Some Distance, Entropy Measures of Single-valued Complex Neutrosophic Sets and their Applications in Multi-criteria Decision Making

Xu Dongsheng and Bi Tengwu

***Abstract*—single-valued complex neutrosophic sets (SVCNSs) expand upon the concept of neutrosophic sets, offering a framework for handling uncertainty and inconsistency in periodic data. Distance and entropy measures are pivotal tools for managing information characterized by inherent uncertainties. However, research on entropy in the context of single-valued complexes is currently limited. Hence, this paper introduces a range of distance measures and presents an entropy calculation method based on these measures specifically tailored for SVCNS. To start, we provide a definition of single-valued complex neutrosophic sets and expound upon their set-theoretic properties. Following this, we introduce normalized distance formulas and propose an axiomatic definition for SVCNS entropy. Finally, to showcase the practicality and effectiveness of our proposed entropy measure, we include a real-world example involving the selection of green providers. Furthermore, we conduct a comparative analysis with existing methods, highlighting the valuable utility of our approach in addressing uncertainty and inconsistency in data analysis.**

***Index Terms*—single-valued complex neutrosophic sets(SVCNSs); distance measure; entropy; green provider selection.**

I. Introduction

SO as to effectively cope with uncertain information, Zadeh introduced the concept of classical fuzzy sets (FS) in 1965 [1]. Since then, various extensions of classical fuzzy sets have been presented, such as intuitionistic fuzzy sets (IFS) [2], type-2 fuzzy sets (T2FS) [3], etc. Classical fuzzy sets can handle uncertain data. However, they face limitations when dealing with periodic data. To address this issue, Ramot proposed the complex fuzzy set (CFS) by combining fuzzy sets and complex numbers [4]. CFS is valuable in various situations involving uncertainty and imprecision. Unlike traditional fuzzy membership degrees, CFS membership degrees are not limited to the [0, 1] range; instead, they extend to the unit circle in the complex plane. While fuzzy sets can handle uncertain information, they encounter challenges when modeling data with incomplete information. To tackle this problem, Smarandache introduced the neutrosophic set (NS) by incorporating an uncertain membership function into IFS [5]. NS extends IFS with completely independent truth, indeterminacy, and falsity membership functions located in the true criteria [0, 1] or the non-criteria interval $]0, 1[$. However, implementing NS in real-life applications can be challenging. To make NS more practical, Wang et al. presented single-valued neutrosophic sets (SVNS) and interval-valued neutrosophic sets (IVNS) as special cases of NS to address scientific and engineering questions [6], [7]. Building on these developments, Ali and Smarandache introduced the complex neutrosophic set (CNS) by combining CFS and SVNS and applied it to signal processing, demonstrating its effectiveness in handling complex and uncertain data [8].

In engineering applications, distance and entropy measures play a crucial role in handling uncertain, inconsistent, and incomplete information. So far, many distance and entropy measures for classical fuzzy sets (FS), interval-valued fuzzy sets (IVFS), complex fuzzy sets (CFS), and complex interval-valued fuzzy sets (CIFS) have been introduced, and they are widely utilized across various domains, including cluster analysis, pattern identification, and multi-criteria decision-making applications [9], [10], [11]. Furthermore, Zhang [12] presented some entropy measures based on distance for IVIFS and explored the interaction between the entropy, distance, and similarity metric. Bi and Zeng [13] introduced two new methods for calculating entropy in the context of cubic fuzzy sets (CFS) and investigated their rotational invariance properties. For single-valued neutrosophic sets (SVNS) and interval-valued neutrosophic sets (IVNS), several similarity and distance measures have been proposed for SVNS [14] and IVNS [15], [16]. Additionally, the concept of information entropy for neutrosophic sets (NS) has been thoroughly examined from multiple perspectives [17], [18], [19], [20], [21], [22]. Subsequently, Ye [23] presented several distance-based entropy measure formulas for IVNS, enhancing our understanding of this framework. Thao and Smarandache [24] introduced a novel concept of entropy for single-valued neutrosophic sets (SVNS) and explored similarity measures derived from this entropy measure. Şahin [25] provided two different definitions of cross-entropy for interval-valued neutrosophic sets (IVNS) by transforming them into fuzzy sets (FS) and SVNS, respectively. Tan and Zhang [26] defined a novel axiomatic definition of the entropy measure for refined single-valued neutrosophic sets (RSVNS) and further discussed the relationship between distance measures and entropy measures. These diverse distance and entropy measures, along with their applications, make significant contributions to effectively handling uncertainty and inconsistency in engineering and decision-making processes.

However, there has been limited research on entropy and distance measures for single-valued complex neutrosophic sets (SVCNS). Building on prior work, particularly the studies in references [12] and [26], this paper aims to introduce innovative distance and entropy measures for SVCNS and

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apply them in the context of greener provider selection. The paper's structure is as follows: In chapter 2, we provide a brief introduction to the definitions of both neutrosophic sets (NS) and single-valued complex neutrosophic sets (SVCNS). In chapter 3, we present some novel distance formulas for SVCNS. In Chapter 4, we propose an axiomatic definition of entropy for SVCNS and include a numerical example to demonstrate the validity of the presented entropy measure. In Chapter 5, a multi-decision problem is introduced, and a comparative analysis of existing method is conducted to verify the effectiveness of the suggested approach. The conclusions of the paper are provided in chapter 6. The primary goal of this study is to expand the investigation of entropy and distance metrics within the framework of SVCNS and employ them in real-world decision-making scenarios, with a specific emphasis on the selection of environmentally friendly suppliers.

II. Preliminaries

A. Single-valued Neutrosophic Set

Definition 2.1 [5] Suppose θ is a fixed limited space that has a universal object over θ indicated by μ . A NS Ψ is indicated by $\Psi = \{< \mu, T_\Psi(\mu), I_\Psi(\mu), F_\Psi(\mu) > | \mu \in \theta\}$ where T, F, and I are truth, indeterminacy, and falsity membership functions, all of which are real criteria or noncriteria subsets of $]0^-, 1^+[$. That is to say $T \rightarrow]0^-, 1^+[$, $I \rightarrow]0^-, 1^+[$, $F \rightarrow]0^-, 1^+[$. Hence, the condition $0^- \leq \sup T_\Psi(\mu) + \sup I_\Psi(\mu) + \sup F_\Psi(\mu) \leq 3^+$ is satisfied for every point $\mu \in \theta$.

Definition 2.2 [6] Suppose θ is a fixed limited space, with a universal object over θ indicated by μ . A SVNS τ can be represented as follows:

$$\tau = \{< \mu, T_\tau(\mu), I_\tau(\mu), F_\tau(\mu) > | \mu \in \theta\}$$

where $T_\tau(\mu), I_\tau(\mu), F_\tau(\mu)$ are three mapping functions from the real numbers to the interval $[0, 1]$. They indicate truth, indeterminacy and falsity membership degrees of τ, respectively. Therefore, τ fulfills the following addition $0 \leq T(\mu) + I(\mu) + F(\mu) \leq 3$ for every point $\mu \in \theta$. A SVNS τ can take different forms for different sets:

If θ is continuous, a SVNS τ for every $\mu \in \theta$ has the form below:

$$\tau = \int_\theta \langle T_\tau(\mu), I_\tau(\mu), F_\tau(\mu)\rangle / \mu$$

If θ is a crisp set, a SVNS τ for every $\mu \in \theta$ has the form below:

$$\tau = \sum_\theta \langle T_\tau(\mu), I_\tau(\mu), F_\tau(\mu)\rangle / \mu$$

Definition 2.3 [6] Suppose k_1 and k_2 are two SVNSs, which follow the following operational laws.

(1) $k_1 \subset k_2$ iff $T_{k_1}(\mu) \leq T_{k_2}(\mu)$, $I_{k_1}(\mu) \geq I_{k_2}(\mu)$, and $F_{k_1}(\mu) \geq F_{k_2}(\mu)$ for every point $\mu \in \theta$.

(2) $k_1 = k_2$ iff $k_1 \subset k_2$, $k_2 \subset k_1$.

(3) ${k_1}^c = \{< \mu, F_{k_1}(\mu), 1 - I_{k_1}(\mu), T_{k_1}(\mu) > | \mu \in \theta\}$, ${k_1}^c$ is the complement of k_1.

(4) $k_1 \cup k_2 =< \max(T_{k_1}(\mu)), T_{k_2}(\mu)), \min(I_{k_1}(\mu), I_{k_2}(\mu)), \min(F_{k_1}(\mu), F_{k_2}(\mu)) >$ for every object $\mu \in \theta$.

(5) $k_1 \cap k_2 =< \min(T_{k_1}(\mu)), T_{k_2}(\mu)), \max(I_{k_1}(\mu), I_{k_2}(\mu)), \max(F_{k_1}(\mu), F_{k_2}(\mu)) >$ for every object $\mu \in \theta$.

B. Single-valued Complex Neutrosophic Set

Definition 2.4 [8] Suppose θ is a fixed limited space , with a universal element in θ indicated by μ. A single-valued complex neutrosophic set (SVCNS) Υ over θ is constituted by $T_\Upsilon(\mu)$, $I_\Upsilon(\mu)$, and $F_\Upsilon(\mu)$. They represent the truth membership degree, the indeterminacy membership degree, and the falsity membership degree of Υ, respectively. $F_\Upsilon(\mu)$ assigns a compound value to the rank of $T_\Upsilon(\mu)$, $I_\Upsilon(\mu)$, and $F_\Upsilon(\mu)$ for every $\mu \in \theta$. Hence, a SVCNS Υ can be constructed in the following form:

$$\Upsilon = \{\langle \mu, T_\Upsilon(\mu), I_\Upsilon(\mu), F_\Upsilon(\mu)\rangle | \mu \in \theta\}$$

where both the numerical values of $T_\Upsilon(\mu)$, $I_\Upsilon(\mu)$, $F_\Upsilon(\mu)$ and their total are in the complex plane of the unit circle. Hence, the former has the form below:

$$T_\Upsilon(\mu) = t_\Upsilon(\mu).e^{j\omega_\Upsilon(\mu)}, I_\Upsilon(\mu) = i_\Upsilon(\mu).e^{j\psi_\Upsilon(\mu)}$$
$$F_\Upsilon(\mu) = f_\Upsilon(\mu).e^{j\phi_\Upsilon(\mu)}$$

where $\sqrt{j} = -1$, $t_\Upsilon(\mu)$, $i_\Upsilon(\mu)$, $f_\Upsilon(\mu)$ and $\omega_\Upsilon(\mu)$, $\psi_\Upsilon(\mu)$, $\phi_\Upsilon(\mu)$ are real-value functions, and $t_\Upsilon(\mu)$, $i_\Upsilon(\mu)$, $f_\Upsilon(\mu)$ are three mapping functions from the real numbers to the interval $[0, 1]$. Therefore, $0 \leq t_\Upsilon(\mu) + i_\Upsilon(\mu) + f_\Upsilon(\mu) \leq 3$.

From the present, the collection of all SVCNSs over the universe of discourse θ will be denoted by $\gamma(\theta)$.

Definition 2.5 [8] Let Υ^c be the complement of a SVCNS Υ. Then, Υ^c can be indicated as follows:

$$\Upsilon^c = \{\langle \mu, T_{\Upsilon^c}(\mu), I_{\Upsilon^c}(\mu), F_{\Upsilon^c}(\mu)\rangle | \mu \in \theta\}$$

where

$$T_{\Upsilon^c}(\mu) = t_{\Upsilon^c}(\mu).e^{j\omega_{\Upsilon^c}(\mu)}$$
$$I_{\Upsilon^c}(\mu) = i_{\Upsilon^c}(\mu).e^{j\psi_{\Upsilon^c}(\mu)}$$
$$F_\Upsilon(\mu) = f_{\Upsilon^c}(\mu).e^{j\phi_{\Upsilon^c}(\mu)}$$

here, the transformation functions are defined as:

$$t_{\Upsilon^c}(\mu) = f_\Upsilon(\mu)$$
$$\omega_{\Upsilon^c}(\mu) = 2\pi - \omega_\Upsilon(\mu)$$
$$i_{\Upsilon^c}(\mu) = 1 - i_\Upsilon(\mu)$$
$$\psi_{\Upsilon^c}(\mu) = 2\pi - \psi_\Upsilon(\mu)$$
$$f_{\Upsilon^c}(\mu) = \omega_\Upsilon(\mu)$$
$$\phi_{\Upsilon^c}(\mu) = 2\pi - \phi_\Upsilon(\mu)$$

These transformations are applied to the components of the SVCNS Υ to derive the components of its complement Υ^c.

Definition 2.6 [27] Suppose k_1 and k_2 are two SVCNSs. Then, the SVCNS operating rules are listed below:

(1) The sum of k_1 and k_2, indicated as $k_1 + k_2$, is listed below:

$$T_{k_1+k_2}(\mu) = (t_{k_1}(\mu) + t_{k_2}(\mu) - t_{k_1}(\mu)t_{k_2}(\mu)) \times e^{j2\pi\left(\frac{\omega_{k_1}(\mu)}{2\pi} + \frac{\omega_{k_2}(\mu)}{2\pi} - \frac{\omega_{k_1}(\mu)\omega_{k_2}(\mu)}{(2\pi)^2}\right)}$$

$$I_{k_1+k_2}(\mu) = (i_{k_1}(\mu)\, i_{k_2}(\mu)).e^{j2\pi\left(\frac{\psi_{k_1}(\mu)}{2\pi}.\frac{\psi_{k_2}(\mu)}{2\pi}\right)}$$

$$F_{k_1+k_2}(\mu) = (f_{k_1}(\mu)\, f_{k_2}(\mu)).e^{j2\pi\left(\frac{\phi_{k_1}(\mu)}{2\pi}.\frac{\phi_{k_2}(\mu)}{2\pi}\right)}$$

(2) The product of k_1 and k_2, indicated as $k_1 \times k_2$, is listed below:

$$T_{k_1 \times k_2}(\mu) = (t_{k_1}(\mu) t_{k_2}(\mu)) .e^{j2\pi\left(\frac{\omega_{k_1}(\mu)}{2\pi} \cdot \frac{\omega_{k_2}(\mu)}{2\pi}\right)}$$

$$I_{k_1 \times k_2}(\mu) = (i_{k_1}(\mu) + i_{k_2}(\mu) - i_{k_1}(\mu) i_{k_2}(\mu)) \times e^{j2\pi\left(\frac{\psi_{k_1}(\mu)}{2\pi} + \frac{\psi_{k_2}(\mu)}{2\pi} - \frac{\psi_{k_1}(\mu)\psi_{k_2}(\mu)}{(2\pi)^2}\right)}$$

$$F_{k_1 \times k_2}(\mu) = (f_{k_1}(\mu) + f_{k_2}(\mu) - f_{k_1}(\mu) f_{k_2}(\mu)) \times e^{j2\pi\left(\frac{\phi_{k_1}(\mu)}{2\pi} + \frac{\phi_{k_2}(\mu)}{2\pi} - \frac{\phi_{k_1}(\mu)\phi_{k_2}(\mu)}{(2\pi)^2}\right)}$$

(3) The scalar multiplication of k_1 is a SVCNS indicated as $k_3 = \lambda k_1 (\lambda > 0)$ and is listed below:

$$T_{k_3}(\mu) = \left(1 - (1 - t_{k_1}(\mu))^{\lambda}\right) .e^{j2\pi\left(1 - (1 - \frac{\omega_{k_1}(\mu)}{2\pi}\right)^{\lambda}}$$

$$I_{k_3}(\mu) = \left((i_{k_1}(\mu))^{\lambda}\right) .e^{j2\pi\left(\frac{\psi_{k_1}(\mu)}{2\pi}\right)^{\lambda}}$$

$$F_{k_3}(\mu) = \left((f_{k_1}(\mu))^{\lambda}\right) .e^{j2\pi\left(\frac{\phi_{k_1}(\mu)}{2\pi}\right)^{\lambda}}$$

(4) The power of k_1 is indicated as $k_4 = (k_1)^{\lambda} (\lambda > 0)$, and is listed below:

$$T_{k_4}(\mu) = \left((t_{k_1}(\mu))^{\lambda}\right) .e^{j2\pi\left(\frac{\omega_{k_1}(\mu)}{2\pi}\right)^{\lambda}}$$

$$I_{k_4}(\mu) = \left(1 - (1 - i_{k_1}(\mu))^{\lambda}\right) .e^{j2\pi\left(1 - \left(1 - \frac{\psi_{k_1}(\mu)}{2\pi}\right)^{\lambda}\right)}$$

$$F_{k_4}(\mu) = \left(1 - (1 - f_{k_1}(\mu))^{\lambda}\right) .e^{j2\pi\left(1 - \left(1 - \frac{\phi_{k_1}(\mu)}{2\pi}\right)^{\lambda}\right)}$$

Definition 2.7 [27] Suppose that k_1 is a SVCNN. Then, the score function $S(k_1)$ and the accuracy function $T(k_1)$ of k_1 are designated by the functions:

$$S(k_1) = \frac{1}{6}((2 + T_{k_1} - I_{k_1} - F_{k_1}) + \frac{1}{2\pi}(4\pi + \omega_{k_1} - \psi_{k_1} - \phi_{k_1}))$$

$$T(k_1) = \frac{1}{2}\left((T_{k_1} - F_{k_1}) + \frac{1}{2\pi}(\omega_{k_1} - \phi_{k_1})\right)$$

Definition 2.8 [27] Suppose α and β are two SVCNNs. S is the score function, and T is the accuracy function. If $S(\alpha) < S(\beta)$, this implies that $\alpha < \beta$; if $S(\alpha) = S(\beta)$, then

(1) If $T(\alpha) < T(\beta)$, this implies that $\alpha < \beta$;

(2) If $T(\alpha) = T(\beta)$, this implies that $\alpha = \beta$.

Definition 2.9 Suppose $b_l = \left(t_l.e^{j\omega_l}, i_l.e^{j\psi_l}, f_l.e^{j\phi_l}\right)$ is a collection of SVCNNs, the single-valued complex neutrosophic weighted averaging (SVCNWA) operator is designated by the function:

$$SVCNWA_w(b_1, b_2, ..., b_m) = \sum_{l=1}^{m} w_l b_l$$

where $w = (w_1, w_2, w_3, ..., w_m)$ is the weight vector of $b_l (l = 1, 2, 3, ..., m)$, with $0 \leq w_l \leq 1$ and $\sum_{l=1}^{n} w_l = 1$. The SVCNWA operator is denoted as:

$$SVCNWA_w(b_1, b_2, ..., b_m)$$

$$= \begin{pmatrix} 1 - \prod_{l=1}^{m}(1 - t_l)^{w_l} .e^{j2\pi\left(1 - \prod_{l=1}^{m}\left(1 - \frac{\omega_l}{2\pi}\right)^{w_l}\right)}, \\ \prod_{l=1}^{m}(i_l)^{w_l} .e^{j2\pi\left(\prod_{l=1}^{m}\left(\frac{\psi_l}{2\pi}\right)^{w_l}\right)}, \\ \prod_{l=1}^{m}(f_l)^{w_l} .e^{j2\pi\left(\prod_{l=1}^{m}\left(\frac{\phi_l}{2\pi}\right)^{w_l}\right)} \end{pmatrix} \quad (1)$$

When the weight vector is $w = \left(\frac{1}{m}, \frac{1}{m}, ..., \frac{1}{m}\right)$, the SVCNWA operator shall be simplified to a single-valued complex neutrosophic average (SVCNA) operator.

III. Distance measures between SVCNSs

Definition 3.1 [28] Suppose θ is a fixed limited space. A distance measure of the single-valued complex neutrosophic set is a function $d : \gamma(\theta) \times \gamma(\theta) \to [0, 1]$, which fulfills the four properties below: for any $k_1, k_2, k_3 \in \gamma(\theta)$

(1) $0 \leq d(k_1, k_2) \leq 1$;

(2) $d(k_1, k_2) = 0$ if and only if $k_1 = k_2$;

(3) $d(k_1, k_2) = d(k_2, k_1)$;

(4) $d(k_1, k_3) \leq d(k_1, k_2) + d(k_2, k_3)$;

In the following we will introduce some normalized distance formulas for single-valued complex neutrosophic sets.

Suppose that k_1 and k_2 are two SVCNSs in $\theta = \{\mu_1, \mu_2, ..., \mu_m\}$. The four normalized distance formulas for k_1 and k_2 are as follows:

Definition 3.2 The normalized Hamming distance:

$$d_{Hm}(k_1, k_2) = \frac{1}{6m}\sum_{j=1}^{m}\Big\{\left|t_{k_1}(\mu_j) - t_{k_2}(\mu_j)\right| + \left|i_{k_1}(\mu_j) - i_{k_2}(\mu_j)\right| + \left|f_{k_1}(\mu_j) - f_{k_2}(\mu_j)\right| + \frac{1}{2\pi}\left|\widehat{\omega_{k_1}}(\mu_j) - \widehat{\omega_{k_2}}(\mu_j)\right| + \left|\widehat{\psi_{k_1}}(\mu_j) - \widehat{\psi_{k_2}}(\mu_j)\right| + \frac{1}{2\pi}\left|\widehat{\phi_{k_1}}(\mu_j) - \widehat{\phi_{k_2}}(\mu_j)\right|\Big\} \quad (2)$$

Definition 3.3 The normalized Euclidean distance:

$$d_{Eu}(k_1, k_2) = \frac{1}{6m}\sum_{j=1}^{m}\Bigg\{\left|t_{k_1}(\mu_j) - t_{k_2}(\mu_j)\right|^2 + \left|i_{k_1}(\mu_j) - i_{k_2}(\mu_j)\right|^2 + \left|f_{k_1}(\mu_j) - f_{k_2}(\mu_j)\right|^2 + \frac{1}{2\pi}\left(\left|\widehat{\omega_{k_1}}(\mu_j) - \widehat{\omega_{k_2}}(\mu_j)\right|^2 + \left|\widehat{\psi_{k_1}}(\mu_j) - \widehat{\psi_{k_2}}(\mu_j)\right|^2 + \left|\widehat{\phi_{k_1}}(\mu_j) - \widehat{\phi_{k_2}}(\mu_j)\right|^2\right)\Bigg\}^{\frac{1}{2}} \quad (3)$$

Definition 3.4 The normalized Hausdorff distance:

$$d_{Ha}(k_1, k_2) = \frac{1}{m}\sum_{j=1}^{m}\max\Big\{\max\Big\{\left|t_{k_1}(\mu_j) - t_{k_2}(\mu_j)\right|, \left|i_{k_1}(\mu_j) - i_{k_2}(\mu_j)\right|, \left|f_{k_1}(\mu_j) - f_{k_2}(\mu_j)\right|\Big\}, \frac{1}{2\pi}\max\Big\{\left|\widehat{\omega_{k_1}}(\mu_j) - \widehat{\omega_{k_2}}(\mu_j)\right|, \left|\widehat{\psi_{k_1}}(\mu_j) - \widehat{\psi_{k_2}}(\mu_j)\right|, \left|\widehat{\phi_{k_1}}(\mu_j) - \widehat{\phi_{k_2}}(\mu_j)\right|\Big\}\Big\} \quad (4)$$

Definition 3.5 The normalized Fifth distance measure:

$$d_{Fd}(k_1,k_2)=\frac{1}{4m}\sum_{j=1}^{m}\Bigg\{\bigg(\left|t_{k_1}(\mu_j)-t_{k_2}(\mu_j)\right| + \left|i_{k_1}(\mu_j)-i_{k_2}(\mu_j)\right|+\left|f_{k_1}(\mu_j)-f_{k_2}(\mu_j)\right|\bigg)/3 + \max\bigg(\left|t_{k_1}(\mu_j)-t_{k_2}(\mu_j)\right|,\left|i_{k_1}(\mu_j)-i_{k_2}(\mu_j)\right|, \left|f_{k_1}(\mu_j)-f_{k_2}(\mu_j)\right|\bigg)+\frac{1}{2\pi}\Bigg\{\bigg(\left|\widehat{\omega_{k_1}}(\mu_j)-\widehat{\omega_{k_2}}(\mu_j)\right| + \left|\widehat{\psi_{k_1}}(\mu_j)-\widehat{\psi_{k_2}}(\mu_j)\right|+\left|\widehat{\phi_{k_1}}(\mu_j)-\widehat{\phi_{k_2}}(\mu_j)\right|\bigg)/3 + \max\bigg(\left|\widehat{\omega_{k_1}}(\mu_j)-\widehat{\omega_{k_2}}(\mu_j)\right|,\left|\widehat{\psi_{k_1}}(\mu_j)-\widehat{\psi_{k_2}}(\mu_j)\right|, \left|\widehat{\phi_{k_1}}(\mu_j)-\widehat{\phi_{k_2}}(\mu_j)\right|\bigg)\Bigg\} \tag{5}$$

where $0\le\widehat{\omega}\le 2\pi$ and $\begin{cases}\widehat{\omega}=\omega+2\,(c+1)\,\pi & \omega\prec 0\\ \widehat{\omega}=\omega-2c\pi & \omega\succ 2\pi\end{cases}$ in which c is a positive integer and $c=\left[\left|\frac{\omega}{2\pi}\right|\right]$. Similarly, $\widehat{\psi}$ and $\widehat{\phi}$ satisfy the same conditions, respectively. We shall now prove that d_{Hm}, d_{Eu}, d_{Ha}, d_{Fd} fulfill the four conditions mentioned above.

Proof: Taking Hamming distance $d_{Hm}(k_1,k_2)$ as an example, we could easy to see that $d_{Hm}(k_1,k_2)$ fulfills the conditions (1) and (3). Thus, we just go to justify the condition (2) and (4).

For the condition (2), $k_1=k_2\Rightarrow d_{Hm}(k_1,k_2)=0$ is easy to see.

We shall prove below that $d_{Hm}(k_1,k_2)=0\Rightarrow k_1=k_2$. Firstly, by $d_{Hm}(k_1,k_2)=0$, we can conclude that: $t_{k_1}(\mu_j)=t_{k_2}(\mu_j)$, $i_{k_1}(\mu_j)=i_{k_2}(\mu_j)$, $f_{k_1}(\mu_j)=f_{k_2}(\mu_j)$, $\widehat{\omega_{k_1}}(\mu_j)=\widehat{\omega_{k_2}}(\mu_j)$, $\widehat{\psi_{k_1}}(\mu_j)=\widehat{\psi_{k_2}}(\mu_j)$ and $\widehat{\phi_{k_1}}(\mu_j)=\widehat{\phi_{k_2}}(\mu_j)\,(j=1,2,...,m)$.

Then, we can obtain: $\omega_{k_1}=\omega_{k_2}\pm 2c\pi$ in which c is an integer.

Obviously, $e^{j\omega_{k_1}}=e^{j\left(\omega_{k_2}\pm 2c\pi\right)}=e^{j\omega_{k_2}}$, hence, $e^{j\omega_{k_1}}=e^{j\omega_{k_2}}$. Similarly, ψ and ϕ satisfy the same conditions, respectively.

Thus, $k_1=k_2$.

For the condition (4), let $k_1,k_2,k_3\in\gamma(\theta)$. By the triangular inequality, we can obtain:

$$\left|t_{k_1}(\mu_j)-t_{k_2}(\mu_j)\right|+\left|t_{k_2}(\mu_j)-t_{k_3}(\mu_j)\right| \ge\left|t_{k_1}(\mu_j)-t_{k_3}(\mu_j)\right|$$

$$\left|i_{k_1}(\mu_j)-i_{k_2}(\mu_j)\right|+\left|i_{k_2}(\mu_j)-i_{k_3}(\mu_j)\right| \ge\left|i_{k_1}(\mu_j)-i_{k_3}(\mu_j)\right|$$

$$\left|f_{k_1}(\mu_j)-f_{k_2}(\mu_j)\right|+\left|f_{k_2}(\mu_j)-f_{k_3}(\mu_j)\right| \ge\left|f_{k_1}(\mu_j)-f_{k_3}(\mu_j)\right|$$

$$\left|\widehat{\omega_{k_1}}(\mu_j)-\widehat{\omega_{k_2}}(\mu_j)\right|+\left|\widehat{\omega_{k_2}}(\mu_j)-\widehat{\omega_{k_3}}(\mu_j)\right| \ge\left|\widehat{\omega_{k_1}}(\mu_j)-\widehat{\omega_{k_3}}(\mu_j)\right|$$

$$\left|\widehat{\psi_{k_1}}(\mu_j)-\widehat{\psi_{k_2}}(\mu_j)\right|+\left|\widehat{\psi_{k_2}}(\mu_j)-\widehat{\psi_{k_3}}(\mu_j)\right| \ge\left|\widehat{\psi_{k_1}}(\mu_j)-\widehat{\psi_{k_3}}(\mu_j)\right|$$

$$\left|\widehat{\phi_{k_1}}(\mu_j)-\widehat{\phi_{k_2}}(\mu_j)\right|+\left|\widehat{\phi_{k_2}}(\mu_j)-\widehat{\phi_{k_3}}(\mu_j)\right| \ge\left|\widehat{\phi_{k_1}}(\mu_j)-\widehat{\phi_{k_3}}(\mu_j)\right|$$

Hence, we can obtain $d\,(k_1,k_2)+d\,(k_2,k_3)\ge d\,(k_1,k_3)$. The proof of the Definition 3.1. is completed.

IV. Entropy of a SVCNS

A. *Axiomatic Definition of the Entropy for SVCNS*

This chapter introduced the definition of SVCNS entropy, drawing inspiration from previous works [12] and [26].

Definition 4.1: A real-value function $E:\gamma(\theta)\to[0,1]$ is known as an entropy measure for a SVCNS M over the universe of discourse θ, if E fulfills the following conditions:

(1) $E(M)=0$ if M is a crisp set;

(2) $E(M)=1$ if and only if $M=N=$

$$\left\{\left\langle\mu_i,0.5e^{j\pi},0.5e^{j\pi},0.5e^{j\pi}\right\rangle\mid\mu_i\in\theta\right\};$$

(3) If $d(M,N)\ge d(P,N)$, this means that $E(M)\le E(P)$ for every point $M,P\in\gamma(\theta)$, where d is one of the four proposed distance formulas for SVCNS;

(4) $E(M)=E(M^c)$. The calculated entropy value of the complement of M is the same as the calculated entropy value of M.

In the following, we shall present some new entropy measure formulas for SVCNS that are consistent with all the conditions mentioned above.

Theorem 1: Given the four distance measures for SVCNSs, denoted as d_{Hm},d_E,d_{Ha},d_{Fd}, for any $M\in\gamma(\theta)$, the entropy measures of M can be expressed as $E_k\,(M)=1-2d_k\,(M,N)$, where k takes values from the set $k=Hm,Eu,Ha,Fd$.

Based on Theorem 1 and the four proposed distance formulas for SVCNS, we can derive four entropy value calculation formulas, denoted as $(E_{Hm},E_{Hu},E_{Ha},E_{Fd})$ for a SVCNS M in $\theta=\{\mu_1,\mu_2,...,\mu_m\}$. These entropy formulas take the following form:

$$E_{Hm}(M)=1-2d_{Hm}(M,N)= 1-\frac{1}{3m}\sum_{j=1}^{m}\Bigg\{\left|t_M(\mu_j)-0.5\right|+\left|i_M(\mu_j)-0.5\right| +\left|f_M(\mu_j)-0.5\right|\Bigg\}\quad+\frac{1}{2\pi}\sum_{j=1}^{m}\Bigg\{\left|\widehat{\omega_M}(\mu_j)-\pi\right| +\left|\widehat{\psi_M}(\mu_j)-\pi\right|+\left|\widehat{\phi_M}(\mu_j)-\pi\right|\Bigg\} \tag{6}$$

$$E_{Eu}(M)=1-2d_E(M,N)= 1-\frac{1}{3m}\sum_{j=1}^{m}\Bigg\{\left|t_M(\mu_j)-0.5\right|^2+\left|i_M(\mu_j)-0.5\right|^2 +\left|f_M(\mu_j)-0.5\right|^2+\frac{1}{(2\pi)^2}\left|\widehat{\omega_M}(\mu_j)-\pi\right|^2 +\left|\widehat{\psi_M}(\mu_j)-\pi\right|^2+\left|\widehat{\phi_M}(\mu_j)-\pi\right|^2\Bigg\}^{1/2} \tag{7}$$

$$E_{Ha}(M) = 1 - 2d_{Ha}(M,N) = 1 - \frac{2}{m}\sum_{j=1}^{m}\max\left\{\max\left\{|t_M(\mu_j) - 0.5|, |i_M(\mu_j) - 0.5|, |f_M(\mu_j) - 0.5|\right\}, \frac{1}{2\pi}\max\left\{|\widehat{\omega_M}(\mu_j) - \pi|, \left|\widehat{\psi_M}(\mu_j) - \pi\right|, \left|\widehat{\phi_M}(\mu_j) - \pi\right|\right\}\right\} \tag{8}$$

$$E_{Fd}(M) = 1 - 2d_{Fd}(M,N) = 1 - \sum_{j=1}^{m}\left\{\frac{\left(\left|t_M(\mu_j) - \frac{1}{2}\right| + \left|i_M(\mu_j) - \frac{1}{2}\right| + \left|f_M(\mu_j) - \frac{1}{2}\right|\right)}{6m} + \frac{\max\left(|t_M(\mu_j) - 0.5|, |i_M(\mu_j) - 0.5|, |f_M(\mu_j) - 0.5|\right)}{2m} + \frac{\left(|\widehat{\omega_M}(\mu_j) - \pi| + \left|\widehat{\psi_M}(\mu_j) - \pi\right| + \left|\widehat{\phi_M}(\mu_j) - \pi\right|\right)}{6m\pi} + \frac{\max\left(|\widehat{\omega_M}(\mu_j) - \pi|, \left|\widehat{\psi_M}(\mu_j) - \pi\right|, \left|\widehat{\phi_M}(\mu_j) - \pi\right|\right)}{2m\pi}\right\} \tag{9}$$

We shall prove below that $E_{Hm}, E_{Hu}, E_{Ha}, E_{Fd}$ fulfill the four conditions mentioned above.

Proof: We will take $E_{Hm}(M)$ as an example and prove that E_{Hm} fulfills the four criteria in Definition 4.1.

For condition (1), if M is a crisp set, i.e., $M = \left\{\left\langle \mu_j, e^{j2\pi}, 0, 0\right\rangle | \mu_j \in \theta\right\}$ or $\left\{\left\langle \mu_j, 0, 0, e^{j2\pi}\right\rangle | \mu_j \in \theta\right\}$, by equation (6), we can obtain:

$$E_{Hm}(M) = 1 - 2d_{Hm}(M,N) = 0.$$

For condition (2), $M = N \Rightarrow E_{Hm}(M) = 1$ is easy to see.

Now, we prove below that $E_{Hm}(M) = 1 \Rightarrow M = N$.

First of all, by $E(M) = 1$, we can obtain: $d_{Hm} = 0$. Then, by equation (6), we can obtain:

$$\widehat{\omega_M}(\mu_j) = \pi$$
$$\widehat{\psi_M}(\mu_j) = \pi$$
$$\widehat{\phi_M}(\mu_j) = \pi$$

since

$$\omega_M(\mu_j) = \widehat{\omega_M}(\mu_j) \pm 2c_1\pi$$
$$\psi_M(\mu_j) = \widehat{\psi_M}(\mu_j) \pm 2c_2\pi$$
$$\phi_M(\mu_j) = \widehat{\phi_M}(\mu_j) \pm 2c_3\pi$$

where c_1, c_2 and c_3 are integers.

Hence, $M = N$.

In conclusion, $M = \left\{\left\langle \mu_j, \frac{1}{2}e^{j\pi}, \frac{1}{2}e^{j\pi}, \frac{1}{2}e^{j\pi}\right\rangle | \mu_j \in \theta\right\}$.

For condition (3), if $d(M,N) \geq d(P,N)$, since $E(M) = 1 - 2d(M,N)$, we can obtain: $E(M) \leq E(P)$.

For condition (4), by $\omega_M(\mu_j) = \widehat{\omega_M}(\mu_j) \pm 2c_1\pi$ and $\omega_{M^c}(\mu_j) = \widehat{\omega_{M^c}}(\mu_j) \pm 2c_2\pi$, where c_1, c_2 are integer, and since $\omega_M(\mu_j) + \omega_{M^c}(\mu_j) = 2\pi$, we can deduce that $\widehat{\omega_M}(\mu_j) + \widehat{\omega_{M^c}}(\mu_j) = 2c_3\pi$ where c_3 is an integer.

Considering that $0 \leq \widehat{\omega} \leq 2\pi$, we can conclude that:

$$\widehat{\omega_M}(\mu_j) + \widehat{\omega_{M^c}}(\mu_j) = 0/2\pi/4\pi$$

When $\widehat{\omega_M}(\mu_j) + \widehat{\omega_{M^c}}(\mu_j) = 0$, it implies that $\widehat{\omega_M}(\mu_j) = \widehat{\omega_{M^c}}(\mu_j) = 0$. Similarly, $\widehat{\psi}$ and $\widehat{\phi}$ satisfy the same conditions, respectively. Hence, $E(M) = E(M^c)$

When $\widehat{\omega_M}(\mu_j) + \widehat{\omega_{M^c}}(\mu_j) = 2\pi$, we can deduce:

$$|\widehat{\omega_M}(\mu_j) - \pi| = |2\pi - \widehat{\omega_{M^c}}(\mu_j) - \pi| = |\widehat{\omega_{M^c}}(\mu_j) - \pi|$$

In a similar manner, $\widehat{\psi}$ and $\widehat{\phi}$ satisfy the same conditions, respectively. Hence, in this case as well, we have $E(M) = E(M^c)$.

When $\widehat{\omega_M}(\mu_j) + \widehat{\omega_{M^c}}(\mu_j) = 4\pi$, since $0 \leq \widehat{\omega} \leq 2\pi$, we can conclude that $\widehat{\omega_M}(\mu_j) = \widehat{\omega_{M^c}}(\mu_j) = 2\pi$. Similarly, $\widehat{\psi}$ and $\widehat{\phi}$ satisfy the same conditions, respectively. Thus, we also have $E(M) = E(M^c)$.

Summarizing, we can deduce $E(M) = E(M^c)$. The proof is completed.

B. Numerical Example

Suppose $\delta = \left\langle \mu, t_\delta(\mu)e^{j\omega_\delta(\mu)}, i_\delta(\mu)e^{j\psi_\delta(\mu)}, f_\delta(\mu)e^{j\phi_\delta(\mu)}\right\rangle$. For any positive real number m, the SVCNS $\eta = \delta^m$ can be derived as follows:

$$\begin{aligned} T_\eta(\mu) &= \left((t_\delta(\mu))^m\right) \cdot e^{j2\pi\left(\frac{\omega_\delta(\mu)}{2\pi}\right)^m} \\ I_\eta(\mu) &= \left(1 - (1 - i_\delta(\mu))^m\right) \cdot e^{j2\pi\left(1 - \left(1 - \frac{\psi_\delta(\mu)}{2\pi}\right)^m\right)} \\ F_\eta(\mu) &= \left(1 - (1 - f_\delta(\mu))^m\right) \cdot e^{j2\pi\left(1 - \left(1 - \frac{\phi_\delta(\mu)}{2\pi}\right)^m\right)} \end{aligned} \tag{10}$$

Now we consider the SVCNS δ on $\theta = \{1, 2, 3, 4, 5\}$ as follows:

$$\delta = \left\{\begin{array}{l} \left\langle 1, 0.1e^{j2\pi(0.35)}, 0.7e^{j2\pi(0.45)}, 0.6e^{j2\pi(0.55)}\right\rangle \\ \left\langle 2, 0.3e^{j2\pi(0.4)}, 0.6e^{j2\pi(0.45)}, 0.5e^{j2\pi(0.45)}\right\rangle \\ \left\langle 3, 0.6e^{j2\pi(0.5)}, 0.5e^{j2\pi(0.4)}, 0.5e^{j2\pi(0.45)}\right\rangle \\ \left\langle 4, 0.8e^{j2\pi(0.55)}, 0.4e^{j2\pi(0.35)}, 0.3e^{j2\pi(0.35)}\right\rangle \\ \left\langle 5, 0.9e^{j2\pi(0.6)}, 0.2e^{j2\pi(0.4)}, 0.1e^{j2\pi(0.3)}\right\rangle \end{array}\right\}$$

Taking into account the quality of the linguistic variables, we classify the variables δ in as "GOOD".

δ^2 can be considered as "Very GOOD";

δ^3 can be considered as "Quite very GOOD";

δ^4 can be considered as "Very very GOOD".

By Equation (10), we can generate the following SVCNSs:

$$\delta^2 = \left\{\begin{array}{l} \left\langle 1, 0.01e^{j2\pi(0.1225)}, 0.91e^{j2\pi(0.6975)}, 0.84e^{j2\pi(0.75)}\right\rangle \\ \left\langle 2, 0.09e^{j2\pi(0.16)}, 0.84e^{j2\pi(0.6975)}, 0.75e^{j2\pi(0.6975)}\right\rangle \\ \left\langle 3, 0.36e^{j2\pi(0.25)}, 0.75e^{j2\pi(0.64)}, 0.75e^{j2\pi(0.6975)}\right\rangle \\ \left\langle 4, 0.64e^{j2\pi(0.3025)}, 0.64e^{j2\pi(0.5775)}, 0.51e^{j2\pi(0.5775)}\right\rangle \\ \left\langle 5, 0.81e^{j2\pi(0.36)}, 0.36e^{j2\pi(0.64)}, 0.19e^{j2\pi(0.51)}\right\rangle \end{array}\right\}$$

$$\delta^3 =$$

TABLE I
AMBIGUITY COMPARISON WITH VARIOUS ENTROPY MEASUREMENTS FOR SVCNS

SVCNS	E_{Hm}	E_{Eu}	E_{Ha}	E_{Fd}
δ	0.7267	0.6385	0.4400	0.6533
δ^2	0.5520	0.4947	0.3370	0.4855
δ^3	0.3720	0.3272	0.2140	0.3114
δ^4	0.2580	0.2230	0.1167	0.2159

$$\left\{\begin{array}{l} \left\langle 1, 0.001e^{j2\pi(0.0429)}, 0.973e^{j2\pi(0.8336)}, 0.936e^{j2\pi(0.875)} \right\rangle \\ \left\langle 2, 0.027e^{j2\pi(0.064)}, 0.936e^{j2\pi(0.8336)}, 0.875e^{j2\pi(0.8336)} \right\rangle \\ \left\langle 3, 0.216e^{j2\pi(0.125)}, 0.875e^{j2\pi(0.784)}, 0.875e^{j2\pi(0.8336)} \right\rangle \\ \left\langle 4, 0.512e^{j2\pi(0.1664)}, 0.784e^{j2\pi(0.7254)}, 0.657e^{j2\pi(0.7254)} \right\rangle \\ \left\langle 5, 0.729e^{j2\pi(0.216)}, 0.488e^{j2\pi(0.784)}, 0.271e^{j2\pi(0.657)} \right\rangle \end{array}\right\}$$

$$\delta^4 =$$

$$\left\{\begin{array}{l} \left\langle 1, 0.0001e^{j2\pi(0.015)}, 0.9919e^{j2\pi(0.9085)}, 0.9744e^{j2\pi(0.9375)} \right\rangle \\ \left\langle 2, 0.081e^{j2\pi(0.0256)}, 0.9744e^{j2\pi(0.9085)}, 0.9375e^{j2\pi(0.9085)} \right\rangle \\ \left\langle 3, 0.1296e^{j2\pi(0.0625)}, 0.9375e^{j2\pi(0.8704)}, 0.9375e^{j2\pi(0.9085)} \right\rangle \\ \left\langle 4, 0.4096e^{j2\pi(0.0915)}, 0.8704e^{j2\pi(0.8215)}, 0.7599e^{j2\pi(0.8215)} \right\rangle \\ \left\langle 5, 0.6561e^{j2\pi(0.1296)}, 0.5904e^{j2\pi(0.8704)}, 0.3439e^{j2\pi(0.7599)} \right\rangle \end{array}\right\}$$

We can calculate the values of $E_k\,(k = Hm, Eu, Ha, Fd)$ using equations (6-9), as shown in Table I.

From both a human intuition and mathematical operations perspective, these entropy measurements for SVCNS should be sorted as follows: $E_k(\delta) \succ E_k(\delta^2) \succ E_k(\delta^3) \succ E_k(\delta^4)$. Therefore, the entropy values computed in Table I demonstrate that $E_k\,(k = Hm, Eu, Ha, Fd)$ exhibit favorable properties with respect to structured linguistic variables.

V. DECISION-MAKING EXAMPLE

A. An Approach for MAGDM

In this section, we utilize the operational guidelines and the previously defined entropy measures for SVCNS to present a multi-attribute group decision-making (MAGDM) approach.

Let's consider a scenario where there is a committee of l_1 experts $(E_h, h = 1, 2, ..., l_1)$ tasked with evaluating l_2 alternatives $(A_p, p = 1, 2, ..., l_2)$ across l_3 selection criteria $(C_q, q = 1, 2, ..., l_3)$. In this evaluation, the performance ratings for these alternatives are represented as single-valued complex neutrosophic numbers (SVCNNs). The process for the MAGDM method is outlined as follows:

Step 1: Constructing the evaluation matrix M_λ for expert E_h.

In this step, we construct the evaluation matrix M_λ represented as $M_\lambda = \left[m_{ij}^\lambda\right]_{l_2 \times l_3}$, where $i = 1, 2, ..., l_2$, $j = 1, 2, ..., l_3$, and $\lambda = 1, 2, ..., l_1$. Each element m_{ij}^λ is defined as:

$$m_{ij}^\lambda = \langle T_{ij}^\lambda, I_{ij}^\lambda, F_{ij}^\lambda \rangle = \left\langle t_{ij}^\lambda.e^{j\omega_{ij}^\lambda}, i_{ij}^\lambda.e^{j\psi_{ij}^\lambda}, f_{ij}^\lambda.e^{j\phi_{ij}^\lambda} \right\rangle$$

where m_{ij}^λ is represented by a single-valued complex neutrosophic number (SVCNN) and contains the single-valued complex neutrosophic message for alternative A_P regarding attribute C_q as assessed by expert E_h.

Step 2: Normalization of the decision matrix.

$$m_{ij}^\lambda = \begin{cases} m_{ij}^\lambda & j \in benefit \quad attributes \\ \left(m_{ij}^\lambda\right)^c & j \in \cos t \quad attributes \end{cases}$$

Step 3: Aggregation of information for alternatives and criteria

Using the decision-making matrix $M_\lambda = \left[m_{ij}^\lambda\right]_{l_2 \times l_3}$, we can obtain the comprehensive decision matrix $M = [m_{ij}]_{l_2 \times l_3} = \langle T_{ij}, I_{ij}, F_{ij} \rangle$, which contains aggregated information. Here, m_{ij} is defined as follows:

$$\begin{aligned} m_{ij} &= \sum_{\lambda=1}^{l_1} \frac{1}{l_1} m_{ij}^\lambda \\ &= \frac{1}{l_1} m_{ij}^1 + \frac{1}{l_1} m_{ij}^2 + \ldots + \frac{1}{l_1} m_{ij}^{l_1} \\ &= \frac{1}{l_1} \left(m_{ij}^1 + m_{ij}^2 + \ldots + m_{ij}^{l_1}\right) \end{aligned} \tag{11}$$

where

$$T_{ij} = \left(1 - \prod_{\lambda=1}^{l_1} \left(1 - t_{ij}^\lambda\right)^{\frac{1}{l_1}}\right).e^{j2\pi\left(1 - \prod_{\lambda=1}^{l_1} \left(1 - \frac{\omega_{ij}^\lambda}{2\pi}\right)^{\frac{1}{l_1}}\right)}$$

$$I_{ij} = \left(\prod_{\lambda=1}^{l_1} \left(i_{ij}^\lambda\right)^{\frac{1}{l_1}}\right).e^{j2\pi\left(\prod_{\lambda=1}^{l_1} \left(\frac{\psi_{ij}^\lambda}{2\pi}\right)^{\frac{1}{l_1}}\right)}$$

$$F_{ij} = \left(\prod_{\lambda=1}^{l_1} \left(f_{ij}^\lambda\right)^{\frac{1}{l_1}}\right).e^{j2\pi\left(\prod_{\lambda=1}^{l_1} \left(\frac{\phi_{ij}^\lambda}{2\pi}\right)^{\frac{1}{l_1}}\right)}$$

Step 4: Calculating the entropy of alternatives in the aggregated decision matrix $M = [m_{ij}]_{l_2 \times l_3}$.

In this step, we compute the entropy values for each alternative in the aggregated decision matrix $M = [m_{ij}]_{l_2 \times l_3}$ by equation(6-9).

Step 5: Ranking the alternatives

By applying the proposed entropy measures in a multi-attribute decision problem, we can effectively rank the alternatives. When the entropy value of one alternative is lower than that of others, it indicates that the decision maker can obtain more valuable information from that alternative. Consequently, we can establish the priority and identify the optimal solution by sorting all alternatives in ascending order based on the proposed entropy values for SVCNSs.

B. Numerical Example

In this section, we will illustrate the application of the presented MAGDM method with a numerical example for optimal target selection. We have adapted a multi-criteria decision problem from Xu [27] for this demonstration.

A small printing and transportation company is in the process of selecting a green supplier to purchase a new vehicle for its aftermarket business. The company intends to evaluate three potential suppliers, denoted as S_1, S_2, S_3. A management committee comprising three decision makers, namely $D1$, $D2$, and $D3$, each with different expertise, is responsible for conducting the evaluation. In the selection process, five attributes are taken into consideration: price/cost $(Q1)$, quality $(Q2)$, delivery $(Q3)$, relationship closeness $(Q4)$, and environmental management systems $(Q5)$. It's important to note that all these attributes are of benefit type. The three decision makers assess the suitability of the three potential suppliers in relation to these attributes using a linguistic rating set $R = \{C, B, F, G, T\}$, where:

$$C = \text{Catastrophic} = \left(0.2e^{j2\pi(0.4)}, 0.8e^{j2\pi(0.5)}, 0.7e^{j2\pi(0.55)}\right)$$

$$B = Bad = \left(0.4e^{j2\pi(0.45)}, 0.6e^{j2\pi(0.45)}, 0.6e^{j2\pi(0.5)}\right)$$

$$F = Fair = \left(0.6e^{j2\pi(0.5)}, 0.5e^{j2\pi(0.4)}, 0.5e^{j2\pi(0.45)}\right)$$

$$G = Good = \left(0.8e^{j2\pi(0.55)}, 0.4e^{j2\pi(0.35)}, 0.4e^{j2\pi(0.4)}\right)$$

$$T = Terrific = \left(0.9e^{j2\pi(0.6)}, 0.2e^{j2\pi(0.3)}, 0.1e^{j2\pi(0.35)}\right)$$

The entire green supplier selection process is outlined in the following steps:

Step 1: Constructing the evaluation matrix M_λ.

Three decision makers $(D1, D2, D3)$ will access three potential suppliers (S_1, S_2, S_3) based on five attributes $(Q1, Q2, Q3, Q4, Q5)$. The evaluation matrix $M_\lambda\,(\lambda = 1, 2, 3)$ is constructed as follows:

$$M_1 = \begin{bmatrix} G & F & B & G & B \\ F & T & G & F & G \\ T & T & F & G & G \end{bmatrix}$$

$$M_2 = \begin{bmatrix} F & F & F & F & F \\ F & G & G & F & G \\ G & G & G & T & F \end{bmatrix}$$

$$M_3 = \begin{bmatrix} G & F & B & G & B \\ G & G & G & F & T \\ T & G & F & G & G \end{bmatrix}$$

Step 2: Because all criteria are benefit attributes, they do not need to be standardized.

Step 3: Aggregation of information for alternatives and criteria.

We may derive the aggregated decision matrix M by equation (11) and show it in Table II.

Step 4: Calculating the entropy values of alternatives, as displayed in Table III.

Step 5: Ranking the alternatives.

If an alternative has a relatively small entropy value compared to all other alternatives, it is considered a better choice. Therefore, the alternative with the lowest entropy value is given significant priority and importance. In this case, when we examine the entropy values for all alternatives in Table III using the presented entropy measures for SVCNS (E_k where $k = Hm, Eu, Ha, Fd$), we observe that the order of entropy values for all alternatives is $E_k(S_1) \succ E_k(S_2) \succ E_k(S_3)$. As a result, we can determine the priority order of all potential providers to be $S_3 \succ S_2 \succ S_1$, with S_3 being the top-ranked provider. This aligns with the findings in [27], demonstrating the validity and reasonableness of the proposed entropy measures for SVCNS in decision-making applications.

C. Comparative Analysis

To further validate the proposed method, we compare it with seven existing methods from Xu[29], Ye[30], Cui et al.[31], and Peng et al.[32] to rank the examples mentioned above.

Since there are limited studies on single-valued complex neutrosophic decision-making in academia at present, we adapt the decision-making method for single-valued neutrosophic sets by incorporating the distance formula and score function proposed earlier. We apply this adapted method to solve the single-valued complex neutrosophic decision-making problem. For instance, in the case of SVCNSWA, we first obtain indicator weights using the maximum discretization difference method. Then, we aggregate the decision matrix using WA operator. Finally, we utilize the similarity equation $S(x, y) = 1 - d(x, y)$ for final ranking, where d represents the distance equation presented earlier. Similar procedures are followed for the remaining methods. The sorting results from all methods are obtained and displayed in Table IV. It is evident from the results that different methods yield consistent rankings and optimal choices, underscoring the effectiveness and practicality of the proposed methodology.

VI. Conclusion

Compared to general neutrosophic sets, SVCNS offers the advantage of effectively handling periodic data. While entropy plays a crucial role in fuzzy set theory, there has been relatively little research on entropy measures specifically designed for SVCNS. Therefore, the objective of this paper is to introduce innovative methods for computing entropy values tailored to SVCNS. In this regard, we first present novel normalized distance formulas for SVCNS by incorporating phase term transformations. Subsequently, we introduce entropy formulas that are based on these distance metrics. Finally, we validate and demonstrate the applicability of the proposed entropy measure within the context of green supplier selection through an illustrative example and comparison with existing methods. The entropy measure introduced in this paper exhibits the following strengths and weaknesses:

Advantages:

(1) Since $\{\langle 0.5e^{j\pi}, 0.5e^{j\pi}, 0.5e^{j\pi}\rangle\}$ represents the fuzziest balance point, the entropy value of SVCNS is higher when the distance between SVCNS and equilibrium points is smaller. This alignment with human cognitive perception makes the proposed axiomatic definition of entropy for SVCNS intuitively consistent.

(2) The simplicity of the entropy formula , which relies on straightforward distance metrics, makes it highly computationally efficient. Furthermore, its adaptability allows for the straightforward derivation of new entropy formulas should novel distance metrics be introduced in the future. This flexibility enhances the utility and versatility of the entropy measures for SVCNS proposed in this study.

Disadvantages:

(1) The proposed SVCNS entropy is based on distance measurements, which implies that the effectiveness of the entropy formula is highly sensitive to the appropriate selection of distance metrics.

TABLE II
THE AGGREGATED DECISION MATRIX M.

Attribute	Alternative	Aggregated Values
$Q1$	S_1	$(0.748e^{j2\pi(0.5339)}, 0.4309e^{j2\pi(0.3659)}, 0.4309e^{j2\pi(0.416)})$
	S_2	$(0.6825e^{j2\pi(0.5173)}, 0.4642e^{j2\pi(0.3826)}, 0.4642e^{j2\pi(0.4327)})$
	S_3	$(0.874e^{j2\pi(0.584)}, 0.252e^{j2\pi(0.3158)}, 0.1587e^{j2\pi(0.3659)})$
$Q2$	S_1	$(0.60e^{j2\pi(0.50)}, 0.50e^{j2\pi(0.40)}, 0.50e^{j2\pi(0.45)})$
	S_2	$(0.8413e^{j2\pi(0.5673)}, 0.3175e^{j2\pi(0.3325)}, 0.252e^{j2\pi(0.3826)})$
	S_3	$(0.8413e^{j2\pi(0.5673)}, 0.3175e^{j2\pi(0.3325)}, 0.252e^{j2\pi(0.3826)})$
$Q3$	S_1	$(0.4759e^{j2\pi(0.4672)}, 0.5646e^{j2\pi(0.4327)}, 0.5646e^{j2\pi(0.4827)})$
	S_2	$(0.80e^{j2\pi(0.55)}, 0.40e^{j2\pi(0.35)}, 0.40e^{j2\pi(0.40)})$
	S_3	$(0.6825e^{j2\pi(0.5173)}, 0.4642e^{j2\pi(0.3826)}, 0.4642e^{j2\pi(0.4327)})$
$Q4$	S_1	$(0.748e^{j2\pi(0.5339)}, 0.4309e^{j2\pi(0.3659)}, 0.4309e^{j2\pi(0.416)})$
	S_2	$(0.60e^{j2\pi(0.50)}, 0.50e^{j2\pi(0.40)}, 0.50e^{j2\pi(0.45)})$
	S_3	$(0.8413e^{j2\pi(0.5673)}, 0.3175e^{j2\pi(0.3325)}, 0.252e^{j2\pi(0.3826)})$
$Q5$	S_1	$(0.4759e^{j2\pi(0.4672)}, 0.5646e^{j2\pi(0.4327)}, 0.5646e^{j2\pi(0.4827)})$
	S_2	$(0.8413e^{j2\pi(0.5673)}, 0.3175e^{j2\pi(0.3325)}, 0.252e^{j2\pi(0.3826)})$
	S_3	$(0.748e^{j2\pi(0.5339)}, 0.4309e^{j2\pi(0.3659)}, 0.4309e^{j2\pi(0.416)})$

TABLE III
THE ENTROPY VALUES OF ALTERNATIVES

SVCNS	E_{Hm}	E_{Eu}	E_{Ha}	E_{Fd}
S_1	0.8621	0.8186	0.7077	0.8083
S_2	0.7497	0.6865	0.4940	0.6781
S_3	0.5100	0.6269	0.4052	0.6173

TABLE IV
COMPARISON OF DIFFERENT METHODS

Methods	Rinking	Optimal Option
SVCNSWA[30]	$S_3 \succ S_2 \succ S_1$	S_3
SVCNSWG[30]	$S_3 \succ S_2 \succ S_1$	S_3
TOPSIS[29]	$S_3 \succ S_2 \succ S_1$	S_3
MABAC[29]	$S_3 \succ S_2 \succ S_1$	S_3
TOPSIS-MABAC[31]	$S_3 \succ S_2 \succ S_1$	S_3
Similarity Measure 1[29]	$S_3 \succ S_2 \succ S_1$	S_3
Similarity Measure 2 [32]	$S_3 \succ S_2 \succ S_1$	S_3
Our Method	$S_3 \succ S_2 \succ S_1$	S_3

(2) The construction of cross-entropy measures for SVCNS, often used in practical applications such as multi-attribute decision making and pattern recognition, is not easily achieved using the provided entropy equation for SVCNS.

(3) The presented entropy measures for SVCNS do not take into account the significance or importance of individual elements or objects, which may limit their applicability in scenarios where this information is crucial.

Overall, the introduced entropy measures for SVCNS offer valuable insights and computational simplicity but may require careful consideration of distance metric selection and may not cover all aspects of the complexity of real-world decision-making processes. Further research and refinement are needed to address these limitations.

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