Semi Coloring of General Vague Graph

Naseer Ahmed, Khalifa AlShaqsi, Suganthi

***Abstract*—This paper aims to show an optimum solution for the real-time examination timetabling problem with a fixed number of sessions and a set of difficulty level of each course. The uncertainty and haziness around the problem are taken into account by defining the weighted precedence general vague graph. In this paper, the scheduling part of the examination timetabling problem is approached through novel greedy coloring technique for the weighted precedence general vague graph. As per necessity, the new variant of vertex coloring, namely semi coloring of a graph is defined, such as, for the given graph $G(V, E)$ and the positive integers $r \leq |E|$ and $k < \chi(G)$, the graph G can have the proper $k-$coloring by removing any r edges from the graph. Finally, the greedy algorithm set out for obtaining the semi coloring of the weighted precedence general vague graph and the minimum number of edge removal.**

***Index Terms*—Coloring, Fuzzy Graph, Vague Graph, Examination Timetabling Problem**

I. Introduction

THE graph-theoretic model with uncertainty and haziness is tackled through the use of fuzzy sets, interval valued fuzzy sets, intuitionistic fuzzy sets, and vague sets, along with graph theory basics. While fuzzy sets and interval-valued fuzzy sets do not consider as evidence the value against x in the vague sets, intuitionistic fuzzy sets provide advantages by considering both membership and non-membership degrees to indicate how much a member of the universe of discourse belongs to a subset of it. Additionally, interval-valued fuzzy sets and intuitionistic fuzzy sets are not the same as vague sets, although vague relation is a generalization of a fuzzy relation. A vague graph is defined as similar to an intuitionistic fuzzy graph but also takes into account the hesitation region. The $\mu-$vague graph, a generalized vague graph for a given fuzzy membership function, is defined by considering the similarity between the definitions of fuzzy graph and vague graph. Furthermore, bounded, regular, strong regular, anti $\mu-$vague graph, and complement of $\mu-$vague graph are defined, and some of their properties are established. The purpose of studying the nature of $\mu-$vague graph is to account for both uncertainty and haziness in the decision-making problem. This can be applied to the classical examination timetabling problem by defining the weighted precedence $\mu-$vague graph and using it to find the best feasible solution for the given constraints.

Manuscript received November 29, 2022; revised September 7, 2023. This work was supported by The Research Council, Sultanate of Oman through the research grant program (BFP/RGP/CBS/19/226).

Naseer Ahmed is a Head of Mathematics Section of the Information Technology Department, University of Technology and Applied Sciences, Nizwa, Sultanate of Oman (e-mail: naseer.shukoor@utas.edu.om)

Khalifa AlShaqsi is a Head of Mathematics Department of the Department of Mathematical and Physical Sciences, University of Nizwa, Nizwa, Sultanate of Oman (e-mail: khalifa.alshaqsi@unizwa.edu.om)

Suganthi is a lecturer of the Information Technology Department, University of Technology and Applied Sciences, Nizwa, Sultanate of Oman (e-mail: suganthi.gopal@utas.edu.om)

In 1965, Fuzzy Set was introduced by Zadeh [21]. Fuzzy set is defined as a class of objects with a continuum of grades of membership. In 1975, Interval-valued fuzzy sets were introduced independently by Zadeh [20], Grattan-Guiness [9], Jahn [10], and Sambuc [19]. Atanassov generalized the concept of a fuzzy set by using two membership functions for the elements of the universe of discourse in 1983. After three years, the English version appeared in [1]. In a vague set, an interval-valued membership value is assigned to each element by considering both the evidence for x and the evidence against x. In interval-valued fuzzy sets, evidence against x is not being considered by the decision maker. Even vague relation is a generalization of a fuzzy relation, the interval-valued fuzzy sets are not vague sets. There is a major difference in the choice on the degree of membership function between them. The bacteria classification problem is approached to examine their capabilities in encountering uncertainty in medical pattern recognition by using the identified similarity measures of fuzzy sets and intuitionistic fuzzy sets [11].

In graph theory, the fuzzy graph was first introduced by Rosenfeld [18] in 1975. The structure of fuzzy graphs was developed by considering fuzzy relations between fuzzy sets, which led to the creation of analogs for several graph theoretical concepts. Subsequently, some comments on fuzzy graphs and operations on fuzzy graphs were introduced [3], [15]. Zadeh not only introduced the fuzzy sets; he also discussed the concept of a convex fuzzy set in [21]. Later R. Lowan gathered some elementary known results about convex fuzzy sets and completes the convex fuzzy set theory by introducing the necessary concepts in 1980 [12]. Although vague relations are a more general type of fuzzy relations, interval-valued fuzzy sets are not considered vague sets. There is a significant difference in how the degree of membership is determined between these two types of sets. In vague sets, an interval-valued membership value is assigned to each element, taking into account both the evidence for and against x. However, in interval-valued fuzzy sets, the evidence against x is not considered by the decision maker. In [22], the authors demonstrated the use of vague sets in the medical diagnosis of four types of cancer. To accomplish this, they utilized four new operations of a vague graph: maximal product, rejection, symmetric difference, and residue product.

The diversification of scheduling and assignment problems increased significantly in the past two decades. The timetabling problem, a very complex problem, is not only limited with academic purposes like class timetabling and exam timetabling, more so, it is growing in sports, hospital, transport, air fleet, etc. The examination timetabling problem (ETTP) is one of the mostly experimented general timetabling problems which is NP-Hard [6]. It can be defined as assigning a set of examinations, each associated with a number of registered students with a fixed number of slots, depending on satisfying a number of predefined hard and soft constraints.

The ETTP has been a long-time research topic that uses hard constraints, such as, two or more exams can not be scheduled at the same slot for the student and assigning the invigilators, rooms, labs, chief invigilators for all of the sessions with limited given resources. As such, this becomes an exceptionally difficult problem. If resource allocation is taken into account and the number of students taking the examination is limited by the capacity of the room or the laboratory, then the problem is a capacitated ETTP, otherwise, it is incapacitated. Further, each university or the institution has its own set of rules to conduct the examination. To be met with such rules, more number of constraints are described. Due to this, the higher range of solution techniques are provided for use with the mathematical model, which is well-studied in the literature [16], [13], [14].

This study focuses on solving the university examination timetabling problem, which involves assigning resources such as classrooms, laboratories, and academic staff as invigilators, while accounting for uncertainty and haziness. To address this, the authors proposed a new variant of graph coloring and develop a mathematical model. They then presented a three-stage heuristic solution procedure for solving a real instance of this problem at the University of Technology and Applied Sciences, Nizwa, in the Sultanate of Oman. Their goal is to find an optimal solution that is better than what could be achieved manually by the timetabling committee, using less computational time. While there are existing solution strategies in literature, the authors found that the direct heuristics based on successive assignments were insufficient, and instead, recommended a new technology.

The ETTP was formulated as a graph-theoretic model by characterizing the weighted precedence $\mu-$ vague graph decomposed into three stages; scheduling exams using semi coloring, auto room assignment, and invigilator and chief invigilator allocation with uniform work load distribution. After applying the greedy coloring algorithm proposed in this paper, the scheduling process was completed by postponing / re-scheduling the least number of examinations for affected students, and coming up with single room or laboratory assignments for multiple courses to minimize invigilation duties, a room or lab assignment has planned by allowing multiple courses in a single venue that minimizes the total number of invigilation duties. Therefore, no student group experienced any clash in their examination schedules. Furthermore, academic staff members saved time, and this could be spent doing research, as the resource usage was optimized for the examination. In this paper, the described first phase of ETTP, examination scheduling, is handled by defining the weighted precedence $\mu-$ vague graph.

II. Structure of Vague Graph

To have a better understanding on how the vague graph and the fuzzy graph were defined from the vague set and the fuzzy set, several preliminary definitions have been studied from the existing literature as described in [21],[1],[8],[4],[17],[23],[5],[2],[7].

The similarities are observed in the definition of the fuzzy graph and the vague graph. In the fuzzy graph $G = (\alpha, \mu)$, the membership function μ defined for all the edges $uv \in E$ is defined such that $\mu(uv) \in [0, \min\{\alpha(u), \alpha(v)\}]$ over the closed interval $[0, 1]$. In the same way, the vague set (t_B, f_B) for the edges $uv \in E$ for all $u, v \in V$ is defined such that $t_B(uv) \in [0, \min\{t_A(u), t_A(v)\}]$ and $f_B(uv) \in [\max\{f_A(u), f_A(v)\}, 1]$ (i.e. $1 - f_B(uv) \in [0, \min\{1 - f_A(u), 1 - f_A(v)\}]$) over the closed interval $[0, 1]$. From this, it can be simply stated that $[t_B(uv), 1 - f_B(uv)] \subseteq [0, \min\{1 - f_A(u), 1 - f_A(v)\}]$. Also, the anti-fuzzy graph defined by the membership function μ for the edges $uv \in E$ for all $u, v \in V$ is defined such that $\mu(uv) \in [\max\{\alpha(u), \alpha(v)\}, 1]$ over the closed interval $[0, 1]$. Similarly, the general structure of vague graph and anti-vague graph are defined as in Definition 1 and 2, respectively.

Definition 1. *For a graph $G^* = (V, E)$, a triple $G = (\mu, A, B)$ is known as $\mu-$vague graph on G^* where μ is a fuzzy membership function on E, $A = (t_A, f_A)$ is a vague set on V and $B = (t_B, f_B)$ is a vague set on $E \subseteq V \times V$ such that for each $xy = e \in E$,*

$$t_B(e) \leq \mu(e)(t_A(x) \wedge t_A(y)) + (1 - \mu(e))(t_A(x) \vee t_A(y)),$$
and
$$f_B(e) \geq \mu(e)(f_A(x) \vee f_A(y)) + (1 - \mu(e))(f_A(x) \wedge f_A(y)),$$

Two vertices x and y are strongly adjacent, if

$$t_B(e) \geq \frac{1}{2} [\mu(e)(t_A(x) \wedge t_A(y)) + (1 - \mu(e))(t_A(x) \vee t_A(y))],$$

and

$$f_B(e) \leq \frac{1}{2} [\mu(e)(f_A(x) \vee f_A(y)) + (1 - \mu(e))(f_A(x) \wedge f_A(y))],$$

and it is called weakly adjacent otherwise.

A $\mu-$vague graph $G = (A, B)$ is strong, if

$$t_B(e) = \mu(e)(t_A(x) \wedge t_A(y)) + (1 - \mu(e))(t_A(x) \vee t_A(y)),$$
and
$$f_B(e) = \mu(e)(f_A(x) \vee f_A(y)) + (1 - \mu(e))(f_A(x) \wedge f_A(y)),$$

for every $e \in E$.

A $\mu-$vague graph $G = (\mu, A, B)$ is complete, if

$$t_B(e) = \mu(e)(t_A(x) \wedge t_A(y)) + (1 - \mu(e))(t_A(x) \vee t_A(y)),$$
and
$$f_B(e) = \mu(e)(f_A(x) \vee f_A(y)) + (1 - \mu(e))(f_A(x) \wedge f_A(y)),$$

for every $x, y \in V$.

If $\mu = 1$, then a triple $G = (1, A, B)$ is known to be a vague graph.

Definition 2. *For a graph $G^* = (V, E)$, a triple $G = (\mu, A, B)$ is called $\mu-$anti vague graph on G^* where μ is fuzzy membership function on E, $A = (t_A, f_A)$ is vague set on V and $B = (t_B, f_B)$ is vague set on $E \subseteq V \times V$ such that for each $xy = e \in E$,*

$$t_B(e) \geq (1-\mu(e))(t_A(x) \wedge t_A(y)) + \mu(e)(t_A(x) \vee t_A(y)),$$
$$\text{and}$$
$$f_B(e) \leq (1-\mu(e))(f_A(x) \vee f_A(y)) + \mu(e)(f_A(x) \wedge f_A(y)),$$

If $\mu = 1$, a triple $G = (1, A, B)$ is said to be an anti-vague graph.

Definition 3. *An anti-vague graph is defined to be a pair $G = (A, B)$, where $A = (t_A, f_A)$ is a vague set on V and $B = (t_B, f_B)$ is a vague set on $E \subseteq V \times V$ such that for each $uv \in E$,*

$$t_B(uv) \geq t_A(x) \vee t_A(y), \text{ and } f_B(uv) \leq f_A(x) \wedge f_A(y).$$

The anti-vague set (t_B, f_B) for the edges $uv \in E$ for all $u, v \in V$ is defined such that

$$t_B(uv) \in [\max\{t_A(u), t_A(v)\}, 1]$$

and

$$f_B(uv) \in [0, \min\{f_A(u), f_A(v)\}]$$

(i.e. $1 - f_B(uv) \in [\max\{1 - f_A(u), 1 - f_A(v)\}, 1]$)

Over the closed interval $[0, 1]$, it can be stated simply that

$$[t_B(uv), 1 - f_B(uv)] \subseteq [\max\{1 - f_A(u), 1 - f_A(v)\}, 1].$$

Definition 4. *For a graph $G^* = (V, E)$, a triple $G = (\mu, A, B)$ is called a $\mu-$bounded vague graph on G^* or general bounded vague graph where μ is a fuzzy membership function on E, $A = (t_A, f_A)$ is a vague set on V and $B = (t_B, f_B)$ is a vague set on $E \subseteq V \times V$ such that for each $xy = e \in E$,*

$$(t_A(x) \wedge t_A(y) \leq t_B(xy) \leq (1-\mu(e))(t_A(x) \wedge t_A(y)) + \mu(e)(t_A(x) \vee t_A(y)),$$
$$\text{and}$$
$$(f_A(x) \vee f_A(y) \geq f_B(e) \geq (1-\mu(e))(f_A(x) \vee f_A(y)) + \mu(e)(f_A(x) \wedge f_A(y)),$$

If $\mu = 1$, a triple $G = (1, A, B)$ is said to be a bounded vague graph.

Definition 5. *The $\mu-$vague graph $G = (\mu, A, B)$ is said to be regular if,*

$$\sum_{\substack{x \in V \\ x \neq y}} t_B(xy) = \text{constant and} \sum_{\substack{x \in V \\ x \neq y}} t_B(xy) = \text{constant}$$

Moreover, it is called strong regular if, for all $xy \in E$,

1) $t_B(xy) = t_A(x) \wedge t_A(y)$ *and* $f_B(xy) = f_A(x) \vee f_A(y)$.
2) $\sum_{\substack{x \in V \\ x \neq y}} t_B(xy) = \text{constant and} \sum_{\substack{x \in V \\ x \neq y}} t_B(xy) = \text{constant.}$

Definition 6. *The complement of a $\mu-$vague graph $G = (A, B)$ is a vague graph $\bar{G} = (\bar{A}, \bar{B})$, where $A = \bar{A}$ and B is described as follows:*

The true and false membership values for the edges of G are given below.

$$t_{\bar{B}}(xy) = \mu(xy)(t_A(x) \wedge t_A(y)) + (1-\mu(xy))(t_A(x) \vee t_A(y)) - t_B(xy)$$
$$f_{\bar{B}}(xy) = f_B(xy) - \mu(xy)(f_A(x) \vee f_A(y)) - (1-\mu(xy))(f_A(x) \wedge f_A(y)),$$

for all $x, y \in V$.

III. APPLICATION

The structure of the $\mu-$vague graph, known as the generalized vague graph, manages datasets that have certain and ambiguous data, which is often present in real-world problems. This type of data can be inconsistent or indeterminate, and traditional fuzzy graphs and vague graphs may not be able to adequately address these factors or obtain the desired results.

Let us consider the mechanism of the University to prepare for the succeeding semester's workload of staff based on the current students' results:

As an example, the University of Technology and Applied Sciences assumes that all students passed the courses they are currently studying in preparation for the succeeding semester's class timetabling of courses. Obviously, the failure rate may vary from 0% to 40% according to the nature of difficulty of the course. Hence, the assumption fails to support the pre-planning done for the succeeding semester. At the same time, it is impossible to assume that every course may have a fixed percentage of failure too. In this regard, the information that is required is the student's status - whether pass or fail - halfway in the current semester, based on the progression of the course. This information provides the motivation to predict the student's final mark for the course, which is generally the sum of the continuous assessment and the final examination. As a form of formative assessment, students are subjected to continuous assessments in their courses throughout the semester. Though there is positive correlation between the student performance in continuous assessment components and the final exam, it is difficult and cumbersome to collect all continuous assessment components for all the students in the middle of the semester. There are few missing data in the collection of the marks of continuous assessments that will be used to predict the final total mark.

A simple model which scales up existing known marks to out of hundred could be adopted, but it is not going to be accurate either. The aim of the authors is to propose a model that takes into consideration a few other external factors that may affect individual student performance using fuzzy membership value measures to predict the final total marks and identify whether a particular student passes or fails a particular course. It is easy to describe a functions with range $(0, 1)$ for minimum and maximum possible mark that the student can attain, say a and b. These two values are the true membership and the false membership values. Simply stated, students could just provide the marks that will determine whether they will pass or fail, and these will be a and b. To find the fixed value instead of the range for each and every student, the course teacher has to provide the fuzzy membership value μ which measure the confidence level of the teacher on the student performance. This model identifies and measure the student performance more specific using $\mu-$vague graph.

ETTP has been used in real-time experiments during the past two academic years in UTAS, Nizwa. This model was developed to semi-automate the process and increase the satisfaction level of the various University stakeholders. During the early implementation of the project, the ETTP scheduling is done through graph-coloring of conflict graph. The minimum number of colors required to color the conflict graph is either 12 or 13. Later after pandemic, the constraint is to schedule the examinations within 10 sessions, though it requires 12 or 13. How this could be possible? Obviously, it is impossible to have the proper coloring which requires the number of colors is lesser than the chromatic number. This situation motivated the authors to look for the minimum edge removal of the conflict graph to make it contain 10 or less chromatic numbers. Moreover, the student satisfaction level may increase only by having the proper gap between examination schedules in order to give them more time preparing for their examinations. Majority of students have common understanding about the difficulty of the courses they are currently registered, and thus, they are asked to define α and β for the course as percentages easy and difficult in terms of how they prepare for their examinations. Here $\alpha + \beta$ is not necessarily be equal to one, because of the incomplete data due the haziness of the student in the choice of selecting the outcomes as either difficult or easy in terms of percentage. The ratios $\dfrac{\alpha}{\alpha+\beta}$ and $\dfrac{\beta}{\alpha+\beta}$ are the values that lie in $[0, 1]$. The sum $\alpha + \beta \leq 1$. If the student may not take the decision for some of the outcomes, whether they are easy or difficult, such outcomes are with missing data. The ETTP is added with additional constraint that is to optimize the gap between the two courses which has lower α and higher β values.

If the model is able to find a coloring, then, the level of student satisfaction in regard to the timetable schedule will also increase among students. Hence, every vertex of the conflict graph should be assigned with the true membership and false membership values based on students choice of the values α and β. For this model,the conflict graph is considered as a vertex and edge-weighted vague graph. The investigation has taken place to get the optimum stratification of stakeholders on the solution of the ETTP. Further, this work requires to consider the fuzzy measure of the difficulty level of courses in terms of the course teacher. According to the membership value assigned for the courses by the course teachers, it could be further generalized. If the model considers the additional fuzzy membership function μ that defines the choice of confidence or acceptance of the course teacher on the decision made by the students, then it is easy to obtain the optimum scheduling by considering it as $\mu-$vague graph for investigation.

IV. Examination Timetabling Problem with Uncertainty

The basis of this timetabling problem includes assigning exams to a given number of days within a defined examination session with provided higher satisfaction of all hard and maximum number of soft constraints. By considering the purpose of timetabling, a feasible solution is one, which all hard constraints are met, but the soft constraints are also possibly attained. The university ETTP includes major administrative activities. The quality of the obtained solution of university examination timetabling problem is measured by means of the higher percentage of soft constraint satisfaction, because the provided conditions aim to satisfy all hard constraints. An increasing number of student enrollment, wide range of courses, fixed available resources, and verity of soft constraints increase the complication of scheduling and assignment process. The examination timetabling problem involves a set of constraints, and there have been numerous approaches proposed in the literature. Here, the uncertainty is taken in to account. Consider the courses are outcome based. The courses are classified based on the student ability to learn or prepare for the examination. If the student feels the outcome is difficult, then map it with 1. If the student feels the outcome is easy, then map it with 0. The student may feel that only few outcomes can be mapped with values that are between 0 and 1 in terms of difficulty. Also, they may not map the value for few of the outcomes due to haziness in evaluating them.

A real-world examination timetabling problem in the University of Technology and Applied Sciences is handled with these uncertainty measures defined for each course. Particularly, the data presented here is real data of final examination for Semester II, Academic year 2022. The data presented here has been processed excluding courses that have no examinations. Further, the original data is modified, replacing the appropriate exams accordingly. In this data, the cardinals are 283 courses, 818 exams groups with 14047 students, and 75857 enrollments. The number of exam days and sessions are 5 and 15, respectively.

Considering the examination timetabling problem with few uncertainty measures requires the $\mu-$vague graph for further investigation. It is the novel approach for the scheduling and assignment problem such as ETTP. One of the important soft constraints is to increase the gap between the courses that are treated as difficult by the student in the examination schedule. In terms of graph theory, the variant of the coloring has to be defined for weighted precedence $\mu-$vague graph for the scheduling. In this paper, a method is proposed to infer the edge weight of the conflict graph of ETTP scheduling. Every vertex of the precedence graph is mapped with a weight and vague interval. This weight denotes the number of students that have conflicts with the course being scheduled. If we draw a loop for every single student, then this number represents the number of loops on the respective vertices. Similarly, the edge weight is the number of students that has a conflict between the two courses that are associated with the vertices of the edge. If we draw a parallel edge for every single student who has conflict, then the edge weight is nothing but the number of parallel edges between the courses associated with the vertices. Simply, in the weighted precedence $\mu-$vague graph, the weight function ω is defined to map the weight for vertices as well as edges. In other words, vertex weight is the number of students enrolled in the course mapped with the vertex. Edge weight is the enrolled students in both the courses mapped with the vertices. To illustrate the definition of the weighted precedence $\mu-$vague graph, information about five courses is listed in the table below to provide an example

Table I: Course mapping with true and false membership function

Vertex	Course Code	Student	t_A	f_A
v_1	ITNT101	5	0.7	0.2
v_2	MATH1202	304	0.8	0
v_3	ITIS101	40	0.5	0.3
v_4	ENTW1200	271	0	0.9
v_5	ITDB101	17	0.4	0.6

The t_A for the course is determined by dividing the number of difficult outcomes selected by students by the total number of outcomes. Similarly, the f_A for the course is determined by dividing the number of easy outcomes chosen by students by the total number of outcomes.

Table II shows the clash list between courses. The edge weight is mapped with the enrolled students in the respective two courses.

Table II: Clash list and its true and false membership function

Edge	Course1	Course2	#	t_A	f_A
v_1v_2	ITNT101	MATH1202	3	0.6	0.3
v_2v_3	MATH1202	ITIS101	26	0.5	0.35
v_3v_4	ITIS101	ENTW1200	34	0	0.9
v_4v_5	ENTW1200	ITDB101	17	0	0.9
v_5v_1	ITDB101	ITNT101	3	0.2	0.7
v_4v_1	ENTW1200	ITNT101	4	0	1

Figure 1 illustrates the weighted precedence vague graph.

The student's measures t_A and f_A for every course are defined and considered for elevating the student's satisfaction to schedule the two courses v_i and v_j. The validation of the student evaluation could be done using another fuzzy membership function μ defined for every pair of courses by the teacher. Instead of considering the minimum $\{t_A(v_i), t_A(v_j)\}$ as the upper bound for $t_B(v_iv_j)$ by only considering the student input, the value $\mu(v_iv_j)$ is used to increase the upper bound of $t_B(v_iv_j)$. Primarily, this value is used to evaluate the difficulty level of having two courses v_i and v_j as continuous exams. Adding this fuzzy measure μ increases the accuracy in modeling the real time examination timetabling problem in terms of uncertainty involved in it. For a vertex v of graph G, $N(v)$, the open neighborhood of v, is the set of adjacent vertices of v and the closed neighborhood of v, $N[v]$, is $N(v) \cup v$.

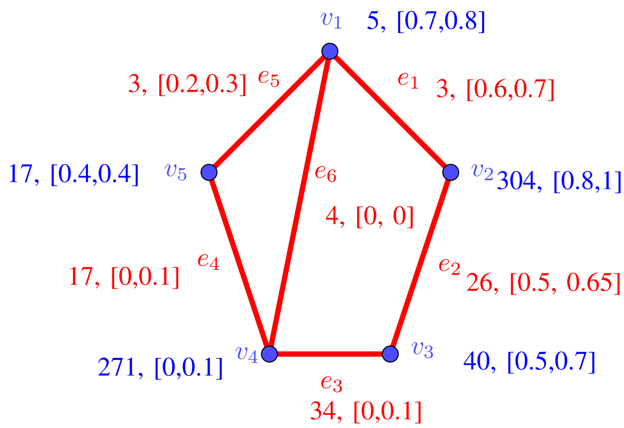

Figure 1: Weighted precedence vague graph

Definition 7. *For a vertex v of properly colored graph G and given color i, the open $i-$neighborhood of G, denoted by $N_i(v)$, is the set of adjacent vertices of v which belong to the color class $C[i]$. i.e., $N_i(v) = \{u : uv \in E(G)$ and $u \in C[i]\}$*

Figure 2 illustrates the weighted precedence $\mu-$vague graph.

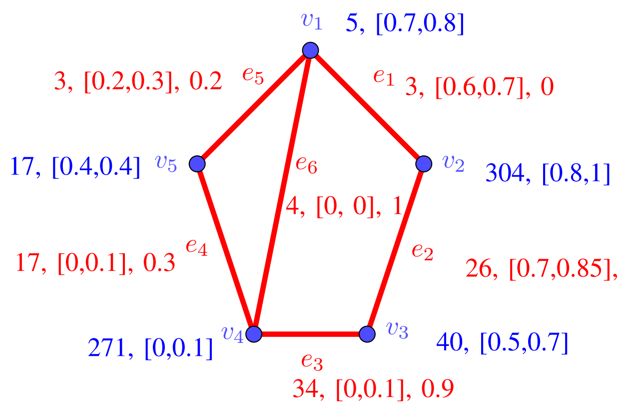

Figure 2: Weighted precedence $\mu-$vague graph

Definition 8. *For a subset $S \in V$ of weighted graph G, the weight of S, denoted by $w(S)$, is the sum of weights of each vertices of S.*
For a subset $R \in E$ of weighted graph G, the weight of R, denoted by $w(R)$, is the sum of weights of each edges of R.

Definition 9. *For a vertex v of properly colored graph G and given color i, the edge incidence of v, denoted by $I(v)$, is the set of all incidence edges of v whose other end vertex belongs to the color class $C[i]$.*
i.e., $I_i(v) = \{uv : uv \in E(G)$ and $u \in C[i]\}$

Definition 10. *For a graph G with chromatic number χ, $r-$semi $k-$coloring of the graph for $k < \chi(G)$ is defined as the possible $k-$coloring after removing $s \geq r$ edges from the graph G.*

A graph G with chromatic number χ is called $r-$semi $k-$colorable graph for $k < \chi(G)$ if the graph has $r-$semi $k-$coloring.

In other words, the graph G is $k-$colorable after removing minimum of $s \geq r$ edges from the graph G.

Let us consider the graph G. Given r, the graph $G - R$ where $R \subseteq E$ can have the proper $k-$coloring such that $|R| \leq r$. Based on the assumption, we can say that the unweighted graph has the weight for each edge 1.

In the weighted graph, the edge removal can be done in such a way that $w(R) \leq s$. Minimum of such possible all $w(R)$ is defined as the density of $r-$semi $k-$colorable graph.

ALGORITHM

A linear runtime algorithm can be used to perform a greedy coloring for a given ordering of vertices. This algorithm processes the vertices in a specific descending sequence based on the sum of weights of all edges incident to each vertex. It assigns the smallest possible available color to each vertex as it processes them. The k colors are represented by the numbers $\{0, 1, 2, \ldots, k-1\}$.

To find the smallest available color for the vertex v_j, we may use the $k \times 2$ dimensional array to have the sum of the weights of the edges of neighbors of each color i, and replace the values of first column with $w(I_i(v_j))$ for the index i, then replace the values of the second column with $w(I_{i-1}(v_j)) + w(I_{i+1}(v_j))$ and then scan the array to find the index of its smallest in the first column. If we have multiple such smallest values in the first column then scan the array to find the index of the smallest value of the second column. After assigning the color to the vertex v_j, we can repeat the process for v_{j+1}.

Algorithm 1: Greedy Coloring Algorithm

Input: Weighted precedence vague graph G and number of colors k
Output: Vertex coloring of G and list of edge removal
Data: Vertices $V = \{v_1, v_2, ..., v_n\}$, Edges $E = \{e_1, e_2, \ldots, e_m\}$, and the weight $w(v_i, v_j) = 0$ if there is no edge $v_i v_j$ exists in G

for $j = 0$ *to* $n-1$ **do**
  $w[j][0] := j + 1$ // labelling each vertex v_{j+1}
  $w[j][1] := \sum_{i=1}^{n} w(v_i v_{j+1})$ // assigning weight for each vertex v_{j+1}
  $w[j][2] := \frac{t_A(v_{j+1}) + f_A(v_{j+1})}{2}$ // assigning vague value for each vertex v_{j+1}
Sort the array w in descending order of second column and name it as W
for $j = 0$ *to* $n-1$ **do**
  if $j = 0$ **then**
    $color := 0$
  else
    for $i = 0$ *to* $k-1$ **do**
      if $i = 0$ **then**
        $a := W[j][0]$
        $c[i][0] := w(I_i(v_a))$
        $c[i][1] := w(I_{i+1}(v_a))$
      else if $i = k-1$ **then**
        $a := W[j][0]$
        $c[i][0] := w(I_i(v_a))$
        $c[i][1] := w(I_{i-1}(v_a))$
      else
        $a := W[j][0]$
        $c[i][0] := w(I_i(v_a))$
        $c[i][1] := w(I_{i-1}(v_a)) + w(I_{i+1}(v_a))$
      $multicolor := list[min(c[i][0])]$
      if *length(multicolor)=1* **then**
        $color := multicolor$
      else
        $color :=$ smallest i in which the $min(c[i][0])$ and $min(c[i][1])$ exist
  $W[j][1] :=$ *color* a least possible color from $\{0, 1, 2, \ldots, k-1\}$ // Assign the color $W[j][1]$ to the vertex $W[j][0]$
  return $v_{W[j][0]} \leftarrow [j][1]$ // printing color for each vertex v_{j+1}
  if $min(c[i][0])] > 0$ **then**
    The required edge removal : **return** $I_{color}(v_{W[j][0]})$

OUTPUT:

The output of the algorithm for the figure 1 is $v_3 \leftarrow 0$, $v_4 \leftarrow 1$, $v_2 \leftarrow 1$, $v_5 \leftarrow 0$, $v_1 \leftarrow 0$

The required edge removal : $I_0(v_1)$ ie., $\{v_1 v_5\}$

RESULTS:

The scheduling and assignment for the academic year 2021–2022, ETTP at UTAS, Nizwa, were prepared by employing the semi-coloring proposed algorithm. The main impact of this application is that:

- the list of students who have conflict in the examination schedule is obtained before the publication of the examination timetable to the students. Previously, the number of clashes are found after the publication, if the students report it. The number of the students with conflict in examination is minimized in the schedule.
- the number of continuous exams has been kept to minimum, fulfilling one of the objectives of this research.
- the exams are distributed with the maximum gap between the exams.

It is not feasible to compare the solution obtained by the proposed greedy algorithm with the existing algorithms in the literature, because the proposed semi-coloring is allowing the minimum possible conflict to reduce the number of examination schedules. Hence, few observations are listed below in terms of graph theory:

- Any graph G with chromatic number χ is $a-$semi $b-$colorable if $b < \chi$ and $a \geq \omega$
- For any graph G with $\chi-$colorable $k > 1$ components, the graph G is $a-$semi $b-$colorable iff each component of graph G is $a-$semi $b-$colorable.
- For every graph G with chromatic number χ, there exists a sequence of numbers $a_1, a_2, ..., a_\chi$ such that the graph G is a_i-semi $\chi - i-$colorable.

V. Conclusion and Future Work

The approach comprises three stages. Initially, a heuristic algorithm is utilized, which is based on a proposed variant of graph coloring, to determine the color classes of the exams. In the second stage, this algorithm is combined with another algorithm that orders courses to assign exams to specific rooms. In the final stage, pairs of invigilators are assigned to each room or laboratory. Throughout all three stages, the set of soft and hard constraints are considered to achieve a feasible and optimal solution, although these constraints may be uncertain. Prior to these three stages, ETTP-UTAS follows a set of predefined steps to collect necessary data. This data is categorized as static or dynamic, with the dynamic data changing for each exam, such as the exam dates.

In this work, a novel greedy coloring algorithm is presented for the precedence graph of the examination timetabling problem with uncertainty. Out of the main three stages, scheduling courses in different slots by minimizing the number of clashes included the uncertainty. The other two might be automated with other algorithms in the future work. For the scheduling, we propose the edge decomposition from the actual precedence graph to have lesser number of coloring than the actual required. Such coloring process is defined as semi-coloring. The novel greedy algorithm is developed to get the semi coloring by considering weights of edges that are actually mapped with the measures of uncertainty.

The primary objective of the ETTP of UTAS, Nizwa is the number of days for examination schedule in ten slots with the minimum of conflicts. Also, minimizing the resource assignment and maximize the satisfaction for the student and invigilators on their schedule and assigned resources are considered as additional objectives. To the programmers of the educational technological center of UTAS, Nizwa, the Examination Management System proposes to develop the required reports generation and to display the proposed scheduling to ensure the smooth functioning of university examinations by improving the existing examination scheduling practices. The required details and reports can be accessed online easily and securely by the admin staff, academic staff, and students as per their requirement.

References

[1] K Atanassov. Intuitionistic fuzzy sets. *Fuzzy Sets and Systems*, 20:87–96, 1986.

[2] L Bhaskar, P Narsimhaswamy, and T Srinivas. Some results on an anti vague field. *International Journal of Pure and Applied Mathematics*, 117(13):127–133, 2017.

[3] P Bhattacharya. Some remarks on fuzzy graphs. *Pattern Recognition Letters*, 6:297–302, 1987.

[4] R Biswas. Vague groups. *International Journal of Comput. Cognition*, 4(2):20–23, 2006.

[5] Rajab Ali Borzooei and Hossein Rashmanlou. Domination in vague graphs and its applications. *Journal of Intelligent & Fuzzy Systems*, 29(5):1933–1940, 2015.

[6] Tim B. Cooper and Jeffrey H. Kingston. The complexity of timetable construction problems. In Edmund Burke and Peter Ross, editors, *Practice and Theory of Automated Timetabling*, pages 281–295, Berlin, Heidelberg, 1996. Springer Berlin Heidelberg.

[7] Arindam Dey, Le Hoang Son, P K Kishore Kumar, Selvachandran Ganeshsree, and Shio Gai Quek. New concepts on vertex and edge coloring of simple vague graphs. *Symmetry*, 10(373):1–18, 2018.

[8] W L Gau and D J Buherer. Vague sets. *IEEE Transactions On Systems, Man and cybernetics*, 23:610–614, 1993.

[9] I Grattan-Guiness. Fuzzy membership mapped onto interval and many-valued quantities. *Mathematical Logic*, 22:149–160, 1975.

[10] K U Jahn. Intervall-wertige mengen. *Math. Nach.*, 68:115–132, 1975.

[11] Vahid Khatibi and Gholam Ali Montazer. Intuitionistic fuzzy set vs. fuzzy set application in medical pattern recognition. *Artificial Intelligence in Medicine*, 47:43–52, 2009.

[12] R Lowan. Convex fuzzy sets. *Fuzzy Sets and Systems*, 3(3):291–310, 1980.

[13] Ashis Kumar Mandal, M. N. M. Kahar, and Graham Kendall. Addressing examination timetabling problem using a partial exams approach in constructive and improvement. *Computation*, 8(2), 2020.

[14] Mozhgan Mokhtari, Majid Vaziri Sarashk, Milad Asadpour, Nadia Saeidi, and Omid Boyer. Developing a model for the university course timetabling problem: A case study. *Complexity*, 2021, 2021.

[15] J N Mordeson and C S Peng. Operations on fuzzy graphs. *Information Sciences*, 79:159–170, 1994.

[16] Nelishia Pillay and Rong Qu. *Examination Timetabling Problems*, pages 75–82. Springer International Publishing, Cham, 2018.

[17] N Ramakrishna. Vague graphs. *International Journal of Computational Cognition*, 7:51–58, 2009.

[18] A Rosenfeld, L A Zadeh, K S Fu, and M Shimura. Fuzzy sets and their applications. *Academic Press*, pages 77–95, 1975.

[19] R Sambuc. *Application laide au diagnostic en pathologie thyroidienne*. PhD thesis, Univ. Marseille, France, 1975.

[20] L Zadeh. The concept of a linguistic variable and its application to approximate reasoning. *Information Science*, 8:199—249, 1975.

[21] L A Zadeh. Fuzzy sets. *Information and Control*, 8:338–353, 1965.

[22] Shao Zehui, Kosari Saeed, Shoaib Muhammad, and Rashmanlou Hossein. Certain concepts of vague graphs with applications to medical diagnosis. *Frontiers in Physics*, 8(373):1–18, 2020.

[23] T Zelalem. I-vague sets and i-vague relations. *International Journal of Computational Cognition*, 8(4):102–109, 2010.