Two Types of Failures M/M/1 Queueing System with One Standby Service Station and Start-up Time

Shengli Lv, *Member, IAENG*, Huiling Fu and Jingbo Li

***Abstract*—This paper, based on the M/M/1 queueing system, discusses two types of failures with standby service station and start-up time. In this system, two types of failures could occur in a normal working service station: complete failures and incomplete failures. During the incomplete failure, the service station serves the customer at a lower rate, and during the complete failure, the service station loses service capability. It leads to a standby service station, which does not fail and is immediately available, needs to be activated to continue service. In this paper, the steady-state state transfer diagram of this system is drawn firstly, then the steady-state equilibrium condition and the steady-state probability vector of the system are derived using the matrix geometry solution method, and the steady-state queueing length of the system is obtained. Finally, the conclusions are analyzed numerically by Matlab.**

***Index Terms*—standby service station; start-up time; failure; matrix geometric solution.**

I. Introduction

In our daily life, the queueing problems are very common where congestion problems are likely to occur if a server fails, especially in hospitals, restaurants, scenic spots, airports, supermarkets, and other public places. Therefore, many scholars paid much attention to the situation that the server station may fail in the queueing system. Choudhury and Deka [1] studied a two-stage queueing system containing server failures and Bernoulli vacation, which obtained some reliability indices for the model by methods such as probabilistic generating functions and Laplace Stieltjes transform. Karthick and Suvitha [2] studied a queuing system in which the servers had vacation and service rates were different. Yu et al. [3] studied a service station with complete and incomplete failures. They considered both fully observable and fully unobservable cases. Xu and Wang [4] analyzed the queuing model containing two types of parallel customers through reasonable utility function. The model will completely fail and the start-up time is interruptible, and they analyzed the influence of system parameters on customer behavior strategy through numerical simulation. Yang [5] introduced the start-up time, the working vacation and the working breakdown in the repairable M/M/1/N queueing system by using matrix geometry, the elementary array and the covariance matrix theory, the system performance metrics were derived. Vijayashree and Anjuka [6] analyzed a fluid queue with catastrophes, server failures and repairs. Jing and Tao [7] studied a queueing system with server failures, working vacations and Bernoulli vacation interruption. He obtained the probability production function of the queue and the queue length by matrix analysis and supplementary variable method. Finally, the cost analysis of the model are presented.

In many cases the system cannot be stopped, otherwise it will cause serious property damage or even dangerous conditions, such as the electrical system in hospitals, the cashier in supermarkets, and the machines in factories. So in case of machine failure, it is necessary to start the standby station to maintain the normal operation. Klimenok [8] studied multi-server queuing systems with backup servers. Xu et al. [9] added a standby server station based on the Geom/Geom/1 queueing model, and derived expressions for the performance metrics. Huang and Ye [10] further studied the service capacity supplement strategy based on the existing customer queueing behavior decision model, and the simulation results showed that the probability of customer retention was reduced and the queueing length was maintained at a low and steady level after the standby service station was turned on. Hu et al. [11] considered an M/M/1 queueing systems of vacation that with the start-up time, the standby attendants, and the customers with positive and negative. Ma et al. [12] added a standby attendant to the repairable queuing system with vacation.

In this paper, the standby service station is added based on two types of failures that can occur in the system, and the system can still maintain normal operation in the case of complete failure of the service station to avoid security accidents and property losses.

II. Model Description

1) The process of customer arrival to the system follows the Poisson process with λ. Customers service is according to the first-come, first-served mechanism.

2) There is one standard service station and one standby service station in the system. The service times both obey negative exponential distribution with parameters μ_1 and μ_2 ($\mu_2 < \mu_1$) respectively, and the standard service station provides service first.

3) The complete failure and incomplete failure may occur in the standard service station. In the event of incomplete

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Shengli Lv is an associate professor in the School of Science, Yanshan University, Qinhuangdao, Hebei 066004, PR China. (e-mail: qhdddlsl@163.com).

Huiling Fu is a postgraduate student in the School of Science, Yanshan University, Qinhuangdao, Hebei 066004, PR China. (corresponding author to provide phone: +86-158-330-52680; e-mail: fuhuiling929@163.com).

Jingbo Li is an associate professor in the School of Hebei Normal University of Science and Technology, Qinhuangdao, Hebei 066004, PR China. (e-mail: lijingbo668@126.com).

failure of the standard service station, the system still retains part of the service capacity and continues to serve customers at a low rate μ_v ($\mu_v < \mu_2 < \mu_1$). The standard service station loses its service capacity in the event of complete failure and activates the standby service station for service immediately. The process of incomplete fault occurrence and the process of complete fault occurrence are both Poisson processes, and the parameters are ϵ_1 and ϵ_2 respectively. For the standard service station where an incomplete failure occurs, the time spend on the repair process is an exponential distribution with ζ.

4) When there are no customers in this system, the standard service station enters the shutdown period, at which the arriving customer cannot be served immediately and needs to go through a start-up period. The start-up time obeys an exponential distribution with parameter s. After the start-up time, the service station enters a regular busy period.

5) Assume that the waiting process of customers, the service process of the service station, the process of incomplete failure occurrence, and the process of complete failure occurrence and repair time are independent with each other.

$\{Q_{(\mathrm{t})}, J_{(t)}\}$ refers to the state of system at moment t, $Q_{(t)}$ refers to the number of customers in the system at moment t, $J_{(t)}$ is defined as the service state of the standard service station.

$$J_{(\mathrm{t})} = \begin{cases} 0, & \text{Closing or start} - \text{up period}, \\ 1, & \text{Normal working period}, \\ 2, & \text{Total failure period} \\ & \text{(The standby service station is activated)}, \\ 3, & \text{Incomplete failure period.} \end{cases}$$

The state transition space for $\{Q(t), J(t)\}$ is

$$\Omega = \{(0,0),(0,3)\} \cup \{(k,j), k \geq 1, j = 0,1,2,3\}.$$

By ordering the states of the model in lexicographical order, the state transition of the system is shown in Figure 1.

In lexicographical order, the infinitesimal generating matrix $\mathcal{Q}$ can be written as the following partitioned tridiagonal matrix.

$$\mathcal{Q} = \begin{pmatrix} A_0 & C_0 & & & \\ B_1 & A & C & & \\ & B & A & C & \\ & & B & A & C \\ & & & \ddots & \ddots & \ddots \end{pmatrix},$$

where

$$A_0 = \begin{pmatrix} -\lambda & 0 \\ \xi & -(\lambda+\xi) \end{pmatrix}, C_0 = \begin{pmatrix} \lambda & 0 & 0 & 0 \\ 0 & 0 & 0 & \lambda \end{pmatrix},$$

$$B_1 = \begin{pmatrix} 0 & 0 \\ \mu_1 & 0 \\ \mu_2 & 0 \\ 0 & \mu_v \end{pmatrix}, B = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \mu_1 & 0 & 0 \\ 0 & 0 & \mu_2 & 0 \\ 0 & 0 & 0 & \mu_v \end{pmatrix},$$

$$A = \begin{pmatrix} -(\lambda+\mathrm{s}) & s & 0 & 0 \\ 0 & \mathrm{A}_{21} & \varepsilon_2 & \varepsilon_1 \\ 0 & 0 & -(\lambda+\mu_2) & 0 \\ 0 & 0 & 0 & -(\lambda+\mu_v) \end{pmatrix},$$

$$C = \begin{pmatrix} \lambda & 0 & 0 & 0 \\ 0 & \lambda & 0 & 0 \\ 0 & 0 & \lambda & 0 \\ 0 & 0 & 0 & \lambda \end{pmatrix},$$

where

$$\mathrm{A}_{21} = -(\lambda + \varepsilon_1 + \varepsilon_2 + \mu_1).$$

By observing the matrix $\mathcal{Q}$, we can see that $\{Q_{(\mathrm{t})}, J_{(t)}\}$ is a quasi-birth-and-death (QBD). Therefore, the minimum non-negative solution of the matrix equation

$$R^2 B + RA + C = 0 \tag{1}$$

will be obtained. This solution R is called rate matrix commonly.

Theorem 1. When $\rho_2 = \frac{\lambda}{\mu_2} < 1, \rho_v = \frac{\lambda}{\mu_v} < 1$, the minimum non-negative solution of the matrix equation (1) is

$$R = \begin{pmatrix} \frac{\lambda}{\lambda+s} & \frac{sr}{\lambda+s-\mu_1 r} & r_{13} & r_{14} \\ 0 & r & \frac{\varepsilon_2 r}{\mu_2(1-r)} & \frac{\varepsilon_1 r}{\mu_v(1-r)} \\ 0 & 0 & \frac{\lambda}{\mu_2} & 0 \\ 0 & 0 & 0 & \frac{\lambda}{\mu_v} \end{pmatrix}, \tag{2}$$

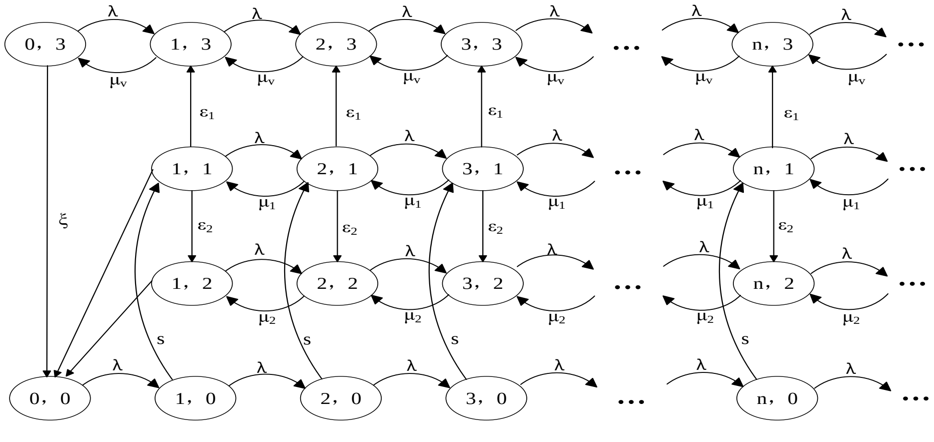

Fig. 1. State transfer diagram.

where

$$r_{13} = \frac{(\lambda+s)\varepsilon_2 r}{\mu_2(1-r)(\lambda+s-\mu_1 r)},$$

$$r_{14} = \frac{(\lambda+s)\varepsilon_1 r}{\mu_v(1-r)(\lambda+s-\mu_1 r)}.$$

Among them $0 < r < 1$, and

$$r = \frac{1}{2\mu_1}(\lambda+\varepsilon_1+\varepsilon_2+\mu_1-\sqrt{(\lambda+\varepsilon_1+\varepsilon_2+\mu_1)^2-4\lambda\mu_1}).$$

Proof The matrices A, B and C in Eq. (1) are all upper triangular matrices, so set R be an upper triangular matrix, i.e.

$$R = \begin{pmatrix} r_{11} & r_{12} & r_{13} & r_{14} \\ 0 & r_{22} & r_{23} & r_{24} \\ 0 & 0 & r_{33} & r_{34} \\ 0 & 0 & 0 & r_{44} \end{pmatrix}.$$

Taking R^2 and R into the Eq. (1), the following system of equations can be obtained

$$\begin{cases} \lambda - r_{11}(\lambda+s) = 0, \\ \mu_1(r_{11}r_{12}+r_{12}r_{22}) + sr_{11} - r_{12}(\lambda+\varepsilon_1+\varepsilon_2+\mu_1) = 0, \\ \mu_2(r_{11}r_{13}+r_{12}r_{23}+r_{13}r_{33}) + \varepsilon_2 r_{12} - r_{13}(\lambda+\mu_2) = 0, \\ \mu_v(r_{11}r_{14}+r_{12}r_{24}+r_{13}r_{34}+r_{14}r_{44}) + \varepsilon_1 r_{12} \\ \quad - r_{14}(\lambda+\mu_v) = 0, \\ \mu_1 {r_{22}}^2 - r_{22}(\lambda+\varepsilon_1+\varepsilon_2+\mu_1) + \lambda = 0, \\ \mu_2(r_{22}r_{23}+r_{23}r_{33}) + \varepsilon_2 r_{22} - r_{23}(\lambda+\mu_2) = 0, \\ \mu_v(r_{22}r_{24}+r_{23}r_{34}+r_{24}r_{44}) + \varepsilon_1 r_{22} - r_{24}(\lambda+\mu_v) = 0, \\ \mu_2 {r_{33}}^2 - r_{33}(\lambda+\mu_2) + \lambda = 0, \\ \mu_v(r_{33}r_{34}+r_{34}r_{44}) - r_{34}(\lambda+\mu_v) = 0, \\ \mu_v {r_{44}}^2 - r_{44}(\lambda+\mu_v) + \lambda = 0. \end{cases} \tag{3}$$

To obtain the minimum non-negative solution of Eq. (1), in the fifth equation of the Eq.(3), take $r_{22} = r$ (the other root is greater than 1). The following proves that $0 < r < 1$, letting

$$f(r_{22}) = \mu_1 {r_{22}}^2 - (\lambda+\varepsilon_1+\varepsilon_2+\mu_1)r_{22} + \lambda,$$

f is a quadratic continuous function, and $f(0) = \lambda > 0, f(1) = -\varepsilon_1 - \varepsilon_2 < 0$. By the zero point theorem there must exist $0 < r < 1$ such that $f(r) = 0$. By solving the Eq. (3) we get

$$\begin{aligned} r_{11} &= \frac{\lambda}{\lambda+s}, r_{12} = \frac{sr}{\lambda+s-\mu_1 r}, \\ r_{13} &= \frac{\varepsilon_2 r(\lambda+s)}{\mu_2(\lambda+s-\mu_1 r)(1-r)}, \\ r_{14} &= \frac{\varepsilon_1 r(\lambda+s)}{\mu_v(\lambda+s-\mu_1 r)(1-r)}, r_{23} = \frac{\varepsilon_2 r}{\mu_2(1-r)}, \\ r_{24} &= \frac{\varepsilon_1 r}{\mu_v(1-r)}, r_{33} = \frac{\lambda}{\mu_2}, r_{34} = 0, r_{44} = \frac{\lambda}{\mu_v}. \end{aligned} \tag{4}$$

Since r satisfies the equation $\mu_1 {r_{22}}^2 - r_{22}(\lambda+\varepsilon_1+\varepsilon_2+\mu_1)+\lambda = 0$, dividing both sides of the equation by r gives

$$\frac{\lambda}{r} = \lambda+\varepsilon_1+\varepsilon_2+\mu_1(1-r). \tag{5}$$

Theorem 2. QBD process $\{Q_{(t)}, J_{(t)}\}$ is positive recurrent if and only if $\rho < 1$.

Proof According to the literature [13], QBD process $\{Q_{(t)}, J_{(t)}\}$ is positive recurrent when the spectral radius of R that $SP(R) < 1$, and the system of equations $(x_0, x_1, x_2, x_3, x_4, x_5)B[R] = 0$ has a positive solution, and from equation (3), we know that

$$B[R] = \begin{pmatrix} A_0 & C_0 \\ B_1 & RB+A \end{pmatrix} =$$

$$\begin{pmatrix} -\lambda & 0 & \lambda & 0 & 0 & 0 \\ \xi & -(\lambda+\xi) & 0 & 0 & 0 & \lambda \\ 0 & 0 & -(\lambda+s) & \frac{s(\lambda+s)}{\lambda+s-\mu_1 r} & B_{35} & B_{36} \\ \mu_1 & 0 & 0 & -\frac{\lambda}{r} & \frac{\varepsilon_2}{1-r} & \frac{\varepsilon_1}{1-r} \\ \mu_2 & 0 & 0 & 0 & -\mu_2 & 0 \\ 0 & \mu_v & 0 & 0 & 0 & -\mu_v \end{pmatrix}, \tag{6}$$

where

$$B_{35} = \frac{\varepsilon_2 r(\lambda+s)}{(\lambda+s-\mu_1 r)(1-r)}, B_{36} = \frac{\varepsilon_1 r(\lambda+s)}{(\lambda+s-\mu_1 r)(1-r)}.$$

$B[R]$ is an irreducible, aperiodic, finite state generator, so $(x_0,x_1,x_2,x_3,x_4,x_5,)B[R] = 0$ has a positive solution and the process $\{Q_{(t)}, J_{(t)}\}$ is positive recurrent if and only if

$$SP(R) = max\left\{\frac{\lambda}{\lambda+s}, r, \frac{\lambda}{\mu_2}, \frac{\lambda}{\mu_v}\right\} < 1 .$$

Since $0 < \frac{\lambda}{\lambda+s} < 1$ and $0 < r < 1$, we have $\frac{\lambda}{\mu_2} < 1$ and $\frac{\lambda}{\mu_v} < 1$.

III. Steady-state probability distribution

Define the steady-state probability as

$$\pi_{kj} = \lim_{t\to\infty} P\{Q(t) = k, J(t) = j\}, (k,j) \in \Omega.$$

When $\frac{\lambda}{\mu_2} < 1$ and $\frac{\lambda}{\mu_v} < 1$, let Π be the steady-state probability vector of $\{Q_{(t)}, J_{(t)}\}$, then

$$\Pi = (\pi_0, \pi_1, \pi_2, \cdots),$$

where

$$\pi_k = \begin{cases} (\pi_{00}, \pi_{01}), k = 0, \\ (\pi_{k0}, \pi_{k1}, \pi_{k2}, \pi_{k3}), k \geq 1. \end{cases}$$

By means of matrix geometry solution

$$\begin{cases} (\pi_{00},\pi_{03},\pi_{10},\pi_{11},\pi_{12},\pi_{13})B[R] = 0, \\ \pi_0 e + \pi_1(I-R)^{-1}e = 1. \end{cases} \tag{7}$$

Substituting the Eq. (2), Eq. (5) and Eq. (6) into the Eq. (7), we have

$$\begin{cases} -\lambda\pi_{00} + \xi\pi_{03} + \mu_1\pi_{11} + \mu_2\pi_{12} = 0, \\ -(\lambda+\xi)\pi_{03} + \mu_v\pi_{13} = 0, \\ \lambda\pi_{00} - (\lambda+s)\pi_{10} = 0, \\ \frac{s(\lambda+s)}{\lambda+s-\mu_1 r}\pi_{10} - \frac{\lambda}{r}\pi_{11} = 0, \\ \frac{\varepsilon_2 r(\lambda+s)}{(\lambda+s-\mu_1 r)(1-r)}\pi_{10} + \frac{\varepsilon_2}{1-r}\pi_{11} - \mu_2\pi_{12} = 0, \\ \lambda\pi_{03} + \frac{\varepsilon_1 r(\lambda+s)}{(\lambda+s-\mu_1 r)(1-r)}\pi_{10} + \frac{\varepsilon_1}{1-r}\pi_{11} \\ \quad - \mu_v\pi_{13} = 0. \end{cases} \tag{8}$$

Setting $\pi_{00} = K$, we obtain

$$(\pi_{00},\pi_{03},\pi_{10},\pi_{11},\pi_{12},\pi_{13}) = K(1, \frac{\varepsilon_1 r(\lambda+s)}{\xi(1-r)(\lambda+s-\mu_1 r)}, \frac{\lambda}{\lambda+s}, \frac{sr}{\lambda+s-\mu_1 r}, \frac{\varepsilon_2 r(\lambda+s)}{\mu_2(1-r)(\lambda+s-\mu_1 r)}, \frac{\varepsilon_1 r(\lambda+s)(\lambda+\xi)}{\xi\mu_v(1-r)(\lambda+s-\mu_1 r)}\Bigg), \tag{9}$$

where

$$K = \Bigg[\frac{\lambda+s}{s} + \left(\frac{\varepsilon_1}{\mu_v-\lambda} + \frac{\varepsilon_2}{\mu_2-\lambda}\right) \frac{s^2 r^2 + \lambda r(\lambda+sr-\lambda r)}{s(1-r)^2(\lambda+s-\mu_1 r)} + \frac{\lambda+s}{(1-r)(\lambda+s-\mu_1 r)} \left(r + \frac{\varepsilon_1 r(\lambda+\xi)}{\xi(\mu_v-\lambda)} + \frac{\varepsilon_2 r}{\mu_2-\lambda} + \frac{\varepsilon_1 r}{\xi}\right)\Bigg]^{-1}. \tag{10}$$

When $k \geq 1$,

$$(\pi_{k0},\pi_{k1},\pi_{k2},\pi_{k3}) = (\pi_{10},\pi_{11},\pi_{12},\pi_{13})R^{k-1}.$$

IV. Performance indicators of the system in steady-state

1) The mean queueing length

$$\begin{aligned}E(L) &= \sum_{n=1}^{\infty} n\pi_n e = \pi_1 e + 2\pi_2 e + 3\pi_3 e + \cdots \\ &= \sum_{n=1}^{\infty} n\pi_1 e R^{n-1} e = \pi_1 \frac{1}{(I-R)^2} e \\ &= \frac{r^2 b}{(1-r)^3(s+\lambda-\mu_1 r)} + \frac{rc(s+\lambda)}{(1-r)(s+\lambda-\mu_1 r)} \\ &\quad + \frac{(s+\lambda-r\lambda)(ar\lambda+sd+\lambda d)}{s^2(1-r)^2(s+\lambda-\mu_1 r)}.\end{aligned}$$

2) The mean sojourn time

$$\begin{aligned}E(W) &= E(L)/\lambda \\ &= \frac{r^2 b}{\lambda(1-r)^3(s+\lambda-\mu_1 r)} + \frac{rc(s+\lambda)}{\lambda(1-r)(s+\lambda-\mu_1 r)} \\ &\quad + \frac{(s+\lambda-r\lambda)(ar\lambda+sd+\lambda d)}{\lambda s^2(1-r)^2(s+\lambda-\mu_1 r)},\end{aligned}$$

where

$$\begin{aligned}a &= \frac{\varepsilon_1\left[(s+\lambda)(\mu_v-\lambda)+\mu_v s\right]}{(\mu_v-\lambda)^2} + \frac{\varepsilon_2\left[(s+\lambda)(\mu_2-\lambda)+\mu_2 s\right]}{(\mu_2-\lambda)^2}, \\ b &= \frac{\varepsilon_1\left[\mu_v(\lambda+2s-rs)-\lambda(\lambda+s)\right]}{(\mu_v-\lambda)^2} + \frac{\varepsilon_2\left[\mu_2(\lambda+2s-rs)-\lambda(\lambda+s)\right]}{(\mu_2-\lambda)^2}, \\ c &= \frac{\mu_v\varepsilon_1(\lambda+\xi)}{\xi(\mu_v-\lambda)^2} + \frac{\mu_2\varepsilon_2}{(\mu_2-\lambda)^2}, \\ d &= \lambda(1-r)^2 + sr.\end{aligned}$$

3) The availability of the system

$$\begin{aligned}A &= \pi_{00} + \sum_{k=1}^{\infty}\pi_k\eta_1 \\ &= \pi_{00} + \pi_1(I-R)^{-1}\eta_1 \\ &= \left[1 + \frac{\lambda}{s} + \frac{r(s+\lambda)}{(s+\lambda-\mu_1 r)(1-r)}\right]K,\end{aligned}$$

where $\eta_1 = \begin{pmatrix}1\ 1\ 0\ 0\end{pmatrix}^T$.

4) The probability that the service station is in a complete fault state

$$\begin{aligned}P_2 &= P(Y=2) = \sum_{k=1}^{\infty}\pi_{k2} \\ &= \sum_{k=1}^{\infty}\pi_1 R^{k-1}\eta_2 = \pi_1(I-R)^{-1}\eta_2 \\ &= \frac{\varepsilon_2 r[\lambda(s+\lambda-\lambda r)+s^2 r+s(s+\lambda)(1-r)]}{s(1-r)^2(\mu_2-\lambda)(s+\lambda-\mu_1 r)}K,\end{aligned}$$

where $\eta_2 = \begin{pmatrix}0\ 0\ 1\ 0\end{pmatrix}^T$.

5) The probability that the service station is in an incomplete fault state

$$\begin{aligned}P_3 &= (Y=3) = \sum_{k=0}^{\infty}\pi_{k3} \\ &= \pi_{03} + \sum_{k=1}^{\infty}\pi_1 R^{k-1}\eta_3 = \pi_{03} + \pi_1(I-R)^{-1}\eta_3 \\ &= \frac{\varepsilon_1 r(\lambda+s)[\xi(sr-\lambda r+\lambda)+s(1-r)(\mu_v+\xi)]}{s\xi(1-r)^2(\mu_v-\lambda)(s+\lambda-\mu_1 r)}K,\end{aligned}$$

where $\eta_3 = \begin{pmatrix}0\ 0\ 0\ 1\end{pmatrix}^T$.

V. Numerical analysis

In this section, numerical simulation experiments are used to observe the impact of system parameters on each performance indexes.

In Figure 2 and Figure 3, we study the impact of service rate μ_1 on the queueing length and sojourn time by the change of arrival rate λ. Set parameters $\mu_2 = 3.0$, $\mu_v = 2.5$, $\varepsilon_1 = 0.1$, $\varepsilon_2 = 0.08$, $s = 5$ and $\xi = 2$. The mean queueing length and sojourn time are calculated by Matlab with the change of service rate μ_1. $E(L)$ and $E(W)$ increase with the increase of arrival rate λ, as more and more customers enter the system, the queueing length and sojourn time both increase gradually; when λ is a fixed value, $E(L)$ and $E(W)$ decrease as μ_1 increases. As the service rate of the standard service station increases, the mean queueing length and sojourn time decreases, and the analysis results are consistent with the real life situation.

In Figure 4, set parameters $\mu_1 = 5.5$, $\mu_2 = 3.0$, $\mu_v = 2.5$, $\lambda = 2$, $\varepsilon_2 = 0.01$ and $s = 4$. We investigate the relationship between mean queueing length $E(L)$ and repair rate ξ by changing the incomplete failure rate ε_1. When ξ is constant, $E(L)$ shows an increasing trend with the increase of the incomplete failure rate ε_1, and when ε_1 is fixed, $E(L)$ decreases with the increase of ξ.

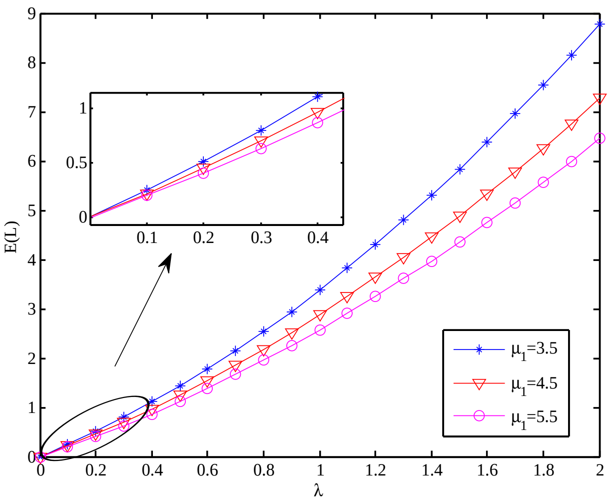

Fig. 2. The effect of λ on $E(L)$ when μ_1 takes different values .

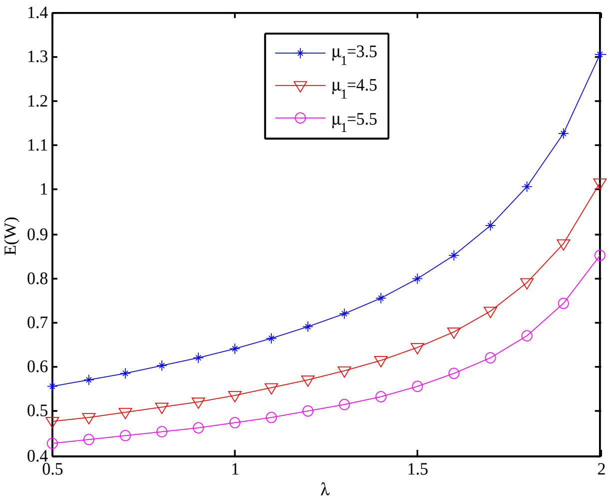

Fig. 3. The effect of λ on $E(W)$ when μ_1 takes different values .

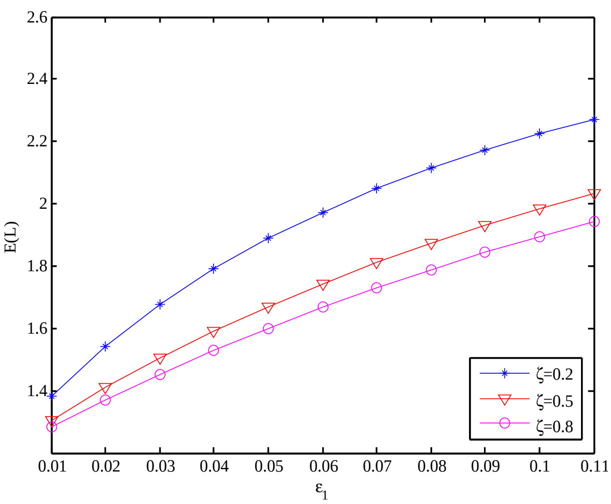

Fig. 4. The effect of $\varepsilon_1$on $E(L)$ when ξ takes different values .

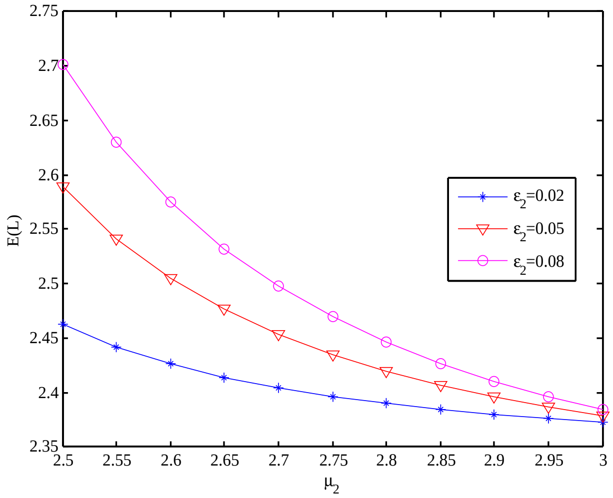

Fig. 5. The effect of μ_2 on $E(L)$ when ε_2 takes different values .

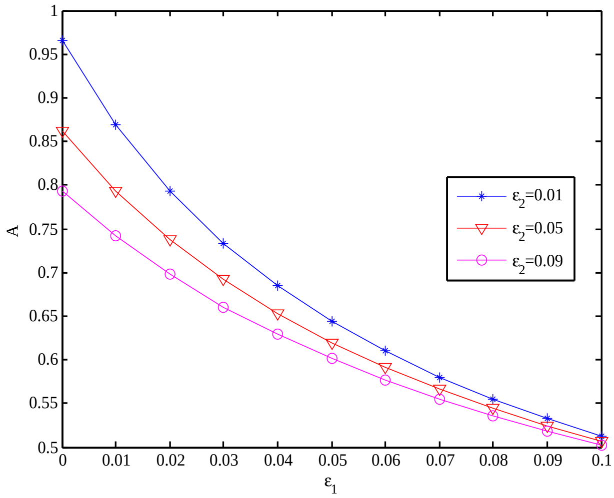

Fig. 6. The effect of ε_1 on availability when ε_2 takes different values .

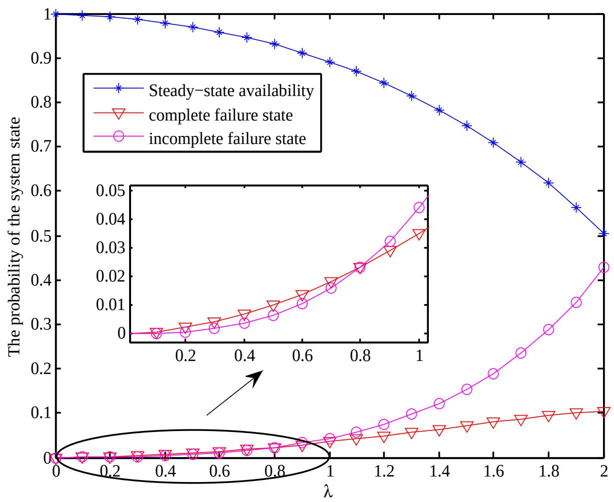

Fig. 7. The trend of availability and probability of two types of failure with λ.

In Figure 5, set parameters $\mu_1 = 4$, $\mu_v = 2.5$, $\lambda = 2$, $\varepsilon_2 = 0.1$, $s = 4$ and $\xi = 2$. We find the trend of the mean queueing length with the change of the standby service station service rate μ_2 and the complete failure rate ε_2. When ε_2 is constant, the queueing length decreases with the increase of μ_2. The greater the failure rate ε_2, the more obvious the downward trend is. Therefore, the higher the probability of complete failure, the more necessary it is to set up a standby service station.

In Figure 6, we observe the impact of incomplete failure rate and complete failure rate on availability. Set parameter $\mu_1 = 4.0$, $\mu_2 = 3.5$, $\mu_v = 3$, $\lambda = 2$, $s = 0.2$ and $\xi = 2$. When the complete failure rate is constant, availability A decreases as the the incomplete failure rate ε_1 increase. When ε_1 is constant, the availability A decreases with the increase of the complete failure rate ε_2. When the failure rate increases, the possibility of service station failure increases, and the availability of the system decreases.

Figure 7 depicts the trend of the availability and the steady-state probability of two faults with λ. Set parameters $\mu_1 = 4.0$, $\mu_2 = 3.5$, $\mu_v = 2.5$, $\varepsilon_1 = 0.1$, $\varepsilon_2 = 0.08$, $s = 0.2$ and $\xi = 2$. As we can see in the figure, the setting meet the steady-state condition. With the increase of arrival rate λ, the availability gradually decreases, and the steady-state probability of failure gradually increases. When $0.8 > \lambda > 0$, the steady-state probability of complete failure is greater than the steady-state probability of incomplete failure. When $\lambda > 0.8$, the steady-state probability of complete failure is less than the steady-state probability of incomplete failure, and the gap is increasing.

VI. Summary

In this paper, a standby service station is added to a service system with two fault characteristics, so that the system can run continuously in the event of a service station failure. This service model is closer to the actual situation. The stationary probability distribution is solved by matrix geometry solutions, and then the performance indexes are obtained. The impacts of system parameters on the performance index are obtained by numerical analysis. In real life, the service rate of the standby service station can be reasonably set according to the actual arrival rate and the probability of machine failure, which can reduce the waste of resources while ensuring that too many customers are not hoarded in the system. The obtained results are significant for optimizing supermarket queueing systems, hospital visit systems and various other systems.

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