Prescribed Performance Adaptive Control for Nonlinear Systems with Unmodeled Dynamics via Event-triggered

Yaobang Zang, Nannan Zhao, Xinyu Ouyang, Jiangnan Zhao

Abstract—**A prescribed performance neural network adaptive control scheme based on event-triggered mechanism is presented for a class of strict-feedback nonlinear systems with unmodeled dynamics. First, in order to improve the performance of system, finite-time performance function is introduced. The unknown nonlinear functions are approximated by radial basis function (RBF) neural networks. Then, an adaptive event-triggered controller based on back-stepping is designed, which guarantees that all signals of the closed-loop system are semi-globally uniformly ultimately bounded (SGUUB). Meanwhile, the tracking error can converge to a prescribed range, and the Zeno-behavior can be avoided. Finally, simulation verifies the effectiveness of the proposed method.**

Index Terms—**Nonlinear systems, finite-time prescribed performance, event-triggered, adaptive control, unmodeled dynamics.**

I. Introduction

In recent years, adaptive back-stepping control combined with RBF neural networks (NNs) or fuzzy logic systems (FLS) has been widely applied to solve the problem of uncertain nonlinear systems. By using the approximation property of NNs and FLS, the virtual control law derivation process is simplified, and the unknown smooth function can be approximated, so that a good control effect can be obtained. Currently, NNs or FLS adaptive back-stepping control methods for strict-feedback nonlinear systems have been adopted by many scholars [1], [2], [3], [4]. NNs and FLs have also become important in dealing with nonlinear systems.

In the practical control systems, there exist usually parts of the unmodeled dynamics that originate from ignored higher-order state terms. Moreover, the stability of the control system can be affected and the difficulty of controller design is increasing by unmodeled dynamics. If unmodeled dynamics are simply ignored during modeling, the expected effects in practical control will be difficult to achieve. Therefore, unmodeled dynamics can not be simply ignored. In order to deal with the nonlinear system with unmodeled dynamics, there are scholars have conducted a lot of researches and obtained lots of results [5], [6], [7], [8], [9], [10], [11], [12], [13]. In [5], dynamic signal was introduced for the first time to deal with dynamic disturbances, the problem of unmodeled dynamics was solved. In [6], the small gain theorem and input-to-state practically stable (ISPS) theory were introduced, and adaptive back-stepping control was designed. In [8], [9], [10], [11], [12], [13], a large number of scholars have proposed different control approaches for nonlinear systems with unmodeled dynamics by applying NNs or FLS.

For any control system, how to ensure both transient and steady-state performance is a problem worth pondering. Therefore, in [14], prescribed performance control (PPC) approach has been proposed for the first time. After more than a decade of development, PPC has been researched by many scholars [15], [16], [17], [18], [19], [20], [21], [22]. In [15], for quadrotor unmanned aerial vehicles with uncertainties and input constraints, prescribed performance function was introduced, and in order to deal with uncertainties and disturbances, an extended state observer was constructed, then, back-stepping controller was designed by using dynamic surface control and extended state observer, which ensures transient and steady-state performance and overcame complexity explosion issues, and improves the robustness of system. In [16], a class of electro-hydraulic actuator model manipulators were considered, the tracking error was constrained by prescribed performance constraint technology, and the NNs adaptive control method was presented. In [17], a class of strict-feedback nonlinear systems with actuator failures, component failures and unknown control directions was considered, and a prescribed performance fault-tolerant controller was designed. However, traditional prescribed performance control can not satisfy the requirement of high accuracy control. Therefore, the finite-time performance function (FTPF) is first proposed in [18], the tracking error can converge to a bounded range during the tuning time. Then, in [19], an unknown nonlinear system was considered, an adjustable finite-time prescribed performance function was proposed, and an adaptable finite-time prescribed performance function is presented, and an fuzzy adaptive controller with finite-time prescribed performance is designed.

On the other hand, in order to solve the problem of limited network resources, in literature [23], the event-triggered was proposed. Then, several event-triggered control (ETC) shames were proposed for nonlinear systems in [24], [25], [26], [27], [28], [29], [30], [31], [32], [33], [34]. In [25], Xing et al. used adaptive compensation mechanism to improve the traditional control methods, which provided a

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firm foundation for the extensive development of subsequent event-triggered. Since then, many scholars have successfully researched a number of important results based on the event-triggered mechanism. In [26], the stochastic systems with actuator failures was studied. literature [27] researched nonstrict-feedback stochastic systems with external disturbances. In [28], the event-triggered finite-time stabilization of nonlinear switched systems was studied by Deng et al. In [29], non-fragile reliable passive event-triggered control for switched systems was researched. Furthermore, the fixed time method was used in literature [30], [31], and feedback linearization was applied by [32], pure-feedback was researched in [33], output-feedback controller was designed in [34]. The event-triggered mechanisms in the above literature mainly include fixed threshold and relative threshold strategies. Compared with the fixed threshold strategy, the relative threshold strategy is more flexible. Moreover, the considered nonlinear systems with unmodeled dynamics is more consistent with the practical systems, and in [18], the finite-time performance function can only change the convergence time to the boundary, but cannot change the convergence speed within the convergence time. In addition, to the best of our knowledge, there has not been any work considering the finite-time prescribed performance adaptive NNs event-triggered controller design for nonlinear systems with unmodeled dynamics. Therefore, in this paper, the event-triggered mechanism and the improved finite-time prescribed performance are used in the design of the controller, and the impact of unmodelled dynamics on the system is fully considered.

Inspired by the above literatures, in this paper, a novel finite-time prescribed performance event-triggered adaptive control scheme for a class of nonlinear systems with unmodeled dynamics is proposed. By comparing with the existing results, the contributions of this paper are summarized below:

(1) The improved FTPF is first proposed, different form [18], the improved performance function can not only ensure converge to the bounded range during the settling time, but also change the convergence speed.
(2) Comparing to [4] and [30], the event-triggered mechanism can improve communication utilization, and the relative threshold strategy is more flexible than fixed threshold strategy.

II. System Description and Preliminaries

Consider an uncertain nonlinear systems with unmodeled dynamics as follows:

$$\begin{cases} \dot{\xi} = p(\xi, x) \\ \dot{x}_1 = g_1(x_1)x_2 + f_1(\bar{x}_1) + \Delta_1(x, \xi, t) \\ \dot{x}_i = g_i(x_i)x_{i+1} + f_i(\bar{x}_i) + \Delta_i(x, \xi, t) \\ \dot{x}_n = g_n(x_n)u + f_n(\bar{x}_n) + \Delta_n(x, \xi, t) \\ y = x_1 \end{cases} \tag{1}$$

where $\bar{x}_i = [x_1, x_2, \ldots, x_i]^T \in R^i$ represent the system state vector; $u \in R$ and $y \in R$ is the system input and output respectively, $\xi \in R^{n_0}$ is the unmodeled dynamics in (1), $\Delta_i(\cdot)$ denotes uncertain dynamic disturbance, $f_i(\cdot)$ and $g_i(\cdot)$ are smooth unknown nonlinear functions, $\Delta_i(\cdot)$ and $\xi(\cdot)$ are uncertain Lipschitz functions.

Assumption 1 [6]: For $i = 1 \ldots n$, there exists an unknown constant b_m satisfying $0 < b_m \leq |g_i(\bar{x}_i)| < \infty$.

Assumption 2 [7]: The reference signal y_r is known smooth and bounded function.

Assumption 3 [8]: For $i = 1 \ldots n$, there exist the non-negative monotone increasing smooth functions $\varphi_{i1}(\cdot)$ and $\varphi_{i2}(\cdot)$ satisfying

$$|\Delta_i(x, \xi, t)| \leq \varphi_{i1}(|\bar{x}_i|) + \varphi_{i2}(|\xi|) \tag{2}$$

Assumption 4 [5]: For system (1) is supposed to be exponentially input-to-state practically stable(Exp-ISPS). Then, there exists a Lyapunov function $V(\xi)$ which satisfying

$$\nu_1(|\xi|) \leq V(\xi) \leq \nu_2(|\xi|) \tag{3}$$

$$\frac{\partial V(\xi)}{\partial \xi} p(\xi, x) \leq -\Lambda_0 V(\xi) + \eta(|x|) + d_0 \tag{4}$$

where ν_1, ν_2 and η are three class K_∞ functions, and Λ_0, $d_0 > 0$ are constants.

Lemma 1 [5]: If there exists an Exp-ISPS Lyapunov function V, i.e. (3) and (4) holds , then for $\forall \Lambda \in (0, \Lambda_0)$, with initial value $\xi_0 = \xi_0(0)$ and function $\bar{\eta}(x_1) \geq \eta(|x_1|)$, there exists a finite time $T_0 = T_0(\Lambda, r_0, \xi_0), D(t) \geq 0$ for all $t \geq 0$ and a signal expressed by

$$\dot{r} = -\Lambda r + \eta(x_1(t)) + d_0, r(0) = r_0 \tag{5}$$

such that $D(t) = 0$ for all $t \geq T_0$

$$\mathrm{V}(\xi(t)) \leq r(t) + D(t) \tag{6}$$

For all $t > 0$, the solutions are defined. Generally, we can choose a smooth function satisfying $\bar{\eta}(s) = s^2 \eta(s^2)$. Then, (6) can be written as follow:

$$\dot{r} = -\Lambda r + x_1^2 \eta_0(x_1^2) + d_0, r(0) = r_0 \tag{7}$$

where $\eta_0 \geq 0$ is a smooth function.

Lemma 2 [4]: For $\forall \chi \in R$ and $\ell > 0$, one has

$$0 \leq |\chi| - \chi \tanh\left(\frac{\chi}{\ell}\right) < \beta \ell, \beta \leq 0.2785 \tag{8}$$

Lemma 3 [35]: Defined set $\Omega_{z_1} = \{z_1 | |z_1| < 0.8814 \upsilon\}$, where $\upsilon > 0$ is constant. When $\forall z_1 \notin \Omega_{z_1}$, the inequality $\left[1 - \tanh^2(z_1/\upsilon)\right] < 0$ holds.

Lemma 4 [30]: For $\forall x, y \in R$, there exists

$$xy \leq \frac{\beta^p}{p}|x|^p + \frac{1}{q\beta^q}|y|^q \tag{9}$$

where β is an non-negative constant, $p > 1$, $q > 1$, furthermore p and q need to satisfy $(p-1)(q-1) = 1$.

A. RBF Neural Network

The RBF neural networks are used to approach the continuous functions $F(Z): R^q \rightarrow R$

$$F(Z) = W^{*T} S(Z) \tag{10}$$

where $Z \in \Omega_Z \subset R^q$ is the input vector and q denotes the RBF NNs input dimension, the desired weight vector is expressed by $W^* = [W_1^*, W_2^* \ldots W_l^*]^T \in R^l$, where $l > 1$ is the NNs node number, and $S(Z) =$

$[S_1(Z), S_2(Z) \ldots S_l(Z)]^T$ is the basis function vector of Gaussian function. Generally, $S_i(Z)$ can be expressed as

$$S_i(Z) = \exp\left[-\frac{(Z-p_i)^T (Z-p_i)}{G_i^2}\right], i = 1, 2 \ldots j \quad (11)$$

where $p_i = [p_{i1}, p_{i2} \ldots p_{iq}]^T$ represents the center of the receptive field, and the width of Gaussian function is expressed by G_i. For given accuracy $\delta > 0$, the unknown function $F(Z)$ can be approached as follows

$$F(Z) = W^{*T} S(Z) + \delta(Z) \quad (12)$$

where the weight vector W^* is

$$W^* := \arg\min_{W \in R^j} \left\{ \sup_{Z \in \Omega_Z} \left| F(Z) - W^T S(Z) \right| \right\} \quad (13)$$

and $\delta(Z)$ represents the approximation error and $\delta(Z) < \bar{\delta}$, where $\bar{\delta}$ is a positive constant.

B. Prescribed Performance

The system state errors can be expressed by the following form:

$$\begin{cases} z_1 = x_1 - y_r \\ z_i = x_i - \alpha_{i-1} \end{cases} \quad (14)$$

where y_r represents the desired signal, $\alpha_i \ (i = 2, 3 \ldots n)$ is the virtual control law.

In order to improve the performance of system and ensure bounded tracking error z_1, the finite-time performance function is defined as

$$\rho(t) = \begin{cases} \left(\rho_0 - \frac{t}{T_s}\right) e^{\frac{lt}{t-T_s}} + \rho_\infty & t \in [0, T_s) \\ \rho_\infty & t \in [T_s, +\infty) \end{cases} \quad (15)$$

where ρ_0 is the initial value of $\rho(t)$, ρ_∞ is the maximum allowable range of z_1 at the steady state, T_s is the settling time, l is convergence speed and $\rho_0, \rho_\infty, , T_s, l$ are positive parameters.

To achieve the output tracking error can guarantee bounded and constrained the transient, the error transformation is selected as

$$\zeta = tan\left(\frac{\pi z_1}{2\rho}\right) \quad (16)$$

Further, the time-derivative of ζ is given by

$$\dot{\zeta} = H\left(f_1 + g_1 x_2 + \Delta_1 - \dot{y}_r - \frac{2}{\pi}\dot{\rho}\arctan z_1\right) \quad (17)$$

where $H = \pi\left(1 + \zeta^2(t)\right)/2\rho$.

III. Controller Design

Now, an n-order adaptive back-stepping controller is designed. Here's the design process.

STEP 1: Consider the system (1) when $i = 1$ and noting

$$\dot{\xi} = p(\xi, x) \quad (18)$$

$$\dot{\zeta} = H\left(f_1 + g_1 x_2 + \Delta_1 - \dot{y}_r - \frac{2}{\pi}\dot{\rho}\arctan z_1\right) \quad (19)$$

Consider a Lyapunov function as

$$V_1 = \frac{1}{2}\zeta^2 + \frac{1}{\lambda_0} r + \frac{b_m}{2\gamma_1}\tilde{\theta}_1^2 \quad (20)$$

where $\tilde{\theta}_i = \theta_i - \hat{\theta}_i, i = 1, 2, ..., n$ represent the estimation error, and λ_0, γ_1 are the design positive parameters.

According to Assumption 3 and equation (7), the time-derivative of V_1 as

$$\begin{aligned} \dot{V}_1 \leq & \zeta H\left(f_1 + g_1 x_2 + \Delta_1 - \dot{y}_r - \frac{2}{\pi}\dot{\rho}\arctan z_1\right) \\ & + |\zeta H| \varphi_{11}(x_1) + |\zeta H| \varphi_{12}(\xi) - \frac{\Lambda}{\lambda_0} r \\ & - \frac{b_m}{\gamma_1}\tilde{\theta}_1\dot{\hat{\theta}}_1 + \frac{1}{\lambda_0}\left(x_1^2 \eta_0\left(x_1^2\right) + d_0\right) \end{aligned} \quad (21)$$

Then, handling $|\zeta H| \varphi_{11}(x_1)$ and $|\zeta H| \varphi_{12}(\xi)$. Based on Lemma 2, one has

$$\begin{aligned} |\zeta H| \varphi_{11}(|x_1|) \leq & \zeta H \varphi_{11}(|x_1|) \tanh\left(\frac{\zeta H \varphi_{11}(|x_1|)}{\tau_{11}}\right) \\ & + 0.2785\tau_{11} \\ \leq & \zeta H \varphi_{11}^* + \tau_{11}^* \end{aligned} \quad (22)$$

where $\varphi_{11}^* = \varphi_{11}(|x_1|) \tanh(\zeta \varphi_{11}(|x_1|)/\tau_{11}), \tau_{11}^* = 0.2785\tau_{11}$.

Due to ν_1 is K_∞ function, and ν_1^{-1} is increasing function, one has

$$|\xi| \leq \nu_1^{-1}(r(t) + D(t)) \quad (23)$$

Based on Assumption 3 and φ_{12} is a non-negative incremental function, one gets

$$\varphi_{12}(|\xi|) \leq \varphi_{12}\left[\nu_1^{-1}(r(t) + D(t))\right] \quad (24)$$

Let $\varphi_{12} \circ \nu_1^{-1}(r(t) + D(t)) = \varphi_{12}\left[\nu_1^{-1}(r(t) + D(t))\right]$, one obtains

$$\varphi_{12}(|\xi|) \leq \varphi_{12} \circ \nu_1^{-1}(r(t) + D(t)) \quad (25)$$

For any t, one has

$$\min\{2r(t), 2D(t)\} \leq r(t) + D(t) \leq \max\{2r(t), 2D(t)\} \quad (26)$$

By equation (27) and the Young's inequality, it yields

$$\begin{aligned} |\zeta H| \varphi_{12}(\xi) = & |\zeta H| \varphi_{12} \circ \nu_1^{-1}(r(t) + D(t)) \\ \leq & |\zeta H| \varphi_{12} \circ \nu_1^{-1}(2r(t)) \\ & + |\zeta H| \varphi_{12} \circ \nu_1^{-1}(2D(t)) \end{aligned} \quad (27)$$

By equation (27) and Young's inequality, one has

$$\begin{aligned} |\zeta H| \varphi_{12}(\xi) \leq & \zeta H \bar{\varphi}_{12} + \frac{1}{4}\zeta^2 H^2 + d_1(t) \\ \leq & \zeta H \bar{\varphi}_{12} \tanh\left(\frac{\zeta H \bar{\varphi}_{12}}{\tau_{12}}\right) + 0.2785\tau_{12} \\ & + \frac{1}{4}\zeta^2 H^2 + d_1(t) \\ \leq & \zeta H \varphi_{12}^* + \tau_{12}^* + \frac{1}{4}\zeta^2 H^2 + d_1(t) \end{aligned} \quad (28)$$

where $d_1(t) = \left(\varphi_{12} \circ \nu_1^{-1}(2D(t))\right)^2$, $\tau_{12}^* = 0.2785\tau_{12}$, and $\varphi_{12}^* = \bar{\varphi}_{12} \tanh(|\zeta| \bar{\varphi}_{12}/\tau_{12})$, $\bar{\varphi}_{12} = \varphi_{12} \circ \nu_1^{-1}(2r(t))$.

Substituting (22) and (28) into (21), it produces

$$\begin{aligned} \dot{V}_1 \leq & \zeta\left(H f_1 + H g_1 x_2 - H\dot{y}_r - H\frac{2}{\pi}\dot{\rho}\arctan z_1\right) \\ & + \zeta H \varphi_{11}^* + \zeta H \varphi_{12}^* + \zeta\frac{x_1^2 \eta_0\left(x_1^2\right)}{\lambda_0 \zeta} + \frac{1}{4}\zeta^2 H^2 \\ & + \tau_{11}^* + \tau_{12}^* + d_1(t) + \frac{d_0}{\lambda_0} - \frac{\Lambda}{\lambda_0} r - \frac{b_m}{\gamma_1}\tilde{\theta}_1\dot{\hat{\theta}}_1 \end{aligned} \quad (29)$$

when $\zeta = 0$, $x_1^2\eta_0(x_1^2)/(\lambda_0\zeta)$ is discontinuous function, and can not be approximated by RBF NNs. Therefore the hyperbolic tangent function $\tanh(\zeta/\ell)$ is used to solve this problem. Then, (29) can be written as

$$\begin{aligned}\dot{V}_1 \leq& \zeta H g_1 z_2 + \zeta H g_1 \alpha_1 + \zeta F_1(Z_1) - \frac{1}{2}\zeta^2 + \tau_{11}^* \\ &+ \tau_{12}^* + d_1(t) + \frac{d_0}{\lambda_0} - \frac{\Lambda}{\lambda_0}r - \frac{b_m}{\gamma_1}\tilde{\theta}_1\dot{\hat{\theta}}_1 + \Upsilon\end{aligned} \tag{30}$$

where $\Upsilon = \left(1 - 2\tanh^2\left(\frac{\zeta}{\ell}\right)\right)\frac{x_1^2\eta_0(x_1^2)}{\lambda_0}$, and $F_1(Z_1)$ is expressed by the following form

$$\begin{aligned}F_1(Z_1) =& H\left(f_1 - \dot{y}_r - \frac{2}{\pi}\dot{\rho}\arctan z_1 + \varphi_{11}^* + \varphi_{12}^*\right) \\ &+ \frac{1}{2}\zeta + \frac{1}{4}\zeta H^2 + \frac{2}{\zeta}\tanh^2\left(\frac{\zeta}{\ell}\right)\frac{x_1^2\eta_0(x_1^2)}{\lambda_0}\end{aligned} \tag{31}$$

Due to $F_1(Z_1)$ is unknown nonlinear smooth and continuous function, Therefore, $F_1(Z_1)$ can be approached through RBFNN $W_1^{*T}S_1(Z_1)$, one has

$$F_1(Z_1) = W_1^{*T}S_1(Z_1) + \delta_1(Z_1), |\delta_1(Z_1)| \leq \bar{\delta}_1 \tag{32}$$

According to the Young's inequality, it produces

$$\begin{aligned}\zeta F_1(Z_1) &= \zeta\left(W_1^{*T}S_1(Z_1) + \delta_1(Z_1)\right) \\ &\leq \zeta\left(\|W_1^*\| \|\|S_1(Z_1)\| + \bar{\delta}_1\right) \\ &\leq \frac{b_m}{2a_1^2}\zeta^2\theta_1 S_1^T S_1 + \frac{a_1^2}{2} + \frac{\bar{\delta}_1^2}{2} + \frac{\zeta^2}{2}\end{aligned} \tag{33}$$

where $\theta_1 = \|W_1^*\|^2/b_m$ is unknown constant.

Setting a virtual control law α_1 as

$$\alpha_1 = -\frac{\zeta}{H}\left(c_1 + \frac{1}{2a_1^2}\hat{\theta}_1 S_1^T S_1\right) \tag{34}$$

where $c_1 > 0$ is design parameter, and $\dot{\hat{\theta}}_1$ is adaptive updating law as

$$\dot{\hat{\theta}}_1 = \frac{\gamma_1}{2a_1^2}\zeta^2 S_1^T S_1 - \mu_1\hat{\theta}_1 \tag{35}$$

where γ_1, μ_1 are design positive parameters.

Substituting (33)-(35) into (30), it yields

$$\begin{aligned}\dot{V}_1 \leq& \zeta H g_1 z_2 - c_1 b_m\zeta^2 + \tau_{11}^* + \tau_{12}^* + d_1(t) + \frac{a_1^2}{2} \\ &+ \frac{\bar{\delta}_1^2}{2} + \frac{d_0}{\lambda_0} - \frac{\Lambda}{\lambda_0}r + \frac{\mu_1 b_m}{\gamma_1}\tilde{\theta}_1\hat{\theta}_1 + \Upsilon\end{aligned} \tag{36}$$

Applying the Young's inequality gives

$$\begin{aligned}\frac{\mu_1 b_m}{\gamma_1}\tilde{\theta}_1\hat{\theta}_1 &\leq \frac{\mu_1 b_m}{\gamma_1}\tilde{\theta}_1\left(\theta_1 - \tilde{\theta}_1\right) \\ &\leq -\frac{\mu_1 b_m}{2\gamma_1}\tilde{\theta}_1^2 + \frac{\mu_1 b_m}{2\gamma_1}\theta_1^2\end{aligned} \tag{37}$$

Finally, the expression for $\dot{V}_1$ is as follows

$$\begin{aligned}\dot{V}_1 \leq& \zeta H g_1 z_2 - k_1\zeta^2 - \frac{\mu_1 b_m}{2\gamma_1}\tilde{\theta}_1^2 + B_1 \\ &+ \frac{d_0}{\lambda_0} - \frac{\Lambda}{\lambda_0}r + \Upsilon\end{aligned} \tag{38}$$

where $k_1 = c_1 b_m, B_1 = \tau_{11}^* + \tau_{12}^* + d_1(t) + \frac{a_1^2}{2} + \frac{\bar{\delta}_1^2}{2} + \frac{\mu_1 b_m}{2\gamma_1}\theta_1^2$.

STEP 2 : $z_2 = x_2 - \alpha_1$, so $\dot{z}_2$ can be obtained

$$\begin{aligned}\dot{z}_2 &= \dot{x}_2 - \dot{\alpha}_1 \\ &= f_2 + g_2 x_3 + \Delta_2 - \frac{\partial\alpha_1}{\partial x_1}\Delta_1 - \Gamma_1\end{aligned} \tag{39}$$

where $\Gamma_1 = \frac{\partial\alpha_1}{\partial x_1}(f_1 + g_1 x_2) + \frac{\partial\alpha_1}{\partial\rho}\dot{\rho} + \frac{\partial\alpha_1}{\partial\hat{\theta}_1}\dot{\hat{\theta}}_1 + \frac{\partial\alpha_1}{\partial r}\dot{r} + \sum_{j=0}^{1}\frac{\partial\alpha_1}{\partial y_r^{(j)}}y_r^{(j+1)}$.

Consider a Lyapunov function as

$$V_2 = V_1 + \frac{1}{2}z_2^2 + \frac{b_m}{2\gamma_2}\tilde{\theta}_2^2 \tag{40}$$

The time-derivative of V_2 is as follows

$$\begin{aligned}\dot{V}_2 \leq& \zeta H g_1 z_2 - k_1\zeta^2 - \frac{\mu_1 b_m}{2\gamma_1}\tilde{\theta}_1^2 + B_1 + \frac{d_0}{\lambda_0} - \frac{\Lambda}{\lambda_0}r \\ &+ z_2(f_2 + g_2 x_3 + \Delta_2^* - \Gamma_1) - \frac{b_m}{\gamma_2}\tilde{\theta}_2\dot{\hat{\theta}}_2 + \Upsilon\end{aligned} \tag{41}$$

where $\Delta_2^* = \Delta_2 - (\partial\alpha_1/\partial x_1)\Delta_1$, $\Gamma_1 = \frac{\partial\alpha_1}{\partial x_1}(f_1 x_2 + g_1) - \frac{\partial\alpha_1}{\partial\rho}\dot{\rho} - \frac{\partial\alpha_1}{\partial\hat{\theta}_1}\dot{\hat{\theta}}_1 + \sum_{j=0}^{1}\frac{\partial\alpha_1}{\partial y_r^{(j)}}y_r^{(j+1)} + \frac{\partial\alpha_1}{\partial r}\dot{r}$.

Based on Assumption 3, it can obtain

$$\begin{aligned}|z_2\Delta_2^*| &\leq \left|z_2\left(|\Delta_2| + \left|\frac{\partial\alpha_1}{\partial x_1}\right||\Delta_1|\right)\right| \\ &\leq |z_2|\left(\varphi_{21}(|\bar{x}_2|) + \left|\frac{\partial\alpha_1}{\partial x_1}\right|\varphi_{11}(|x_2|)\right) \\ &\quad + |z_2|\left(\varphi_{22}(|\xi|) + \left|\frac{\partial\alpha_1}{\partial x_1}\right|\varphi_{12}(|\xi|)\right)\end{aligned} \tag{42}$$

According to Lemma 2, it can get

$$\begin{aligned}&|z_2|\left(\varphi_{21}(|\bar{x}_2|) + \left|\frac{\partial\alpha_1}{\partial x_1}\right|\varphi_{11}(|x_2|)\right) \\ &\leq z_2\varphi_{21}^*(x_2) + \tau_{21}^*\end{aligned} \tag{43}$$

$$\begin{aligned}&|z_2|\left(\varphi_{22}(|\xi|) + \left|\frac{\partial\alpha_1}{\partial x_1}\right|\varphi_{12}(|\xi|)\right) \\ &\leq z_2\varphi_{22}^* + \tau_{22}^* + d_2(t) + \frac{z_2^2}{4}\left[1 + \left|\frac{\partial\alpha_1}{\partial x_1}\right|^2\right]\end{aligned} \tag{44}$$

where $d_2(t) = \left(\varphi_{22}\circ\nu_1^{-1}(2D(t))\right)^2$, $\tau_{21}^* = 0.2785\tau_{21}$ and $\tau_{22}^* = 0.2785\tau_{22}$, $\varphi_{21}^* = (\varphi_{21} + |\partial\alpha_1/\partial x_1|\varphi_{21}) \times \tanh(z_2(\varphi_{11} + |\partial\alpha_1/\partial x_1|\varphi_{11})/\tau_{21})$, $\varphi_{22}^* = \bar{\varphi}_{22}\tanh(z_2\bar{\varphi}_{22}/\tau_{22})$, $\bar{\varphi}_{22} = |\partial\alpha_1/\partial x_1|\varphi_{12}\circ\nu_1^{-1}(2r) + \varphi_{22}\circ\nu_1^{-1}(2r)$.

Substituting (43) and (44) into (41) gets

$$\begin{aligned}\dot{V}_2 \leq& -k_1\zeta^2 - \frac{\mu_1 b_m}{2\gamma_1}\tilde{\theta}_1^2 + B_1 + z_2(g_2 x_3 + F_2(Z_2)) \\ &- \frac{1}{2}z_2^2 + \tau_{21}^* + \tau_{22}^* + d_2(t) - \frac{\Lambda}{\lambda_0}r + \frac{d_0}{\lambda_0} \\ &- \frac{b_m}{\gamma_2}\tilde{\theta}_2\dot{\hat{\theta}}_2 + \Upsilon\end{aligned} \tag{45}$$

where $F_2(Z_2) = \frac{1}{2}z_2 + f_2 + \zeta H g_1 - \Gamma_1 + \varphi_{21}^* + \varphi_{22}^* + \frac{z_2}{4}\left[1 + \left|\frac{\partial\alpha_1}{\partial x_1}\right|^2\right]$.

Similarly, the unknown function $F_2(Z_2)$ can be approximated by the RBF neural network $W_2^{*T}S_2(Z_2)$, it can get

$$F_2(Z_2) = W_2^{*T}S_2(Z_2) + \delta_2(Z_2), |\delta_2(Z_2)| \leq \bar{\delta}_2 \tag{46}$$

According to Young's inequality, one obtains

$$\begin{aligned}z_2 F_2(Z_2) =& z_2\left(W_2^{*T}S_2(Z_2) + \delta_2(Z_2)\right) \\ \leq& z_2\left(\|W_2^*\|\|S_2(Z_2)\| + \bar{\delta}_2\right) \\ \leq& \frac{b_m}{2a_2^2}z_2^2\theta_2 S_2^T S_2 + \frac{1}{2}a_2^2 + \frac{1}{2}z_2^2 + \frac{1}{2}\bar{\delta}_2^2\end{aligned} \tag{47}$$

where $\theta_2 = \|W_2^*\|/b_m$ is unknown constant.

Setting the virtual control law α_2 as

$$\alpha_2 = -c_2 z_2 - \frac{1}{2a_2^2} z_2 \hat{\theta}_2 S_2^T S_2 \tag{48}$$

where $c_2 > 0$ is design parameter, and the adaptive updating law $\dot{\hat{\theta}}_i$ is as follows

$$\dot{\hat{\theta}}_2 = \frac{\gamma_2}{2a_2^2} z_2^2 S_2^T S_2 - \mu_2 \hat{\theta}_2 \tag{49}$$

where γ_2, μ_2 are positive constants.

Bring (47)-(49) into (45), it yields

$$\begin{aligned}\dot{V}_2 \leq & -k_1\zeta^2 - c_2 b_m z_2^2 - \frac{\mu_1 b_m}{2\gamma_1}\tilde{\theta}_1^2 + B_1 + z_2 g_2 z_3 \\ & + \frac{1}{2}a_2^2 + \frac{1}{2}\bar{\delta}_2^2 + \tau_{21}^* + \tau_{22}^* + d_2(t) + \frac{d_0}{\lambda_0} \\ & - \frac{\Lambda}{\lambda_0} r + \frac{\mu_2 b_m}{\gamma_2}\tilde{\theta}_2\hat{\theta}_2 + \Upsilon\end{aligned} \tag{50}$$

Similar to Step 1, according to the Young's inequality, it yields

$$\begin{aligned}\frac{\mu_2 b_m}{\gamma_2}\tilde{\theta}_2\hat{\theta}_2 \leq & \frac{\mu_2 b_m}{\gamma_2}\tilde{\theta}_2\left(\theta_2 - \tilde{\theta}_2\right) \\ & \leq -\frac{\mu_2 b_m}{2\gamma_2}\tilde{\theta}_2^2 + \frac{\mu_2 b_m}{2\gamma_2}\theta_2^2\end{aligned} \tag{51}$$

Finally, the time-derivative of V_2 is expressed by

$$\begin{aligned}\dot{V}_2 \leq & -k_1\zeta^2 - k_2 z_2^2 - \sum_{j=1}^{2}\frac{\mu_j b_m}{2\gamma_j}\tilde{\theta}_j^2 + \sum_{j=1}^{2} B_j \\ & + z_2 g_2 z_3 + \frac{d_0}{\lambda_0} - \frac{\Lambda}{\lambda_0} r + \Upsilon\end{aligned} \tag{52}$$

where $k_2 = c_2 b_m$, $B_2 = \tau_{21}^* + \tau_{22}^* + d_2(t) + \frac{a_2^2}{2} + \frac{\bar{\delta}_2^2}{2} + \frac{\mu_2 b_m}{2\gamma_2}\theta_2^2$.

STEP i ($3 \leq i \leq n-1$) : The derivative of $z_i = x_i - \alpha_{i-1}$ is expressed by the following form

$$\begin{aligned}\dot{z}_i &= \dot{x}_i - \dot{\alpha}_{i-1} \\ &= f_i + g_i x_{i+1} + \Delta_i - \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}\Delta_j + \Gamma_{i-1}\end{aligned} \tag{53}$$

where $\Gamma_{i-1} = \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}\left(f_j + g_j x_{j+1}\right) + \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial y_r^{(j)}} y_r^{(j+1)} + \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\hat{\theta}_j}\dot{\hat{\theta}}_j + \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\rho^{(j)}}\rho^{(j+1)} + \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial r}\dot{r}$.

Select a Lyapunov function as

$$V_i = V_{i-1} + \frac{1}{2}z_i^2 + \frac{b_m}{2\gamma_i}\tilde{\theta}_i^2 \tag{54}$$

The time-derivative of V_i is as follows

$$\begin{aligned}\dot{V}_i \leq & -k_1\zeta^2 - \sum_{j=2}^{i-1} k_j z_j^2 - \sum_{j=1}^{i-1}\frac{\mu_j b_m}{2\gamma_j}\tilde{\theta}_j^2 + \sum_{j=1}^{i-1} B_j \\ & + z_i\left(f_i + g_i x_{i+1} + z_{i-1}g_{i-1} + \Delta_i^* - \Gamma_{i-1}\right) \\ & + \frac{d_0}{\lambda_0} - \frac{\Lambda}{\lambda_0} r - \frac{b_m}{\gamma_i}\tilde{\theta}_i\dot{\hat{\theta}}_i + \Upsilon\end{aligned} \tag{55}$$

where $\Delta_i^* = \Delta_i - \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}\Delta_j$, and $k_j = c_j b_m, B_j = \tau_{n1}^* + \tau_{i2}^* + d_j(t) + \frac{a_j^2}{2} + \frac{\delta_j^2}{2} + \frac{\mu_j b_m}{2\gamma_j}\theta_j^2, j = 1, 2, \ldots, i-1$.

By Assumption 3, one has

$$\begin{aligned}|z_i\Delta_i^*| \leq & |z_i|\left(\varphi_{i1}\left(|\bar{x}_i|\right) + \sum_{j=1}^{i-1}\left|\frac{\partial\alpha_{i-1}}{\partial x_j}\right|\varphi_{j1}\left(|x_j|\right)\right) \\ & + |z_i|\left(\varphi_{i2}\left(|\xi|\right) + \sum_{j=1}^{i-1}\left|\frac{\partial\alpha_{i-1}}{\partial x_j}\right|\varphi_{j2}\left(|\xi|\right)\right)\end{aligned} \tag{56}$$

According to Lemma 2, one obtains

$$\begin{aligned}|z_i|\left(\varphi_{i1}\left(|\bar{x}_i|\right) + \sum_{j=1}^{i-1}\left|\frac{\partial\alpha_{i-1}}{\partial x_j}\right|\varphi_{j1}\left(|x_j|\right)\right) \\ \leq z_i\varphi_{i1}^* + \tau_{i1}^*\end{aligned} \tag{57}$$

$$\begin{aligned}&|z_i|\left(\varphi_{i2}(|\xi|) + \sum_{j=1}^{i-1}\left|\frac{\partial\alpha_{i-1}}{\partial x_j}\right|\varphi_{j2}(|\xi|)\right) \\ &\leq z_i\varphi_{i2}^* + \tau_{i2}^* + \frac{z_i^2}{4}\left[1 + \sum_{j=1}^{i-1}\left(\frac{\partial\alpha_{i-1}}{\partial x_j}\right)^2\right] + d_i(t)\end{aligned} \tag{58}$$

where the expression functions of φ_{i1}^*, φ_{i2}^* are as follows

$$\begin{aligned}\varphi_{i1}^* = & \left(\varphi_{i1} + \sum_{j=1}^{i-1}\left|\frac{\partial\alpha_{i-1}}{\partial x_j}\right|\varphi_{j1}\right) \\ & \times\tanh\left(z_i\left(\varphi_{i1} + \sum_{j=1}^{i-1}\left|\frac{\partial\alpha_{i-1}}{\partial x_j}\right|\varphi_{j1}\right)/\tau_{i1}\right)\end{aligned} \tag{59}$$

$$\varphi_{i2}^* = \left(\bar{\varphi}_{i2}\right)\times\tanh\left(\frac{z_i\bar{\varphi}_{i2}}{\tau_{i1}}\right) \tag{60}$$

where $d_i(t) = \sum_{j=1}^{i}\left[\varphi_{j2}\circ\nu_1^{-1}\left(2D\left(t\right)\right)\right]^2$, $\tau_{i1}^* = 0.2785\tau_{i1}$, $\tau_{i2}^* = 0.2785\tau_{i2}$, $\bar{\varphi}_{i2} = \sum_{j=1}^{i-1}\left|\partial\alpha_{i-1}/\partial x_j\right|\varphi_{j2}\circ\nu_1^{-1}\left(2r\right) + \varphi_{i2}\circ\nu_1^{-1}\left(2r\right)$. .

Substituting (59) and (60) into (55), it yields

$$\begin{aligned}\dot{V}_i = & -k_1\zeta^2 - \sum_{j=2}^{i-1} k_j z_j^2 - \sum_{j=1}^{i-1}\frac{\mu_j b_m}{\gamma_j}\tilde{\theta}_i^2 - \frac{\Lambda}{\lambda_0} r \\ & + z_i\left(g_i x_{i+1} + F_i\left(Z_i\right)\right) - \frac{1}{2}z_i^2 + \tau_{i1}^* + \tau_{i2}^* \\ & + d_i(t) + \frac{d_0}{\lambda_0} - \frac{\Lambda}{\lambda_0} r - \frac{b_m}{\gamma_i}\tilde{\theta}_i\dot{\hat{\theta}}_i + \Upsilon\end{aligned} \tag{61}$$

where $F_i\left(z_i\right) = \frac{1}{2}z_i + f_i + z_{i-1}g_{i-1} - \Gamma_{i-1} + \varphi_{i1}^* + \varphi_{i2}^* + \frac{z_i}{4}\left[1 + \sum_{j=1}^{i-1}\left(\frac{\partial\alpha_{i-1}}{\partial x_i}\right)^2\right]$.

Similar to Step 1, the unknown nonlinear function $F_i\left(Z_i\right)$ can be approximated by the RBF neural network $W_i^* S_i\left(Z_i\right)$, it can get

$$F_i\left(Z_i\right) = W_i^* S_i\left(Z_i\right) + \delta_i\left(Z_i\right), \left|\delta_i\left(Z_i\right)\right| \leq \bar{\delta}_i \tag{62}$$

According to Young's inequality, one has

$$\begin{aligned}z_i F_i\left(Z_i\right) = & z_i\left(W_i^{*T} S_i\left(Z_i\right) + \delta_i\left(Z_i\right)\right) \\ \leq & |z_i|\left(\|W_i^*\|\left\|S_i\left(Z_i\right)\right\| + \bar{\delta}_i\right) \\ \leq & \frac{b_m}{2a_i^2} z_i^2\theta_i S_i^T S_i + \frac{1}{2}a_i^2 + \frac{1}{2}z_i^2 + \frac{1}{2}\bar{\delta}_i^2\end{aligned} \tag{63}$$

where $\theta_i = \|W_i^*\|^2 / b_m$ is unknown constant.

Setting the virtual control law α_i as

$$\alpha_i = -c_i z_i - \frac{1}{2a_i^2} z_i \hat{\theta}_i S_i^T S_i \tag{64}$$

where $c_i > 0$ is design parameter, and the adaptive updating law $\dot{\hat{\theta}}_i$ as

$$\dot{\hat{\theta}}_i = \frac{\gamma_i}{2a_i^2} z_i^2 S_i^T S_i - \mu_i \hat{\theta}_i \tag{65}$$

where γ_i, μ_i are design positive parameters.

Substituting (63) - (65) into (61), it yields

$$\begin{aligned}\dot{V}_i =& -k_1\zeta^2 - \sum_{j=2}^{i-1} k_j z_j^2 - \sum_{j=1}^{i-1} \frac{\mu_j b_m}{\gamma_j} \tilde{\theta}_j^2 + \sum_{j=1}^{i-1} B_j \\ &+ z_i g_i z_{i+1} - c_i b_m z_i^2 + \tau_{i1}^* + \tau_{i2}^* + d_i(t) \\ &+ \frac{1}{2} a_i^2 + \frac{1}{2} \bar{\delta}_i^2 + \frac{d_0}{\lambda_0} - \frac{\Lambda}{\lambda_0} r + \frac{\mu_i b_m}{\gamma_i} \tilde{\theta}_i \hat{\theta}_i + \Upsilon\end{aligned} \tag{66}$$

According to the Young's inequality, it produces

$$\begin{aligned}\frac{\mu_i b_m}{\gamma_i} \tilde{\theta}_i \hat{\theta}_i \leq& \frac{\mu_i b_m}{\gamma_i} \tilde{\theta}_i \left(\theta_i - \tilde{\theta}_i\right) \\ \leq& -\frac{\mu_i b_m}{2\gamma_i} \tilde{\theta}_i^2 + \frac{\mu_i b_m}{2\gamma_i} \theta_i^2\end{aligned} \tag{67}$$

Finally, $\dot{V}_i$ is written as follows

$$\begin{aligned}\dot{V}_i =& -k_1\zeta^2 - \sum_{j=2}^{i} k_j z_j^2 - \sum_{j=1}^{i} \frac{\mu_j b_m}{2\gamma_j} \tilde{\theta}_i^2 + \sum_{j=1}^{i} B_j \\ &+ z_i g_i z_{i+1} + \frac{d_0}{\lambda_0} - \frac{\Lambda}{\lambda_0} r + \Upsilon\end{aligned} \tag{68}$$

where $k_i = c_i b_m$, $B_i = \tau_{i1}^* + \tau_{i2}^* + d_i(t) + \frac{a_i^2}{2} + \frac{\bar{\delta}_i^2}{2} + \frac{\mu_i b_m}{2\gamma_i} \theta_i^2$.

STEP n : From $z_n = x_n - \dot{\alpha}_{n-1}$, the derivative of z_n is as follows

$$\begin{aligned}\dot{z}_n &= \dot{x}_n - \dot{\alpha}_{n-1} \\ &= f_n + g_n u + \Delta_n - \sum_{j=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_j} \Delta_j - \Gamma_{n-1}\end{aligned} \tag{69}$$

where $\Gamma_{n-1} = \sum_{j=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial y_r^{(j)}} y_r^{(j+1)} + \sum_{j=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial \hat{\theta}_j} \dot{\hat{\theta}}_j + \sum_{j=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_j} (f_j + g_j x_{j+1}) + \sum_{j=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial \rho^{(j)}} \rho^{(j+1)} + \sum_{j=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial r} \dot{r}$.

Then, choose a Lyapunov function as follows

$$V_n = V_{n-1} + \frac{1}{2} z_n^2 + \frac{b_m}{2\gamma_n} \tilde{\theta}_n^2 \tag{70}$$

The time-derivative of V_n is expressed by

$$\begin{aligned}\dot{V}_n \leq& -k_1\zeta^2 - \sum_{j=2}^{n-1} k_j z_j^2 - \sum_{j=1}^{n-1} \frac{\mu_j b_m}{2\gamma_j} \tilde{\theta}_j^2 + \sum_{j=1}^{n-1} B_j \\ &+ z_n (f_n + g_n u + z_{n-1} g_{n-1} + \Delta_n^* - \Gamma_{n-1}) \\ &+ \frac{d_0}{\lambda_0} - \frac{\Lambda}{\lambda_0} r - \frac{b_m}{\gamma_n} \tilde{\theta}_n \dot{\hat{\theta}}_n + \Upsilon\end{aligned} \tag{71}$$

where $\Delta_n^* = \Delta_n - \sum_{j=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_j} \Delta_j$.

By Assumption 3, one obtains

$$\begin{aligned}|z_n \Delta_n^*| \leq& |z_n| \left(\varphi_{n1}(|\bar{x}_n|) + \sum_{j=1}^{n-1} \left| \frac{\partial \alpha_{n-1}}{\partial x_j} \right| \varphi_{j1}(|x_j|) \right) \\ &+ |z_n| \left(\varphi_{n2}(|\xi|) + \sum_{j=1}^{n-1} \left| \frac{\partial \alpha_{n-1}}{\partial x_j} \right| \varphi_{j2}(|\xi|) \right)\end{aligned} \tag{72}$$

According to Lemma 2, it yields

$$\begin{aligned}|z_n| \left(\varphi_{n1}(|\bar{x}_n|) + \sum_{j=1}^{n-1} \left| \frac{\partial \alpha_{n-1}}{\partial x_j} \right| \varphi_{j1}(|x_j|) \right) \\ \leq z_n \varphi_{n1}^* + \tau_{n1}^*\end{aligned} \tag{73}$$

$$\begin{aligned}&|z_n| \left(\varphi_{n2}(|\xi|) + \sum_{j=1}^{n-1} \left| \frac{\partial \alpha_{n-1}}{\partial x_j} \right| \varphi_{j2}(|\xi|) \right) \\ &\leq z_i \varphi_{n2}^* + \tau_{n2}^* + \frac{z_n^2}{4} \left[1 + \sum_{j=1}^{i-1} \left(\frac{\partial \alpha_{n-1}}{\partial x_j} \right)^2 \right] + d_n(t)\end{aligned} \tag{74}$$

where the functions of φ_{n1}^* and φ_{n2}^* are expressed by

$$\begin{aligned}\varphi_{n1}^* =& \left(\varphi_{n1} + \sum_{j=1}^{n-1} \left| \frac{\partial \alpha_{n-1}}{\partial x_j} \right| \varphi_{j1} \right) \\ &\times \tanh \left(z_n \left(\varphi_{n1} + \sum_{j=1}^{n-1} \left| \frac{\partial \alpha_{n-1}}{\partial x_j} \right| \varphi_{j1} \right) / \tau_{n1} \right)\end{aligned} \tag{75}$$

$$\varphi_{n2}^* = (\bar{\varphi}_{n2}) \times \tanh \left(\frac{z_n \bar{\varphi}_{n2}}{\tau_{i1}} \right) \tag{76}$$

where $d_n(t) = \sum_{j=1}^{n} \left[\varphi_{j2} \circ \nu_1^{-1}(2D(t)) \right]^2$, $\tau_{n1}^* = 0.2785\tau_{n1}$, $\bar{\varphi}_{n2} = \varphi_{n2} \circ \nu_1^{-1}(2r) + \sum_{j=1}^{n-1} |\partial \alpha_{n-1} / \partial x_j| \varphi_{j2} \circ \nu_1^{-1}(2r)$, $\tau_{n2}^* = 0.2785\tau_2$.

Substituting (73) and (74) into (71), it yields

$$\begin{aligned}\dot{V}_n =& -k_1\zeta^2 - \sum_{j=2}^{n-1} k_j z_j^2 - \sum_{j=1}^{n-1} \frac{\mu_j b_m}{\gamma_j} \tilde{\theta}_n^2 - \frac{\Lambda}{\lambda_0} r \\ &+ z_n (g_n u + F_n(Z_n)) - \frac{1}{2} z_n^2 + \tau_{n1}^* + \tau_{n2}^* \\ &+ d_n(t) + \frac{d_0}{\lambda_0} - \frac{\Lambda}{\lambda_0} r - \frac{b_m}{\gamma_n} \tilde{\theta}_n \dot{\hat{\theta}}_n + \Upsilon\end{aligned} \tag{77}$$

where $F_n(z_n) = \frac{1}{2} z_n + f_n + z_{n-1} g_{n-1} - \Gamma_{n-1} + \varphi_{n1}^* + \varphi_{n2}^* + \frac{z_n}{4} \left[1 + \sum_{j=1}^{n-1} \left(\frac{\partial \alpha_{n-1}}{\partial x_n} \right)^2 \right]$.

Similarly, the unknown nonlinear function $F_n(Z_n)$ can be approximated by the RBF neural network $W_n^* S_n(Z_n)$, it can get

$$F_n(Z_n) = W_n^* S_n(Z_n) + \delta_n(Z_n), |\delta_n(Z_n)| \leq \bar{\delta}_n \tag{78}$$

Applying Young's inequality again to obtain

$$\begin{aligned}z_n F_n(Z_n) =& z_n \left(W_n^{*T} S_n(Z_n) + \delta_n(Z_n) \right) \\ \leq& z_n \left(\|W_n^*\| \|S_n(Z_n)\| + \bar{\delta}_n \right) \\ \leq& \frac{b_m}{2a_n^2} z_n^2 \theta_n S_n^T S_n + \frac{1}{2} a_n^2 + \frac{1}{2} z_n^2 + \frac{1}{2} \bar{\delta}_n^2\end{aligned} \tag{79}$$

where $\theta_n = \|W_n^*\|^2/b_m$ is unknown constant.

Therefore, the virtual controller law α_n is defined as follows

$$\alpha_n = -c_n z_n - \frac{1}{2a_n^2} z_n \hat{\theta}_n S_n^T S_n \tag{80}$$

where $c_n > 0$ is design parameter, and the adaptive updating law $\dot{\hat{\theta}}_n$ as

$$\dot{\hat{\theta}}_n = \frac{\gamma_n}{2a_n^2} z_n^2 S_n^T S_n - \mu_n \hat{\theta}_n \tag{81}$$

where γ_n, μ_n are design positive parameters.

Substituting (79) - (81) into (77), it follows that

$$\begin{aligned}\dot{V}_n =& -k_1\zeta^2 - \sum_{j=2}^{n-1} k_j z_j^2 - \sum_{j=1}^{n-1} \frac{\mu_j b_m}{\gamma_j}\tilde{\theta}_j^2 + \sum_{j=1}^{n-1} B_j \\ &+ z_n g_n (u-\alpha_n) - c_n b_m z_n^2 + \tau_{n1}^* + \tau_{n2}^* + d_n(t) \\ &+ \frac{1}{2}a_n^2 + \frac{1}{2}\bar{\delta}_n^2 + \frac{\mu_n b_m}{\gamma_n}\tilde{\theta}_n\hat{\theta}_n + \frac{d_0}{\lambda_0} - \frac{\Lambda}{\lambda_0} r + \Upsilon\end{aligned} \tag{82}$$

Applying the Young's inequality, it follows that

$$\begin{aligned}\frac{\mu_n b_m}{\gamma_n}\tilde{\theta}_n\hat{\theta}_n \le& \frac{\mu_n b_m}{\gamma_n}\tilde{\theta}_n\left(\theta_i - \tilde{\theta}_n\right) \\ \le& -\frac{\mu_n b_m}{2\gamma_n}\tilde{\theta}_n^2 + \frac{\mu_n b_m}{2\gamma_n}\theta_n^2\end{aligned} \tag{83}$$

Finally, $\dot{V}_n$ is written as follows

$$\begin{aligned}\dot{V}_n =& -k_1\zeta^2 - \sum_{j=2}^{n} k_j z_j^2 - \sum_{j=1}^{n} \frac{\mu_j b_m}{2\gamma_j}\tilde{\theta}_i^2 + \sum_{j=1}^{n} B_j \\ &+ z_n g_n (u-\alpha_n) + \frac{d_0}{\lambda_0} - \frac{\Lambda}{\lambda_0} r + \Upsilon\end{aligned} \tag{84}$$

where $k_n = c_n b_m$, $B_n = \tau_{n1}^* + \tau_{n2}^* + d_n(t) + \frac{a_n^2}{2} + \frac{\bar{\delta}_n^2}{2} + \frac{\mu_n b_m}{2\gamma_n}\theta_n^2$.

In order to reduce the waste of computing and communication resources, this paper introduces an event-triggered controller as follows

$$\begin{cases} w(t) \\ = -(1+\kappa)\left[\alpha_n \tanh\left(z_n\alpha_n/\varepsilon\right) + \bar{m}\tanh\left(z_n\bar{m}/\varepsilon\right)\right] \\ u(t) = w\left(t_k\right) \quad \forall t \in \left[t_k, t_{k+1}\right) \\ t_{k+1} = \inf\left\{t \in R \,|\, |e(t)| \ge \kappa|u(t)| + m\right\} \end{cases} \tag{85}$$

where t_k, $k \in Z^+$, ε, $\kappa(0<\kappa<1)$, $\bar{m} > m/(1-\kappa)$ are positive design parameters. And $e(t) = \omega(t) - \omega(t_k)$ represents the measurement error.

According to (85), in the interval $[t_k, t_{k+1})$, it can be get

$$\omega(t) = u + \varsigma_1(t)\kappa u(t) + \varsigma_2(t) m \tag{86}$$

where the time-varying parameter $\varsigma_1(t), \varsigma_2(t)$, and $|\varsigma_1(t)| \le 1$, $|\varsigma_2(t)| \le 1$.

Thus, it can be obtained that

$$u(t) = \omega(t)/(1+\varsigma_1(t)\kappa) - \varsigma_2(t) m/(1+\varsigma_1(t)\kappa) \tag{87}$$

Then, one has

$$z_n g_n (u-\alpha_n) = z_n g_n \left(\frac{\omega - \varsigma_2 m}{1+\varsigma_1\kappa} - \alpha_n\right) \tag{88}$$

Since $\forall m \in R, \varepsilon > 0, -m\tanh(m/\varepsilon) \le 0$, it follows that $z_n\omega < 0$. As mentioned earlier, $\varsigma_{1,2}(t) \in [-1,1]$, we have $z_n\omega/(1+\varsigma_1\kappa) \le z_n\omega/(1+\kappa)$, and $|\varsigma_2 m/(1+\varsigma_1\kappa)| \le |m/(1-\kappa)|$, it yields

$$z_n g_n (u-\alpha_n) \le g_n\left(\frac{z_n\omega}{1+\kappa} - \left|\frac{z_n m}{1-\kappa}\right| - z_n\alpha_n\right) \tag{89}$$

Based on Lemma 2, and bring (85) into (89), it follows that

$$\begin{aligned}z_n g_n (u-\alpha_n) \le& g_n\left[|z_n\alpha_n| - z_n\alpha_n\tanh\left(\frac{z_n\alpha_n}{\varepsilon}\right)\right] \\ &+ g_n\left[|z_n\bar{m}| - |z_n\bar{m}|\tanh\left(\frac{|z_n\bar{m}_1|}{\varepsilon}\right)\right] \\ \le& 0.557 b_m\varepsilon\end{aligned} \tag{90}$$

Then, (84) can be rewritten by

$$\begin{aligned}\dot{V}_n =& -k_1\zeta^2 - \sum_{j=2}^{n} k_j z_j^2 - \sum_{j=1}^{n} \frac{\mu_j b_m}{2\gamma_j}\tilde{\theta}_i^2 \\ &+ \sum_{j=1}^{n} B_j + \frac{d_0}{\lambda_0} - \frac{\Lambda}{\lambda_0} r + 0.557 b_m\epsilon + \Upsilon\end{aligned} \tag{91}$$

IV. STABILITY ANALYSIS

Theorem 1: Consider a strict-feedback nonlinear system (1) satisfying Assumption 1-4, with the virtual control law (34), (48), (64), (80), the adaptive updating law (35), (49), (65), (81), and the event-triggered controller (85), then the following properties are hold:

(1) All signals of the closed-loop control system are SGUUB.
(2) The tracking error can converge to the bounded range during settling time and always be within it.
(3) There exists a time $t^* > 0$ satisfying $t_{k+1} - t_k > t^*, \forall k \in z^+$ such that Zeno-behavior can be avoided.

Proof: (1) Let $a_0 = \min\{2k_i, \Lambda, \mu_i; i = 1,2,\ldots,n\}$, $b_0 = d_0/\lambda_0 + \sum_{j=1}^{n} B_j + 0.557 b_m\varepsilon$.

Then, (91) can be rewritten as

$$\dot{V} \le -a_0 V + b_0 + \left(1 - 2\tanh^2\left(\frac{\zeta}{\ell}\right)\right)\frac{x_1^2\eta_0\left(x_1^2\right)}{\lambda_0} \tag{92}$$

Due to the sign of $\left[1-2\tanh^2(\zeta/\ell)\right]\left[x_1^2\eta_0\left(x_1^2\right)/\lambda_0\right]$ is unknown, so it needs to be discussed in different cases.

Case 1: When $\zeta \in \Omega_\zeta = \{\zeta \,|\, |\zeta| < 0.8814\ell\}$ for $\forall\ell$ in (30). According to (14), the reason why x_1 is bounded is that z_1 and y_d are bounded. Due to $\eta_0\left(x_1^2\right) \ge 0$ is a smooth function, $\left[1-2\tanh^2(\zeta/\ell)\right]\left[x_1^2\eta_0\left(x_1^2\right)/\lambda_0\right]$ is bounded and its boundary is defined as c_0. By (92), we have

$$\dot{V}(t) \le -a_0 V + h_0 \tag{93}$$

where $h_0 = b_0 + c_0$. Then, (92) satisfies

$$0 \le V_n \le \left(V(0) - \frac{h_0}{a_0}\right)e^{-a_0 t} + \frac{h_0}{a_0} \tag{94}$$

Case 2: When $\zeta \notin \Omega_\zeta$, According to Lemma 2 and $\eta_0\left(x_1^2\right) \ge 0$, it follows that

$$\left(1 - 2\tanh^2\left(\frac{\zeta}{\ell}\right)\right)\frac{x_1^2\eta_0\left(x_1^2\right)}{\lambda_0} \le 0 \tag{95}$$

Therefore, (92) can be written as

$$\dot{V}_n(t) \le -a_0 V + b_0 \tag{96}$$

According to (96), one obtains

$$0 \le V_n(t) \le \left(V(0) - \frac{b_0}{a_0} \right) e^{-a_0 t} + \frac{b_0}{a_0} \tag{97}$$

Finally, by comparing (94) and (97), it yields

$$0 \le V_n \le V(0) + \frac{h_0}{a_0} \tag{98}$$

Therefore, all signals of the closed-loop control system are SGUUB.

(2) In order to prove the tracking error z_1 is bounded by prescribed performance, bring V_1 to (98), it can get

$$\frac{1}{2}\zeta^2 = \frac{1}{2}tan^2\left(\frac{\pi z_1}{2\rho}\right) \le V(0) + \frac{h_0}{a_0} \tag{99}$$

Finally, the boundary of z_1 is restricted by

$$|z_1| \le \frac{2\rho}{\pi} \arctan \sqrt{2\left(V(0) + \frac{h_0}{a_0}\right)} < \rho \tag{100}$$

Therefore, the tracking error can converge to the bounded range during settling time and always be within it.

(3) In order to prove there exists a time $t^* > 0$ satisfying $t_{k+1} - t_k > t^*, \forall k \in z^+$, let $e(t) = \omega(t) - u(t), \forall t_k \in [t_k, t_{k+1})$, one has

$$\frac{d}{dt}|e| = \frac{d}{dt}|e * e|^{\frac{1}{2}} = sign(e)\dot{e} \le |\dot{\omega}| \tag{101}$$

From (86), all signals included by $\dot{\omega}$ are SGUUB. Hence, there exists a positive constant s satisfying $|\dot{\omega}| < s$. From $e(t_k) = 0$ and $\lim\limits_{t \to t_k} e(t) = \kappa |u(t)| + m$, the lower bound of interval time $t^* \ge (\kappa |u(t)| + m)/s$ can be obtained. In addition, the Zeno-behavior can be avoided. ■

V. SIMULATION RESULTS

Consider the following second-order strict-feedback nonlinear system with unmodeled dynamics as follows

$$\begin{cases} \dot{\xi} = -\xi + x_1^2 + 0.5 \\ \dot{x}_1 = x_1^2 e^{-0.1x_1} + \left(1 + x_1^2\right) x_2 + \xi \sin\left(x_1^2\right) \\ \dot{x}_2 = x_1 x_2^2 + \left(1 + \frac{x_2^2}{x_1^2 + x_2^2}\right) x_2 + \xi x_1^2 \\ y = x_1 \end{cases} \tag{102}$$

Since Assumptions 1-3 is easy to establish, we focus on checking Assumption 4 holds for ξ-subsystem in (102). We select Lyapunov function $V_\xi(\xi) = \xi^2$, then it follows that

$$\begin{aligned} \dot{V}_\xi(\xi) =& 2\xi\left(-\xi + x_1^2 + 0.5\right) \\ \le& -2\xi^2 + \frac{1}{4\iota}(2\xi)^2 + \iota x_1^4 + \frac{\iota}{4} + \frac{\xi^2}{\iota} \end{aligned} \tag{103}$$

Then, by selecting $\iota = 2.5$, it yields

$$\dot{V}_\xi(\xi) \le -1.2\xi^2 + 2.5x_1^4 + 0.625 \tag{104}$$

By selecting $v_1(|\xi|) = 0.5\xi^2$, $v_2(|\xi|) = 2\xi^2$, $\Lambda_0 = 1.2$, $b_0 = 0.625$, and $\eta(|x_1|) = 2.5x_1^4$, Assumption can be satisfied. Then we define $\Lambda = 1 \in (0, \Lambda_0)$ and dynamic signal r is as

$$\dot{r} = -\xi^2 + 2.5x_1^4 + 0.625 \tag{105}$$

To verify the tracking performance of the designed controller, the reference signal is selected as $y_r = sin(t)$. The design parameters are taken as follows: $c_1 = 15, c_2 = 10, a_1 = 1, a_2 = 1, \gamma_1 = 0.15, \gamma_2 = 0.1, \mu_1 = 1, \mu_2 = 1, m = 0.3, \bar{m}_1 = 1, \epsilon = 10, \kappa = 0.15$. The initial condition of system $\left[\xi(0), x_1(0), x_2(0), \hat{\theta}_1(0), \hat{\theta}_2(0), r(0)\right]$ are chose as $[0, 0.5, 0.2, 0, 0, 0]$.

The finite-time performance function is selected as

$$\rho(t) = \begin{cases} \left(1 - \frac{t}{2}\right) e^{-\frac{2t}{2-t}} + 0.045 & t \in [0, 2) \\ 0.045 & t \in [2, +\infty) \end{cases} \tag{106}$$

In Fig.1, 'PM' is the proposed method, 'FTPF' is the finite-time performance function method in [18], 'PPC' is traditional prescribed perfoemance method. The system tracking errors are shown by Fig.1. The system tracking signal are shown by Fig.2. And Fig.3 shows the system state x_2. Fig.4 displays event-triggered control signal and adaptive signal. Triggering instant is shown by Fig.5, and triggering time interval is displayed by Fig.6.

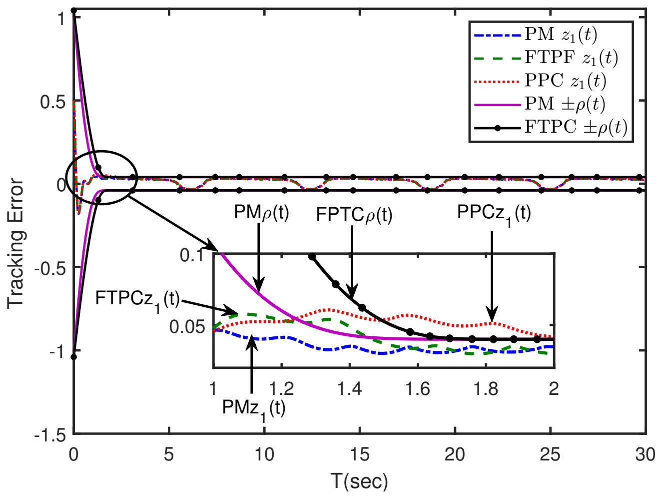

Fig. 1. Tracking errors in the proposed method (PM), FTPC and PPC.

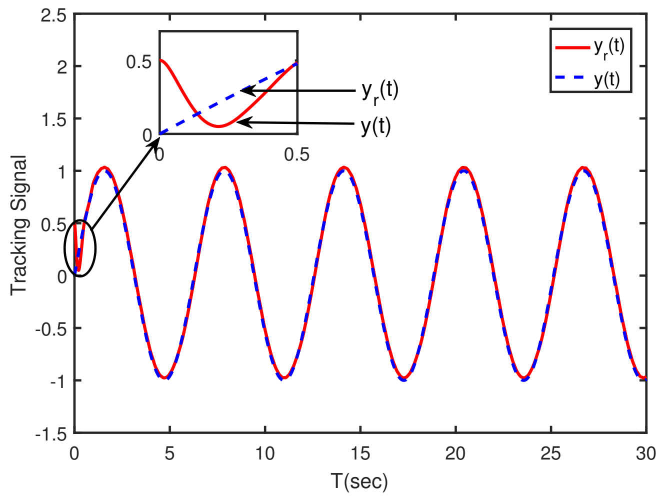

Fig. 2. Tracking signal in the proposed method.

Form Fig.1, it can be seen that the tracking error of proposed method can faster convergence to the bounded range than FTPF [18] and PPC, it is greatly meaningful for some high-precision control systems. After the control signal is processed by an event-triggered controller, it is transformed a discrete signal, thereby improving resource

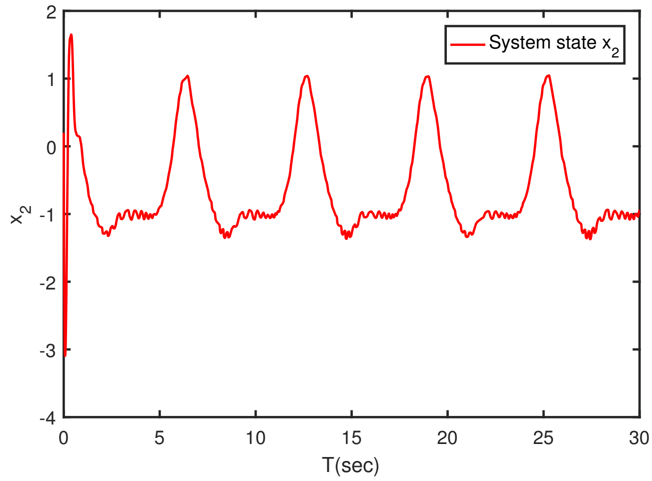

Fig. 3. System state x_2.

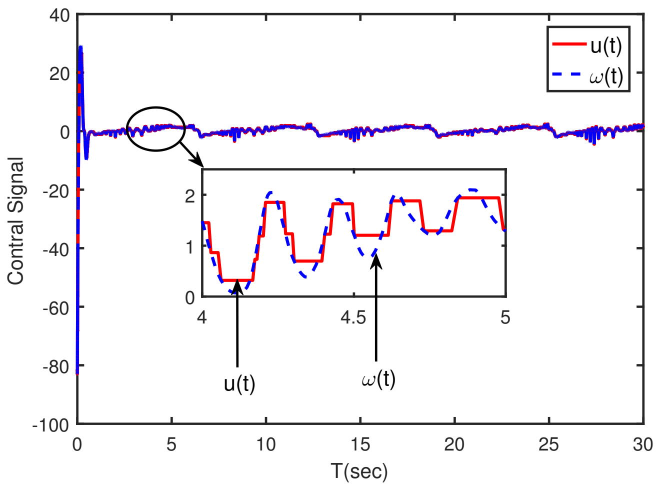

Fig. 4. Event-triggered control signal $u(t)$ and adaptive signal $\omega(t)$.

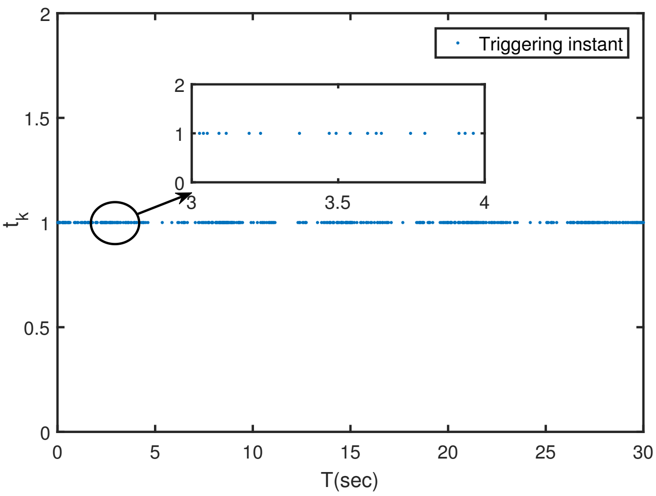

Fig. 5. Triggering instant t_k.

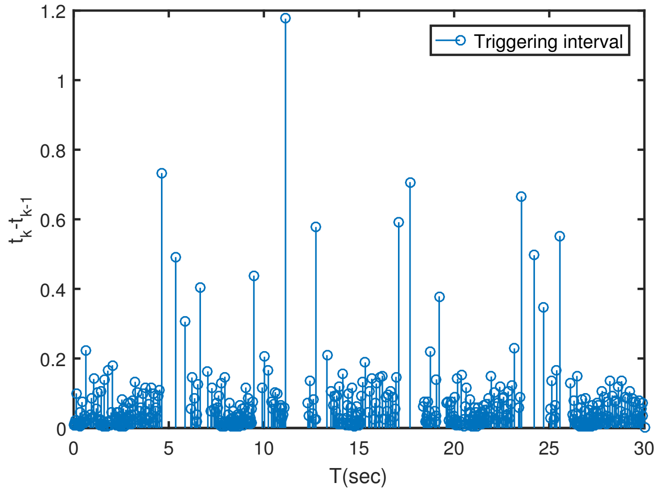

Fig. 6. Triggering interval $t_k - t_{k-1}$.

utilization, in addition, the transformed control signal can also ensure the performance of the control system.

VI. CONCLUSION

The problem of prescribed performance adaptive NNs event-triggered control for a class of nonlinear systems with unmodeled dynamics has been solved. Dynamic disturbances have been suppressed by dynamic signal and the finite-time prescribed performance method has been applied which guarantees the boundedness of the tracking error and also improves the control accuracy. In order to simplify the calculation, the unknown nonlinear functions have been approximated by NNs. Then, a novel finite-time prescribed performance event-triggered adaptive NNs controller has been designed, which ensures that all signals of the close-loop control system are SGUUB, and the Zeno-behavior has been avoided. In the future, we can consider more complex systems, e.g. MIMO or multiagent systems, etc.

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