

# Bipolar Fuzzy Influence Graph and Its Chemical Application

Juanjuan Lu, Linli Zhu, Wei Gao

**Abstract**—As a critical field of theoretical chemistry, chemical graph theory works as a key part in the model topology calculation and prediction based on chemical molecular structures. The paper can contribute to the research in the following aspects: (1) We propose the concept of bipolar fuzzy influence graph, using positive and negative membership functions to describe positive and negative uncertainties respectively; (2) Based on the new concept, we give a hypothesis to explain the memory of water. The hypothesis is based on two salient facts of the bipolar fuzzy influence graph. Initially, the non-collapsibility of the positive and negative membership functions ensures that the information after using fuzzy coding will not change with the material layer. Then, the influence pairs connect the vertices and edges between different components in the graph through bipolar uncertain influence. At the end of this paper, the conditions for the establishment of the hypothesis and the facts to be verified in the future are analyzed.

**Index Terms**—water, fuzzy system, membership function, fuzzy graph, fuzzy influence graph, bipolar fuzzy influence graph

## I. INTRODUCTION

AS a structured representation tool, graph models have been broadly applied to network and chemical molecular science. As a result, graph related computing theory and algorithms are used in the decision making of graph-based structured data framework. In particular, in the field of chemical molecules, vertices and edges are utilized to represent the chemical bonds between atoms, and the molecular structure of the whole compound is represented by a graph. In this way, graph theory is introduced into chemical molecular science as a useful tool. Furthermore, if there is uncertainty in the graph model, then the uncertainty of vertices and edges is characterized by membership functions (MFs), following which the whole model the whole model comes to be a fuzzy graph. For various applications of fuzzy graphs, refer to Gao et al. [1], Ganesan et al [2], Gayathri et al. [3], Akram et al. [4], Muhiuddin et al. [5], Li et al. [6], Nie et al. [7], Khan et al. [8], Maneckshaw and Mahapatra [9], and

Lang et al. [10].

The membership function cannot meet the complementary law. Therefore, a single membership function cannot depict the uncertainty of objectives from both positive and negative perspectives. For fuzzy graphs, at least two vertex membership functions are required to fully characterize the uncertainty of vertices, and at least two edge membership functions are also required to characterize the positive and negative uncertainties on edges. For this aim, bipolar fuzzy graphs are introduced into fuzzy graph theory and play a role in chemistry.

In recent years, the studies on fuzzy graphs and bipolar fuzzy graphs in various settings have become a hot topic. Yuan and Wang [11] proposed a fuzzy-based complete learning approach to obtain bipartite graphs. Golcuk et al. [12] raised a multiple attribute decision making framework by means of fuzzy graph expression. Das et al. [13] suggested a new trick to control medicine resources using a picture fuzzy threshold graph. Perumal [14] determined a fuzzy graph based intelligent framework for document clustering. He et al. [15] designed a technology to partition graph data into some overlapping subgraph data in view of fuzzy clustering. Long et al. [16] presented a trick to use fuzzy knowledge graph pairs rather than a single pair of the standard framework. Ullah et al. [17] analyzed the competition graph by using a novel and prevailing technique of complex q-rung orthopair fuzzy setting. Xue et al. [18] investigated the distributed fuzzy H infinity filtering problem for Takagi-Sugeno (T-S) fuzzy model-based nonlinear systems interconnected over an undirected graph. Arunkumar et al. [19] re-examined and re-detected the fake images or videos in light of the fuzzy fisher face capsule dual graph. Lu et al. [20] introduced a cyclic connectivity index and its average version for bipolar fuzzy incidence graph, and applied it to measure anti-aging-related drugs. Khan et al. [21] introduced bipolar picture fuzzy graphs and determined several fundamental characteristics. Khan et al. [22] defined and discussed characteristics of the Cayley picture fuzzy graphs, and applied it to interconnected networks. Taouti and Khan [23] studied the fuzzy subnear-semirings and fuzzy soft subnear-semirings. Meanwhile, the fuzzy soft anti-homomorphism of fuzzy soft near-semirings and fuzzy soft R-homomorphisms of fuzzy soft R-subsemigroups are further investigated. More advances on fuzzy graphs and their applications can be referred to Josy et al. [24], Liu et al. [25], Islam and Pal [26], Poulik and Ghorai [27], Zhu et al. [28], Luo and Gao [30], and Gong et al. [30].

As an extended fuzzy incidence graph, the fuzzy influence graph has attracted much attention in recent years (see Mathew and Mordeson [31] and Gayathri et al. [3]). Although a few literatures on this topic have obtained rich

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results, the definition and application of fuzzy influence graphs in other settings are still open. This encourages us to extend the concept to the bipolar fuzzy framework and get the corresponding theory. It is noteworthy that our new results can be applied in molecular science.

Our contributions are summarized in two-fold:

- (1) We defined the bipolar fuzzy influence graph and analyzed its basic characteristics from the perspectives of graph topology and membership functions, providing its structural type theorem.
- (2) We used the obtained theory to explain the phenomenon of water having memory, and provide a possible hypothesis from a hypothetical perspective.

It is imperative to explain that the example provided in this article is analyzed under the following assumption. Imagine that each object has a primitive image in high-dimensional space, and through some mapping, different levels of images can be obtained, and the object we can see is the outermost layer of this multi-level image. And there are multiple hierarchical structures within the image that have an impact on the object.

The remainder of the paper is arranged in the following way. First, we define the new concepts, and introduce the notations and terminologies in the next section. Then, the main theoretical results on bipolar fuzzy influence graphs are presented. Finally, we show the application in chemistry, where a hypothesis is proposed to explain water's ability to remember.

## II. FROM BIPOLAR FUZZY INCIDENCE GRAPH TO BIPOLAR FUZZY INFLUENCE GRAPH

We call  $(x, xx')$  an incidence pair. Let  $G = (\mathcal{G}^P, \mathcal{G}^N, \zeta^P, \zeta^N, \Xi^P, \Xi^N)$  be a bipolar fuzzy incidence graph with  $(\mathcal{G}^P, \mathcal{G}^N): V \rightarrow [0, 1] \times [-1, 0]$ ,  $(\zeta^P, \zeta^N): E \rightarrow [0, 1] \times [-1, 0]$ , and  $(\Xi^P, \Xi^N): V \times E \rightarrow [0, 1] \times [-1, 0]$ , where  $(\Xi^P, \Xi^N)$  is a bipolar fuzzy incidence of  $G$ . Set  $\mathcal{G}^*$ ,  $\zeta^*$  and  $\Xi^*$  as the vertex set, edge set and incidence pair set of  $G$ , respectively. The positive incidence strength is denoted by the minimum value of  $\Xi^P(x, xx')$  and negative incidence strength is represented by the maximum value of  $\Xi^N(x, xx')$ . In bipolar fuzzy incidence graph setting,  $\Xi^P$  and  $\Xi^N$  express the positive and negative relations between atoms and chemical bonds. A bipolar fuzzy incidence graph  $G = (\mathcal{G}^P, \mathcal{G}^N, \zeta^P, \zeta^N, \Xi^P, \Xi^N)$  is complete if  $\Xi^P(x, xx') = \mathcal{G}^P(x) \wedge \zeta^P(xx')$  and  $\Xi^N(x, xx') = \mathcal{G}^N(x) \vee \zeta^N(xx')$  for any  $(x, xx') \in \Xi^*$ .

We say  $(x, x'x'')$  is an influence pair if  $x$ ,  $x'$  and  $x''$  are distinct. A bipolar fuzzy influence graph (BFIG) is a bipolar fuzzy incidence graph  $G = (\mathcal{G}^P, \mathcal{G}^N, \zeta^P, \zeta^N, \Xi^P, \Xi^N)$  with at least one influence pair. A path  $P$  is an influence path if it contains at least one influence pair, and its bipolar influence is denoted by

$$i(P) = (i^P(P), i^N(P)),$$

where  $i^P(P) = \wedge \{\Xi^P(x, x'x'') : (x, x'x'') \text{ is an influence pair}\}$  and  $i^N(P) = \vee \{\Xi^N(x, x'x'') : (x, x'x'') \text{ is an influence pair}\}$ . The bipolar influence connectivity or bipolar strength of influence between  $x, y \in \mathcal{G}^* \cup \zeta^*$  is represented

$$ICONN_G(x, y) = (ICONN_G^P(x, y), ICONN_G^N(x, y)),$$

where  $ICONN_G^P(x, y) = \vee \{i^P(P) : P \text{ is an influence pair between } x \text{ and } y\}$ ,  $ICONN_G^N(x, y) = \wedge \{i^N(P) : P \text{ is an influence pair between } x \text{ and } y\}$ .

An influence pair  $(x, x'x'')$  is considered as a

- positive strong influence pair if  $\Xi^P(x, x'x'') \geq ICONN_{G-(x, x'x'')}^P(x, x'x'')$ ;
- positive strongest or positive  $\alpha$ -strong pair if  $\Xi^P(x, x'x'') > ICONN_{G-(x, x'x'')}^P(x, x'x'')$ ;
- negative strong influence pair if  $\Xi^N(x, x'x'') \leq ICONN_{G-(x, x'x'')}^N(x, x'x'')$ ;
- negative strongest or negative  $\alpha$ -strong pair if  $\Xi^N(x, x'x'') < ICONN_{G-(x, x'x'')}^N(x, x'x'')$ ;
- bipolar strong influence pair if it is both positive strong influence pair and negative strong influence pair;
- bipolar  $\alpha$ -strong pair if it is both positive  $\alpha$ -strong pair and negative  $\alpha$ -strong pair;
- positive  $\beta$ -strong influence pair if

$$\Xi^P(x, x'x'') = ICONN_{G-(x, x'x'')}^P(x, x'x'');$$

- negative  $\beta$ -strong influence pair if
- bipolar  $\beta$ -strong pair if it is both positive  $\beta$ -strong pair and negative  $\beta$ -strong pair;
- positive  $\delta$ -pair or positive weak influence pair if

$$\Xi^P(x, x'x'') < ICONN_{G-(x, x'x'')}^P(x, x'x'');$$

- negative  $\delta$ -pair or negative weak influence pair if
- bipolar  $\delta$ -pair (resp. bipolar weak influence pair) if it is both positive  $\delta$ -pair (resp. positive weak influence pair) and negative  $\delta$ -pair (resp. negative weak influence pair);
- positive effective influence pair if

$$\Xi^P(x, x'x'') = \mathcal{G}^P(x) \wedge \zeta^P(x'x'');$$

- negative effective influence pair if
- bipolar effective influence pair if it is both positive effective influence pair and negative effective influence pair.

A vertex  $x \in \mathcal{G}^*$  of a bipolar fuzzy influence graph is called a

- positive influence cutvertex if  $ICONN_{G-x}^P(x', x'') < ICONN_G^P(x', x'')$  for some  $x', x'' \in \mathcal{G}^* \cup \zeta^*$ ;
- negative influence cutvertex if  $ICONN_{G-x}^N(x', x'') > ICONN_G^N(x', x'')$  for some  $x', x'' \in \mathcal{G}^* \cup \zeta^*$ ;

- bipolar influence cutvertex if  $x$  is both positive influence cutvertex and negative influence cutvertex.

An edge  $xy \in \zeta^*$  is called a

- positive influence bridge if  $ICONN_{G-xy}^P(x', x'') < ICONN_G^P(x', x'')$  for some  $x', x'' \in \mathcal{G}^* \cup \zeta^*$ ;
  - negative influence bridge if  $ICONN_{G-xy}^N(x', x'') > ICONN_G^N(x', x'')$  for some  $x', x'' \in \mathcal{G}^* \cup \zeta^*$ ;
  - bipolar influence bridge if it is both a positive influence bridge and negative influence bridge;
- $(x, yz) \in \Xi^*$  is called a
- positive influence cutpair if  $ICONN_{G-(x,yz)}^P(x', x'') < ICONN_G^P(x', x'')$  for some  $x', x'' \in \mathcal{G}^* \cup \zeta^*$ ;
  - negative influence cutpair if  $ICONN_{G-(x,yz)}^N(x', x'') > ICONN_G^N(x', x'')$  for some  $x', x'' \in \mathcal{G}^* \cup \zeta^*$ ;
  - bipolar influence cutpair if it is both a positive influence cutpair and negative influence cutpair.

A complete bipolar fuzzy influence graph is a complete bipolar fuzzy incidence graph

$G = (\mathcal{G}^P, \mathcal{G}^N, \zeta^P, \zeta^N, \Xi^P, \Xi^N)$  such that for any  $x \in \mathcal{G}^*$  influences any  $x' \in \zeta^*$ , we have  $\Xi^P(x, xx') = \mathcal{G}^P(x) \wedge \zeta^P(xx')$  and  $\Xi^N(x, xx') = \mathcal{G}^N(x) \vee \zeta^N(xx')$ .

A bipolar fuzzy influence graph  $G = (\mathcal{G}^P, \mathcal{G}^N, \zeta^P, \zeta^N, \Xi^P, \Xi^N)$  becomes a bipolar fuzzy influence forest when there is an influence subgraph  $F = (\mathcal{G}^P, \mathcal{G}^N, \zeta^P, \zeta^N, \Psi^P, \Psi^N)$  (its corresponding crisp graph is a forest) such that for any  $(x, yz) \in \Xi^* \setminus \Psi^*$ , we have  $ICONN_F^P(x, yz) > \Xi^P(x, yz)$  and  $ICONN_F^N(x, yz) < \Xi^N(x, yz)$ . A bipolar fuzzy influence graph  $G = (\mathcal{G}^P, \mathcal{G}^N, \zeta^P, \zeta^N, \Xi^P, \Xi^N)$  becomes a bipolar fuzzy influence tree when there is an influence subgraph  $F = (\mathcal{G}^P, \mathcal{G}^N, \zeta^P, \zeta^N, \Psi^P, \Psi^N)$  (its corresponding crisp graph is a tree) such that for any  $(x, yz) \in \Xi^* \setminus \Psi^*$ , we have  $ICONN_F^P(x, yz) > \Xi^P(x, yz)$  and  $ICONN_F^N(x, yz) < \Xi^N(x, yz)$ . A bipolar fuzzy influence graph  $G = (\mathcal{G}^P, \mathcal{G}^N, \zeta^P, \zeta^N, \Xi^P, \Xi^N)$  has  $(\text{supp}(\mathcal{G}^P), \text{supp}(\mathcal{G}^N), \text{supp}(\zeta^P), \text{supp}(\zeta^N), \text{supp}(\Xi^P), \text{supp}(\Xi^N))$  which is a cycle where

$$\begin{aligned} \text{supp}(\mathcal{G}^P) &= \{x \in \mathcal{G}^* : \mathcal{G}^P(x) > 0\}, \\ \text{supp}(\mathcal{G}^N) &= \{x \in \mathcal{G}^* : \mathcal{G}^N(x) < 0\}, \\ \text{supp}(\zeta^P) &= \{xy \in \zeta^* : \zeta^P(xy) > 0\}, \\ \text{supp}(\zeta^N) &= \{xy \in \zeta^* : \zeta^N(xy) < 0\}, \\ \text{supp}(\Xi^P) &= \{(x, yz) \in \Xi^* : \Xi^P(x, yz) > 0\}, \\ \text{supp}(\Xi^N) &= \{(x, yz) \in \Xi^* : \Xi^N(x, yz) < 0\}. \end{aligned}$$

The bipolar fuzzy cycle  $G$  becomes a bipolar fuzzy influence cycle if it doesn't have unique  $(x, yz) \in \text{supp}(\Xi^P) \cap \text{supp}(\Xi^N)$  such that

$$\Xi^P(x, yz) = \wedge \{\Xi^P(w, w'w'') : (w, w'w'') \in \text{supp}(\Xi^P)\}$$

and

$$\Xi^N(x, yz) = \vee \{\Xi^N(w, w'w'') : (w, w'w'') \in \text{supp}(\Xi^N)\}.$$

A bipolar fuzzy influence graph is called a

- positive fuzzy influence block if it doesn't have positive influence cutvertex;
- negative fuzzy influence block if it has no negative influence cutvertex;
- bipolar fuzzy influence block if it doesn't have bipolar influence cutvertex.

For a positive strong (resp. negative strong or bipolar strong) pair  $(x, yz)$  in bipolar fuzzy influence graph, the vertex  $x$  and the edge  $yz$  are considered as positive strong (resp. negative strong or bipolar strong) neighbors. A positive fuzzy influence end vertex (resp. negative fuzzy influence end vertex or bipolar fuzzy influence end vertex) in BFIG  $G$  is a vertex  $x$  with a unique positive strong fuzzy influence neighbor (resp. negative strong fuzzy influence neighbor or bipolar strong fuzzy influence neighbor). A positive fuzzy influence end edge (resp. negative fuzzy influence end edge or bipolar fuzzy influence end edge) in bipolar fuzzy influence graph  $G$  is edge  $xy$  with a unique positive strong fuzzy influence neighbor (resp. negative strong fuzzy influence neighbor or bipolar strong fuzzy influence neighbor).

An influence pair  $(x, yz)$  in bipolar fuzzy influence graph  $G$  is a

- positive influence bound if  $ICONN_{G-(x,yz)}^P(c, d) < ICONN_G^P(c, d)$  for some  $c, d \in \mathcal{G}^* \cup \zeta^*$  with at least one of  $c$  and  $d$  different from  $x$  and  $yz$ ;
- negative influence bound if  $ICONN_{G-(x,yz)}^N(c, d) > ICONN_G^N(c, d)$  for some  $c, d \in \mathcal{G}^* \cup \zeta^*$  with at least one of  $c$  and  $d$  different from  $x$  and  $yz$ ;
- bipolar influence bound if  $ICONN_{G-(x,yz)}^P(c, d) < ICONN_G^P(c, d)$  and  $ICONN_{G-(x,yz)}^N(c, d) < ICONN_G^N(c, d)$  for some  $c, d \in \mathcal{G}^* \cup \zeta^*$  with at least one of  $c$  and  $d$  different from  $x$  and  $yz$ .

An influence pair  $(x, yz)$  in bipolar fuzzy influence graph  $G$  is a

- positive influence cutbound if  $ICONN_{G-(x,yz)}^P(c, d) < ICONN_G^P(c, d)$  for some  $c, d \in \mathcal{G}^* \cup \zeta^*$  with both  $c$  and  $d$  different from  $x$  and  $yz$ ;
- negative influence cutbound if  $ICONN_{G-(x,yz)}^N(c, d) > ICONN_G^N(c, d)$  for some  $c, d \in \mathcal{G}^* \cup \zeta^*$  with both  $c$  and  $d$  different from  $x$  and  $yz$ ;
- bipolar influence cutbound if  $ICONN_{G-(x,yz)}^P(c, d) < ICONN_G^P(c, d)$  and  $ICONN_{G-(x,yz)}^N(c, d) < ICONN_G^N(c, d)$

for some  $c, d \in \mathcal{G}^* \cup \zeta^*$  with both  $c$  and  $d$  are different from  $x$  and  $yz$ .

In all, a bipolar fuzzy influence graph is a special bipolar fuzzy incidence graph with at least one influence pair.

### III. THEORETICAL RESULTS AND PROOFS

The following part aims to showcase the conclusions, and we always assume that  $G = (\mathcal{G}^P, \mathcal{G}^N, \zeta^P, \zeta^N, \Xi^P, \Xi^N)$  is a bipolar fuzzy influence graph. The theorems obtained in this section can be regarded as an extension to Gayathri et al. [3], and we omit the repetitive proof process as these results can be deduced directly by the proofing tricks presented in [3]. To be frank, by means of tricks in Gayathri et al. [3], the negative parts can be verified and hence the whole conclusions are determined. The examples are given to illustrate the results.

**Theorem 1.** If  $G = (\mathcal{G}^P, \mathcal{G}^N, \zeta^P, \zeta^N, \Xi^P, \Xi^N)$  has  $(\text{supp}(\mathcal{G}^P), \text{supp}(\mathcal{G}^N), \text{supp}(\zeta^P), \text{supp}(\zeta^N), \text{supp}(\Xi^P), \text{supp}(\Xi^N))$  and its crisp graph is a cycle, then

- the edge shared by two positive influence cutpair is a positive influence bridge;
- the edge shared by two negative influence cutpair is a negative influence bridge;
- the edge shared by two bipolar influence cutpair is a bipolar influence bridge.

**Theorem 2.** Let  $(x, yz)$  be an influence pair in bipolar fuzzy influence graph  $G$ . We have the following three facts:

- $(x, yz)$  is a positive influence cutpair  
 $\Leftrightarrow$  no cycle has  $(x, yz)$  as the positive weakest influence pair  
 $\Leftrightarrow \text{ICONN}_{G-(x,yz)}^P(x, yz) < \Xi^P(x, yz)$ ;
- $(x, yz)$  is a negative influence cutpair  
 $\Leftrightarrow$  no cycle has  $(x, yz)$  as the negative weakest influence pair  
 $\Leftrightarrow \text{ICONN}_{G-(x,yz)}^N(x, yz) > \Xi^N(x, yz)$ ;
- $(x, yz)$  is a bipolar influence cutpair  
 $\Leftrightarrow$  no cycle has  $(x, yz)$  as the bipolar weakest influence pair  
 $\Leftrightarrow \text{ICONN}_{G-(x,yz)}^P(x, yz) < \Xi^P(x, yz)$  and  $\text{ICONN}_{G-(x,yz)}^N(x, yz) > \Xi^N(x, yz)$ .

**Proof of Theorem 2.** We choose to solely prove the negative part (i.e., the second part), and the remainder parts can be coped with in the same fashion.

If  $(x, yz)$  is a negative influence cutpair, then

$$\text{ICONN}_{G-(x,yz)}^N(u, v) > \Xi^N(u, v)$$

for some vertices (or edges)  $u$  and  $v$  in  $G$ . We suppose a cycle is the negative weakest influence pair, then we convert any negative influence path between  $u$  and  $v$  which is negative strongest and involves  $(x, yz)$  to a  $u-v$  negative strongest influence path without  $(x, yz)$  by putting the  $x-yz$  path in the resting segment of the cycle, i.e.,  $(x, yz)$  can't be a negative influence cutpair, which leads to contradictions.

Suppose that there isn't a cycle having  $(x, yz)$  as the bipolar weakest influence pair, but  $\text{ICONN}_{G-(x,yz)}^N(x, yz) \leq \Xi^N(x, yz)$ . Then, there is a  $x-yz$  negative strongest influence path  $\Theta$  which doesn't contain  $(x, yz)$  with negative strength at most  $\Xi^N(x, yz)$ . Hence,  $\Theta$  connecting  $(x, yz)$  results in cycle with  $(x, yz)$  as its weakest negative influence pair.

Now, suppose  $\text{ICONN}_{G-(x,yz)}^N(x, yz) > \Xi^N(x, yz)$ , and we aim to prove that  $(x, yz)$  is a negative influence cutpair. Otherwise, the removing of  $(x, yz)$  makes no difference in the negative influence connectivity among any set of elements in  $G$  (the vertex set union the edge set). Specifically,  $\text{ICONN}_{G-(x,yz)}^N(x, yz)$

$$= \text{ICONN}_{G-(x,yz)}^N(x, yz) \leq \Xi^N(x, yz),$$

which contradicts the hypothesis.  $\square$

**Theorem 3.** Let  $G$  be a BFIG.

- If  $(x, yz)$  is a positive influence cutpair of  $G$ , then  $\text{ICONN}_G^P(x, yz) = \Xi^P(x, yz)$ ;
- If  $(x, yz)$  is a negative influence cutpair of  $G$ , then  $\text{ICONN}_G^N(x, yz) = \Xi^N(x, yz)$ ;
- If  $(x, yz)$  is a bipolar influence cutpair of  $G$ , then  $\text{ICONN}_G^P(x, yz) = \Xi^P(x, yz)$  and  $\text{ICONN}_G^N(x, yz) = \Xi^N(x, yz)$ .

**Proof of Theorem 3.** Similarly, we only prove the negative part.

As there exists a path  $x, (x, yz), yz$  between  $x$  and  $yz$ , and  $\text{ICONN}_G^N(x, yz)$  is the lowest value of negative influence strength of all  $x-yz$  negative influence paths, we infer  $\text{ICONN}_G^N(x, yz) \leq \Xi^N(x, yz)$ .

If  $\text{ICONN}_G^N(x, yz) < \Xi^N(x, yz)$ , then there is a negative strongest  $x-yz$  influence path  $\Theta$  with negative strength  $< \Xi^N(x, yz)$ . Hence, each negative influence pair of  $\Theta$  has the  $\Xi^N$  value strictly smaller than  $\Xi^N(x, yz)$ . Thus,  $\text{ICONN}_{G-(x,yz)}^N(x, yz) < \Xi^N(x, yz)$ . In this way,  $(x, yz)$  connecting  $\Theta$  produces a cycle with  $(x, yz)$  as the extreme negative pair, which implies  $(x, yz)$  cannot be a negative influence cutpair.  $\square$

**Theorem 4.** Let  $G$  be a BFIG.

- If  $(x, yz)$  and  $(x, y'z')$  are positive influence cutpairs of  $G$ , then  $x$  is a positive influence cutvertex of  $G$ ;
- If  $(x, yz)$  and  $(x, y'z')$  are negative influence cutpairs of  $G$ , then  $x$  is a negative influence cutvertex of  $G$ ;
- If  $(x, yz)$  and  $(x, y'z')$  are bipolar influence cutpairs of  $G$ , then  $x$  is a bipolar influence cutvertex of  $G$ .

The following instance explains that the converse statement of Theorem 4 is not hold.

**Example 1.** As manifested in Fig. 1, a bipolar fuzzy influence graph with  $V = \{x_1, x_2, x_3\}$ , assumes that  $\mathcal{G}^P(x) = 1$  and  $\mathcal{G}^N(x) = -1$  for any  $x \in V$ . Define

$$\begin{aligned} \zeta^P(x_1x_2) &= 0.4, \zeta^N(x_1x_2) = -0.4, \zeta^P(x_2x_3) = 0.6, \\ \zeta^N(x_2x_3) &= -0.6, \zeta^P(x_3x_1) = 0.5, \zeta^N(x_3x_1) = -0.5, \\ \Xi^P(x_1, x_1x_2) &= \Xi^P(x_2, x_2x_1) = \Xi^P(x_2, x_2x_3) \\ &= \Xi^P(x_3, x_3x_2) = \Xi^P(x_3, x_3x_1) \\ &= \Xi^P(x_1, x_1x_3) = 0.2, \\ \Xi^N(x_1, x_1x_2) &= \Xi^N(x_2, x_2x_1) = \Xi^N(x_2, x_2x_3) \\ &= \Xi^N(x_3, x_3x_2) = \Xi^N(x_3, x_3x_1) \\ &= \Xi^N(x_1, x_1x_3) = -0.2, \end{aligned}$$

$\Xi^P(x_1, x_2x_3) = 0.4$  and  $\Xi^N(x_1, x_2x_3) = -0.4$ . Since there exist  $x_2, x_3 \in \mathcal{G}^*$  satisfying

$$\begin{aligned} 0 &= ICONN_{G-x_1}^P(x_2, x_3) < ICONN_G^P(x_2, x_3) = 0.4, \\ 0 &= ICONN_{G-x_1}^N(x_2, x_3) > ICONN_G^N(x_2, x_3) = -0.4, \end{aligned}$$

$x_1$  is a bipolar influence cutvertex of  $G$ . But  $x_1$  is a vertex not belonging to two bipolar influence cutpairs.

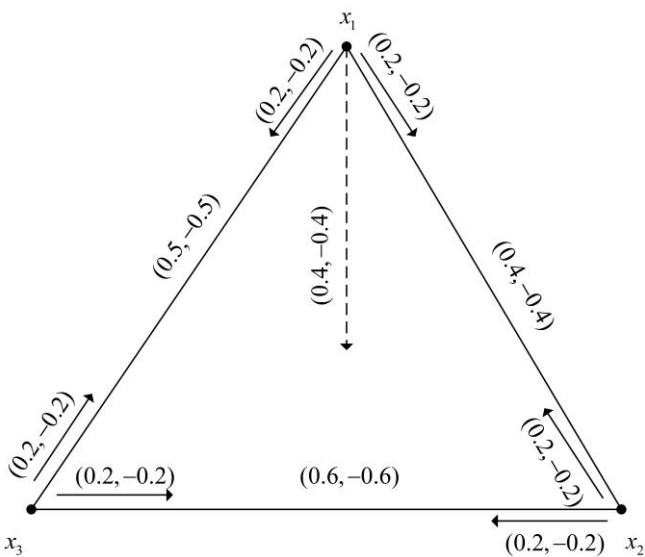


Fig. 1. A bipolar fuzzy influence graph with influence cutvertex.

**Theorem 5.** Suppose the support  $(\text{supp}(\mathcal{G}^P), \text{supp}(\mathcal{G}^N), \text{supp}(\zeta^P), \text{supp}(\zeta^N), \text{supp}(\Xi^P), \text{supp}(\Xi^N))$  of a bipolar fuzzy influence tree  $G$  fails to be a tree, then  $G$  contains at least a single influence pair  $(x, yz)$  such that  $ICONN_G^P(x, yz) > \Xi^P(x, yz)$  and  $ICONN_G^N(x, yz) < \Xi^N(x, yz)$ .

**Proof of Theorem 5.** Now, we show the correctness of negative segment.

To be a negative influence tree,  $G$  contains  $F$ , which is a negative influence subgraph with the characteristics with negative support as a tree and  $ICONN_F^N(x, yz) < \Xi^N(x, yz)$  for each pair  $(x, yz)$ . Moreover,

$$ICONN_G^N(x, yz) \leq ICONN_F^N(x, yz).$$

According to the hypothesis, there exists at least a pair  $(x, yz) \notin F$  such that

$$ICONN_G^N(x, yz) \leq ICONN_F^N(x, yz) < \Xi^N(x, yz).$$

Therefore, the result follows.  $\square$

**Theorem 6.** If an influence pair  $(x, yz)$  in bipolar fuzzy influence graph  $G = (\mathcal{G}^P, \mathcal{G}^N, \zeta^P, \zeta^N, \Xi^P, \Xi^N)$  satisfies

- $\Xi^P(x, yz) = \vee\{\Xi^P(u, vw) : (u, vw) \in \Xi^*\}$ , then  $(x, yz)$  is a positive strong pair;
- $\Xi^N(x, yz) = \wedge\{\Xi^N(u, vw) : (u, vw) \in \Xi^*\}$ , then  $(x, yz)$  is a negative strong pair;
- $\Xi^P(x, yz) = \vee\{\Xi^P(u, vw) : (u, vw) \in \Xi^*\}$  and  $\Xi^N(x, yz) = \wedge\{\Xi^N(u, vw) : (u, vw) \in \Xi^*\}$ , then  $(x, yz)$  is a bipolar strong pair.

However, the converse of Theorem 6 is not hold, and the following instance is used to illustrate it.

**Example 2.** Consider the bipolar fuzzy influence graph presented in Fig. 2, where  $V = \{v_1, v_2, v_3, v_4\}$ , the value of  $\mathcal{G}^P$  for each vertex is assumed to be 1, and value of  $\mathcal{G}^N$  for each vertex is supposed to be  $-1$ . Set

$$\begin{aligned} \zeta^P(v_1v_2) &= \zeta^P(v_1v_3) = 0.4, \\ \zeta^N(v_1v_2) &= \zeta^N(v_1v_3) = -0.4, \end{aligned}$$

$$\begin{aligned} \Xi^P(v_1, v_1v_2) &= \Xi^P(v_2, v_2v_1) = \Xi^P(v_1, v_1v_3) \\ &= \Xi^P(v_2, v_2v_1) = \Xi^P(v_1, v_1v_3) \\ &= \Xi^P(v_3, v_3v_1) = 0.2, \end{aligned}$$

$$\begin{aligned} \Xi^N(v_1, v_1v_2) &= \Xi^N(v_2, v_2v_1) = \Xi^N(v_1, v_1v_3) \\ &= \Xi^N(v_3, v_3v_1) = -0.2, \end{aligned}$$

$$\Xi^P(v_2, v_3v_1) = 0.4, \Xi^N(v_2, v_3v_1) = -0.4,$$

$$\Xi^P(v_4, v_2v_1) = 0.3 \text{ and } \Xi^N(v_4, v_2v_1) = -0.3.$$

In  $G$ ,  $\vee\{\Xi^P(u, vw) : (u, vw) \in \Xi^*\} = 0.4$  and  $\wedge\{\Xi^N(u, vw) : (u, vw) \in \Xi^*\} = -0.4$ , while  $ICONN_{G-(v_4, v_1v_2)}^P(v_4, v_1v_2) = ICONN_{G-(v_4, v_1v_2)}^N(v_4, v_1v_2) = 0$ . It implies that  $(v_4, v_1v_2)$  is a bipolar strong pair, but  $\Xi^P(v_4, v_1v_2) < \vee\{\Xi^P(u, vw) : (u, vw) \in \Xi^*\}$  and  $\Xi^N(v_4, v_1v_2) > \wedge\{\Xi^N(u, vw) : (u, vw) \in \Xi^*\}$ .

**Theorem 7.** Let  $G = (\mathcal{G}^P, \mathcal{G}^N, \zeta^P, \zeta^N, \Xi^P, \Xi^N)$  be a bipolar fuzzy influence graph such that  $G - (x, yz)$  is connected for an influence pair  $(x, yz)$ . Then

- $(x, yz)$  is positive strong  $\Leftrightarrow \Xi^P(x, yz) = \vee\{\Xi^P(u, vw) : (u, vw) \in \Xi^*\}$ ;
- $(x, yz)$  is negative strong  $\Leftrightarrow \Xi^N(x, yz) = \wedge\{\Xi^N(u, vw) : (u, vw) \in \Xi^*\}$ ;
- $(x, yz)$  is bipolar strong  $\Leftrightarrow \Xi^P(x, yz) = \vee\{\Xi^P(u, vw) : (u, vw) \in \Xi^*\}$  and  $\Xi^N(x, yz) = \wedge\{\Xi^N(u, vw) : (u, vw) \in \Xi^*\}$ .

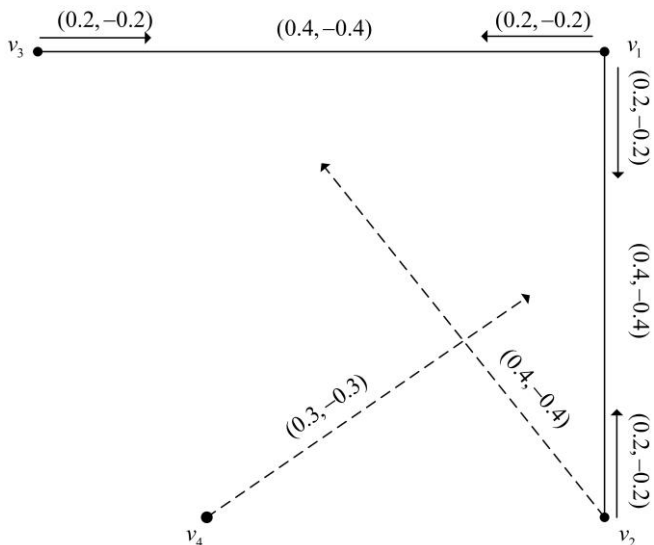


Fig. 2. A bipolar fuzzy influence graph for Example 2.

**Proof of Theorem 7.** We only manifest the proof of the negative part.

Assume that  $(x, yz)$  is negative strong, then

$$ICONN_{G-(x,yz)}^N(x, yz) \geq \Xi^N(x, yz).$$

Hence, any  $x - yz$  negative paths have a negative strength of negative influence  $\Xi^N(x, yz)$ . It reveals from the connectedness of  $G - (x, yz)$  that a negative incidence path exists between any vertex and edge in  $G - (x, yz)$ . Thus, in terms of any negative influence pair, we get a negative influence  $x - yz$  path. Hence, any negative influence pair has its  $\Xi^N$  value  $\geq \Xi^N(x, yz)$ . Therefore,  $(x, yz)$  has minimum  $\Xi^N$  value among all negative influence pairs.

The converse segment can be obtained in light of the negative part of Theorem 6.  $\square$

It is noteworthy that there exists a positive effective pair (resp. negative effective pairs, or bipolar effective pairs) which is not positive strong (resp. negative strong or bipolar strong), which is showcased below.

**Example 3.** Consider the bipolar fuzzy influence graph in Fig. 3, we have  $V = \{x_1, x_2, x_3\}$ , and assume that

$\mathcal{G}^P(x) = 1$  and  $\mathcal{G}^N(x) = -1$  for any  $x \in V$ . Define  $\zeta^P(x_1x_2) = 0.3$ ,  $\zeta^N(x_1x_2) = -0.3$ ,  $\zeta^P(x_2x_3) = 0.1$ ,  $\zeta^N(x_2x_3) = -0.1$ ,  $\zeta^P(x_3x_1) = 0.2$ ,  $\zeta^N(x_3x_1) = -0.2$ ,

$$\begin{aligned} \Xi^P(x_1, x_1x_2) &= \Xi^P(x_2, x_2x_1) = \Xi^P(x_2, x_2x_3) \\ &= \Xi^P(x_3, x_3x_2) = \Xi^P(x_3, x_3x_1) \\ &= \Xi^P(x_1, x_1x_3) = 0.1, \end{aligned}$$

$$\begin{aligned} \Xi^N(x_1, x_1x_2) &= \Xi^N(x_2, x_2x_1) = \Xi^N(x_2, x_2x_3) \\ &= \Xi^N(x_3, x_3x_2) = \Xi^N(x_3, x_3x_1) \\ &= \Xi^N(x_1, x_1x_3) = -0.1, \end{aligned}$$

$\Xi^P(x_2, x_1x_3) = 0.2$ ,  $\Xi^N(x_2, x_1x_3) = -0.2$ ,  $\Xi^P(x_3, x_1x_2) = 0.3$  and  $\Xi^N(x_3, x_1x_2) = -0.3$ . It can be checked that  $(x_2, x_1x_3)$  is a bipolar effective pair, but is not bipolar strong.

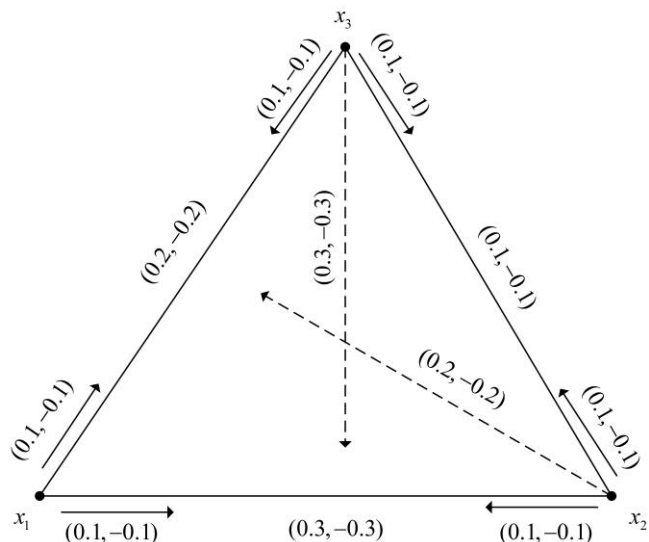


Fig. 3. A bipolar fuzzy influence graph for Example 3.

**Theorem 8.** Let  $G = (\mathcal{G}^P, \mathcal{G}^N, \zeta^P, \zeta^N, \Xi^P, \Xi^N)$  be a bipolar fuzzy influence graph and  $(x, yz)$  be an influence pair.

- $(x, yz)$  is positive strong  
 $\Leftrightarrow \Xi^P(x, yz) = ICONN_G^P(x, yz);$
- $(x, yz)$  is negative strong  
 $\Leftrightarrow \Xi^N(x, yz) = ICONN_G^N(x, yz);$
- $(x, yz)$  is bipolar strong  
 $\Leftrightarrow \Xi^P(x, yz) = ICONN_G^P(x, yz)$  and  
 $\Xi^N(x, yz) = ICONN_G^N(x, yz).$

**Proof of Theorem 8.** We only verify the negative part.

First, assume  $(x, yz)$  is negative strong, then there exist  $\Theta : x, (x, yz), yz$  as a  $x - yz$  negative influence path and hence  $\Xi^N(x, yz) \geq ICONN_G^N(x, yz)$ . Once  $\Theta$  is the unique path from  $x$  to  $yz$ , then  $\Xi^N(x, yz) = ICONN_G^N(x, yz)$ . Or else, we think about another  $x - yz$  negative influence path  $\Phi$ . If  $(x, yz) \in \Phi$ , then  $i^N(\Phi) \geq \Xi^N(x, yz)$ . If  $(x, yz) \notin \Phi$ , then  $G - (x, yz)$  involves the  $x - yz$  negative influence path  $\Phi$ . Since  $(x, yz)$  is negative strong,

$$\Xi^N(x, yz) \leq ICONN_{G-(x,yz)}^N(x, yz)$$

and

$$i^N(\Phi) \geq ICONN_{G-(x,yz)}^N(x, yz).$$

Hence, we have  $\Xi^N(x, yz) \leq i^N(\Phi)$ , which indicates the negative strength of influence of any  $x - yz$  negative influence path is at least  $\Xi^N(x, yz)$ . Thus,  $\Xi^N(x, yz) \leq ICONN_G^N(x, yz)$  and therefore,  $\Xi^N(x, yz) = ICONN_G^N(x, yz)$ .

Second, we assume  $\Xi^N(x, yz) = ICONN_G^N(x, yz)$ . Since  $G - (x, yz)$  is a negative influence subgraph of  $G$ , we deduce  $ICONN_{G-(x,yz)}^N(x, yz) \geq ICONN_G^N(x, yz)$ . So,

$ICONN_{G-(x,yz)}^N(x,yz) \geq \Xi^N(x,yz)$ . Therefore,  $(x,yz)$  is negative strong.  $\square$

**Theorem 9.** The pair  $(x,yz)$  in bipolar fuzzy influence graph  $G$  is

- a positive influence cutpair  $\Leftrightarrow$  it is a positive  $\alpha$ -strong pair;
- a negative influence cutpair  $\Leftrightarrow$  it is a negative  $\alpha$ -strong pair;
- a bipolar influence cutpair  $\Leftrightarrow$  it is a bipolar  $\alpha$ -strong pair.

**Proof of Theorem 9.** We only verify the negative part.

Suppose  $(x,yz)$  a negative influence cutpair, then we verify

$$\Xi^N(x,yz) < ICONN_{G-(x,yz)}^N(x,yz)$$

which implies  $(x,yz)$  is a negative  $\alpha$ -strong pair.

Now, suppose  $(x,yz)$  is a negative  $\alpha$ -strong pair, and hence  $\Xi^N(x,yz) < ICONN_{G-(x,yz)}^N(x,yz)$ . It means there is a unique strongest  $x-yz$  negative influence path obtained by  $P: x, (x,yz), yz$ , and the removing of  $(x,yz)$  reduces the  $x-yz$  negative influence path, which uncovers  $(x,yz)$  is a negative influence cutpair.  $\square$

**Theorem 10.** The BFIG  $G = (\mathcal{G}^P, \mathcal{G}^N, \zeta^P, \zeta^N, \Xi^P, \Xi^N)$  is a bipolar fuzzy influence forest if and only if a pair  $(x,yz)$  exists in any bipolar fuzzy cycle  $C$  of  $G$  satisfying  $\Xi^P(x,yz) < ICONN_{G-(x,yz)}^P(x,yz)$  and  $\Xi^N(x,yz) > ICONN_{G-(x,yz)}^N(x,yz)$ .

**Proof of Theorem 10.** We only verify the negative part.

Suppose that  $G$  meets the demands of bipolar fuzzy influence forest. Given a negative fuzzy cycle  $C$ , there is a  $(x,yz) \in C$  not belonging to bipolar influence forest  $F$  with  $\Xi^N(x,yz) > ICONN_F^N(x,yz) \geq ICONN_{G-(x,yz)}^N(x,yz)$ .

On the other hand, we suppose to search an  $(x,yz)$  in a fixed bipolar fuzzy cycle  $C$  such that  $\Xi^N(x,yz) > ICONN_{G-(x,yz)}^N(x,yz)$ . Among all such pairs with such characteristics, select the pair with the largest  $\Xi^N$  value and remove this selected pair from  $C$ . This action is repeated on all such cycles, and we confirm that the remaining bipolar influence graph is a negative influence forest.  $\square$

**Theorem 11.** If a BFIG  $G$  without any bipolar fuzzy influence cycles is connected, then we call it a bipolar fuzzy influence tree.

**Theorem 12.** Suppose bipolar fuzzy influence graph  $G$  is connected, and  $G$  is a bipolar fuzzy influence tree if and only if no bipolar  $\beta$ -strong pair belongs to  $G$ .

**Theorem 13.** Suppose a BFIG  $G = (\mathcal{G}^P, \mathcal{G}^N, \zeta^P, \zeta^N, \Xi^P, \Xi^N)$  is complete, then for any  $x \in \mathcal{G}^*$ , we have

$$\Xi^P(x,yz) \leq \Xi^P(y,yz) = \Xi^P(z,yz),$$

$$\Xi^N(x,yz) \geq \Xi^N(y,yz) = \Xi^N(z,yz).$$

Furthermore,

$$\bullet \Xi^P(x,yz) = \Xi^P(y,yz) = \Xi^P(z,yz)$$

$$\Leftrightarrow \mathcal{G}^P(x) \geq \mathcal{G}^P(y) \wedge \mathcal{G}^P(z);$$

$$\bullet \Xi^N(x,yz) = \Xi^N(y,yz) = \Xi^N(z,yz)$$

$$\Leftrightarrow \mathcal{G}^N(x) \leq \mathcal{G}^N(y) \vee \mathcal{G}^N(z).$$

**Theorem 14.** There are at least two positive (resp. negative) fuzzy influence end vertices for any nontrivial bipolar fuzzy influence tree.

However, the following example shows that the reverse of Theorem 14 is not true.

**Example 4.** As can be seen in Fig. 4,  $V = \{x_1, x_2, x_3, x_4\}$ ,

and assume that  $\mathcal{G}^P(x) = 1$  and  $\mathcal{G}^N(x) = -1$  for any  $x \in V$ . Define  $\zeta^P(x_1x_2) = 0.3$ ,  $\zeta^N(x_1x_2) = -0.3$ ,

$$\zeta^P(x_2x_3) = \zeta^P(x_4x_1) = 0.2, \zeta^N(x_2x_3) = \zeta^N(x_4x_1) = -0.2,$$

$$\zeta^P(x_3x_4) = 0.4, \zeta^N(x_3x_4) = -0.4,$$

$$\Xi^P(x_1, x_1x_2) = \Xi^P(x_2, x_2x_1) = \Xi^P(x_2, x_2x_3)$$

$$= \Xi^P(x_3, x_3x_2) = \Xi^P(x_3, x_3x_4)$$

$$= \Xi^P(x_4, x_4x_3) = \Xi^P(x_4, x_4x_1)$$

$$= \Xi^P(x_1, x_1x_4) = 0.1,$$

$$\Xi^N(x_1, x_1x_2) = \Xi^N(x_2, x_2x_1) = \Xi^N(x_2, x_2x_3)$$

$$= \Xi^N(x_3, x_3x_2) = \Xi^N(x_3, x_3x_4)$$

$$= \Xi^N(x_4, x_4x_3) = \Xi^N(x_4, x_4x_1)$$

$$= \Xi^N(x_1, x_1x_4) = -0.1,$$

$$\Xi^P(x_1, x_3x_4) = \Xi^P(x_3, x_1x_2) = 0.1, \Xi^N(x_1, x_3x_4) = \Xi^N(x_3,$$

$$x_1x_2) = -0.1, \Xi^P(x_2, x_4x_1) = \Xi^P(x_4, x_2x_3) = 0.2$$

$$\text{and } \Xi^N(x_2, x_4x_1) = \Xi^N(x_4, x_2x_3) = -0.2. \text{ Clearly, } G \text{ has two bipolar fuzzy influence end vertices, but } G \text{ is not an influence tree.}$$

**Theorem 15.** Any two members in  $\mathcal{G}^* \cup \zeta^*$  of a complete bipolar fuzzy influence graph  $G = (\mathcal{G}^P, \mathcal{G}^N, \zeta^P, \zeta^N, \Xi^P, \Xi^N)$  can be connected by a bipolar strong influence path.

As shown by the example below, if the complete BFIG with a connected bipolar fuzzy influence graph is replaced, then Theorem 15 will not be true.

**Example 5.** As depicted in Fig. 5,  $V = \{x_1, x_2, x_3, x_4, x_5\}$ ,

and assume that  $\mathcal{G}^P(x) = 1$  and  $\mathcal{G}^N(x) = -1$  for any  $x \in V$ . Define

$$\zeta^P(x_1x_2) = \zeta^P(x_3x_4) = \zeta^P(x_4x_5) = 0.3,$$

$$\zeta^N(x_1x_2) = \zeta^N(x_3x_4) = \zeta^N(x_4x_5) = -0.3,$$

$$\Xi^P(x_1, x_1x_2) = \Xi^P(x_2, x_2x_1) = \Xi^P(x_3, x_3x_4)$$

$$= \Xi^P(x_4, x_4x_3) = \Xi^P(x_4, x_4x_5)$$

$$= \Xi^P(x_5, x_5x_4) = 0.1,$$

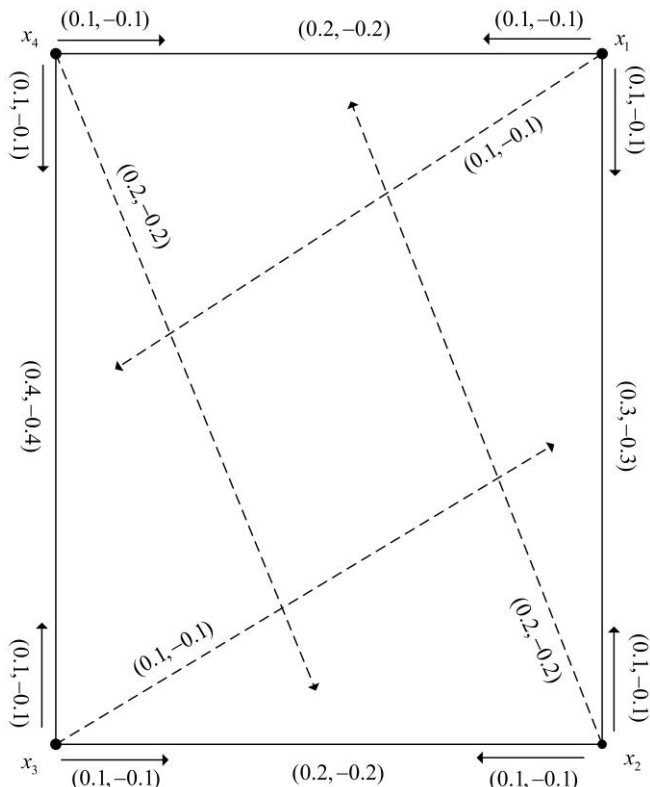


Fig. 4. A bipolar fuzzy influence graph for Example 4.

$$\begin{aligned} \Xi^N(x_1, x_1x_2) &= \Xi^N(x_2, x_2x_1) = \Xi^N(x_3, x_3x_4) \\ &= \Xi^N(x_4, x_4x_3) = \Xi^N(x_4, x_4x_5) \\ &= \Xi^N(x_5, x_5x_4) = 0.1, \end{aligned}$$

$\Xi^P(x_2, x_3x_4) = 0.2$  and  $\Xi^N(x_2, x_3x_4) = -0.2$ . It can be acquired that  $x_4 - x_5$ , the bipolar influence path is bipolar strong.

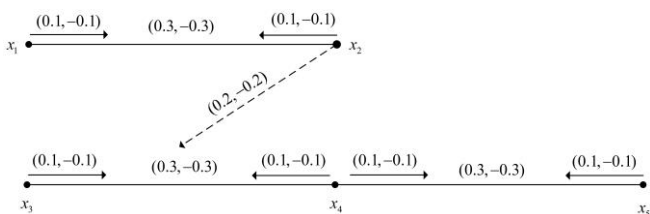


Fig. 5. A bipolar fuzzy influence graph for Example 5.

**Theorem 16.** Weak positive (resp. negative) influence pairs do not exist in a complete BFIG if and only if  $\Xi^P(x, yz)$  (resp.  $\Xi^N(x, yz)$ ) is a constant for any  $(x, yz) \in \Xi^*$ .

**Theorem 17.** An influence pair  $(x, yz)$  in bipolar fuzzy influence graph  $G$  is a

- positive influence bound if and only if it is a positive influence cutpair;
- negative influence bound if and only if it is a negative influence cutpair;
- bipolar influence bound if and only if it is a bipolar influence cutpair.

**Theorem 18.** A bipolar fuzzy influence block has no bipolar influence cutbonds.

#### IV. CODING CONJECTURE ON THE MEMORY OF WATER

In recent years, materials with memory function have become an intriguing topic in the field of chemistry and materials, and the related products are popular with the public. The products include mattresses with memory functions, and some materials that can be restored after being deformed by soaking in water, and so on. One of the important mysteries in the scientific community is why water has a memory function, which involves the phenomenon of long-distance transfer of DNA. Unraveling such phenomena leads our understanding of water to new heights.

It reveals that the aqueous solution containing viral DNA is highly diluted to a theoretical state of pure water, that is, the water is free of impurities. The characteristics of the virus DNA that originally existed still can be detected in the diluted pure water, just like pure water with certain characteristics of memory. And the properties of DNA can also be remotely transferred to another pure water sample. This reveals that water not only has memory, but also can transfer characteristics remotely. Specifically, it prepares an aqueous solution that once it contains the virus, and completely gets the DNA inside removed, and then it performs a high dilution. The obtained aqueous solution is subjected to electromagnetic detection, and the test instrument obtains a low-frequency electromagnetic signal. They made the signal into a 6-second audio file and sent it to colleagues in Italy by email. Collaborators received the audio and played it repeatedly over a sample of distilled water, then put this distilled water that "hears" the audio into a machine that can synthesize DNA. The results showed that the machine-generated DNA was highly similar to the original viral DNA (up to 98%). This shows that DNA can be copied remotely, because water plays an intermediary role in it. Therefore, water has a memory function for transmitting and receiving signals. More details can be referred to Montagnier et al. [32-34].

One explanation lies in that information about the water could be digitally encoded and reinserted into another water sample. If we assume that such a digital code exists, then what kind of coding is possible? Here, it is imperative to consider the following questions:

(1) In what way are water molecules combined with other molecular structures? Apparently, fusion between ordinary molecules cannot explain water's memory, because in Montagnier's experiment, virus molecules have been removed and highly sparse operations are performed. It can be considered that such processed pure aqueous solution does not contain virus molecules, but only some kind of "memory". So where does this "memory" come from without molecular structure fusion?

(2) The encoded information will not be weakened by the scarcity of water, so what information does not exist independently of the substance level?

Here, we give an encoding conjecture from the perspective of the BFIG. There is a need to explain how membership functions are essentially different from probability functions. The probability function is collapsible, but the membership function is not. To illustrate, in the human relationship network, each individual is a vertex, and if we define the edge between two vertices as "family", then the whole graph is a probability graph. Suppose a boy is in love with a girl, and the



probability of marriage between them is  $p$ . As soon as they get married or break up, whether there is an edge between them has been determined, this probability  $p$  will be invalid at this time. If we define the edge between two vertices as "like" or "love", then the whole graph becomes a fuzzy graph, because loving is a personal feeling and will not be completely determined directly by a special event.

Due to the non-collapsibility of the membership function, the code formed by the membership function is not at the quantum level, i.e., once it is established, it will not disappear due to the disappearance of the material layer. For example, if one of your relatives dies, you need to delete his corresponding vertex from the corresponding human relationship network. However, at the level of your personal thoughts and emotions, you still retain the memory of this relative, so the membership function of the edge representing "love" still exists, and the code obtained from it will not disappear because of the disappearance of the vertex.

The main idea raised in this paper is also on the ground of the following hypotheses that have not been scientifically proven. It is assumed that matter is closely related to the vibration of the field and the accumulation of energy, and this field space is eternal. It is not just one level, but consists of many levels, and each level is separated according to the energy status. In other words, within space, actual matter is mirrored within field space with distinct layers. But in the process of material changes, there is also a filling to store information, which is called "morphogenetic field", which is a part of the layer. There is also an intermediate layer, called the "hyperlayer", which lies within the field space, between the morphogenetic layers and material layer. The "hyperlayer" will not be limited by the material layer, and it belongs to the category of the consciousness layer, but it influences the material layer by means of certain ways (see Fig. 6).

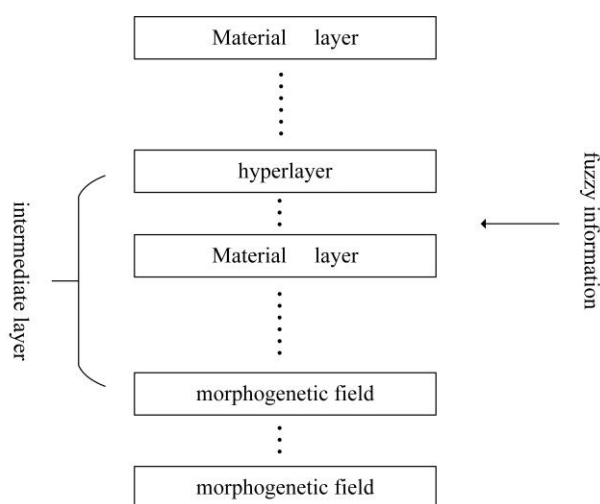


Fig. 6. Material Hierarchy Hypothesis.

Now, we propose our coding conjecture on the memory of water. When the aqueous solution is fused with other viral DNA molecules, the original multiple molecular graphs can be regarded as multiple components on one molecular graph. When the molecular structure is modelled by a bipolar fuzzy influence graph, multiple components are connected by influence pairs (for instance, Fig. 5 is a disconnected graph from the perspective of a bipolar fuzzy incidence graph, but

from the perspective of a BFIG, there is some uncertainty-based influence between vertices and edges between different connected subgraphs, so that multiple components are connected in view of influence pairs). The fuzzy information after fusion is stored in the hyperlayer in light of unknown encoding, so that this information goes beyond the scope of the material layer. When the viral DNA molecule is deleted and the water is diluted, the ambiguous code with the viral DNA information remains in the hyperlayer. Since the intermediate layer connects the morphogenetic field and the material layer, in a certain state, the bipolar fuzzy influence graph information originally retained in the hyperlayer is re-excited in terms of the morphogenetic field and mirrored to the material layer, thereby making water have a memory function. Refer to Fig. 7 for a schematic diagram of the fusion of water molecules and viral DNA molecules via a bipolar fuzzy influence graph.

As a medium, water molecules store the structural information of other molecules by combining with fuzzy graph information. The linking of information between different molecular structures is tackled based on the influence pair of the BFIG. Due to the non-collapsibility of the membership function, when a certain encoding mechanism is applied to encode this bipolar uncertainty, the encoding information will exist independently of the graph structure, and then will be

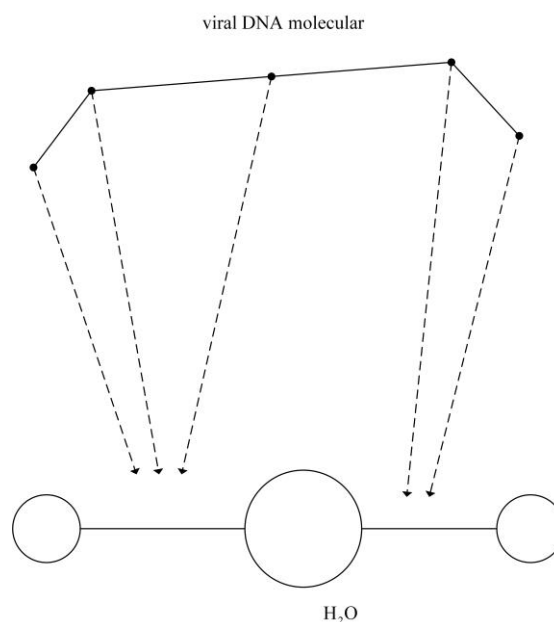


Fig. 7. Water molecule and virus DNA molecule are fused by bipolar fuzzy influence graph.

retained in the hyperlayer by a certain unknown mechanism. Afterwards, it will be stimulated through the morphogenetic field in a certain principle, and then manifested as water having a certain "memory" on the physical layer.

What we must emphasize is that our article only gives a hypothesis on the explanation of the memory of water, but we cannot confirm it from rigorous experiments. If this perspective is to be verified, the following facts must be clarified in the future:

- (1) How to overcome the difficulty of describing the membership function of vertices and edges on the BFIG, in other words, how to describe the uncertainty of atoms and

atomic bonds? Most importantly, how to characterize the bipolar uncertainty represented by the influence pair?

(2) What mechanism does the hyperlayer use to store the topology of the bipolar fuzzy influence graph and the corresponding uncertainty?

(3) What mechanism does the topological information of the graph and the information of the uncertainty of the membership function use to stimulate the morphogenetic field, which in turn affects the material layer?

## V. CONCLUSION

This contribution introduces the concept of BFIGs, and extends the theoretical results from fuzzy influence graphs into bipolar settings. Furthermore, we put forward a hypothesis from the perspective of bipolar fuzzy influence graphs, to explain why water has memory. Our hypothesis is based on the following facts:

(1) The membership function that characterizes the uncertainty is not collapsible, so the encoding with fuzzy information will not be lost due to the loss of the structure in the graph.

(2) The influence pair is used to describe the interaction of vertices and edges between different components.

However, the establishment of this hypothesis is also based on some theories that cannot be verified at present, such as the existence of the intermediate layer of every substance. We hope that this idea will inspire other scholars' related research.

## REFERENCES

- [1] W. Gao, W. F. Wang, L. N. Zheng, "Fuzzy fractional factors in fuzzy graphs," *International Journal of Intelligent Systems*, vol. 37, no. 11, pp. 9886-9903, Aug. 2022.
- [2] B. Ganesan, S. Raman, M. Pal, "Strong domination integrity in graphs and fuzzy graphs," *Journal of Intelligent & Fuzzy Systems*, vol. 43, no. 3, pp. 2619-2632, Aug. 2022.
- [3] G. Gayathri, S. Mathew, J. N. Mordeson, "Fuzzy influence graphs with applications," *Fuzzy Sets and Systems*, vol. 450, pp. 1-26, Dec. 2022.
- [4] M. Akram, U. Ahmad, Rukhsar, S. Samanta, "Threshold graphs under pythagorean fuzzy information," *Journal of Multiple-Valued Logic and Soft Computing*, vol. 38, no. 5-6, pp. 547-574, Apr. 2022.
- [5] G. Muhiuddin, T. Pramanik, A. M. Alanazi, A. Mahboob, M. Pal, "Independent fuzzy graph: a new approach," *Proceedings of the National Academy of Sciences India Section A-Physical Sciences*, vol. 92, no. 3, pp. 373-389, Sep. 2022.
- [6] L. Li, L. Jiang, C. Y. Bu, Y. Zhu, X. D. Wu, "Interval-valued intuitionistic fuzzy decision with graph pattern in big graph," *IEEE Transactions on Emerging Topics in Computational Intelligence*, vol. 6, no. 5, pp. 1057-1067, Oct. 2022.
- [7] F. P. Nie, C. D. Liu, R. Wang, Z. Wang, X. L. Li, "Fast fuzzy clustering based on anchor graph," *IEEE Transactions on Fuzzy Systems*, vol. 30, no. (7): 2375-2387, Jul. 2022.
- [8] W. A. Khan, K. Faiz, A. Taouti, "Cayley picture fuzzy graphs and interconnected networks," *Intelligent Automation and Soft Computing*, vol. 35, no. 3, pp. 3317-3330, Oct. 2022.
- [9] B. Maneckshaw, G. S. Mahapatra, "Novel fuzzy matrix swap algorithm for fuzzy directed graph on image processing," *Expert Systems with Applications*, vol. 193, DOI:10.1016/j.eswa.2021.116291, Jan. 2022.
- [10] Q. Lang, X. D. Liu, W. J. Jia, "AFS graph: multidimensional axiomatic fuzzy set knowledge graph for open-domain question answering," *IEEE Transactions on Neural Networks and Learning Systems*, DOI: 10.1109/TNNLS.2022.3171677, May. 2022.
- [11] Y. Yuan, C. Z. Wang, "Bipartite graph based spectral rotation with fuzzy anchors," *Neurocomputing*, vol. 471, pp. 369-376, Jan. 2022.
- [12] I. Golcuk, E. D. Durmaz, R. Sahin, "Prioritizing occupational safety risks with fuzzy FUCOM and fuzzy graph theory-matrix approach," *Journal of the Faculty of Engineering and Architecture of Gazi University*, vol. 38, no. 1, pp. 57-69, Oct. 2022.
- [13] S. Das, G. Ghorai, Q. Xin, "Picture fuzzy threshold graphs with application in medicine replenishment," *Entropy*, vol. 24, no. 5, DOI: 10.3390/e24050658, May. 2022.
- [14] P. Perumal, "Document clustering using graph based fuzzy association rule generation," *Computer Systems Science and Engineering*, vol. 43, no. 1, pp. 203-218, Apr. 2022.
- [15] W. Q. He, S. H. Liu, W. H. Xu, F. S. Yu, W. T. Li, F. Li, "On rough set based fuzzy clustering for graph data," *International Journal of Machine Learning and Cybernetics*, vol. 13, no. 11, pp. 3463-3490, Nov. 2022.
- [16] C. K. Long, P. V. Hai, T. M. Tuan, L. T. H. Lan, P. M. Chuan, L. H. Son, "A novel fuzzy knowledge graph pairs approach in decision making," *Multimedia Tools and Applications*, vol. 81, no. 18, pp. 26505-26534, Jul. 2022.
- [17] K. Ullah, A. Hussain, T. Mahmood, Z. Ali, A. Alabrah, S. M. M. Rahman, "Complex q-rung orthopair fuzzy competition graphs and their applications," *Electronic Research Archive*, vol. 30, no. 4, pp. 1558-1605, May. 2022.
- [18] X. J. Xue, L. Xu, X. L. Yu, "Distributed H infinity fuzzy filtering for nonlinear systems interconnected over graphs," *International Journal of Adaptive Control and Signal Processing*, vol. 36, no. 5, pp. 1172-1195, May. 2022.
- [19] P. M. Arunkumar, Y. Sangeetha, P. V. Raja, S. N. Sangeetha, "Deep learning for forgery face detection using fuzzy fisher capsule dual graph," *Information Technology and Control*, vol. 51, no. 3, pp. 563-574, Nov. 2022.
- [20] J. J. Lu, L. L. Zhu, W. Gao, "Cyclic connectivity index of bipolar fuzzy incidence graph," *Open Chemistry*, vol. 20, no. 1, pp. 331-341, Apr. 2022.
- [21] W. A. Khan, B. Ali, A. Taouti, "Bipolar picture fuzzy graphs with application," *Symmetry*, vol. 13, no. 8, DOI: 10.3390/sym13081427, Aug. 2021.
- [22] W. A. Khan, K. Faiz, A. Taouti, "Cayley picture fuzzy graphs and interconnected networks," *Intelligent Automation and Soft Computing*, vol. 35, no. 3, pp. 3317-3330, Jan. 2023.
- [23] A. Taouti, W. A. Khan, "Fuzzy subnear-semirings and fuzzy soft subnear-semirings," *AIMS Mathematics*, vol. 6, no. 3, pp. 2268-2286, Jul. 2021.
- [24] A. Josy, S. Mathew, J. N. Mordeson, "Neighborhood connectivity index of a fuzzy graph and its application to human trafficking," *Iranian Journal of Fuzzy Systems*, vol. 19, no. 3, pp. 139-154, Jun. 2022.
- [25] H. Liu, C. M. Wu, C. X. Li, Y. Q. Zuo, "Fast robust fuzzy clustering based on bipartite graph for hyper-spectral image classification," *IET Image Processing*, vol. 16, no. 13, pp. 3634-3647, Nov. 2022.
- [26] S. R. Islam, and M. Pal, "An investigation of edge F-index on fuzzy graphs and application in molecular chemistry," *Complex & Intelligent Systems*, DOI: 10.1007/s40747-022-00896-2, Nov. 2022.
- [27] S. Poulik, G. Ghorai, "Estimation of most effected cycles and busiest network route based on complexity function of graph in fuzzy environment," *Artificial Intelligence Review*, vol. 55, no. 6, pp. 4557-4574, Aug. 2022.
- [28] L. L. Zhu, W. G. Tao, X. Z. Min, W. Gao, "Theoretical characteristics of ontology learning algorithm in multi-dividing setting," *IAENG International Journal of Computer Science*, vol. 43, no. 2, pp. 184-191, 2016.
- [29] K. Luo, W. Gao, "Independent set in bipolar fuzzy graph," *IAENG International Journal of Applied Mathematics*, vol. 52, no. 4, pp. 846-854, 2022.
- [30] S. Gong, X. X. Huang, C. H. Qiu, W. Gao, "Locally ontology relaxed stability analysis in various ontology learning settings," *IAENG International Journal of Applied Mathematics*, vol. 52, no. 2, pp. 432-440, 2022.
- [31] S. Mathew, J. N. Mordeson, "Fuzzy influence graphs," *New Mathematics and Natural Computation*, vol. 13, no. 3, pp. 311-325, Oct. 2017.
- [32] L. Montagnier, J. Aïssa, C. Lavallée, M. Mbamy, J. Varon, H. Chenal, "Electromagnetic detection of HIV DNA in the blood of AIDS patients treated by antiretroviral therapy," *Interdisciplinary Sciences: Computational Life Sciences*, vol. 1, no. 4, pp. 245-253, Dec. 2009.
- [33] L. Montagnier, J. Aïssa, S. Ferris, J. Montagnier, C. Lavallée, "Electromagnetic signals are produced by aqueous nanostructures derived from bacterial DNA sequences," *Interdisciplinary Sciences: Computational Life Sciences*, vol. 1, no. 2, pp. 81-90, Jun. 2009.
- [34] L. Montagnier, E. Del Giudice, J. Aïssa, C. Lavallee, S. Motschwiller, A. Capolupo, A. Polcari, P. Romano, A. Tedeschi, G. Vitiello, "Transduction of DNA information through water and electromagnetic waves," *Electromagnetic Biology and Medicine*, vol. 34, no. 2, pp. 106-112, Jun. 2015.