# Data Pre-processing Method of Explosion Seismic Waves Based on Cross-Correlation Analysis

Qingwei GUO, Wanpeng WANG, Yibo XIONG, Shulei ZHENG

Abstract—Explosion Seismic Waves can easily be affected by random interference generated from the explosion source, propagation medium, and topography, resulting in zero shift or great dispersion. Therefore, data pre-processing is necessary for validity determination. This paper presents a data pre-processing method based on cross-correlation theory, which considers the waves' zero shift and similarity as objects. A data pre-processing flow wherein zero drift assessment precedes similarity assessment is introduced. The results obtained by processing the data collected from six groups of explosion seismic waves reveal that the data pre-processing method is feasible and effective, and can satisfactorily distinguish the waves' zero drift and similarity degree and provide effective data support for subsequent seismic wave post-processing, such as spectrum analysis, wavelet transforms, and Hilbert-Huang transforms.

*Index Terms*—Explosion seismic waves, data pre-processing, cross-correlation analysis, center moving average method, zero drift, similarity

# I. INTRODUCTION

 $E_{\rm by}$  the explosion source, propagation medium characteristics, distance to explosion center, and topographic and geological conditions [1-4]. Diffused waves should be investigated to analyze the explosion seismic effects [5] by considering the contrast in the characteristic parameters. In practice, the peak, duration time, and other parameters in the time domain cannot sufficiently represent the explosion seismic effect [6-8]. Consequently, frequency domain analysis [9–11], wavelet transforms [12–14], Hilbert–Huang transforms [15–17], and so on, are used in explosion seismic wave analysis, and require good time-domain waveforms. When seismic waves are particularly influenced by random interference, zero drift or great dispersion arises in the waveforms, undermining the data processing confidence and precision. Therefore, the pre-processing of seismic waves is needed to estimate the usability by validity determination.

The cross-correlation technique is an efficient tool with increasing importance in the seismological research community, and is widely used in real-time earthquake prediction [18], earthquake discrimination [19–24] (including explosion earthquakes, micro-earthquakes, and

aftershocks), the seismic monitoring of underground nuclear explosions [25–26], seismic source location [27–31], precise estimation of relative earthquake magnitudes [32], the performance monitoring of broadband seismic instruments [33], seismic characterization research [34], and so on. These applications focus on data post-processing and data mining based on recorded seismic waves, but give little attention to waveform contrast and validity determination [35]. This paper proposes a pre-processing approach based on the cross-correlation technique to determine the zero drift and similarity and provide precise data for subsequent post-processing such as frequency domain analysis, wavelet transforms, and Hilbert–Huang transforms.

#### II. FUNDAMENTAL CROSS-CORRELATION THEORY

To investigate the influence of different factors on explosion seismic effects, the critical factor must be controlled to change as designed instead of changing under the effect of other factors or compound factors. In practical operation, data must be selected from the same station and location to make the interference by the propagation path become systematic deviation and effectively improve the consistency of waves. Then, the cross-correlation technique can be used to identify waves with large random interference, quantitatively analyze the similarity of seismic waves under different conditions, and assess the data validity. Finally, the influence of different factors on the explosion seismic effect can be obtained by comparing the valid seismic waves.

#### A. Basic Algorithms

Cross-correlation analysis is based on cross-correlation theory [36–37]. In actual explosion seismic monitoring, the recorded data are a finite discrete array. To analyze the similarity of seismic waves quantitatively, the normalized cross-correlation function is used:

$$\begin{cases} R_{xy}(m) = \frac{\sum_{i=0}^{N-1} x(i) * y(i+m)}{\sqrt{R_{xx}(0)} * \sqrt{R_{yy}(0)}} \\ R_{xx}(0) = \sum_{i=0}^{N-1} x(i) * x(i) \end{cases}$$
(1)

where *x* and *y* are seismic waves sampled at equal intervals of the same station in different experiments; N denotes the array length; *m* denotes the offset cross-correlation function, m=-N+1...-1,0,1...N-1;  $R_{xx}(0)$ , and  $R_{yy}(0)$  are the cross-correlation function value when the offset m of the autocorrelation function of *x* and *y* is zero, and also the maximum value.

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Fig. 2. Cross-correlation function of explosion seismic waves.

Figure 1 shows the comparison of the explosion seismic waveforms in the vertical direction of the two explosion experiments under different conditions with the same equivalent charge, same station, and different explosion sources. Figure 2 shows the corresponding cross-correlation function of the waves in Fig. 1. Both waveforms have small zero drift, particularly the noise part, but have good amplitude similarity, and it is difficult to determine the difference between the two waves through the peak and duration time. In Fig. 2, the cross-correlation function is used to calculate the maximum value of 0.96, indicating the high linear correlation relationship between the two waves, which is consistent with the waveform comparison in Fig. 1. Because there is large fluctuation in the baseline, cross-correlation theory can be used to analyze the similarity of different explosion seismic waves quantitatively, but fails in effectively distinguishing zero drift.

## B. Problems

Considering the demand for seismic wave pre-processing, various problems related to cross-correlation must be solved to make the method feasible.

(1) The maximum value of the cross-correlation function can effectively determine the similarity of two waves, but fails in determining deviation, particularly zero drift.

(2) Seismic waves are broadband random signals whose cross-correlation function consists of rapid oscillation attenuation curves, which introduces the problem of reasonably establishing the relationship between data validity and waves.

(3) A method of setting reasonable determination criteria is



Fig. 3. Cross-correlation function of timed offset.

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needed to assess the validity.

### III. SEISMIC WAVE PRE-PROCESSING METHOD

The key factors impacting data validity are zero drift and curve similarity, both stemming from issues in data pre-processing based on cross-correlation. This study investigated a zero drift determination algorithm and similarity determination algorithm for explosion seismic waves, and developed a data pre-processing method.

# A. Cross-correlation Function Improvement Offset Timing

The offset *m* of the cross-correlation function corresponds to the sampling sequence, and an offset corresponds to a sampling interval  $\Delta T$ . Accordingly, the offset of the cross-correlation function is timed to elucidate the relationship between the cross-correlation function and time, and achieve better correspondence with the subsequent zero drift algorithm. The specific calculation formula is expressed as follows:

$$m_{t} = m * \Delta T \tag{2}$$

According to Eq. (2), Fig. 2 can be converted to Fig. 3.

### Signal Delay Adjustment

As shown in Fig. 3, the maximum value of the cross-correlation function is obtained when the offset time  $m_t$  is 0.002 s, which reveals the time delay dt=0.002 s between seismic waves x and y, that is, there is a sampling interval time lag. To provide better original waveforms, wave adjustment is applied to correct the time delay:

$$y = y(i + dt / \Delta T) \tag{3}$$

## *B. Zero Drift Determination Algorithm Based on Center Moving Average Method*

According to the cross-correlation definition in Eq. (1), if the seismic waves x and y have zero drift, the corresponding cross-correlation has zero drift in the same manner. Specifically, the zero drift of the cross-correlation function is the square of a single waveform, provided that the two waves are similar. Therefore, the zero drift can be determined through the above-mentioned transfer relationship.

# Baseline Calculation Based on Center Moving Average Algorithm

An important zero drift characteristic is that the baseline changes and deviates from the zero line. Because seismic waves are broadband random signals and their cross-correlation function is a rapid oscillation attenuation curve, it is necessary to establish an accurate cross-correlation function baseline using the oscillation curve. For calculation, this study used the center moving average method [38–39], which provides a satisfactory smoothing of the oscillation fluctuations. The basic algorithm is expressed as follows:

$$R_{xybl}(m_t) = \frac{1}{2K+1} \sum_{i=0}^{2K} R_{xy}(m_t - K + i)$$
(4)

where K is the number of points for the moving average before and after  $m_t$ , respectively, amounting to a total of

2K+1 points.

To solve the cross-correlation baseline effectively, K should be sufficiently large to smooth out most fluctuations and obtain a flat baseline. However, if K is too large, this will make the baseline insensitive to the actual data, and K will deviate from the actual baseline. In this study, K was considered to take 1/4 of the sampling frequency  $f_s$ , such that the total number of points is  $f_s/2+1$ , to ensure that the baseline can contain as much information as possible. Considering the waves in Fig. 3 as an example, the baseline calculation shown in Fig. 4 was carried out. The calculation result is in good agreement with the waveforms' variation trend and confirms the effectiveness of the proposed algorithm.

## Least Squares Curves Fitting of Baseline

The magnitude of zero drift is proportional to the deviation from the baseline, which can be represented by the baseline slope. However, the baseline slope is continuous and its comparison with the slope of a certain point is difficult. Because the duration  $T_e$  of the explosion seismic waves is limited, and values greater than zero and less than zero in the cross-correlation function represent the offset of the two waves, respectively, the correlation of seismic signals can be represented in the offset time range  $|m_t| \leq T_e$ , and the function beyond this interval is the correlation of seismic signals and noise. Therefore, it is only necessary to assess the baseline change within the offset time  $|m_t| \leq T_e$ .

The method of least square polynomial fitting curves [40–41] is adopted to fit the cross-correlation baseline in the range  $|m_t| \le T_e$  according to the offset direction, that is,  $m_t < 0$  and  $m_t > 0$ .

$$\begin{cases} R_{yxbl}(m_{t}) = a_{yx} * m_{t} + b_{yx} - T_{e} \le m_{t} < 0 \\ R_{xybl}(m_{t}) = a_{xy} * m_{t} + b_{xy} & 0 < m_{t} \le T_{e} \end{cases}$$
(5)

The proposed algorithm was used for the linear fitting of the baseline in Fig. 4, and the results are shown in Fig. 4 as  $R_{yxbl}$  and  $R_{xybl}$ ; the corresponding slopes are  $a_{yx}$ =0.20 and  $a_{xy}$ =-0.26.

#### Determination Criteria

Based on the transfer relationship between zero drift and the baseline change, the slope of the line fitted by the least square polynomial method applied to the baseline is used in the assessment, because it can accurately represent the zero drift of the main signal of the explosion seismic waves. The criterion is expressed as follows:

$$\begin{cases} \left| a_{yx} \right| > a_{\lim} \cup \left| a_{xy} \right| > a_{\lim}, Yes \\ or , No \end{cases}$$
(6)

where  $a_{lim}$  denotes the threshold of the baseline slope used to determine the zero drift, and generally takes values in the range of 0.05–0.1 based on experiments. Notably, the sensitivity to zero drift is higher with a smaller value. As shown in Fig. 4, both  $a_{yx}$  and  $a_{xy}$  of the baseline fitting lines are greater than 0.1, demonstrating the waves' obvious zero drift.

#### C. Waveform Similarity Determination Algorithm

The maximum value of the cross-correlation function, that is, the cross-correlation coefficient, represents the overall linear correlation degree of seismic waves and can be used as



Fig. 5. Data pre-processing flow block diagram.

the first criterion for waveform similarity, as follows:

$$\begin{cases} |R_{xy\max}| \ge R_{xy\lim}, Yes \\ or , No \end{cases}$$
(7)

where  $R_{xylim}$  denotes the threshold of the cross-correlation coefficient used to determine the similarity, and generally takes values in the range of 0.85–0.95 based on experiments.

From the cross-correlation definition in Eq. (1), the cross-correlation function in the offset time range  $m_i < 0$  and  $m_i > 0$  corresponds to the zero drift of seismic waves *x* and *y*,

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respectively. Therefore, the relationship between the slopes of the baseline fitting lines in the two cases can be used as the second waveform similarity criterion to identify the similarity of the corresponding seismic waves, as follows:

$$\begin{cases} R_{xy \lim} \le \left| \frac{a_{yx}}{a_{xy}} \right| \le \frac{1}{R_{xy \lim}}, Yes \\ or , No \end{cases}$$
(8)

However, when the zero drift is very small or non-existent, the baseline of the cross-correlation function is an irregular random curve close to zero, and the slopes of its fitting lines are also random and small numbers, which fail to represent the waveforms' similarity degree. Consequently, the condition used in Eq. (8) is set to  $|a_{yx}| \& |a_{xy}| > a_{lim}/10$ .

## D. Data Pre-processing Flow Design

The primary objective of seismic wave pre-processing based on cross-correlation analysis is to determine the zero drift and similarity of waveforms, and confirm whether the waves are valid and appropriate for subsequent data analysis. To this end, the data pre-processing flow was designed as shown in Fig. 5.

# IV. VERIFICATION

## A. Explosion Seismic Waves and Pre-processing

To verify the adaptability and accuracy of the data pre-processing method, six groups of explosion seismic waves with different zero drift and similarity degrees were employed, as shown in Fig. 6.

Six groups of seismic waves were pre-processed using the proposed algorithm and process, including Eqs. (4)–(8); the relevant parameters were  $T_e=1$  s,  $a_{lim}=0.1$ , and  $R_{xylim}=0.90$ . Thus, the cross-correlation, baseline and its fitting lines were calculated as presented in Fig. 7 and Table 1.

### B. Results and Discussion

From the data pre-processing results in Fig. 7 and Table I, the following conclusions are drawn:

(1) From the baseline change trend in Fig. 7 and baseline slope in Table 1, seismic wave groups (a), (b), and (c) have obvious zero drift, while group (d) has small zero drift, and groups (e) and (f) have no zero drift, corresponding to the time-domain waveform drift in Fig. 6, respectively.

(2) From the cross-correlation coefficients in Table 1, that is, the maximum value of the cross-correlation function, the values of groups (a) and (e) are the smallest, and the two groups have the worst similarity. Particularly, both waves of group (a) have obvious drift, and those of group (e) have great differences in the waveform change trend and consistency. Zero drift also exists in group (c), but the two waveforms have good consistency and are similar in terms of drift direction and size, as indicated by  $|a_{yx}/a_{xy}|=1.0$ . Group (d)

is affected by small zero drift, and the waveform similarity decreases, but the overall consistency is better. The waveform change trend of group (f) is very consistent with a high degree of similarity. Additionally, the pre-processing results correspond to the characteristics of the time-domain waveform in Fig. 6.

(3) From Table 1, it is concluded that only groups (d) and (f) are effective, while groups (a), (b), and (c) cannot be used owing to the large drift, and group (e) is ruled out because of the poor waveform similarity. These conclusions are consistent with the waveform comparison results in Fig. 6, and confirm the rationality of the proposed data pre-processing method and process.

From the analysis of the above results, the comparison between the data pre-processing results and the waveform characteristics confirm the rationality and efficiency of the proposed seismic wave pre-processing algorithm, which includes a zero drift determination algorithm based on the center moving average method and a waveform similarity determination algorithm. Additionally, it is proven that the proposed data pre-processing flow is feasible.

## V. CONCLUSION

To address the problems of explosion seismic waves with regard to data processing efficiency, this paper proposes a data pre-processing method based on cross-correlation. The proposed method includes the determination of waveform zero drift and similarity.

With consideration to the transfer relationship between seismic waves and the cross-correlation function, the zero drift magnitude is analyzed through the baseline change of the cross-correlation function. The baseline is calculated using the center moving average method, and the slope of its fitting curves is used to determine the zero drift. Two methods are used for similarity determination: cross-correlation coefficient determination and the determination of the slope of the baseline fitting lines. The data pre-processing flow was designed such that zero drift assessment is carried out first, followed by similarity determination.

The results obtained from processing six groups of explosion seismic waves confirm the feasibility of the proposed pre-processing method. Moreover, procedural design can be programmed for automatic assessment, which is convenient for batch data processing, effectively improves the data processing efficiency, and provides accurate data support for subsequent seismic wave post-analysis, such as frequency domain analysis, wavelet transforms, and Hilbert–Huang transforms.

TABLE I RESULTS OF DATA PRE-PROCESSING							
Group	Zero Drift Determination			Similarity Determination			Pre-Processing Conclusion
	$a_{yx}$	$a_{xy}$	Result	<b>R</b> <sub>xymax</sub>	$ a_{yx}/a_{xy} $	Result	
(a)	0.70	-0.70	Yes	0.880	1.00	No	Invalid
(b)	0.28	-0.30	Yes	0.980	0.93	Yes	Invalid
(c)	0.14	-0.14	Yes	0.939	1.00	Yes	Invalid
(d)	0.046	-0.049	No	0.925	0.94	Yes	Valid
(e)	0.0003	0.00006	No	0.833	-	No	Invalid
(f)	0.0012	-0.0009	No	0.977	-	Yes	Valid









Fig. 6. Explosion seismic waves of different states.

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Fig. 7. Results of data pre-processing.

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