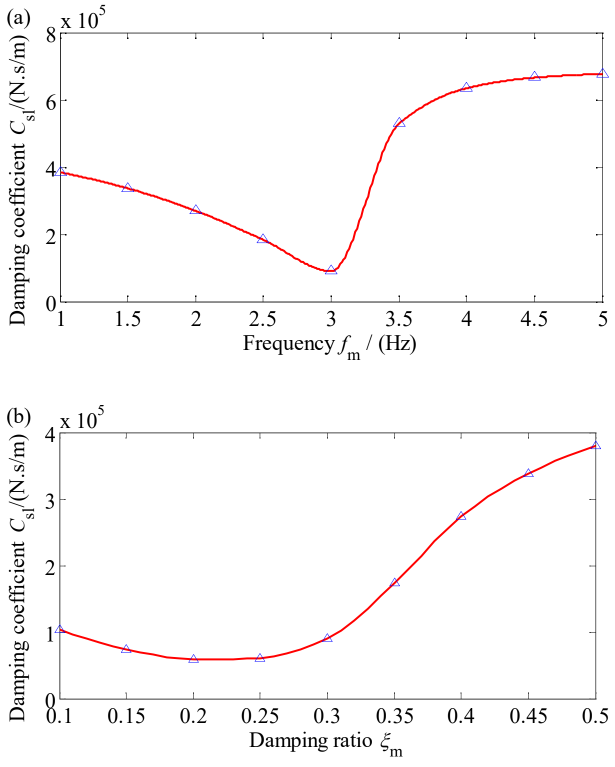

Fig. 3. The influence law of motor suspension parameters: (a) influence of the natural frequency; (b) influence of the damping ratio

From Figure 3, we can see that, the yaw damper critical damping coefficient C_{sl}: decreases first, then increases, and finally tends to be flat with the increase of f_m; decreases first, then increases with the increase of ξ_m. It can be known that, in order to make the power bogie system have a good operation stability,the critical damping coefficient of the yaw damper is different when matching different traction motor suspension parameters. That is to say, the motor system has a significant impact on the design of the yaw damper damping coefficient. This conclusion can not be obtained in the analysis of the trailer bogie systems.

As is well known, the curve negotiation ability of railway vehicles is significantly affected by the yaw damper [1,2]. If the damping coefficient of the designed yaw damper is too large, it will cause excessive steering resistance moment between the car body and the bogie frame, which will reduce the bending performance of railway vehicles, also, the turning resistance moment will cause a trend of increasing gauge, which may lead to train derailment. Therefore, when designing the damping coefficient of the yaw damper, we must ensure that, the vehicle has a high operating speed (i.e. good motion stability), and at the same time, make the train have the best curving performance. Generally, in the design of a bogie system, it is necessary to ensure that the wheel-set do not deviate from the rail. To achieve this design goal, when the train passes through a minimum radius curve, the rotational resistance of the bogie must be less than or equal to 0.08 [14,18], that is

$$\frac{T}{4Qa}\le 0.08 \qquad (13)$$

where, T is the bogie frame rotation resistance moment, Q is the average bogie axle load, a is half of the wheelbase.

Through analysis, it can be concluded that, when a train runs on a curved track, the rotational resistance moment of the bogie is generated by the yaw damper's rotational resistance moment, the secondary lateral damper's rotational resistance moment, and the secondary suspension spring's rotational resistance moment, therefore, the maximum damping coefficient of the yaw damper allowed for railway vehicles running on curved tracks can be obtained according to equation (13), that is

$$C_{su}=\frac{0.64WaRl-C_tb_0^2LV_r-K_{2x}b_2^2Ll}{b_3^2LV_r} \qquad (14)$$

Here, R is the radius of the circular curve of the track, l is the length of the transition curve of the curve track, L is the distance between the centers of two bogies in a single carriage, V_r is the maximum speed at which a train passes through a curved track.

According to the design requirement of the yaw damper, i.e. the railway vehicle should have the best curve negotiation performance possible while ensuring the bogie system has a good hunting stability. Therefore, according to the calculated damping coefficients C_{sl} and C_{su}, the feasible design interval can be established for the yaw damper's damping coefficient.

$$C_s\in(C_{sl},C_{su}) \qquad (15)$$

From equation (15), it can be seen that, by selecting an appropriate damping coefficient C_s within the range of (C_{sl}, C_{su}), a good compromise can be achieved between the bogie's hunting stability and the vehicle's curve passing ability. Therefore, in power bogie system's initial design stage, a appropriate yaw damper damping coefficient value can be selected according to designer's propensity to vehicle's running performance by using equation (15). For example, referring to the golden section design method for the damping coefficient of the trailer bogie system's yaw damper established by our previous research [18], the design value of the power bogie system's yaw damper damping coefficient can be obtained, that is C_s=0.382C_{su}+0.618C_{sl}.

B. Design example for the power bogie yaw damper

Based on the established damping coefficient design method for the power bogie's yaw damper, the train given in section III is designed, and the obtained values of its yaw damper are shown in Table III. Here, R=300 m, L=19 m, V_r=90 km/m, l=60 m, the vehicle design speed is 300km/h, and C_s was obtained by using the golden section method, i.e. C_s=0.382C_{su}+0.618C_{sl}.

TABLE III
DESIGN RESULTS

Damping	C_{sl}/(N·s/m)	C_{su}/(N·s/m)	C_s/(N·s/m)
Value	90 730	2 593 440	1 046 770

In engineering practice, the design results are often first verified through simulation experiments. Based on this, we substituted the design result C_s=1 046 770 N·s/m into the ADAMS/Rail model and simulated the motion of the power bogie system at the operating speeds of v=250 km/h and v=300 km/h, so as to verify the reliability of the established design method. Figures 4 and 5 show the analysis results of the lateral and yaw displacements of the bogie frame at the two operating speeds. During simulation, the external disturbances suffered by each wheel-set are track direction irregularity and horizontal irregularity [22]

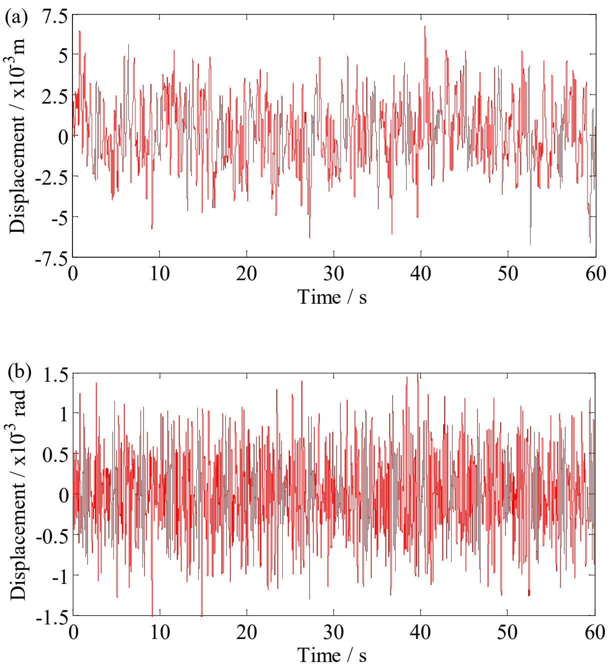

Fig. 4. Responses of the bogie frame under v=250 km/h: (a) bogie lateral displacement; (b) bogie yaw displacement

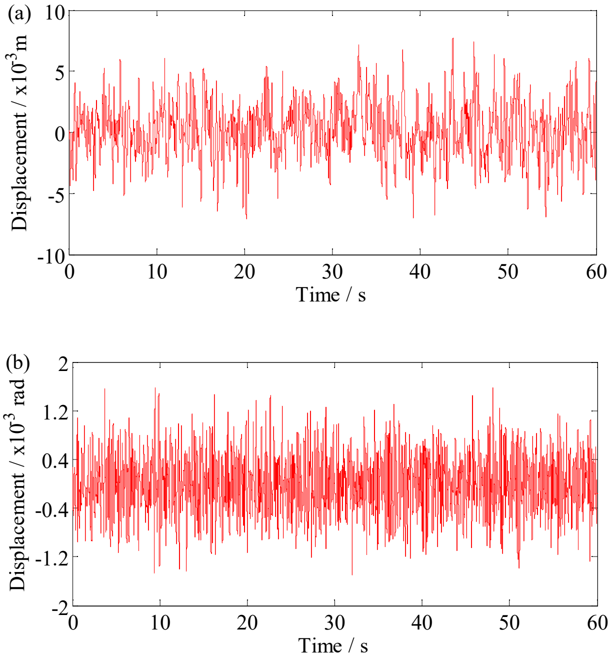

Fig. 5. Responses of the bogie frame under v=300 km/h: (a) bogie lateral displacement; (b) bogie yaw displacement

From Figures 4 and 5, it can be seen that, the lateral displacement and yaw displacement amplitudes of the bogie frame tend to stabilize at the operating speeds of v=250 km/h and v=300 km/h. This implies that, the yaw damper's damping coefficient designed can effectively suppress bogie system's hunting motion and enable the train to have a good curve negotiation performance. In other words, the damping coefficient design method of the yaw damper for power bogie systems established is reliable.

VI. CONCLUSIONS

To facilitate designers to design the power bogie's yaw damper damping coefficient, a power bogie hunting motion model is proposed, in which the traction motor is suspended on the bogie frame, and the effectiveness of the model is validated by ADAMS/Rail software. The proposed model can effectively describe the hunting motion of power bogie under self-excitation. On the basis of the validated hunting motion model, an analytical calculation model of the yaw damper critical damping coefficient for power bogie hunting motion is established, and the variation law of the yaw damper critical damping coefficient of the power bogie with the traction motor suspension parameters is explored, which are benefit for future research. Correspondingly, by using the derived analytical calculation model, a forward design method of the yaw damper damping coefficient is given in terms of the analytical calculation method.

This study can provide a useful reference for the preliminary design of the yaw damper for power bogie systems or for other special purposes, especially in the case where many vehicle parameters are unknown.

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