Stability of Nonlinear Markovian Switched Delay Sub-Fractional Stochastic Systems

Chao Wei, Mengjie Ma, Haoran Qu and Haotian Wang

Abstract—**This article is devoted to investigate the stability of nonlinear Markovian switched delay sub-fractional stochastic systems. Firstly, we study the existence of global unique solution. Secondly, by applying general Itô formula, Gronwall's inequality and Borel-Cantelli lemma, we discuss the stability of sub-fractional stochastic system. Finally, an example is given to verify the validity of our theoretical results.**

Index Terms—**Nonlinear Markovian switched stochastic delay systems; almost sure exponential stability; sub-fractional Brownian motion; existence and uniqueness**

I. Introduction

Many systems do not satisfy the principle of linear superposition. Thence, except for a small part that can be approximately regarded as linear systems, most of them are nonlinear. For instance, simple pendulum systems [13], gravitational three-body systems [16]. The nonlinear system is the essence and the linear system is the approximation or part of the nonlinear system. Therefore, it is necessary to discuss the properties of the nonlinear systems. Recently, nonlinear systems have been discussed by some authors. For example, by applying a terminal sliding mode control, Fei et al. [7] designed a dual-hidden-layer recurrent controller for the nonlinear system. Guo et al. [8] studied the control issue of assurance of cost for event-triggered stochastic systems. Liu et al. [12] presented the adaptive control problem for nonlinear time-varying systems. Wang and Qiao [18] designed the incremental deep pretraining to extract effective features and consider them as the input of the self-organizing fuzzy neural network.

System dynamics can be depicted by some subsystems or dynamic models. There is no jump phenomenon in the continuous state of the system at the moment of switching. The research of switching systems has always been a hot issue in the field of control. For instance, Arteaga et al. [1] derived the regression analysis about the drain voltage data. Cheng et al. [5] discussed the T-S switched system by designing finite time filtering. Jiao et al. [11] addressed the incremental stability for Markovian switched delay stochastic systems. Qi et al. [15] designed the synovial controller for semi-Markovian switched stochastic systems.

Generally, because of uncertain communication environment, the time delay is always unavoidable. Hence, taking the factor of time delay into consideration for stochastic systems is necessary. Chen et al. [4] studied the stability of delay stochastic systems by using aperiodic sampling. Feng et al. [6] discussed the exponential stability for highly nonlinear hybrid delay stochastic systems by using multiple degenerate functionals. By using the multiple linear regression model, Plonis et al. [14] presented the synthesis of delay system. Wei [20] studied the exponential stability of nonlinear stochastic delayed systems driven by G-Brownian motion. Zhao and Zhu [25] discussed boundedness of the unique solution for highly Markovian switching delay systems.

The sub-fractional Brownian motion does not have the stationary increments, the increments are weakly correlated, the covariance decays faster. Hence, we could use sub-fractional Brownian motion to describe non-stationary models with long-range dependence and self-similarity. As the sub-fractional Brownian motion is not martingale, it is difficult to study. In the past few years, some authors have studied the stochastic models driven by sub-fractional Brownian motion. For example, Bian and Li [2] discussed the European option pricing for financial stochastic market. Wei [19] studied the parameter estimation problem for stochastic fractional differential equations with incomplete information. Xiao et al. [22] considered parameter estimation for fractional Vasicek model based on $H > \frac{1}{2}$.

The nonlinear characteristic of the systems make the performance of the systems more complicated, which brings difficulties to the analysis of stability of systems. Stability has always been the most fundamental and core issue in system analysis. In recent years, lots of results about stability has been reported in the literature [3], [21]. For example, Haddad and Lee [9] developed Lyapunov and converse Lyapunov theorems for discrete-time nonlinear stochastic semistable dynamical systems. Han and Chung [10] studied incremental stability of nonlinear stochastic systems driven by Lévy noise. Wang et al. [17] discussed the stochastic stability of nonlinear stochastic system with impulsion. Zhang et al. [24] analyzed the interval stability for linear stochastic delay systems by using Lyapunov-Krasovskii functionals. In recent years, the financial empirical research showed that volatility in financial asset prices shows long-range dependence and self-similarity and the sub-fractional Brownian motion could be used to exhibit these properties. Therefore, it is necessary to investigate the stability of sub-fractional stochastic system. Inspired by the aforementioned works, we discuss the stability of nonlinear Markovian switched delay sub-fractional stochastic systems. The existence of the global unique solution is derived. By using general

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Chao Wei is a Professor of School of Mathematics and Statistics, An'yang Normal University, An'yang, 455000, China (email: aytongji@126.com).

Mengjie Ma is an Undergraduate Student of School of Mathematics and Statistics, An'yang Normal University, An'yang, 455000, China (email: 277565527@qq.com).

Haoran Qu is a Postgraduate Student of School of Mathematics and Statistics, An'yang Normal University, An'yang, 455000, China (email: 15231253133@163.com).

Haotian Wang is a Postgraduate Student of School of Mathematics and Statistics, An'yang Normal University, An'yang, 455000, China (email: 15038594692@163.com).

Itô formula, Borel-Cantelli lemma, Gronwall's inequality, Hölder inequality and Chebyshev inequality, we investigated the almost sure exponential stability of the sub-fractional system.

The rest of this paper is organized as follows. The nonlinear Markovian switched delay sub-fractional stochastic system, some definitions and assumptions are given in Section 2. The existence of global unique solution is derived in Section 3. Moreover, the stability of the system is studied as well. We provide an example in Section 4. In Section 5, we make the conclusion and give some future works.

Denote $(\{\mathcal{F}_t\}_{t\geq 0})$ is a filtration generated by sub-fractional Brownian motion $\{B^H(t), t \geq 0\}$, $H \in (\frac{1}{2}, 1)$, $\mathcal{C}^{1,2}(\mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}_+ \times \mathbb{S}; \mathbb{R}_+)$ is the family of $V(x, y, t, i) > 0$, $r(t), t \geq 0$ is a right-continuous Markov chain.

The nonlinear Markovian switched delay sub-fractional stochastic systems is introduced as follows:

$$\begin{aligned} dx(t) \quad &= f(x(t), x(t-\tau(t)), t, r(t))dt \\ &+g(x(t), x(t-\tau(t)), t, r(t))dB^H(t), \end{aligned} \tag{1}$$

where $0 \leq \tau(t) \leq \tau$, the nonrandom initial data $\{x(t) = \xi(t) : -\tau \leq t \leq 0\} = \xi \in \mathcal{C}([-\tau, 0]; \mathbb{R}^n)$, $r(t), t \geq 0 \in \mathbb{S} = \{1, 2, ..., N\}$ is a right-continuous ergodic Markov chain, $\mathbb{S}$ is a finite state space, $r(0) = r_0 \in \mathbb{S}$, $B^H(t)$, $H \in (\frac{1}{2}, 1)$ is a sub-fractional Brownian motion with one dimension. The mapping rule of f and g is: $\mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}_+ \times \mathbb{S} \to \mathbb{R}^n$.

Firstly, we introduce some assumptions and definitions which are very important in the proof of main results.

Assumption 1: $\exists L_K > 0$, for $\forall t \geq 0$, $|x| \vee |x'| \vee |y| \vee |y'| \leq K$, $i \in S$, $|f(x, y, t, i) - f(x', y', t, i)| \vee |g(x, y, t, i) - g(x', y', t, i)| \leq L_K(|x - x'| + |y - y'|)$.

Assumption 2: $f(0, t, i) \equiv 0, \quad g(0, t, i) \equiv 0$, $\forall i \in \mathbb{S}$.

Assumption 3: $\lim_{|x|\to\infty} \inf_{t\geq 0, i\in S} V(x, y, t, i) = \infty$, $\mathcal{L}^H V(x, y, t, i) \leq -c_1 V(x, y, t, i)$, where $V(x, y, t, i) > 0 \in \mathcal{C}^{1,2}(\mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}_+ \times \mathbb{S}; \mathbb{R}_+)$, $\mathcal{C}^{1,2}(\mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}_+ \times \mathbb{S}; \mathbb{R}_+)$ is continuously once differentiable in t and twice differentiable in x.

Definition 1: For any $x_0 \in \{F\}_0$, $r_0 \in \mathbb{S}$. If

$$\lim_{t\to\infty} \sup \frac{1}{t} \log(|x(t; x_0, r_0)|) < -\lambda,$$

where $\lambda > 0$ is a constant, the sub-fractional system (1) is almost sure exponential stability.

The sub-fractional Brownian process B_t^H is a Gaussian process, $B_0^H = 0$, $\mathbb{E}[B_t^H] = 0$, the covariance

$$\mathbb{E}(B_t^H B_s^H) = s^{2H} + t^{2H} - \frac{1}{2}\{|t-s|^{2H} + |t+s|^{2H}\},$$

where $s, t \geq 0$. As $H = \frac{1}{2}$, B_t^H is the standard Brownian motion.

Moreover, for all $s \leq t$,

$$\mathbb{E}(|B_t^H - B_s^H|^2) = -2^{2H-1}(t^{2H} + s^{2H}) + (t+s)^{2H} - (t-s)^{2H},$$

and for $m \leq n \leq s \leq t$,

$$\begin{aligned} &\mathbb{E}(B_t^H - B_s^H)(B_n^H - B_m^H) \\ &= \frac{1}{2}[(t+m)^{2H} + (t-m)^{2H} + (s+n)^{2H} + (s-n)^{2H} \\ &-(t+n)^{2H} - (t-n)^{2H} - (s+m)^{2H} - (s-m)^{2H}]. \end{aligned}$$

Define $\phi(s, t) = H(2H-1)|s-t|^{2H-2}$ and the operator $\mathcal{L}^H V$

$$\begin{aligned} & \mathcal{L}^H V(x, y, t, i) \\ = \quad & V_t(x, y, t, i) + V_x(x, y, t, i) f(x, y, t, i) \\ + \quad & V_x(x, y, t, i) g(x, y, t, i) \int_0^t \phi(\nu, s) g(x, y, \nu, i) d\nu. \end{aligned}$$

Then,

$$\begin{aligned} dV(x, y, t, i) \quad &= \mathcal{L}^H V(x, y, t, i)dt \\ &+V_x(x, y, t, i) g(x, y, t, i) dB^H(t). \end{aligned}$$

II. Main Results and Proofs

Theorem 1: When Assumptions 1-3 hold, the global unique solution of sub-fractional system (1) exists.

Proof: Let the initial value $|x_0| \leq \xi$. For $m \geq \xi$, $m \in \mathbb{N}$, we suppose that

$$f^{(m)}(x, y, t, i) = f(\frac{|x| \wedge m}{|x|} x, \frac{|y| \wedge m}{|y|} y, t, i), \tag{2}$$

$$g^{(m)}(x, y, t, i) = g(\frac{|x| \wedge m}{|x|} x, \frac{|y| \wedge m}{|y|} y, t, i), \tag{3}$$

when $x = 0$, $(\frac{|x|\wedge m}{|x|} x) = 0$.

We obtain that $f^{(m)}$ and $g^{(m)}$ satisfy the conditions of existence and uniqueness. Thus,

$$\begin{aligned} &dx_m(t) = f^{(m)}(x_m(t), x_m(t-\tau(t)), t, r(t))dt \\ &+g^{(m)}(x_m(t), x_m(t-\tau(t)), t, r(t))dB^H(t), \end{aligned} \tag{4}$$

has the global unique solution.

Let

$$\eta_m = \inf\{t \geq 0 : |x_m(t)| \geq m\}. \tag{5}$$

When $0 \leq t \leq \eta_m$, $x_m(t) = x_{m+1}$. Then, $\{\eta_m\}$ is increasing. Thus, $\exists \eta$ satisfies

$$\eta = \lim_{m\to\infty} \eta_m. \tag{6}$$

Let

$$x(t) = \lim_{m\to\infty} x_m(t), \quad -\tau \leq t < \eta. \tag{7}$$

When $-\tau \leq t < \eta$, we obtain that $x(t)$ is unique.

By using general Itô formula, for $t \geq 0$, we get

$$\begin{aligned} &V(x_m(t \wedge \eta_m), x_m(t \wedge \eta_m - \tau(t \wedge \eta_m)), \\ &t \wedge \eta_m, r(t \wedge \eta_m)) \\ &= V(\xi(0), x_m(-\tau(0)), 0, r_0) \\ &+ \int_0^{t\wedge\eta_m} \mathcal{L}^{H(m)} V(x_m(s), x_m(s-\tau(s)), s, r(s))ds \end{aligned}$$

where $\mathcal{L}^{H(m)} V(x_m(s), x_m(s - \tau(s)), s, r(s)) = \mathcal{L}^H V(x_m(s), x_m(s-\tau(s)), s, r(s))$ when $0 \leq s \leq t \wedge \eta_m$.

Then, we obtain

$$\begin{aligned} &\mathbb{E}[V(x_m(t \wedge \eta_m), x_m(t \wedge \eta_m - \tau(t \wedge \eta_m)), \\ &t \wedge \eta_m, r(t \wedge \eta_m))] \\ &\leq \mathbb{E}[V(\xi(0), x_m(-\tau(0)), 0, r_0)] \\ &+\mathbb{E}[\int_0^{t\wedge\eta_m} \mathcal{L}^{H(m)} V(x_m(s), x_m(s-\tau(s)), s, r(s))ds] \\ &\leq \mathbb{E}[V(\xi(0), x_m(-\tau(0)), 0, r_0)] \\ &+ \int_0^{t\wedge\eta_m} \mathbb{E}[V(x_m(s), x_m(s-\tau(s)), s, r(s))]ds. \end{aligned}$$

According to the Gronwall's inequality, we obtain

$$\begin{aligned}&\mathbb{E}[V(x_m(t\wedge\eta_m),x_m(t\wedge\eta_m-\tau(t\wedge\eta_m)),\\&t\wedge\eta_m,r(t\wedge\eta_m))]\\&\leq\mathbb{E}[V(\xi(0),x_m(-\tau(0)),0,r_0)]e^{(t\wedge\eta_m)}.\end{aligned}\tag{8}$$

Furthermore, as

$$\begin{aligned}&\mathbb{P}\{\eta_m\leq t\}\inf_{|x|\geq m,|y|\geq m,t\geq 0,i\in S}V(x,y,t,i)\\&\leq\int_{\eta_m\leq t}V(x_m(t\wedge\eta_k),x_m(t\wedge\eta_m-\tau(t\wedge\eta_m)),\\&t\wedge\eta_m,r(t\wedge\eta_m))dP\\&\leq\mathbb{E}V(x_m(t\wedge\eta_m),x_m(t\wedge\eta_m-\tau(t\wedge\eta_m)),\\&t\wedge\eta_m,r(t\wedge\eta_m)),\end{aligned}$$

we have

$$\mathbb{P}\{\eta_m\leq t\}\leq\frac{\mathbb{E}[V(\xi(0),x_m(-\tau(0)),0,r_0)]e^{(t\wedge\eta_m)}}{\inf_{|x|\geq m,|y|\geq m,t\geq 0,i\in S}V(x,y,t,i)}.\tag{9}$$

When $t\to\infty$,

$$\mathbb{P}\{\eta\leq t\}=0.\tag{10}$$

Thus,

$$\mathbb{P}\{\eta=\infty\}=1.\tag{11}$$

The proof is complete. ∎

Theorem 2: If $\exists V(x,y,t,i)$ and some constants $a_1,a_2,c_1>0$ satisfy

$$a_1|x|^2\leq V(x,y,t,i)\leq a_2|x|^2,\tag{12}$$

$$\mathcal{L}^H V(x,y,t,i)\leq -c_1V(x,y,t,i),\tag{13}$$

the sub-fractional system (1) is almost sure exponential stability.

Proof: By using general Itô formula, we get

$$\begin{aligned}&e^{\lambda t}V(x(t),x(t-\tau(t)),t,i)-V(x_0,0,i)\\&=\int_0^t\lambda e^{\lambda s}V(x(s),x(s-\tau(s)),s,i)ds\\&+\int_0^t e^{\lambda s}V_s(x(s),x(s-\tau(s)),s,i)ds\\&+\int_0^t e^{\lambda s}V_x(x(s),x(s-\tau(s)),s,i)\\&f(x(s),x(s-\tau(s)),s,i)ds\\&+\int_0^t e^{\lambda s}V_x(x(s),x(s-\tau(s)),s,i)\\&g(x(s),x(s-\tau(s)),s,i)dB^H(s)\\&+\int_0^t e^{\lambda s}V_x(x(s),x(s-\tau(s)),s,i)\\&g(x(s),x(s-\tau(s)),s,i)\\&\int_0^u\phi(\omega,s)g(x(\omega),x(\omega-\tau(\omega)),\omega,i)d\omega ds,\end{aligned}$$

for $\forall i\in S$, $t>0$, $\lambda>0$.

Thus, we obtain

$$\begin{aligned}&e^{\lambda t}V(x(t),x(t-\tau(t)),t,i)\\&=V(x_0,0,i)\\&+\int_0^t e^{\lambda s}[\lambda V(x(s),x(s-\tau(s)),s,i)\\&+\mathcal{L}^H V(x(s),x(s-\tau(s)),s,i)]ds\\&+\int_0^t e^{\lambda s}V_x(x(s),x(s-\tau(s)),s,i)\\&g(x(s),x(s-\tau(s)),s,i)dB^H(s).\end{aligned}$$

Since

$$\begin{aligned}&\mathbb{E}[\int_0^t e^{\lambda s}V_x(x(s),x(s-\tau(s)),s,i)\\&g(x(s),x(s-\tau(s)),s,i)dB^H(s)]=0,\end{aligned}$$

we get

$$\begin{aligned}&\mathbb{E}[e^{\lambda t}V(x(t),x(t-\tau(t)),t,i)]\\&=\mathbb{E}[V(x_0,0,i)]\\&+\mathbb{E}\int_0^t e^{\lambda s}[\lambda V(x(s),x(s-\tau(s)),s,i)\\&+\mathcal{L}^H V(x(s),x(s-\tau(s)),s,i)]ds\\&\leq a_2|x_0|^2\\&+\mathbb{E}\int_0^t e^{\lambda s}[\lambda V(x(s),x(s-\tau(s)),s,i)\\&-c_1V(x(s),x(s-\tau(s)),s,i)]ds\\&=a_2|x_0|^2\\&+(\lambda-c_1)\int_0^t\mathbb{E}[e^{\lambda s}V(x(s),x(s-\tau(s)),s,i)]ds.\end{aligned}\tag{14}$$

Hence, we obtain

$$\mathbb{E}|x(t)|^2\leq\frac{a_2}{a_1}e^{-c_1t}|x_0|^2.\tag{15}$$

It can be easily to check that

$$\begin{aligned}&|x(t)|^2\\&=|x_0+\int_0^t f(x(s),x(s-\tau(s)),s,r(s))ds\\&+\int_0^t g(x(s),x(s-\tau(s)),s,r(s))dB^H(s)|^2\\&\leq 3|x_0|^2+3|\int_0^t f(x(s),x(s-\tau(s)),s,r(s))ds|^2\\&+3|\int_0^t g(x(s),x(s-\tau(s)),s,r(s))dB^H(s)|^2.\end{aligned}$$

For $\forall h>0$ satisfies $h^2L_K^2(1+h^{2H-2})<\frac{1}{3}$ and positive integer κ_0, let $\kappa=\kappa_0,\kappa_0+1,\kappa_0+2,\cdots$. By using the

Hölder inequality, we get

$$\begin{aligned}
&\mathbb{E}[\sup_{\kappa h\le t\le(\kappa+1)h}|x(t)|^2]\\
&\le 3\mathbb{E}[|x(\kappa h)|^2]\\
&+3\mathbb{E}(\int_{\kappa h}^{(\kappa+1)h}|f(x(s),x(s-\tau(s)),s,r(s))|ds)^2\\
&+3\mathbb{E}|\int_0^t g(x(s),x(s-\tau(s)),s,r(s))dB^H(s)|^2\\
&\le 3\mathbb{E}[|x(\kappa h)|^2]\\
&+3\mathbb{E}(h\sup_{\kappa h\le s\le(\kappa+1)h}|f(x(s),x(s-\tau(s)),s,r(s))|)^2\\
&+3\mathbb{E}[\sup_{\kappa h\le s\le(\kappa+1)h}|\int_{\kappa h}^{(\kappa+1)h}\\
&g(x(s),x(s-\tau(s)),s,r(s))dB^H(s)|^2]\\
&\le 3\mathbb{E}[|x(\kappa h)|^2]+3h^2L_K^2\mathbb{E}[\sup_{\kappa h\le s\le(\kappa+1)h}|x(s)|^2]\\
&+3h^{2H}L_K^2\mathbb{E}[\sup_{\kappa h\le s\le(\kappa+1)h}|x(s)|^2]\\
&\le 3\frac{a_2}{a_1}|x_0|^2e^{-c_1\kappa h}\\
&+3h^2L_K^2(1+h^{2H-2})\mathbb{E}[\sup_{\kappa h\le s\le(\kappa+1)h}|x(s)|^2].
\end{aligned}$$

Then, it can be checked that

$$\mathbb{E}[\sup_{\kappa h\le t\le(\kappa+1)h}|x(t)|^2]\le\frac{3\frac{a_2}{a_1}|x_0|^2e^{-c_1\kappa h}}{1-3h^2L_K^2(1+h^{2H-2})}. \tag{16}$$

According to the Chebyshev inequality, we obtain

$$\begin{aligned}
&\mathbb{P}(\sup_{\kappa h\le t\le(\kappa+1)h}|x(t)|>e^{\frac{-c_1\kappa h}{2}})\\
&\le\frac{\mathbb{E}[\sup_{\kappa h\le t\le(\kappa+1)h}|x(t)|^2]}{e^{-c_1\kappa h}}\\
&\le\frac{3\frac{a_2}{a_1}|x_0|^2}{1-3h^2L_K^2(1+h^{2H-2})}.
\end{aligned}$$

From the Borel-Cantelli lemma, we obtain

$$\sup_{\kappa h\le t\le(\kappa+1)h}|x(t)|\le e^{\frac{-c_1\kappa h}{2}}. \tag{17}$$

Thus, for $\kappa h\le t\le(\kappa+1)h$, we get

$$\lim_{t\to\infty}\sup\frac{\log(|x(t)|)}{t}<-\frac{c_1}{2}. \tag{18}$$

The proof is complete. ∎

Remark 1: Under the conditions in Theorem 2, the system (1) is pth moment exponentially stable.

Since

$$\begin{aligned}
&\mathbb{E}[e^{\lambda t}V(x(t),x(t-\tau(t)),t,i)]\\
&\le\mathbb{E}[V(x_0,x(-\tau(0),0)]\\
&+\mathbb{E}\int_0^t e^{c_1s}[\lambda V(x(s),x(s-\tau(s)),s,i)\\
&+\mathcal{L}V(x(s),x(s-\tau(s)),s,i)]ds\\
&\le a_2|x_0|^p.
\end{aligned} \tag{19}$$

Then,

$$\mathbb{E}[V(x(t),x(t-\tau(t)),t,i)]\le e^{-c_1t}a_2|x_0|^p. \tag{20}$$

Hence,

$$a_1\mathbb{E}|x(t)|^p\le e^{-c_1t}a_2|x_0|^p, \tag{21}$$

which implies that

$$\mathbb{E}|x(t)|^p\le\frac{a_2}{a_1}e^{-c_1t}|x_0|^p. \tag{22}$$

Therefore, the system (1) is pth moment exponentially stable.

III. Example

The sub-fractional Brownian motion $B^H(t)$ is one dimensional, $H=0.8$, $m=1$, $L_K=\frac{1}{3}$, $r(t)\in\mathbb{S}=\{1,2\}$, $x_0=2$, $r_0=1$, $\Gamma=(\gamma_{ij})_{2\times2}=$

$$\begin{pmatrix}-0.6 & 0.6\\ 0.2 & -0.2\end{pmatrix}$$

Consider the following nonlinear Markovian switched delay sub-fractional stochastic systems:

$$\begin{aligned}
dx(t) &= f(x(t),x(t-\tau(t)),t,r(t))dt\\
&+g(x(t),x(t-\tau(t)),t,r(t))dB^H(t),
\end{aligned}$$

where

$$f(x(t),x(t-\tau(t)),t,1)=-\frac{1}{4}x(t)+\frac{1}{6}x(t-\tau(t)),$$

$$g(x(t),x(t-\tau(t)),t,1)=\frac{1}{14}x(t),$$

$$f(x(t),x(t-\tau(t)),t,2)=-\frac{1}{3}x(t)+\frac{1}{8}x(t-\tau(t)),$$

$$g(x(t),x(t-\tau(t)),t,2)=\frac{1}{6}x(t),$$

$$\tau(t)=1+0.2\sin(t),$$

Hence, $\tau=1.2$, $m^2L_K^2(1+m^{2H-2})=\frac{2}{9}<\frac{1}{3}$. Let $V(x,y,t,i)=x^2, i=1,2$, we get

$$\mathcal{L}^HV(x,y,t,1)\le-\frac{1}{42}x^2,$$

$$\mathcal{L}^HV(x,y,t,2)\le-\frac{1}{12}x^2.$$

Then, it is obviously that

$$\mathcal{L}^HV(x,y,t,1)\le-\frac{1}{42}V(x,y,t,1),$$

$$\mathcal{L}^HV(x,y,t,2)\le-\frac{1}{12}V(x,y,t,2).$$

IV. Conclusion

This article is concerned with the stability of nonlinear Markovian switched delay sub-fractional stochastic systems. We have proved the existence and uniqueness of the global solution and provided sufficient conditions for the stability. We will consider the stability of nonlinear fractional stochastic hybrid systems with aperiodically intermittent control in future.

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