Efficient Iterative Reconstruction Method for Randomly Sampled Multi-Band Signals

H. Semlali, N. Boumaaz, A. Maali, *S.* Laafar, A. Soulmani, *A. Ghammaz,* J. -F. Diouris

Abstract—**Random sampling provides several benefits for multi-band signals compared to uniform sampling case. These benefits include greater sample frequency flexibility, fewer restrictions on signal filtering, and, in the case of stationary sampling sequences, a decrease or suppression of spectrum aliases. This makes random sampling a preferred option in Software Defined Radio (SDR) systems, which face the challenge of supporting different standards with different sampling frequencies. However, the task of reconstruction becomes more complex with random sampling. In the literature, various reconstruction techniques are provided. In this paper, we focus on employing the ADPW-CG iterative method to address the reconstruction issue with randomly sampled signals within an SDR system context. The effectiveness of the proposed algorithm is evaluated in terms of complexity, reconstructed signal quality, and robustness, and robustness and the findings is compared with the performance of the SVD direct algorithm and other iterative methods. Based on the obtained results, we observed that the proposed approach demonstrates promising performance in comparison to other algorithms.**

Index Terms— **ADPW-GC, Iterative Reconstruction, Random Sampling, Software Defined Radio**

I. Introduction

A multi-mode, multi-standard reconfigurable wireless communication system, known as Software Defined Radio (SDR), relies on software for most of its processing tasks. This approach is seen as a way to enhance the flexibility and adaptability of users, service providers, and manufacturers to various standards. The fundamental idea of a software defined radio system is to digitize close to the antenna while using a straightforward and universal analog block at the receiver head (common to the standards in which we are interested). The rest of the treatment must be performed digitally, in order to facilitate the reconfiguration [1-4].

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The Digital Front-End (DFE) of such systems is the interface between the analog-to-digital converter which digitizes the system band and the digital circuits which will process a particular channel or several channels (Fig. 1).

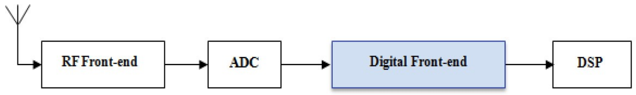

Fig. 1. Software defined radio architecture.

The fundamental design of these communication systems offers a number of benefits but also many technical difficulties, including the requirement for high sample rates and highly selective filters. For SDR systems where constraints on converters are major, the choice of the sampling technique to adopt may help to alleviate the constraints on the converters. Random sampling is used in software radio systems to overcome the restriction of the forbidden bands imposed in the case of a uniform process [5-11] and to reduce the spectrum aliases restrictions of the sampled signal, which reduces the constraints on the various transmission chain components. However, the reconstruction problem is more complex than the uniform case. It is therefore necessary to investigate various reconstruction methods and to evaluate their performances.

In the literature, different reconstruction techniques for signals that were randomly sampled are presented. In this contribution, we present three iterative methods for reconstruction of randomly sampled signals: the Descent method, the Conjugate Gradient (CG) method and the Adaptive Weigths iterative algorithm accelerated by the conjugate gradient (ADPW-CG) method [12-17]. In the following we are particularly interested in the ADPW-CG iterative method to solve the reconstruction of randomly sampled signals problem due to its robustness and convergence speed. The effectiveness of the proposed approach is evaluated in terms of complexity, quality of the reconstructed signal, and robustness and the findings are then compared to the performance of the Singular Value Decomposition (SVD), which is a direct matrix decomposition algorithm [5, 9, 18-21] to illustrate the usefulness of the proposed approach.

The remainder of this paper is structured as follows. An overview of the multi-band signals reconstruction utilizing random sampling is provided in Section 2. Section 3 introduces three iterative methods for reconstruction of randomly sampled signals. In section 4, the convergence speed of the presented iterative methods is examined, while the complexity of the various algorithms studied is evaluated in section 5. In order to show its utility for SDR structures, section 6 evaluates the effectiveness of the proposed approach and contrasts the findings with the performance of the SVD direct algorithm and to other iterative methods. Conclusions are drawn in section 7.

II. Multi-Band Signals Reconstruction using Random Sampling

In the present work, we suppose that the analyzed signal is randomly sampled using a Jittered Random Sampling Sequence (JRS) in view of its popular use [20, 22-24].

Let us consider a multi-band signal $x(t)$ which is characterized by an effective band $I = U\ I_i$ [5]. While U refers for the union operator and I_i represents the signal sub-bands. The samples are identified by the couples $((t_i), xi = x(t_i))$ with i varying from 1 to N. (N is the total samples number collected over the observation time T_0). The following expression defines the reconstructed signal:

$$\hat{x}(t) = \sum_{k=1}^{M} c_k \exp(2j\pi f_k t) \tag{1}$$

The f_k frequencies are selected in the signal bandwidth [5, 9, 25] and the c_k coefficients are established by minimizing the squares error, which is defined as:

$$E_q^2 = \left\| AC - X_s \right\|^2 \tag{2}$$

Where $X_s(t) = [x(t_1), x(t_2), ..., x(t_N)]$ is the sampled signal, C is a vector of length M constructed by the c_k complex coefficients to calculate and A is a matrix of dimension NxM formed by the elements of the matrix A given by [18, 26]:

$$A_{ik} = \exp(2j\pi f_k t_i) \tag{3}$$

The system of linear equations below is solved to determine the minimum of equation (2):

$$A^H AC = A^H X_s \tag{4}$$

A^H is the conjugate transposed matrix of A and A^HA is therefore a square M-dimensional matrix.

The resolution of the equation (4) can be performed by applying the direct matrix factorization algorithms such as the SVD or by iterative methods such as the descent method, the conjugate gradient method and the ADPW-GC method. We will discuss the iterative algorithms in the following paragraph.

The signal can be reconstructed using equation (1) once the c_k coefficients have been determined. Subsequently, it can be compared to the original signal using equation (5) [5, 9]:

$$E_m^2 = \frac{\sum_k \left| x(t_k) - \hat{x}(t_k) \right|^2}{\sum_k \left| x(t_k) \right|^2} \tag{5}$$

The set $\{t_k\}$ consists of a fairly discretization of the time axis during the observation period T_0 and does not necessarily contain the sampling instants.

The signal to noise ratio (SNR) of reconstruction can be defined based on this metric by:

$$SNR = -10 \log_{10}(E_m^2) \tag{6}$$

III. Iterative Reconstruction Methods for Randomly Sampled Multi-band Signals

Various iterative methods have been suggested in the literature to address the reconstruction issue of randomly sampled multi-band signals. In this section we present successively the descent method, the conjugate gradient method and the ADPW-CG iterative method.

A. Descent Method

Let $A^H A^C = A^H X_s$, a system of linear equations to solve. $F=A^HA$ is a square M-dimensional matrix, C is the vector of c_k frequency components ($k = 1, 2, ..., M$) to calculate and X_s the vector of the sampled values.

To formulate this method, we set $E = A^H X_s$. The solution of the equation (4) using the descent method is achieved iteratively by the following steps:

$$r_{it} = E - FC_{it} \tag{7}$$

$$\alpha_{it} = \frac{r_{it}^H r_{it}}{r_{it}^H F r_{it}} \tag{8}$$

$$C_{it+1} = C_{it} + \alpha_{it} r_{it} \tag{9}$$

with C_0 being arbitrary and the index it represents the number of iterations ($it = 0,1,...$).

B. Conjugate Gradient Method

The conjugate gradient method is a modified version of the descent method. This method is frequently employed due to its flexibility and simplicity of computation.

To formulate this method, we set $F = A^HA$ and $E = A^HX_s$. The solution of the equation (4) using the CG method is achieved iteratively by the following steps:

$$p_0 = r_0 = E - FC_0 \tag{10}$$

$$\alpha_{it+1} = \frac{r_{it}^H r_{it}}{p_{it}^H F p_{it}} \tag{11}$$

$$C_{it+1} = C_{it} + \alpha_{it+1} p_{it} \tag{12}$$

$$r_{it+1} = r_{it} - \alpha_{it+1} F p_{it} \tag{13}$$

$$p_{it+1} = r_{it+1} + \frac{r_{it+1}^H r_{it+1}}{r_{it}^H r_{it}} p_{it} \tag{14}$$

with C_0 being arbitrary and the index it represents the number of iterations ($it = 0,1,...$).

The conjugate gradient method can be summarized using the following steps:

1) Step1: begin by selecting the initial estimate C_0 and calculating the residue r_0 and the direction p_0 using the equation (10).
2) Step2: follow the general routine by determining the estimate C_{it} of the it iteration, the residue r_{it} and the direction p_{it}, then calculate C_{it+1}, r_{it+1} and p_{it+1} successively using equations (12), (13) and (14).

C. ADPW-CG Method

This method merges the concept of the ADPW (Adaptive Weights) iterative algorithm with the conjugate gradient approach. The incorporation of the conjugate gradient method accelerates the ADPW algorithm, leading to a reduction in the number of iterations compared to other iterative methods. The ADPW-CG method proves highly efficient for reconstructing randomly sampled signals, owing to its robustness and rapid convergence speed.

Consider a sequence of sampling times $\{t_j\}_{j=1}^{N}$. The reconstruction of a signal x from its samples $x(t_j)$ can be carried out using the ADPW iterative algorithm as follows:

$$\begin{aligned} x_0 &= Sx \\ x_{it+1} &= x_{it} + S(x - x_{it}) \end{aligned} \tag{15}$$

with $it= 0,1,...$ the iteration number and S the approximation operator. S is provided using the equation (16):

$$S_{\omega}x = \sum_{j=1}^{N} x(t_j)\omega_j T_{t_j} \sin c_{BP} \tag{16}$$

$\omega = \{\omega_j\}_{j=1}^{N}$ is the adaptive weight sequence, $x(t_j)$ are the samples values of the signal x, $sincBP$ denotes the well-known sinc-function, BP is the bandwidth and T_{t_j} is a shift operator [13, 17], provided by:

$$T_s x(t) = x(t-s) \tag{17}$$

Based on literature, a recommended approach for determining the weight factors ω_j is as follows: denote the midpoints between samples t_j and t_{j+1} as m_j and define the factors ω_j as [15-17]:

$$\omega_j = m_j - m_{j-1} = \frac{t_{j+1} - t_{j-1}}{2} \tag{18}$$

For a signal x of length N_s, the matrix S has dimensions of $N_s{*}N_s$. This suggests that for lengthy signals, one might encounter computational time or memory constraints. Therefore, it is possible to speed up the ADPW iterative algorithm by implementing the conjugate gradient method without setting the matrix S. Consequently, employing the ADPW-CG method, the signal x can be reconstructed iteratively by the following steps:

$$p_0 = r_0 = E - Sx_0 \tag{19}$$

$$\alpha_{it+1} = \frac{r_{it}^{H} r_{it}}{p_{it}^{H} Sp_{it}} \tag{20}$$

$$x_{it+1} = x_{it} + \alpha_{it+1} p_{it} \tag{21}$$

$$r_{it+1} = r_{it} - \alpha_{it+1} Sp_{it} \tag{22}$$

$$p_{it+1} = r_{it+1} + \frac{r_{it+1}^{H} r_{it+1}}{r_{it}^{H} r_{it}} p_{it} \tag{23}$$

we acknowledge that x_0 is arbitrary, and $it = 0, 1, ...$ denotes the iteration number.

In this scenario, the multiplication of S with a signal x can be performed by [17]:

- Sampling the signal x at the provided sampling positions p_j ($j=1,... ,N$) and for the ADPW method, multiplying the value $x_j(p_j)$ with the weight ω_j requires only N multiplications.
- Then, the operation of S on the vector x can be efficiently executed using two Fast Fourier Transforms (FFTs). Therefore, the complexity in terms of the number of operations for one iteration is $O(N_s log N_s)$ instead of $O(N_s^2)$.

N_s represents the length of the signal $x(t)$. We will explore the convergence speed of the three presented iterative methods in the subsequent section.

IV. Comparative Analysis of Convergence Speed Among the Presented Iterative Methods

To assess the convergence speed of the presented iterative methods, we can compute the error $\|x - \hat{x}_{it}\|_2$ between the original signal x and the approximation $\hat{x}_{it}$ after each iteration. Fig. 2 illustrates the convergence rate for the three iterative methods introduced.

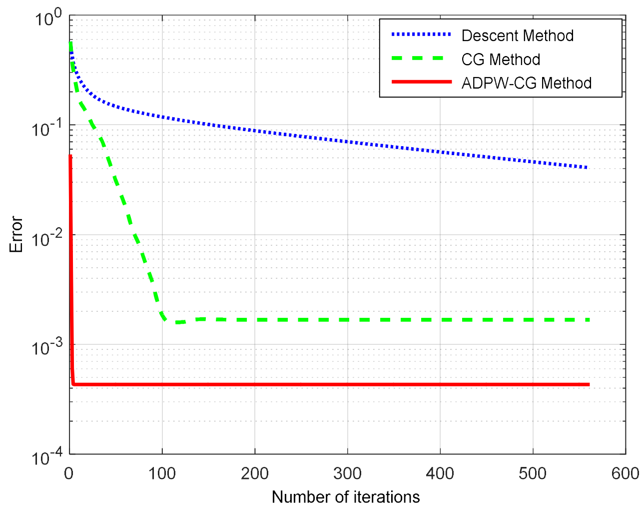

Fig. 2. Comparison of the convergence speed for the three presented methods.

From this figure, we can observe that the descent method is less efficient compared to the other two methods. It requires at least M iterations to reach a solution (in our simulation, the number of frequency components M is equal to 562).

When comparing the CG method with the ADPW-CG method, we see that the ADPW-CG method achieves accuracy much faster, typically within around 5 iterations, compared to 100 iterations for the CG method. Therefore, accelerating the ADPW algorithm with the conjugate gradient method results in faster convergence.

V. Complexity Analysis

In this section, we calculate the number of basic operations in flops (floating point operations) to assess the complexity of the algorithms presented above. Each of the four fundamental operations (addition, subtraction, multiplication, and division) is considered as one flop. Table 1 illustrates the operational complexity for both the direct SVD matrix decomposition algorithm [18, 27] and the three presented iterative methods.

The matrix A has M columns and N rows. We assume that N and M are of similar magnitude (typically, $N \geq M$) to simplify the complexity analysis.

It's worth noting that the number of operations remains constant for direct algorithms, while it varies depending on the number of iterations for iterative methods. The iteration counts are denoted as n_D, n_{CG} and $n_{ADPW-CG}$ for the descent method, the CG method and the ADPW-CG method, respectively. To achieve comparable reconstruction quality across algorithms, we set $n_D = 2M$, $n_{CG} = \frac{M}{5}$ and $n_{ADPW-CG} = \frac{M}{100}$ iterations.

Comparing the number of elementary operations among the analyzed methods, the ADPW-CG method exhibits lower complexity (of order $\frac{M^2}{5}\log M$) compared to the SVD direct algorithm (of order $23M^3$), the descent method (of order $10M^3$), and the conjugate Gradient method (of order $\frac{14}{5}M^3$). Therefore, from a complexity perspective, the ADPW-CG method remains the least costly among the four algorithms discussed.

VI. Application and Simulation Results

A. Generation of the Test Signal

The case-study signal is a multi-band signal constituted of 5 carriers separated by *0.8MHz*, modulated in QPSK then filtered by a raised cosine filter with a roll-off coefficient equal to *0.5*. Each carrier has a symbol rate $R_{sym} = 4.10^5$ *sym/s*. The Fig. 3 illustrates the time and frequency representations of our studied signal which is modulated at a center frequency $f_c = 12.5MHz$.

The non-uniformly sampled signal is generated using a JRS sampling sequence of length N, with an average sampling rate of $f_s = 10MHz$.

For the subsequent simulations, signal reconstruction will be performed for each instant of the discrete time axis. The effectiveness of the reconstruction methods will be evaluated using the test defined by equation (6). Figure 4 depicts the block diagram of the simulation.

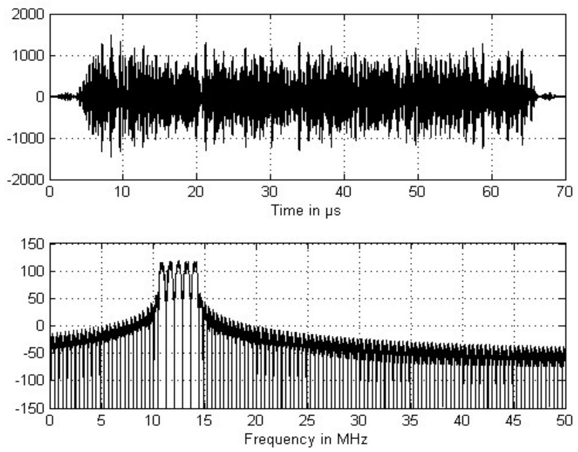

Fig. 3. The time and frequency representations of the studied signal.

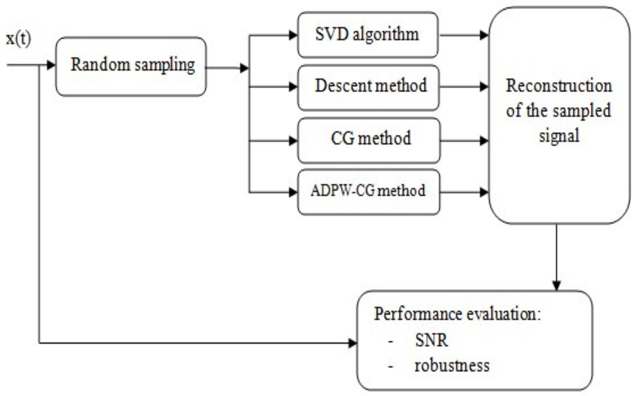

Fig. 4. Simulation Block diagram.

TABLE I
NUMBER OF ELEMENTARY OPERATIONS OF THE DIFFERENT PRESENTED ALGORITHMS

Algorithm	Number of operations
SVD	$\approx 9M^3 + 6N^2M + 8NM^2 + 2M^2 + 2NM + 2M$
Descent method	$\approx n_D(4M^2 + 6M + 1) + 2M^2N + 2M^2 + 2MN + M$
CG method	$\approx n_{CG}(4M^2 + 13M + 2) + 2M^2N + 2M^2 + 2MN + M$
***ADPW-CG** method*	$\approx n_{ADPW-CG}\ (131N + 20N\log N + 3) + 20N\log N + 53N$

B. Effect of the center frequency on the reconstruction process

The objective of this section is to evaluate the reconstruction performance of a randomly sampled multi-band signal (the signal introduced in the preceding subsection).

In this simulation scenario, the reconstruction equations are solved by the direct method using the SVD algorithm and then by the iterative methods outlined earlier. The SNR calculation is performed for central frequencies ranging from *6* to *40MHz*. For the descent method, the number of iterations considered is $n_D=2M$, for the conjugate gradient method it is $n_{CG}=M/5$, and for the ADPW-CG method it is $n_{ADPW\text{-}CG}=M/100$. The performance of each method is analyzed, as depicted in Figure 5 and Figure 6.

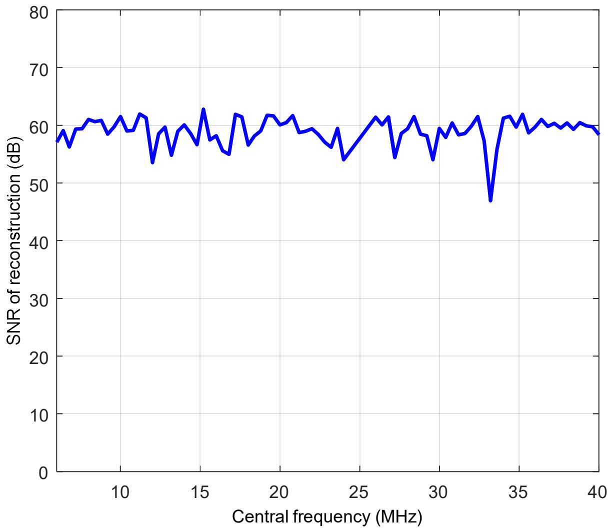

Fig. 5. The SNR of reconstruction versus the center frequency (case of the SVD direct algorithm).

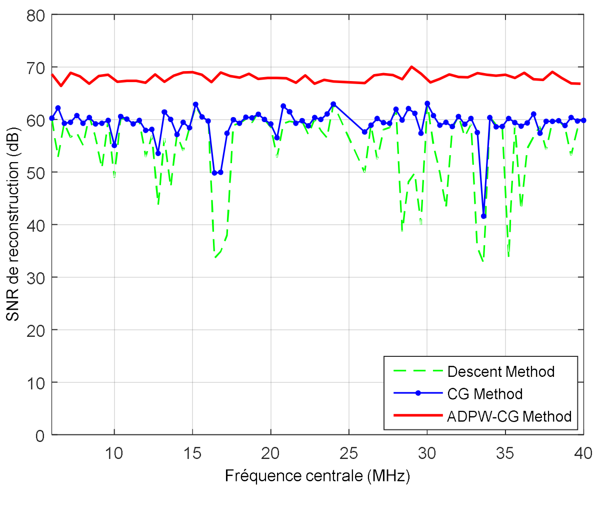

Fig. 6. The SNR of reconstruction versus the center frequency (case of the descent method, the CG method and the ADPW-CG method).

Figures 5 and 6 illustrate that, with a sampling rate slightly higher than the Nyquist rate, the reconstruction process proves efficient (SNR > 30 dB) using various reconstruction algorithms: the SVD direct algorithm and the three iterative algorithms mentioned earlier (descent, CG, and ADPW-CG). It's worth noting that for numerous telecommunication applications, this level of SNR is adequate to render the reconstructed signal noise negligible compared to other sources of noise such as thermal and interference noise [5].

Comparing the various algorithms in terms of reconstruction quality (as shown in Figures 5 and 6), we can observe that the ADPW-CG iterative method presents the highest SNR of reconstruction compared to the other methods, regardless of the center frequency value.

C. Evaluation of the SNR of Reconstruction versus Interference

In Section 2, we established that the signal $x(t)$ can be reconstructed using Equation (1), where the coefficients correspond to the solution that minimizes the quadratic Equation (2). Hence, the channel of interest can be reconstructed based on the c_k coefficients that represent it.

Indeed, let's examine the spectrum of the real signal composed of five bands, as depicted in Figure 7, and let's focus on selecting channel 4. Solving Equation (2) enables us to determine the frequency components within the various signal bands ($c_1 \dots c_p \dots c_q \dots c_{M/2}$ and their conjugates). This allows us to reconstruct channel 4 using Equation (1), utilizing their corresponding coefficients ($c_p \dots c_q$) and their associated frequencies ($f_p \dots f_q$) [28].

In this section, we evaluated the SNR for the reconstruction of the channel of interest under varying levels of interference, using the SVD direct algorithm, the CG iterative method and the ADPW-CG iterative method. As mentioned above, our signal is a randomly sampled multi-band signal. We considered band four as the band of interest (corresponding to the interval of frequencies [*12.9MHz-13.7MHz*]) and we varied the power of adjacent channels (in this simulation, bands *1, 2, 3* and *5*). The formula used to calculate the interference (I) is given by:

$$I(dB) = 10\log_{10}\left[\frac{\textit{Sum of not selected channels power}}{\textit{Useful channel power}}\right] \quad (24)$$

Figure 8 illustrates the evolution of the SNR for the reconstruction of the channel of interest as interference increases. From this figure, we can observe that the SNR of reconstruction decreases with increasing interference. Additionally, the ADPW-CG iterative method demonstrates greater robustness to interference compared to the other two methods (the SVD direct algorithm and the CG method).

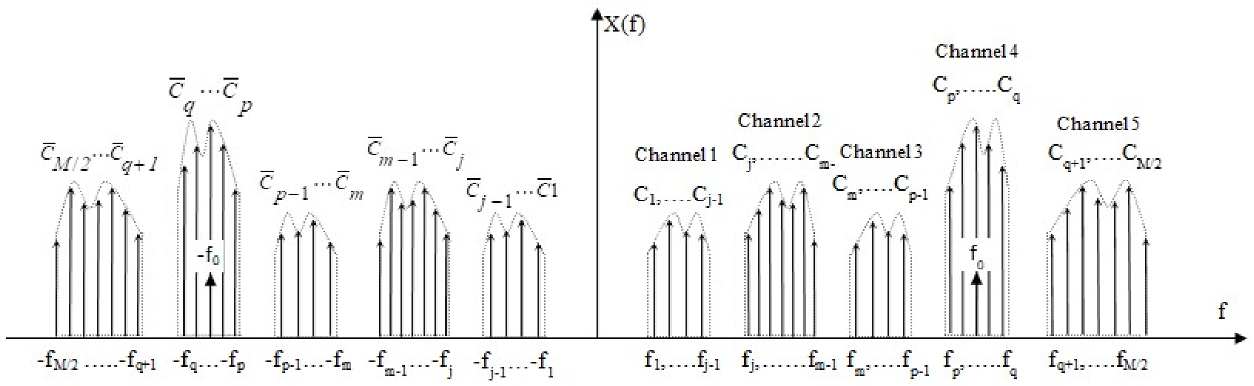

Fig. 7. Example of spectrum of a real 5-band signal.

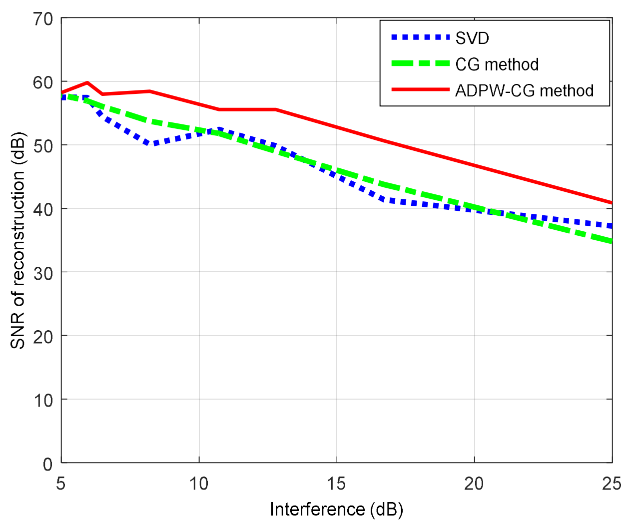

Fig. 8. The SNR of reconstruction of the channel of interest versus interference.

VII. Conclusion

In this contribution, we focused on the reconstruction problem of randomly sampled signals. We presented three iterative methods for the reconstruction of randomly sampled signals (descent method, CG method and the ADPW-CG method) and we were particularly interested in the ADPW-CG iterative method due to its robustness and convergence speed. We assessed the complexity, reconstructed signal quality, and robustness of the performance of the iterative approaches that were provided, and we contrasted the findings with the performance of the SVD direct algorithm.

Based on the obtained results, we observed that the ADPW-CG method demonstrates promising performance compared to other algorithms. It exhibits good reconstruction quality (with an SNR of approximately 67dB), high resilience to interference, and reduced complexity.

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