A Novel Topsis Method for Multi-attribute Decision Making Based on D-anp and Single-valued Neutrosophic Sets Distance Measure

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***Abstract*—Single-valued neutrosophic sets (SVNS) are proficient in conveying fuzzy information within the realm of multiple attribute decision making (MADM), and The TOPSIS method effectively ranks alternatives in MADM. Attribute weight and distance measurement are important steps in TOPSIS method. The focus of this paper is to propose a new attribute weight determination method in the SVNS environment, namely Decision Laboratory and Experimentation Method - Analytic Network Process (D-ANP). A new single-valued neutrosophic distance measure is also defined, namely single-valued neutrosophic Kulcynksi distance, and the effectiveness of the generalized single-valued neutrosophic Kulcynksi distance is proved. Finally, the proposed D-ANP method and the defined Kulcynksi distance are applied to the TOPSIS method of MADM, and the feasibility of the D-ANP method and Kulcynksi distance is verified through application examples.**

***Index Terms*—Single-valued Neutrosophic Set, Kulcynksi distance, D-ANP, TOPSIS, MADM**

I. Introduction

MADM is a common decision problem that entails evaluating and assigning weights to multiple attributes. However, in practical applications, certain attributes are frequently difficult to determine accurately, especially quality evaluation indicators that resist precise representation through deterministic values. To tackle this challenge, Zadeh[1] introduced fuzzy sets, an extension of the traditional set theory that permits elements to belong to sets with varying degrees of membership, effectively capturing the inherent fuzziness and uncertainty prevalent in numerous practical problems. Atanassov[2] extended fuzzy sets by introducing intuitionistic fuzzy sets, where each element's membership and non-membership degrees are separately defined, allowing elements to simultaneously exhibit high membership and non-membership degrees. Smarandache et al.[3][4] introduced neutrosophic sets (NS), an extension of intuitionistic fuzzy sets, featuring independent membership, indeterminacy, and non-membership degrees. However, they are defined on non-standard unit subintervals, rendering them less practical for engineering and scientific applications. To address this limitation, Wang et al.[5] introduced single-valued neutrosophic sets, defined on standard unit subintervals and offer greater convenience for applications in engineering and science. SVNS provide an effective approach for describing and handling uncertain and inconsistent information. Ye[6] formulated a comprehensive distance metric for SVNS and introduced two distance-based similarity measures for these sets. Ye[7] introduced an enhanced cross-entropy measure for SVNS, conducted an analysis of its properties, and expanded its application to cross-entropy measures for INS. Moreover, this cross-entropy measure was applied to address multicriteria decision-making problems, encompassing both singleton and interval intuitionistic fuzzy information. Ye [8] established the Hamming distance and Euclidean distance metrics for INS, drawing from the connection between similarity measures and distances, introduced a similarity measure for INS. Presently, the computation of the distance between two SVNS primarily relies on metrics like Hamming distance, Euclidean distance, Hausdorff distance, and others. Nevertheless, these distance formulas exhibit limitations when determining the distance for specific SVNS instances, thereby diminishing their utility. As a remedy, this paper presents the concept of the single-valued neutrosophic Kulcynksi distance.

In MADM problems, determining attribute weights is crucial for improving decision accuracy and reliability. The DEMATEL (Decision-Making Trial and Evaluation Laboratory) method, introduced by the Battelle Association at the Geneva Center, Georgia University[9], is widely employed for analyzing the interrelationships among factors in decision-making problems and assigning weights to each factor. Researchers like Tian ZP et al.[10] and W. Yang et al.[11][12] have utilized DEMATEL to ascertain attribute weights within diverse neutrosophic contexts and incorporated them with the TOPSIS method to choose optimal solutions for MADM problems. An alternative method, ANP (Analytic Network Process), was introduced by Sssty[13], which computes attribute weights by constructing a network of relationships between attributes. M. Abdel-Basset et al.[14] employed the ANP method to establish attribute weights within the interval neutrosophic context, subsequently applying the TOPSIS method for

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solving supplier selection problems. Yang Y et al.[15] introduced a network analysis approach named D-ANP, which amalgamates DEMATEL and ANP, accounting for factor interdependence and circumventing the laborious pairwise comparisons inherent to ANP. This approach boosts the effectiveness in decision-making. Nevertheless, Research on the application of D-ANP within the context of SVNS is limited. Therefore, this study aims to explore the use of D-ANP in the single-valued neutrosophic environment to determine attribute weights, with the overarching goal of enhancing decision-making efficiency and reliability.

The TOPSIS method is widely utilized in MADM. This method treats each solution as a multidimensional vector and utilizes distance metrics, such as Euclidean or Manhattan distance, to compute the dissimilarities between each solution and the best and worst solutions[16]. These dissimilarities are subsequently utilized for ranking and identifying the optimal solution. The TOPSIS method has found extensive application within the realm of SVNNs, resulting in numerous extensions and adaptations. For instance, Peng et al.[17] extended the TOPSIS method to accommodate the single-valued neutrosophic environment. Liu et al.[18] introduced a novel TOPSIS variation that integrates the relative proximity measure of alternative solutions to the positive reference solution and the degree of similarity to the positive reference solution, introducing the innovative concept of the proximity measure. Xu et al.[19] devised a novel multi-attribute decision-making method by integrating TOPSIS and TODIM approaches within the multi-valued neutrosophic environment. Additionally, Geng et al.[21] presented a method for single-valued intuitionistic linguistic weighted distance measurement (SVNLCWD) within the context of the single-valued neutrosophic linguistic environment. They integrated this method with the TOPSIS approach to formulate the SVNLCWD-TOPSIS method for Multiple Attribute Group Decision Making (MAGDM) problems. Lately, researchers have extensively investigated the fusion of SVNNs with the TOPSIS method. This article presents a novel MADM TOPSIS method, founded on D-ANP and the SVNS Kulcynksi distance. This novel approach seeks to enhance the efficiency and dependability of decision-making within the single-valued neutrosophic environment by incorporating interdependencies between factors through D-ANP and employing the SVNS Kulcynksi distance to compute solution dissimilarities.

This article proposes a method for addressing multi-attribute decision-making within the context of single-valued neutrosophic fuzzy environments. SVNNs are employed to describe both solutions and attributes, while attribute weights are determined through the use of the D-ANP method. Afterward, the TOPSIS method is employed to rank the solutions and ascertain the optimal one. The efficacy of the proposed method is illustrated through example results, highlighting its capability in resolving multi-attribute decision-making challenges.

II. Preliminaries

A. Single-Valued Neutrosophic Set

Definition 1. Assume X is a set of objects, where each element x in X, a NS of X is defined as:

$$A=\left\{\left\langle x,T_A\left(x\right),I_A\left(x\right),F_A\left(x\right)\right\rangle \mid x\in X\right\},$$

where $T_A\left(x\right)$, $I_A\left(x\right)$, $F_A\left(x\right): X\to \left]^{-}0,1^{+}\right[$, the truth degree, indeterminacy degree and false degree are represented by $T_A\left(x\right)$, $I_A\left(x\right)$ and $F_A\left(x\right)$, $^{-}0\le T_A\left(x\right)+I_A\left(x\right)+F_A\left(x\right)\le 3^{+}$.

Definition 2. Let X be a set of objects, x be an element in X, a SVNS of X is defined as:

$$A=\left\{\left\langle x,T_A\left(x\right),I_A\left(x\right),F_A\left(x\right)\right\rangle \mid x\in X\right\},$$

where $T_A\left(x\right)$, $I_A\left(x\right)$ and $F_A\left(x\right)$ are three real numbers between $\left[0,1\right]$, $0\le T_A\left(x\right)+I_A\left(x\right)+F_A\left(x\right)\le 3$.

For any element x in the SVNS A on the object set X, it is called a single-valued neutrosophic number (SVNN) and denoted by $x=\left\langle T_x,I_x,F_x\right\rangle$.

Definition 3. Assume two SVNNs $e=\left\langle T_e,I_e,F_e\right\rangle$ and $f=\left\langle T_f,I_f,F_f\right\rangle$, their arithmetic operations are defined as[20]:

(1) $e\oplus f=\left\langle T_e+T_f-T_eT_f, I_eI_f, F_eF_f\right\rangle$.

(2) $e\otimes f=\left\langle T_eT_f, I_e+I_f-I_eI_f, F_e+F_f-F_eF_f\right\rangle$.

(3) $\lambda e=\left\langle 1-\left(1-T_e\right)^{\lambda},\left(I_e\right)^{\lambda},\left(F_e\right)^{\lambda}\right\rangle,\lambda>0$.

(4) $\left(e\right)^{\lambda}=\left\langle \left(T_e\right)^{\lambda},1-\left(1-I_e\right)^{\lambda},1-\left(1-F_e\right)^{\lambda}\right\rangle,\lambda>0$.

Definition 4. For a SVNN $e=\left\langle T_e,I_e,F_e\right\rangle$, the complement of e is defined as[24]:

$$e^{c}=\left\langle F_e,1-I_e,F_e\right\rangle \tag{1}$$

Definition 5. Assume two SVNs A and B, then $\forall x\in X$, A SVN A is contained in the other SVN B, $A\subseteq B$, if, $T_A\left(x\right)\le T_B\left(x\right)$, $I_A\left(x\right)\ge I_B\left(x\right)$ and $F_A\left(x\right)\ge F_B\left(x\right)$.

Definition 6. For a SVNN $e=\left\langle T_e,I_e,F_e\right\rangle$, the Score Function of e is defined as[25]:

$$S\left(e\right)=\frac{T_e-I_e-F_e+2}{3} \tag{2}$$

Definition 7. For a collection of SVNNs $a_j=\left\langle T_j,I_j,F_j\right\rangle\left(j=1,\cdots,n\right)$, and weight vector $W=\left(\omega_1,\omega_2,\cdots,\omega_n\right)^{T}$. The Single-Valued Neutrosophic Weighted Average (SVNWA) operator of a_j is defined as[26]:

$$SVNWA\left(a_1, a_2, \cdots, a_n\right) = \sum_{j=1}^{n} \omega_j a_j = \left(1 - \prod_{j=1}^{n}\left(1 - T_j\right)^{\omega_j}, \prod_{j=1}^{n}\left(I_j\right)^{\omega_j}, \prod_{j=1}^{n}\left(F_j\right)^{\omega_j}\right) \tag{3}$$

B. The Distance Measures of the SVNS

Definition 8. Assume two SVNNs $e = \langle T_e, I_e, F_e \rangle$ and $f = \langle T_f, I_f, F_f \rangle$,the distance measures between e and f are defined as follows[22][23]:

(1) The SVN Euclidean distance is

$$d_E(e, f) = \left\{\left(T_e - T_f\right)^2 + \left(I_e - I_f\right)^2 + \left(F_e - F_f\right)^2\right\}^{\frac{1}{2}} \tag{4}$$

(2) The SVN Hamming distance is

$$d_H(e, f) = \left|T_e - T_f\right| + \left|I_e - I_f\right| + \left|F_e - F_f\right| \tag{5}$$

(3) The SVN Hausdorf distance is

$$d_{Hd}(e, f) = \max\left(\left|T_e - T_f\right|, \left|I_e - I_f\right|, \left|F_e - F_f\right|\right) \tag{6}$$

Definition 9. Assume two SVNNs $e = \langle T_e, I_e, F_e \rangle$ and $f = \langle T_f, I_f, F_f \rangle$,the SVN Kulcynksi distance between e and f is defined as:

$$d_S(e, f) = \left[\frac{\left|T_e - T_f\right| + \left|I_e - I_f\right| + \left|F_e - F_f\right|}{\min\left(T_e, T_f\right) + \min\left(I_e, I_f\right) + \min\left(F_e, F_f\right)}\right] \tag{7}$$

The generalized SVN Kulcynksi distance between e and f is defined as:

$$d_{gS}(e, f) = \left[\frac{\left|T_e - T_f\right|^{\lambda} + \left|I_e - I_f\right|^{\lambda} + \left|F_e - F_f\right|^{\lambda}}{\min\left(T_e, T_f\right)^{\lambda} + \min\left(I_e, I_f\right)^{\lambda} + \min\left(F_e, F_f\right)^{\lambda}}\right]^{\frac{1}{\lambda}} \tag{8}$$

$\lambda > 0$, when $\lambda = 1$,it is the Kulcynksi distance.

Proposition 1. The above defined the generalized SVN Kulcynksi distance $d_{gS}(e, f)$ between SVNNs e and f satisfies the following properties (1)-(4):

(1) $d_{gS}(e, f) \geq 0$;

(2) $d_{gS}(e, f) = 0$ if $e = f$;

(3) $d_{gS}(e, f) = d_{gS}(f, e)$;

(4) If $e \subseteq f \subseteq g$, g is the other SVNN in X , then $d_{gS}(e, g) = d_{gS}(e, f)$ and $d_{gS}(e, g) = d_{gS}(f, g)$.

Proof. Obviously, d_{gS} satisfies properties (1), (2), and (3). It only needs to be verified that property (4).

If $e \subseteq f \subseteq g$,i.e.,

$T_e \leq T_f \leq T_g$, $I_e \geq I_f \geq I_g$, $F_e \geq F_f \geq F_g$.

From the aforementioned conditions, we obtain

$$\left|T_e - T_g\right| \geq \left|T_e - T_f\right|, \quad \left|T_e - T_g\right| \geq \left|T_f - T_g\right|$$

$$\left|I_e - I_g\right| \geq \left|I_e - I_f\right|, \quad \left|I_e - I_g\right| \geq \left|I_f - I_g\right|$$

$$\left|F_e - F_g\right| \geq \left|F_e - F_f\right|, \quad \left|F_e - F_g\right| \geq \left|F_f - F_g\right|$$

$$T_e + I_g + F_g \leq T_e + I_f + F_f$$

$$T_e + I_g + F_g \leq T_f + I_g + F_g$$

$$\min\left(T_e, T_g\right) = T_e, \quad \min\left(I_e, I_g\right) = I_g$$

$$\min\left(F_e, F_g\right) = F_g, \quad \min\left(T_e, T_f\right) = T_e$$

$$\min\left(I_e, I_f\right) = I_f, \quad \min\left(F_e, F_f\right) = F_f$$

$$\min\left(T_f, T_g\right) = T_f, \quad \min\left(I_f, I_g\right) = I_g$$

$$\min\left(F_f, F_g\right) = F_g$$

According to equation (8), we have

$$d_{gS}(e, g) = \left[\frac{\left|T_e - T_g\right|^{\lambda} + \left|I_e - I_g\right|^{\lambda} + \left|F_e - F_g\right|^{\lambda}}{\left(T_e\right)^{\lambda} + \left(I_g\right)^{\lambda} + \left(F_g\right)^{\lambda}}\right]^{\frac{1}{\lambda}}$$

$$d_{gS}(e, f) = \left[\frac{\left|T_e - T_f\right|^{\lambda} + \left|I_e - I_f\right|^{\lambda} + \left|F_e - F_f\right|^{\lambda}}{\left(T_e\right)^{\lambda} + \left(I_f\right)^{\lambda} + \left(F_f\right)^{\lambda}}\right]^{\frac{1}{\lambda}}$$

$$d_{gS}(f, g) = \left[\frac{\left|T_f - T_g\right|^{\lambda} + \left|I_f - I_g\right|^{\lambda} + \left|F_f - F_g\right|^{\lambda}}{\left(T_f\right)^{\lambda} + \left(I_g\right)^{\lambda} + \left(F_g\right)^{\lambda}}\right]^{\frac{1}{\lambda}}$$

The numerator of $d_{gS}(e, g)$ is greater than or equal to the numerators of $d_{gS}(e, f)$ and $d_{gS}(f, g)$, and the denominator of $d_{gS}(e, g)$ is less than or equal to the denominators of $d_{gS}(e, f)$ and $d_{gS}(f, g)$, according to the above conditions.

We can conclude that

$$d_{gS}(e, g) = d_{gS}(e, f), d_{gS}(e, g) = d_{gS}(f, g).$$

Example 1. For three SVNNs $e = (0.4, 0.5, 0.7)$, $f = (0.4, 0.4, 0.5)$ and $g = (0.4, 0.6, 0.9)$, calculate $d(e, f)$ and $d(e, g)$ utilizing Equations (4), (5), (6), and (7), the results are shown in table I:

Table I
COMPARE

	Euclidean	Hamming	Hausdorff	Kulcynksi
$d(e, f)$	0.1400	0.3000	0.2000	0.2308
$d(e, g)$	0.1400	0.3000	0.2000	0.1875

Referring to the table above, it is evident that the values of $d(e, f)$ and $d(e, g)$ calculated using Equations (4), (5) and (6) are the same, which leads to a lack of certain value in the results. Therefore, it is necessary to establish a new distance metric to overcome the shortcomings of existing metrics. Hence, this paper introduces the Kulcynksi distance.

C. D-ANP Method for SVNS MADM

Step 1: Determine the direct influence matrix $M=\left(m_{ij}\right)_{n\times n}$. Based on the expert scoring method, the strength of influence between elements is determined, and the influence intensity of attribute on attribute is denoted as m_{ij}, $m_{ij}=\left\langle T_{ij},I_{ij},F_{ij}\right\rangle$, $i=j$, $m_{ij}=0$.

Step 2: Transform the direct influence matrix M into a real-valued matrix $M'=\left(m'_{ij}\right)_{n\times n}$ using the score function formula (2) for SVNNs.

Step 3: Normalize the direct influence matrix. Normalize the direct influence matrix M' to obtain the normalized direct influence matrix

$$M''=\left(m''_{ij}\right)_{n\times n}$$

$$m''_{ij}=\frac{m'_{ij}}{a},\ a=\max\left\{\max_{1\le i\le n}\sum_{j=1}^{n}m'_{ij},\max_{1\le i\le n}\sum_{i=1}^{n}m'_{ij}\right\}.$$

Step 4: Calculate the comprehensive influence matrix T using the formula $T=M''\left(I-M''\right)^{-1}$, where I is the identity matrix.

Step 5: Limit Super matrix. Treat real number comprehensive influence matrix T as the unweighted super matrix of ANP. Normalize the columns of T by column vector weights to generate a weighted super matrix $W=\left(w_{ij}\right)_{n\times n}$, where $w_{ij}=\frac{t'_{ij}}{\sum_{i=1}^{n}t'_{ij}}$. Multiply W by itself repeatedly to produce the limit super matrix $\lim_{k\to\infty}W^{k}$, and obtain the weights ω_i, $i=1,2,\cdots n$.

D. Topsis Method for SVNS MADM

Step 1: The decision maker provides the decision matrix $D=\left(a_{ij}\right)_{n\times m}$, which consists of the SVNNs evaluation values of the alternatives with respect to the attributes.

$a_{ij}=\left\langle T_{ij}\left(x\right),I_{ij}\left(x\right),F_{ij}\left(x\right)\right\rangle$ is the evaluation value of alternative A_i under attribute C_j, represented by a SVNS.

Step 2: Weighted aggregation of solution A_i by SVNWA, $\omega=\left(\omega_1,\omega_2,\cdots,\omega_m\right)^{T}$, and the aggregated weighted solution is:

$$A_i'=SVNWA\left(a_{i1,}a_{i2},\cdots,a_{im}\right)=\sum_{j=1}^{m}\omega_j a_{ij}$$

$$=\left(1-\prod_{j=1}^{n}\left(1-T_{ij}\right)^{\omega_j},\prod_{j=1}^{n}\left(I_{ij}\right)^{\omega_j},\prod_{j=1}^{n}\left(F_{ij}\right)^{\omega_j}\right)$$

Step 3: Calculate the positive ideal solution A^{+} and negative ideal solution A^{-}.

$$A^{+}=\left\langle \max_i 1-\prod_{j=1}^{m}\left(1-T_{ij}\right)^{\omega_j},\min_i\prod_{j=1}^{m}\left(I_{ij}\right)^{\omega_j},\min_i\prod_{j=1}^{m}\left(F_{ij}\right)^{\omega_j}\right\rangle \quad (9)$$

$$A^{-}=\left\langle \min_i 1-\prod_{j=1}^{m}\left(1-T_{ij}\right)^{\omega_j},\max_i\prod_{j=1}^{m}\left(I_{ij}\right)^{\omega_j},\max_i\prod_{j=1}^{m}\left(F_{ij}\right)^{\omega_j}\right\rangle \quad (10)$$

Step 4: Calculate the distances $d\left(A'_i,A^{+}\right)$ and $d\left(A'_i,A^{-}\right)$ between the weighted decision matrix A'_i and the positive ideal solution A^{+} and negative ideal solution A^{-}. Compute the relative closeness C_i for each alternative and arrange them in order accordingly. The higher the relative closeness, the better the alternative.

$$C_i=\frac{d\left(A'_i,A^{-}\right)}{d\left(A'_i,A^{-}\right)+d\left(A'_i,A^{+}\right)} \quad (11)$$

III. ILLUSTRATIVE EXAMPLES

A risk investment company plans to invest in an emerging industry and needs to choose a trustworthy company from four companies for investment. The company needs to consider the following four attributes: industry experience $\left(C_1\right)$: the company's experience and professional skill level in the emerging industry. Financial stability $\left(C_2\right)$: the company's financial condition and historical performance. Team strength $\left(C_3\right)$: the quality of the company's management team and employee team. Market prospects $\left(C_4\right)$: the development situation and future trend of the industry market in which the company is located. The decision maker needs to select the most suitable company for investment through comprehensive evaluation of these six companies and four attributes. Let these four companies be the set of schemes $A=\left\{A_1,A_2,A_3,A_4\right\}$, and the set of attributes $C=\left\{C_1,C_2,C_3,C_4\right\}$.

A. D-ANP Method Determining Attribute Weights

Step 1: The decision experts compare the attributes pairwise and obtain the direct influence matrix $M=\left(m_{ij}\right)_{4\times 4}$.

$$M=\begin{bmatrix} 0 & \langle 0.65,0.35,0.30\rangle & \langle 0.75,0.20,0.65\rangle & \langle 0.65,0.35,0.30\rangle \\ \langle 0.65,0.35,0.30\rangle & 0 & \langle 0.50,0.50,0.45\rangle & \langle 0.55,0.10,0.30\rangle \\ \langle 0.50,0.50,0.45\rangle & \langle 0.65,0.35,0.30\rangle & 0 & \langle 0.90,0.10,0.50\rangle \\ \langle 0.50,0.50,0.45\rangle & \langle 0.80,0.20,0.15\rangle & \langle 0.50,0.50,0.45\rangle & 0 \end{bmatrix}$$

Step 2: Convert the direct influence matrix M to a numerical matrix M'.

$$M' = \begin{bmatrix} 0 & 0.667 & 0.633 & 0.667 \\ 0.667 & 0 & 0.517 & 0.717 \\ 0.517 & 0.667 & 0 & 0.767 \\ 0.517 & 0.817 & 0.517 & 0 \end{bmatrix}$$

Step 3: Determine the direct influence matrix M' to obtain matrix M''.

$$M'' = \begin{bmatrix} 0 & 0.310 & 0.294 & 0.310 \\ 0.310 & 0 & 0.240 & 0.333 \\ 0.240 & 0.310 & 0 & 0.357 \\ 0.240 & 0.380 & 0.240 & 0 \end{bmatrix}$$

Step 4: Determine the comprehensive influence matrix T.

$$T = \begin{bmatrix} 1.728 & 2.311 & 1.910 & 2.297 \\ 1.919 & 2.020 & 1.833 & 2.255 \\ 1.908 & 2.296 & 1.668 & 2.308 \\ 1.843 & 2.254 & 1.797 & 1.964 \end{bmatrix}$$

Step 5: Normalize the comprehensive influence matrix T through column weighting to yield the weighted super matrix W. Then, iteratively multiply W by itself to derive the limit super matrix $\lim_{k\to\infty} W^k$.

$$W = \begin{bmatrix} 0.234 & 0.260 & 0.265 & 0.259 \\ 0.259 & 0.228 & 0.255 & 0.256 \\ 0.258 & 0.257 & 0.231 & 0.262 \\ 0.249 & 0.255 & 0.249 & 0.223 \end{bmatrix}$$

$$\lim_{k\to\infty} W^k = \begin{bmatrix} 0.255 & 0.255 & 0.255 & 0.255 \\ 0.249 & 0.249 & 0.249 & 0.249 \\ 0.252 & 0.252 & 0.252 & 0.252 \\ 0.244 & 0.244 & 0.244 & 0.244 \end{bmatrix}$$

The column vectors of the limit supermatrix are the weights vectors of the four attributes

$$\omega = (\omega_1, \omega_2, \omega_3, \omega_4)^T = (0.255, 0.249, 0.252, 0.244)^T$$

B. TOPSIS method Based on Kulcynksi distance measure

Step 1: The evaluator assesses the four enterprises across the four attributes and generates the decision matrix $D = (a_{ij})_{4\times 4}$.

$$D = \begin{bmatrix} \langle 0.80,0.20,0.15\rangle & \langle 0.35,0.65,0.60\rangle & \langle 0.55,0.10,0.30\rangle & \langle 0.55,0.45,0.15\rangle \\ \langle 0.90,0.20,0.25\rangle & \langle 0.25,0.65,0.50\rangle & \langle 0.55,0.20,0.30\rangle & \langle 0.65,0.35,0.30\rangle \\ \langle 0.90,0.20,0.25\rangle & \langle 0.45,0.50,0.70\rangle & \langle 0.55,0.10,0.30\rangle & \langle 0.65,0.30,0.30\rangle \\ \langle 0.80,0.20,0.15\rangle & \langle 0.35,0.60,0.60\rangle & \langle 0.50,0.15,0.25\rangle & \langle 0.65,0.25,0.25\rangle \end{bmatrix}$$

Step 2: Computing the weighted solution utilizing equation (3), and attribute weight vector

$$\omega = (0.255, 0.249, 0.252, 0.244)^T.$$

$$A'_1 = \langle 0.599, 0.275, 0.252\rangle, A'_2 = \langle 0.673, 0.308, 0.325\rangle$$
$$A'_3 = \langle 0.697, 0.233, 0.354\rangle, A'_4 = \langle 0.613, 0.258, 0.273\rangle$$

Step 3: Calculate the positive ideal solution A^+ and negative ideal solution A^- utilizing equation (9) and equation (10).

$$A^+ = \langle 0.697, 0.233, 0.252\rangle, A^- = \langle 0.599, 0.308, 0.354\rangle$$

Step 4: Compute the Kulcynksi distance between each alternative and the positive/negative ideal solutions utilizing equation (7). Then, calculate the relative closeness degree of each alternative utilizing equation (10). the results are shown in table II.

Table II.
THE RELATIVE CLOSENESS DEGREE

	$d(A'_i, A^+)$	$d(A'_i, A^-)$	C_i
A'_1	0.129	0.120	0.482
A'_2	0.140	0.084	0.375
A'_3	0.086	0.146	0.629
A'_4	0.118	0.128	0.520

The ranking order is A_3, A_4, A_1, A_2 and A_3 is the best choice.

Table III
COMPARED RESULTS

weighting method	ranking order	best solution
D-ANP	$A_3 > A_4 > A_1 > A_2$	A_3
DEMATEL	$A_3 > A_4 > A_1 > A_2$	A_3
ANP	$A_3 > A_4 > A_1 > A_2$	A_3

As observed in Table III, the optimal solutions obtained through various weight determination methods are A_3, and the ranking of these solutions remains unchanged. This confirms the viability of the D-ANP method. In future research, we can explore the application of the D-ANP method for weight determination within the context of interval neutrosophic sets or multi-valued neutrosophic sets.

Table IV
COMPARED RESULTS UTILIZING THE DIFFRERENT DISTANCE

distance	ranking order	best solution
Kulcynksi	$A_3 > A_4 > A_1 > A_2$	A_3
Euclidean	$A_3 > A_4 > A_1 > A_2$	A_3
Hamming	$A_3 > A_4 > A_1 > A_2$	A_3
Hausdorff	$A_3 > A_4 > A_1 > A_2$	A_3

Table IV presents a comparison of various distance measures, including Euclidean, Hamming, Hausdorff, and Kulcynksi distances, to assess their influence on decision outcomes. The ranking order indicated that A_3 was the most suitable choice. It appears that alterations in the distance calculation formula do not impact the decision outcomes. This may be attributed to the limitations in the attributes considered in this study and the number of candidate companies. Future research can address certain limitations in the study's findings.

IV. Conclusion

This paper introduces a novel TOPSIS method founded on D-ANP and the Kulcynksi distance, specifically designed for addressing single-valued neutrosophic decision-making problems. Utilizing D-ANP for attribute weight determination allows for comprehensive consideration of interattribute influence and dependencies, eliminating the complex pairwise comparisons inherent to ANP, thereby enhancing the reliability and efficiency of decision-making. Given the limitations of current distance calculation formulas, precise distance measurement holds exceptional importance in the TOPSIS method. Hence, this paper presents the concept of the generalized SVN Kulcynksi distance and provides empirical evidence of its effectiveness. To validate their practical applicability, D-ANP and the Kulcynksi distance are employed in tandem with illustrative examples. Numerous avenues for future research are proposed, encompassing the application of the proposed approach to diverse practical problems across various domains, comparative analysis of the Kulcynksi distance against other distance measures, and exploration of alternative fuzzy sets to enhance uncertainty management.

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