An Efficient Improved Whale Optimization Algorithm for Optimization Tasks

Jiayin Wang, Yukun Wang*

Abstract—**The Whale Optimization Algorithm (WOA) is an efficient meta-heuristic algorithm inspired by the feeding behavior of whales. Although it has successfully solved many optimization problems, it still suffers from premature convergence and poor accuracy when solving complex problems. To address these issues, this paper proposes an improved whale optimization algorithm called EIWOA to improve the search efficiency and accuracy. First, we introduce a new global search mechanism and encircling prey strategy in EIWOA, which utilizes differential evolution, and sine-cosine search strategy to improve the global search efficiency and avoid premature convergence. Second, we use a lévy flight-based spiral update position strategy to improve the local search efficiency of the algorithm, thus improving the convergence speed and accuracy. Again, we introduce a balancing factor with fluctuating decay properties into EIWOA to better balance exploration and exploitation. Finally, we introduce a dynamic opposite learning-based whale-fall strategy in EIWOA to equip the algorithm with the ability to jump out of the local optimum. The qualitative analysis of the algorithm shows that the EIWOA algorithm converges quickly, is highly accurate, and has the ability to jump out of the local optimum. In order to validate the performance of the proposed EIWOA algorithm, we evaluated the algorithm on CEC2017 benchmark function and four real world engineering problems. We also conduct a comparative study of the EIWOA algorithm with EWOA, an excellent variant of WOA, as well as some excellent meta- heuristics developed recently. The results of numerical experiments demonstrate the superiority of the proposed EIWOA, which is further confirmed statistically by the results of Friedman's test and Wilcoxon signed rank test. The proposed EIWOA algorithm is significantly better than WOA and other competing algorithms in terms of convergence speed, accuracy and optimization ability in dealing with complex optimization problems.**

***Index Terms*— whale optimization algorithm, exploration and exploitation, balancing factor, whale-fall strategy**

I. INTRODUCTION

Optimization problems exist widely in social life and scientific research, such as task allocation, path planning, resource scheduling, and so on. In many practical scenarios, optimization problems are more and more dynamic, nonlinear, uncertain and high dimension. Traditional optimization algorithms such as conjugate gradient and variational methods are often powerless to solve these problems. With the progress of society and the continuous development of artificial intelligence, optimization methods based on swarm intelligence have emerged. Swarm Intelligence was a very popular intelligent computing technique in the last decade [1, 2].

Swarm intelligence is the characteristic of individuals with no or simple intelligence that exhibit swarm intelligent behavior through group cooperation and organization. Group intelligence algorithms utilize probabilistic search to find optimal solutions in the solution space without requiring excessive prior knowledge and are well suited for solving non-deterministic polynomia problems. Various meta-heuristic algorithms are based on different natural phenomena and philosophies, such as Genetic Algorithm (GA) [3], Particle Swarm Optimization (PSO) [4], Gravitat-Ional Search Algorithm (GSA) [5], Differential Evolution algorithm (DE) [6], Animal Migration Optimization (AMO) [7], Backtracking Search Optimization Algorithm (BSA) [8], Grey Wolf Optimization Algorithm (GWO) [9], Tree Species Optimization Algorithm (TSA) [10], Whale Optimization Algorithm (WOA) [11], Beluga whale optimization (BWO) [12], Artificial Ecosystem Based Optimizattion Algorithm (AEO) [13], Rat Swarm Optimizer (RSO) [14], Flow Direction Algorithm (FDA) [15], Transit Search (TS) [16], and White Shake Algorithm (WSO)[17], have been proposed and successfully applied in many optimization areas.

WOA is proposed by Mirjalili et al. in 2016. It is a meta-heuristic optimization algorithm that simulates the hunting behavior of humpback whales. The main difference between WOA and other swarm optimization algorithms is the use of stochastic or optimal search agents to simulate hunting behavior and the use of spirals to simulate the humpback whale bubble net attack mechanism. The algorithm has the advantages of the simple mechanism, few parameters, and strong optimization ability, etc. The algorithm requires fewer parameters to be set, is simple to operate, and has strong optimization performance. At present, the algorithm has been successfully applied to cloud resource scheduling, location path planning, optimal power flow in power system, industrial design, engineering field, economic scheduling, optimal control, photovoltaic system, image segmentation and so on.

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Compared with classical optimization algorithms such as PSO and GA, WOA has better global exploration ability, but its local search ability and convergence speed need to be improved. In recent years, researchers have proposed many variants of WOA to enhance the performance of WOA, which can be roughly divided into three categories: (1) parameter optimization; (2) Strategy improvement; (3) Algorithm fusion. Nadimi-Shahraki et al., proposed an enhanced whale optimization algorithm (EWOA) in 2019 [18]. Chen et al. used improved Bernoulilli Shift mapping to initialize the whale population to maintain the diversity of the population and proposed an improved whale algorithm named IWOA. The adaptive convergence factor is introduced to balance the local and global optimization capability of the algorithm[19]. Mafar et al.[20] hybridized the WOA with simulated annealing to solve feature selection problems. Ling et al. [21] proposed an improved WOA algorithm (LWOA) based on lévy behavior in 2017. Tawhid et al proposed an BWOA algorithm for finding optimal minimum subsets in feature selection problems, which combines the rough set method, wrapper method and whale optimization algorithm. A quantum-based whale optimization algorithm (QWOA) [23] for the wrapper feature selection problem was proposed to improve the population diversity and convergence rate of WOA. In the QWOA, a quantum bit representation was proposed to maintain the population diversity, and a quantum rotation gate operator was introduced to balance the search strategies. Debashis et al. proposed the modified whale optimization algorithm named MWOA [24] realizes the application of two-degree-of-freedom fractional order fuzzy proportional integral derivative controller in automatic generation control of multi-region interconnected power system. Zhao et al. [25] proposed a cooperative whale optimization algorithm (CWOA) for energy-efficient sche-duling of the distributed blocking flow-shop with sequence dependent setup time. Wang et al. [26] developed a cross domain algorithm based on a hybrid whale optimization algorithm with simulated annealing (CDWOASA). Got et al. [27] proposed an multi-objective algorithm that hybridi-zzed the filter-wrapper feature selection approach with the WOA. Chen et al. [28] proposed a multi-objective whale optimization algorithm (MONIWOA) to solve the non convex optimized power flow problem. In MONIWOA, the piece wise non-linear strategy, dual dynamic weights mode and constrains-prior Pareto-dominant rule are adopted to enhance the performance of the algorithm.

Inspired by those methods, an efficient improved whale optimization algorithm, named EIWOA, is proposed in this paper. Five improved strategies, including new global search mechanism, new encircle prey method, new spiral updating position strategy, new balancing factor, and a whale-fall strategy are used to balance the exploration and exploitation and to improve the convergence speed and accuracy. The new global search mechanism and encircling prey strategy based on sine-cosine operator and differential evolution strategy improves the global search efficiency so that the algorithm can search as many as desired optimal regions as possible, avoiding the algorithm from falling into local optimum. The new lévy flight-based spiral updating position strategy enhances the whale's ability to search the unknown regions. It can better adapt to the search requirements of different distances and avoid duplicate paths during the search process, thus improving the local search efficiency and accuracy. The new balancing factor has a fluctuating decay property, which is used to balance the exploration and exploitation, so that the new algorithm has a certain global search ability at the late stage of the search. The whale-fall strategy based on dynamic opposite learning equips the algorithm with the ability to jump out of the local optimum and better facilitates the algorithm to converge to the global optimum.

The structure of this paper is as follows: original WOA algorithm is introduced in Section II. and the detailed presentation of the proposed EIWOA is provided in Section III. Section IV shows the results of experiment and statistical analysis. In Section V, EIWOA is used to solve four engineering problems. Finally, the paper gives conclusions in Section VI.

II. Whale optimization algorithm

The model of WOA is constructed mathematically [29]. The specific definition of the model is as follows, assuming that there are n whales in the D-dimensional space, the ith individual of the tth iteration can be expressed as:

$$X_i^t = (x_{i,1}, x_{i,2}, \dots, x_{i,D}), i = 1,2,\dots,t_{\max} \quad (1)$$

Where, t_{max} is the maximum number of iterations. In the process of optimization, the algorithm includes three search mechanisms: the local search of encircling prey, the local search of bubble net attack and the global search of the random learning mechanism.

A. Encircle the prey

This process mimics that of a humpback whale recognizing the location of its prey and encircling it. Since the optimal position in the search space is unknown [30], the WOA algorithm assumes that the current optimal solution is the target prey or a near-optimal solution. After determining the concept of optimal position, other whales swim to the vicinity of the current optimal position to realize the encirclement of the prey. At this stage, the specific update formula for whale position is defined as follows:

$$D = | C \times X^*(t) - X(t) | \quad (2)$$

$$X(t+1) = X^*(t) - A \times D \quad (3)$$

Where t is the current iteration, A and C are the distance adjustment parameters, $|\cdot|$ is the absolute value, and the distance adjustment parameters are defined as follows:

$$A = 2a \times r_1 - a \quad (4)$$

$$D = 2 \times r_2 \quad (5)$$

$$a = 2(1 - \frac{t}{T_{\max}}) \quad (6)$$

Where r_1, r_2 are random numbers between [0,1], a is a parameter that varies with the number of iterations and decreases linearly within [2,0], and $T_{\max}$ is the maximum number of iterations.

B. Bubble-net attacking method

Bubble net attacks mimic the hunting behavior of humpback whales, which swim upward along a spiral path of decreasing radius of curvature and encircle prey by blowing bubbles to form a wall of air. The bubble net attack model consists of two strategies: contraction encircles and spiral position updating.

(1) Contraction encircles. This is achieved by reducing the convergence factor a in Eq.(3). As the value of a decrease, the range of fluctuation of A decreases simultaneously. When $|A| < 1$, the position of each whale gradually approaches the target position as it moves, ultimately realizing the contraction of the prey.

(2) Spiral position updating. Firstly, the distance between the whale and the prey is calculated, and then the spiral swimming of the whale is simulated to capture the prey. The mathematical model is shown as follows:

$$X(t+1) = D^{'} \cdot e^{bl} \cdot \cos(2\pi l) + X^{*}(t) \quad (7)$$

$$D^{'} = |X^{*}(t) - X(t)| \quad (8)$$

Where D represents the distance between each individual and the current optimal solution; b is a constant defining the shape of the logarithmic spiral; and l is a random number in the interval [-1, 1].

When encircling prey along a spiral path, whales also need to narrow their encirclement. To model this effect, the probability p is chosen to narrow the envelope and update the spiral position. The mathematical model is shown below:

$$X(t+1) = \begin{cases} X^{*}(t) - A \cdot D, p < 0.5, |A| < 1 \\ D^{'} \cdot e^{bl} \cdot \cos(2\pi l) + X^{*}(t), p \ge 0.5 \end{cases} \quad (9)$$

Where, p is the probability factor of uniform distribution in the interval [0,1].

C. Random global search mechanism

This mechanism updates the position of each individual by randomly changing the position of whales. When P<0.5 and $|A| \geqslant 1$, individual whales no longer update their position based on the position of the best whale in the current population, but instead randomly select other individual whales and move to them. The expression for position update is as follows:

$$D = |C \cdot X_{rand}(t) - X(t)| \quad (10)$$

$$X(t+1) = X_{rand}(t) - A \cdot D \quad (11)$$

Where, X_{rand} represents the position of the whale randomly selected from the current population.

III. THE PROPOSED EIWOA ALGORITHM

In this section, a new variant of WOA called EIWOA is proposed to balance the exploration and exploitation, avoid the algorithm from falling into local optimums, improve the global and local search efficiency, and increase the convergence speed and accuracy. The block diagram of EIWOA is given in Fig. 1.

A. New global search mechanism

In WOA, random global search mechanism updates individual positions by randomly changing the position of whales. This search method is too random and lacks the guidance of historical search information, so the search efficiency is low, in order to improve the efficiency of the algorithm global search, this paper designs a new global search formula based on the inspiration of PSO algorithm. The new global search formulas are defined as follows:

$$D = (C * X_{rand}(t) - X_{Cbest}) + \left| C * X_{rand}(t) - X_{Pbest}(t) \right| \quad (12)$$

$$X(t+1) = X_{rand}(t) - \psi(A) \cdot A \cdot D \quad (13)$$

Where, X_{Pbest} is the personal best solution of each whale, X_{Cbest} is the best solution of personal best solutions. $\Psi(\cdot)$ is the probability density function of the standard normal distribution.

From Eqs. 11 and 12, it can be seen that the algorithm performs a global search in which the position of the whale is guided by the personal best solutions. It will improve the search efficiency compared to a completely randomized search.

B. New method to encircle prey

The WOA algorithm uses a grid search strategy to encircle the prey. This strategy basically searches around the neighborhood of the current optimal solution and gradually narrows the search range to complete the encirclement of the prey. Since the encirclement operation is guided only by the current best solution, it can accelerate the convergence speed, but it also leads to the rapid loss of population diversity of the WOA algorithm, and the algorithm is prone to fall into a local optimum. To alleviate this shortcoming, a new prey encirclement formula is designed in this paper, as shown in Eq. (14) .

$$X^{t+1}(i,j) = \begin{cases} X^{t}_{Pbest}(r_1, j) + 0.5(X^{t}_{Pbest}(r_2, j) - X^{t}_{Pbest}(r_3, j))\sin(2\pi\theta), & \text{if } j \text{ is odd} \\ X^{t}_{Pbest}(r_1, j) + 0.5(X^{t}_{Pbest}(r_2, j) - X^{t}_{Pbest}(r_3, j))\cos(2\pi\theta), & \text{if } j \text{ is even} \end{cases} \quad (14)$$

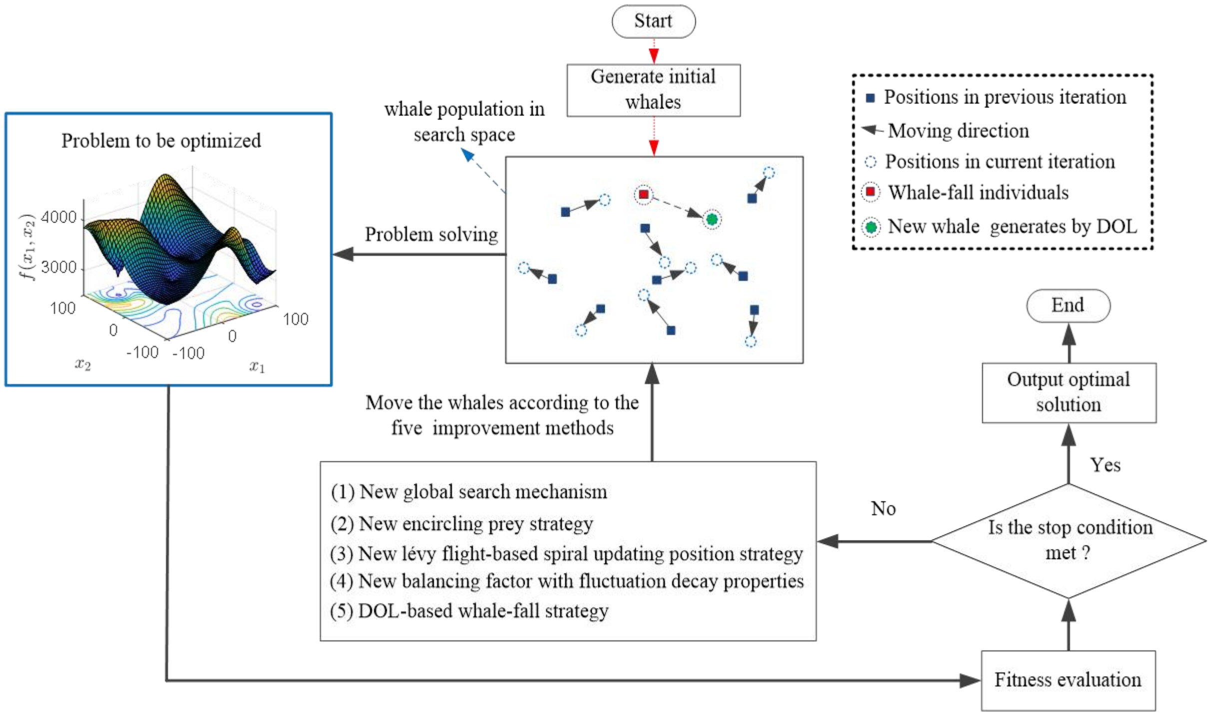


Fig.1 Block diagram of the EIWOA algorithm.

Where r_1, r_2 and r_3 are random integers between 0 and D ($r_1 \neq r_2$, $r_2 \neq r_3$, $r_1 \neq r_3$), respectively. D is the dimension of the problem to be solved.

Eq.14 combines a differential evolution strategy [31] and a sin-cosine search strategy [32], both of which have been shown to be effective in improving the search efficiency of the meta-heuristic algorithms. The new encircle prey strategy is effective in maintaining the diversity of the population and also provides a better search for possible optimal regions.

C. New Spiral updating position strategy

In WOA, during the stage of updating positions, the whale adjusts the moving distance according to the distance between the prey (the current best solution) and its own position, so as to complete the search of the neighborhood of the current best solution, thus improving the convergence accuracy. This simple spiral search pattern approaches the target along a fixed spiral path each time, as shown in Eq. (6). It makes the algorithm prone to fall into local optimum and weakens its local search capability. To address this problem, we propose a new spiral updating position strategy. This strategy dynamically adjusts the search path of the whale during the search process and introduces the personal best solution and lévy flight strategy in the WOA, which enhances the whale's ability to search unknown regions. It can better adapt to the search requirements of different distances and avoid duplicate paths during the search process, thus improving the search efficiency and accuracy of the algorithm. The new formula for updating the spiral updating position is as follows:

$$X^{(t+1)} = X^{*(t)} + e^{l} \cdot \cos 2\pi l \cdot [X_{Pbest}^{(t)}(r_4, pa_1) - X_{Pbest}^{(t)}(r_5, pa_2)] \cdot l\acute{e}vy \tag{15}$$

Where r_4 and r_5 are random integers between 0 and N ($r_4 \neq r_5$), respectively, and N is the population size. *lévy* is a D-dimensional vector, generated by the lévy flight operator. pa_1 and pa_2 are two D-dimensional vectors, respectively, and the value of each dimension is a non-repeating random integer from 1 to D.

D. Design of new balancing factor

Balancing exploration and exploitation is an important element in the improvement of meta-heuristic algorithms. In WOA, whether the whale performs contraction encircles, spiral position updating, or random search search strategy is determined by the balancing factor P (P=0.5 in WOA). This coordination modalities is not conducive to adjusting the algorithm's exploration and exploitation capabilities. We would like the algorithm to perform more exploration operations in the early stages of the search to better traverse the possible optimal regions, and more exploitation operations in the later stages of the search to improve convergence accuracy. Therefore, in this paper, a new balancing factor is designed as shown in Eq. (16).

$$P = \alpha - \left| 2(r_6 - 1)(2 - (T_{\max} - t)\frac{2}{T_{\max}}) \right| \tag{16}$$

Where r_6 is a random vector in [0,1] α is a parameter that needs to be determined manually in advance.

In EIWOA, the algorithm generates a random number r_7, when $r_7 < P$, the algorithm performs contraction encircles and random search operations and otherwise performs spiral position updating operations. Fig.2 gives a curve of the balancing factor P when the value of α is 1.5 and the maximum number of iterations is 1000. From Fig. 2, it can be seen that the value is of P fluctuating and decaying with the increase of iterations. In the early stage, the number of

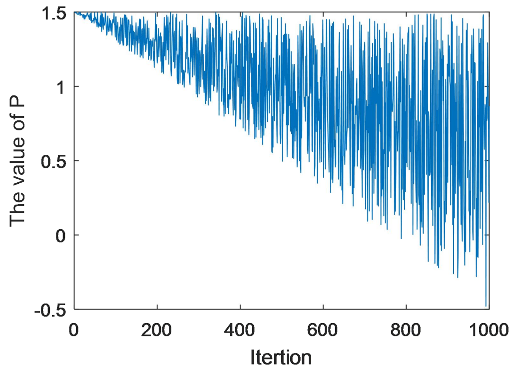


Fig.2 The curve of the balancing factor.

iterations t is small, the value of the balance factor is large, the algorithm has more chances to carry out contraction encircles and global search operations, the exploration ability of the algorithm is stronger, which is conducive to the algorithm searching for more optimal regions with expectations, and with the increase in the number of iterations, the value of P will continue to decay, and the algorithm will have more chances to carry out the operation of spiral updating position, which is conducive to the improvement of the convergence speed and the search accuracy. It can also be seen from Fig. 2 that the EIWOA algorithm still has the opportunity to perform contraction encircles and global search operations in the later stages of the search due to the random fluctuations of P, which helps to avoid the algorithm from falling into local optimum.

E. whale-fall strategy for jumping out of local optimum

Falling into local optimum is the drawback of most meta-heuristic algorithms, and how to help the algorithms jump out of the local optimum is also an important direction for the improvement of meta-heuristic algorithms. In this paper, we introduce the whale-fall strategy to help the EIWOA algorithm jump out of the local optimum.

During migration and foraging, whales are threatened by polar bears and humans. Most whales are smart enough to avoid threats by exchanging information with each other. However, a few whales are not survived and crashed into the deep ocean. This phenomenon is known as whale-fall [33]. This phenomenon is conducive to ensuring the vitality of the population, and its application to the EIWOA algorithm can effectively avoid the algorithm from falling into a local optimum. In order to keep the population size of the algorithm constant, after generating whale-fall, the algorithm will use the DOL strategy to generate new whales based on the location of the whale-fall and the step size of the whale-fall[34,35].

In the EIWOA algorithm, we stipulate that if the whale fails to find new prey (a better personal best solution) over multiple iterations of the search process, it starves to death, producing a whale fall. In order to mathematically simulate the whale fall, we define the whale fall factor β, when a whale fails to find a better personal best solution for a number of times $C>\beta*T_{\max}$, the whale fall is generated, and the formula for the generation of new whale is as follows:

$$X_{new}=X_{fall}+r_8\times[r_9(L_B+U_B-X)-X_{fall}] \quad (17)$$

Where, X_{new} represents a new whale generated by dynamic opposite learning, X_{fall} represents a whale that dies and produces a whale-fall, L_B and U_B are the boundary of the problem. r_8 and r_9 are two random numbers between 0 and 1. The program flowchart of EIWOA is shown in Fig.3.

IV. EXPERIMENTS

In this section, experimental comparative study and statistical analysis are carried out to examine the efficiency, effectiveness and stability of the proposed EIWOA algorithm. To make a fair comparison, all experiments are implemented in MATLAT 2018a and conducted on the PC with i7-9400 CPU @ 2.90GHz, 16GB RAM under Microsoft Windows 10 operating system.

A. Benchmark functions

To measure the performance of the proposed EIWOA, the CEC2017 benchmark functions [36] will be used for verification. The CEC2017 benchmark functions consist of 30 functions with four types. In this paper, f_2 is removed due to its instability in high dimensions.

The reserved functions are unimodal functions (f_1 and f_3,), simple multimodal functions (f_4 to f_{10}), hybrid functions (f_{11} to f_{20}) and composition functions (f_{21} to f_{30}), respectively. Different types of functions can comprehensively and effectively test the optimization ability and stability of the algorithm.

B. Parameters selection of EIWOA

In EIWOA, the parameters α and β need to be determined in advance. These two parameters affect the performance of the algorithm. the value of α affects the exploration and exploitation intensity of the algorithm, while β $a$ffects the probability of whale-fall. In this subsect-

TABLE I
DIFFERENT PARAMETER COMBINATION OF EIWOA ALGORITHM

NO.	Parameter combination	α	β
1	P1	1.5	0.005
2	P2	1.7	0.005
3	P3	1.9	0.005
4	P4	2.1	0.005
5	P5	1.5	0.01
6	P6	1.7	0.01
7	P7	1.9	0.01
8	P8	2.1	0.01
9	P9	1.5	0.025
10	P10	1.7	0.025
11	P11	1.9	0.025
12	P12	2.1	0.025
13	P13	1.5	0.05
14	P14	1.7	0.05
15	P15	1.9	0.05
16	P16	2.1	0.05
17	P17	1.5	0.075
18	P18	1.7	0.075
19	P19	1.9	0.075
20	P20	2.1	0.075

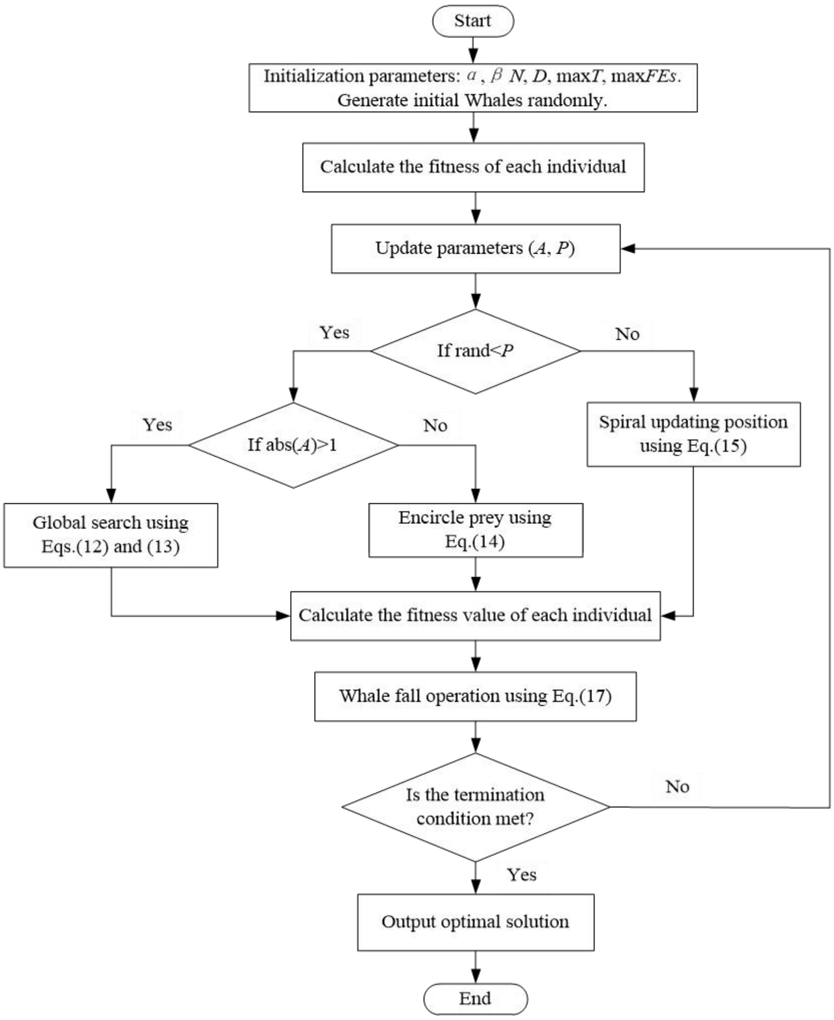


Fig.3 Program flowchart of EIWOA.

ion, we design 20 parameter combinations to investigate the effects of these two parameters on the performance of EIWOA. The specific parameter combinations are shown in Table I. In order to determine the appropriate parameters of the EIWOA algorithm from Table I, we compare the performance of the algorithm under each parameter combination on 12 functions in the CEC2022 benchmark function [37]. The dimensions of these 12 functions are randomly chosen as 10 and 20, as shown in Table Ⅱ.

TABLE II
DIMENSION OF FUNCTIONS USED FOR PARAMETER SELECTION

Type	$f(x)$	Dimension
Unimodal functions	f_1	20
multimodal functions	f_2	10
	f_3	20
	f_4	10
	f_5	10
Hybrid functions	f_6	20
	f_7	20
	f_8	20
Composition functions	f_9	10
	f_{10}	10
	f_{11}	20
	f_{12}	20

When parameterizing the EIWOA, the other parameters of the algorithm are set as follows: populations size N is 50, maximum evaluations maxFEs are 1.0×10^5, maximum iterations T are 2000, and each algorithm independently runs 30 times for each test function. The performance of EIWOA with 20 kinds of parameter combinations is list in Table III. In Table III, "Total rank" is the cumulative result of "rank", and "rank" is the ascending sorting result of the average value of the best results obtained by EIWOA for each test function. The smaller the "Total rank", the better the comprehensive optimization ability of the EIWOA algorithm under this parameter combination. It can be seen from Table III that EIWOA has best comprehensive performance under P9 parameter combination. Although this parameter combination may not be optimal, it is the best of the 20 optional parameters. Therefore, we choose α=1.5, β=0.025 as the initial parameter of EIWOA.

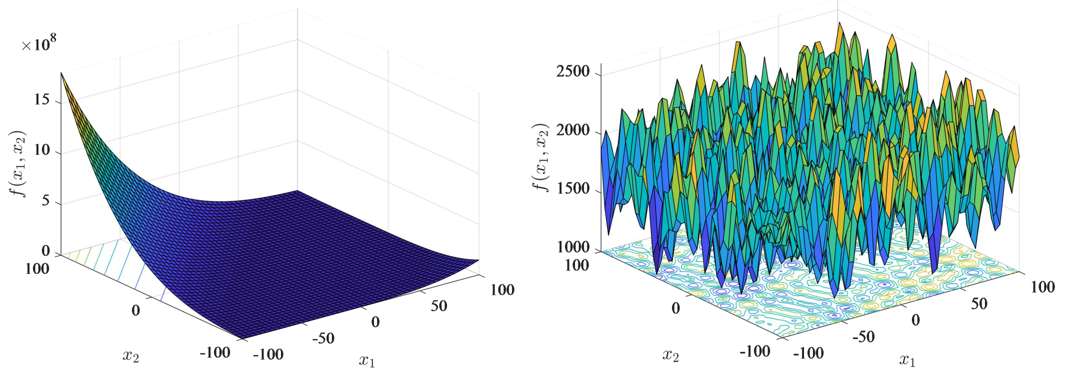
C. Qualitative comparison between WOA and EIWOA

To show the difference of WOA and the proposed EIWOA, this paper compared them qualitatively from the aspects of population diversity and convergence curve on a unimodal function (f_3) and a multimodal function (f_{10}) in CEC2017. Unimodal function can provides a better test of the convergence speed and accuracy of EIWOA and Multi-

TABLE III

PERFORMANCE OF EIWOA UNDER DIFFERENT PARAMETER COMBINATIONS

Index	$f(x)$	Parameter combinations																			
		P1	P1	P3	P4	P5	P6	P7	P8	P9	P10	P11	P12	P13	P14	P15	P16	P17	P18	P19	P20
Rank	f_1	17	18	19	20	8	10	12	15	3	6	11	16	2	5	9	14	1	4	7	13
	f_2	17	18	19	20	11	12	16	5	3	15	10	4	8	6	13	2	9	7	1	14
	f_3	17	18	19	20	16	5	15	6	1	7	2	8	9	3	4	10	14	11	12	13
	f_4	18	17	19	20	13	15	16	14	10	11	7	2	9	5	3	1	12	8	6	4
	f_5	18	17	19	20	3	8	16	14	7	15	11	1	5	9	10	12	2	13	6	4
	f_6	5	18	19	20	11	4	9	7	15	10	3	13	8	6	16	14	12	17	2	1
	f_7	17	18	19	20	15	13	16	14	10	3	2	9	11	4	7	6	12	5	8	1
	f_8	17	18	19	20	3	1	9	12	7	2	8	6	13	4	10	14	5	16	15	11
	f_9	17	18	19	20	16	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	f_{10}	18	17	19	20	15	13	12	3	4	9	7	2	14	6	5	1	8	10	11	16
	f_{11}	17	18	19	20	11	13	16	15	2	6	3	1	8	9	10	12	7	14	4	5
	f_{12}	17	18	19	20	16	15	11	10	8	2	12	7	9	14	3	6	1	4	13	5
Total Rank	—	195	213	228	240	138	110	150	118	74	91	82	76	104	80	100	103	95	122	99	102



(a) f_3 (b) f_{10}

Fig.4. 3D figures of f_3 and f_{10}

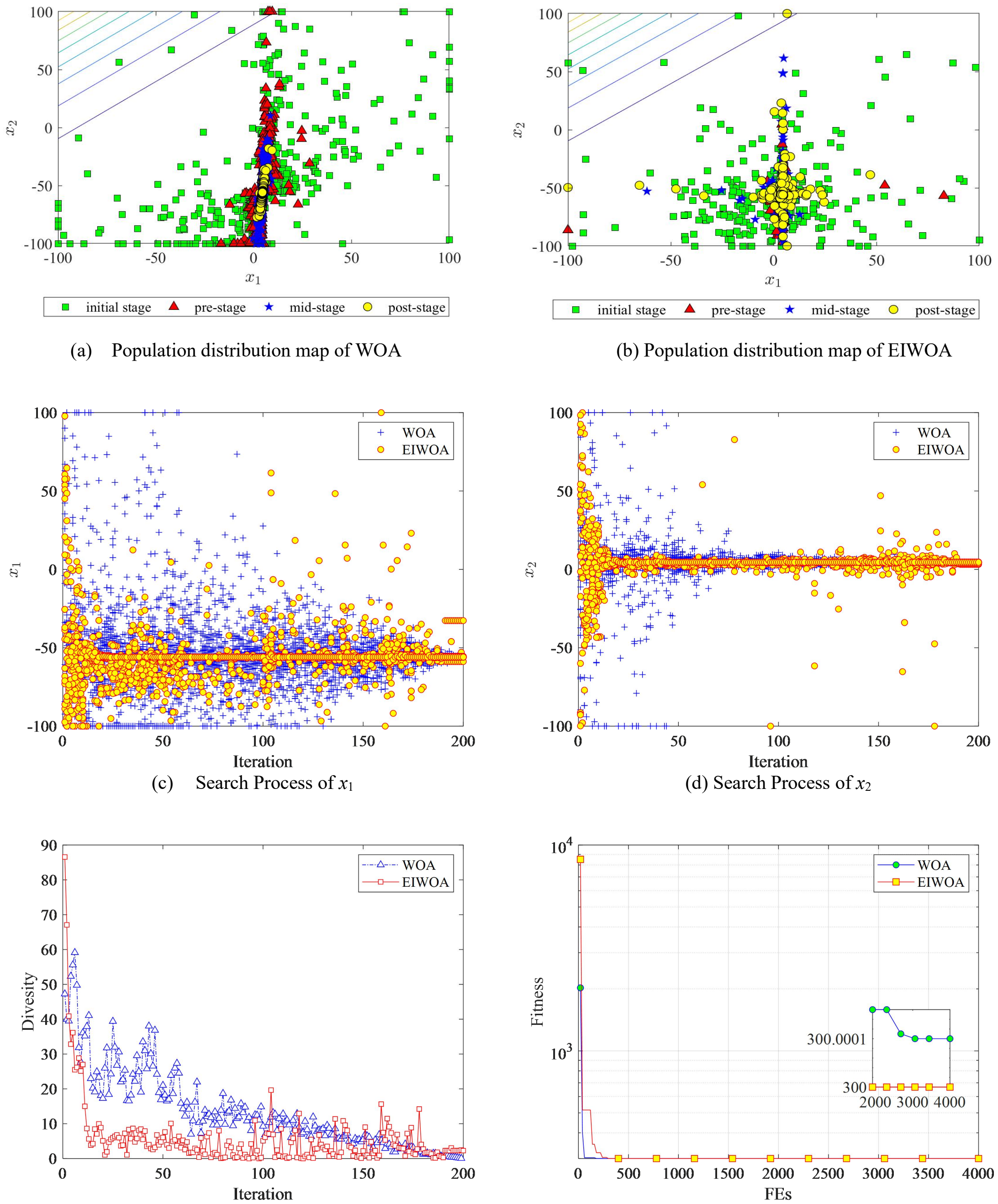
modal function can be a better test of the algorithm's ability to overcome local optima. The 3D figures of the two functions are shown in Fig.4. To observe and compare the results more intuitively, the dimension is set to 2, the maximum iteration T is set to 200, the population size is set to 20, and the diversity of population is defined and shown as Eq. (18).

$$div(t)=\frac{1}{0.5\times N\times\left\|U_B,L_B\right\|}\sum_{i=1}^{N}\left\|x_i(t),\bar{x}(t)\right\| \quad (18)$$

Where t is the current iteration, N is population size, D is dimension, x_i is the ith individual, $\|\cdot\|$ represents european distance, U_B, L_B are the lower and upper bounds of the search space, respectively.

The results of qualitative comparison between WOA and EIWOA are graphically shown in Fig. 5 and Fig.6. In Fig. 5(a) and Fig. 5(b), we divided the whole search process into four stages, i.e., initial stage (t<50), pre-stage (50≤t<100), mid-stage (100≤t<150) and post-stage (150≤t<200). We can see that there is no significant difference in population dispersion between WOA and EIWOA during the whole search process. However, EIWOA has a higher degree of population dispersion in the post-stage, which is mainly due to the whale-fall effect. At the post-stage of the whole search process, EIWOA has obtained the minimum value of f_3, and the fitness values of all individuals cannot continue to be updated and tend to stagnate, at which time EIWOA guides the individuals to search other possible optimal regions. From Fig. 5(c) and Fig. 5(d), it can be seen that for unimodal function, EIWOA can concentrate on the search for the optimal solution, and thus the search is more efficient. From Fig. 5(e), it can be seen that the diversity of EIWOA decreases faster in the early and middle stages of the search, which is conducive to a fast search for the optimal solution of the unimodal function, while in the later stages of the search, the algorithm can automatically regulate the population diversity and guide the individuals to search for the better solution (The present algorithm itself is not aware of the type of the search function.). As can be seen from Fig.5(f), the EIWOA algorithm has a better convergence accuracy than the WOA algorithm.

For function f_{10}, it can be seen from Fig.6(a) and Fig.6(b)



(a) Population distribution map of WOA
(b) Population distribution map of EIWOA
(c) Search Process of x_1
(d) Search Process of x_2
(e) Diversity Curves
(f) Convergence Curves

Fig.5. Results of qualitative comparison on function f_3

that EIWOA has a better population dispersion, which facilitates the algorithm to perform an effective search for each local minimum of multimodal function. From Fig.6(c) and Fig.6(d), it can be seen that the individuals of the EIWOA have a clear jump out of the local optimum operation around the iteration number of 80, while the WOA always converges to a single objective. From Fig. 6(e), it can be seen that the population diversity of EIWOA fluctuates more in the later stages of the search, which indicates that the whale-fall strategy is in its role. From Fig.6(f), it can be seen that EIWOA has the ability to jump out of the local optimum and converge to the global optimal solution, while WOA is stuck in the local optimum and cannot jump out.

From the above comparison results, we can see that EIWOA shows different characteristics and performance from WOA during the search process. The EIWOA is more efficient in searching for unimodal functions with better search accuracy and demonstrates its ability to overcome the local optimum for multimodal function. Therefore, the improvement strategies proposed in this paper are effective.

D. Quantitative comparison with other algorithms

In order to verify the superiority of the proposed EIWOA, this paper compares EIWOA with WOA, EWOA (An excellent variant of WOA), BWO, AEO, RSO, FDA, TS and WSO. The parameters of all the compared algorithms are based on the recommendations of the corresponding references, see Table IV. For the comparison experiments, the population size N of each comparison algorithm is 50, the maximum number of evaluations maxFEs are 1.0×10^5, and each algorithm is run independently for 30 times. The comparative results of the numerical experiments for the 50 and 100 dimensional test functions of CEC2017 are shown in TableV and Table VI, the "Rank" is sorted in ascending order of the "Mean" value, and the "Total Rank" is the cumulative result of the "Rank". "Final Rank" is sorted in ascending order of "Total Rank". As can be seen from TableV and Table VI, the "Final Rank" of EIWOA is ranked first in both 50 and 100 dimensional test functions, and the value of the "total ranking" of the EIWOA algorithm is significantly smaller than that of the other comparison algorithms. In order to intuitively compare the comprehensive performance, stability, and ability to obtain the optimal solution of each algorithm, Table VIII provides detailed statistics on the results of TableV and Table VI.

The "Mean" indicator provides a good analysis of the comprehensive performance of the algorithm. For the "Mean" indicator, Table VII shows that EIWOA obtained 22 best and 4 second best results in 50 dimension, 24 best and 3 second best results in 100 dimension and no worst results. AEO is the second best algorithm in 50 dimension, and it only obtained 4 best and 6 second best results. AEO is also the second best algorithm in 100 dimension and it only obtained 4 best and 10 second best results. Therefore, the EIWOA significantly outperforms the AEO algorithm, and the advantages of the EIWOA algorithm over the WOA and EWOA algorithms are even more significant.

The "Std" indicator can be used to analyze the stability of the algorithm. For the "Std" indicator, Table VII shows that EIWOA obtained 16 best and 6 second best results in 50 dimension, 19 best and 4 second best results in 100 dimension and no worst results. The stability of EIWOA is significantly better than other algorithms.

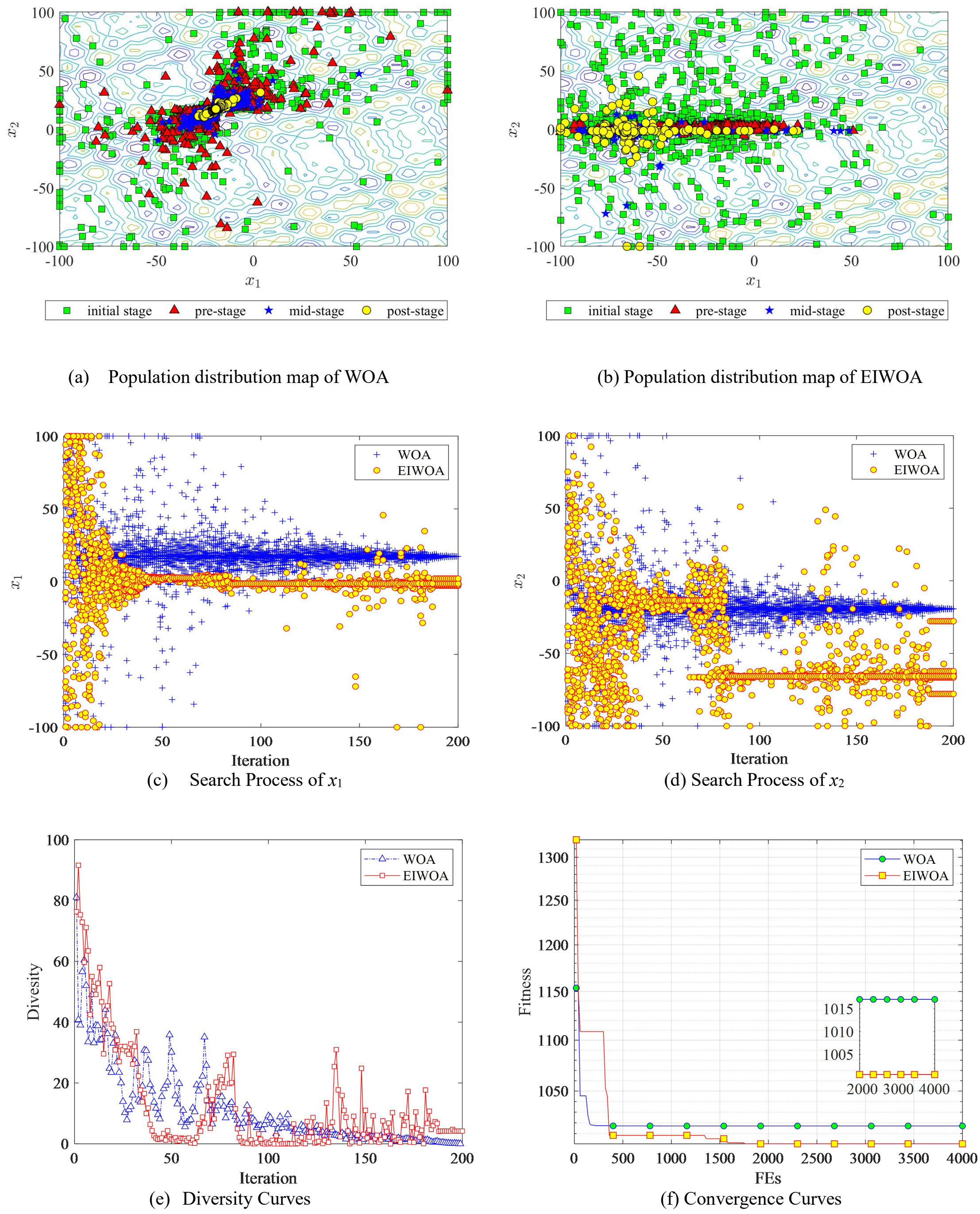


Fig.6. Results of qualitative comparison on function f_{10}

TABLE IV
TABLE IV PARAMETERS SETTINGS OF COMPARISON ALGORITHMS

No.	Name	Parameter settings	Publication year	Literature
1	AEO	a=(1-t/T)*rand;	2020	[13]
2	BWO	W_f decreases linearly from 0.1 to 0.05	2022	[12]
3	RSO	E=2*(1 - t/T)	2020	[14]
4	FDA	α=N (Popsize), β=8	2021	[15]
5	TS	ns = 5 (Number of Stars), SN = 10 (Signal to Noise Ratio)	2022	[16]
6	WSO	p_{min}=0.5, p_{max}=1.5, a_0=6.250, a_1=100, a_2=0.0005	2022	[17]
7	WOA	a decreases linearly from 2 to 0	2016	[11]
8	EWOA	w_{Max} = 0.7, w_{Min} = 0.2, γ, μ is a random number in the range of in [0, 1], and a linearly decreases from 2 to 0.2.	2022	[18]
9	EIWOA	α=1.5, β=0.025, a decreases linearly from 2 to 0	—	—

The ability of the meta-heuristic algorithm to obtain the best value is also very important, and more attention will be paid to the ability of the algorithm to obtain the best value when solving real engineering optimization problems. For the "Best" indicator, Table VII shows that EIWOA obtained 21 best and 1 second best results in 50 dimension, 24 best and 2 second best results in 100 dimension. The other algorithms are far worse than EIWOA in obtaining the best value.

Comparing WOA and EIWOA individually, it can be seen from Tables V and VI that EIWOA is ranked higher than WOA in all the test functions, which indicates that the improvement strategy proposed in this paper is effective. The advantages of EIWOA over the EWOA algorithm are also obvious. Literature [18] has confirmed that the EWOA algorithm outperforms other improved versions of the WOA such as LWOA (2017), CWOA (2018), MWOA (2018), BWOA (2019), WOA-Mm (2020) and HS-WOA+ (2020). Thus, the proposed EIWOA is an excellent variant of the WOA algorithm.

In summary, the improved strategies, i.e., new global search mechanism, new encircling prey strategy, new lévy flight-based spiral updating position strategy, new balancing factor with fluctuation decay properties, and DOL-based whale-fall strategy, are satisfactory and competitive for enhancing the performance of WOA.

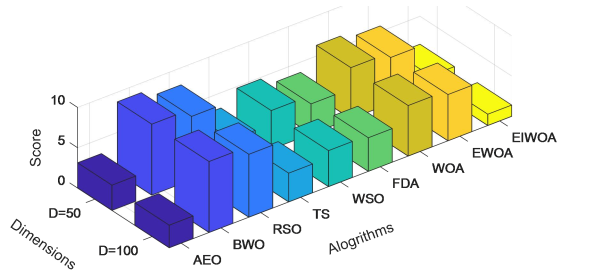
E. Statistic analysis

It would be more scientifically meaningful to statistically validate the advantages of the EIWOA algorithm. In this paper, two non-parametric statistical methods, Friedman's test [38] and Wilcoxon signed rank test [39], which are widely used in data analysis, are used to further analyze the experimental results.

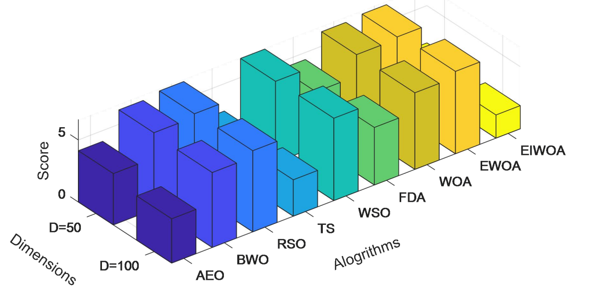
Table VIII shows the Friedman test results of on the indicators of "Mean", "Std" and "Best". For ease of observation, the graphical results are shown in Fig. 7. As can be seen in Table VIII, the p-values of each comparison is much less than 0.05, so the results can be considered significant. From Fig.5, we can see that, for the "Mean", "Std" and "Best" indicators, the Friedman values of EIWOA are obviously smaller than those of comparison algorithms no matter in 50 or 100 dimension. It indicates that statistically, EIWOA significantly outperforms other comparison algorithms in terms of stability, obtaining mean and best values.

In addition, a critical difference graph was used to show the critical difference of the Friedman Rank that combines the above three indicators, and the results are shown in Fig.8. As can be seen from Fig.8, EIWOA has the best performance and is significantly better from the other comparison algorithms.

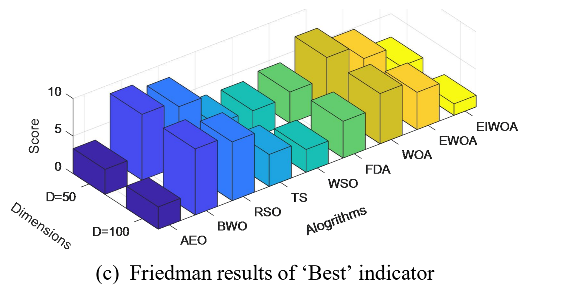
In the Wilcoxon signed rank test, the significance level α is set to 0.05. The results of the test are shown in Table IX. In Table IX, R+ is the value of the rank sum of EIWOA that is superior to the comparison algorithm, and R- indicates that the rank sum of EIWOA is inferior to the comparison algorithm. "+" indicates the number of test functions for which EIWOA outperforms the comparison algorithm. "-" indicates the opposite result and "=" indicates that there is no statistically significant difference



(a) Friedman results of 'Mean' indicator



(b) Friedman results of 'Std' indicator



(c) Friedman results of 'Best' indicator

Fig.7. Results of Friedman results on 50 and 100 Dimension

TABLE V
RESULTS OF COMPARING ALGORITHMS ON THE CEC2017 BENCHMARK FUNCTION (D=50)

$f(x)$	Index	AEO	BWO	RSO	TS	WSO	FDA	WOA	EWOA	MIWOA
f_1	Mean	2.7891E+04	1.1472E+11	8.2434E+10	5.3472E+07	1.4159E+09	1.4826E+09	5.8141E+08	7.8489E+09	7.2321E+03
	Std	7.1701E+04	3.4048E+09	1.4069E+10	1.2122E+07	3.7293E+09	5.4018E+08	3.1633E+08	5.2712E+09	2.9946E+03
	Best	2.6546E+03	1.0605E+11	6.1171E+10	3.3904E+07	4.0558E+02	7.0457E+08	2.5578E+08	1.5515E+04	1.0482E+02
	Rank	2	9	8	3	5	6	4	7	1
f_3	Mean	2.0238E+04	3.1744E+05	1.4908E+05	1.0645E+05	8.5574E+04	6.5644E+04	1.5845E+05	1.9332E+05	1.0832E+05
	Best	3.7997E+03	9.2944E+04	1.6153E+04	1.1269E+04	5.4424E+04	1.3751E+04	4.4966E+04	3.7482E+04	2.0822E+04
	Std	1.2712E+04	2.1036E+05	1.2473E+05	8.8286E+04	1.6945E+04	4.2415E+04	9.2482E+04	1.3025E+05	5.9346E+04
	Rank	1	9	6	4	3	2	7	8	5
f_4	Mean	5.6032E+02	3.8869E+04	1.9114E+04	5.6486E+02	7.7764E+02	6.8954E+02	1.0813E+03	1.6705E+03	4.5288E+02
	Std	5.5015E+01	2.5963E+03	4.8501E+03	3.5305E+01	3.9940E+02	4.3578E+01	1.9017E+02	9.6496E+02	3.5326E+01
	Best	4.4567E+02	3.1563E+04	1.3685E+04	4.9695E+02	5.0508E+02	6.1365E+02	8.5367E+02	5.4109E+02	4.2223E+02
	Rank	2	9	8	3	5	4	6	7	1
f_5	Mean	8.3590E+02	1.2223E+03	1.1213E+03	8.0083E+02	7.0141E+02	8.1354E+02	9.7367E+02	8.8558E+02	5.8187E+02
	Std	5.4601E+01	2.3095E+01	3.9829E+01	1.6384E+01	4.2864E+01	5.2986E+01	7.6482E+01	8.5069E+01	1.9202E+01
	Best	7.5474E+02	1.1801E+03	1.0594E+03	7.6986E+02	5.9651E+02	7.2770E+02	8.7507E+02	7.6514E+02	5.4081E+02
	Rank	5	9	8	3	2	4	7	6	1
f_6	Mean	6.5410E+02	7.0644E+02	6.9585E+02	6.2843E+02	6.0887E+02	6.6305E+02	6.8196E+02	6.5767E+02	6.0000E+02
	Std	7.0885E+00	5.7335E+00	5.8199E+00	5.6505E+00	4.7394E+00	6.4643E+00	1.0159E+01	1.3704E+01	4.9398E-06
	Best	6.3386E+02	6.9451E+02	6.8354E+02	6.1838E+02	6.0188E+02	6.4743E+02	6.6903E+02	6.3242E+02	6.0000E+02
	Rank	4	9	8	3	2	6	7	5	1
f_7	Mean	1.5355E+03	2.0290E+03	1.8938E+03	1.1839E+03	1.1074E+03	1.1822E+03	1.7754E+03	1.2731E+03	8.4577E+02
	Std	1.4849E+02	4.4955E+01	7.7381E+01	6.5920E+01	6.9720E+01	8.9714E+01	9.6328E+01	1.2617E+02	3.0769E+01
	Best	1.2762E+03	1.8896E+03	1.7729E+03	1.0409E+03	1.0006E+03	1.0304E+03	1.6475E+03	1.0635E+03	8.0065E+02
	Rank	6	9	8	4	2	3	7	5	1
f_8	Mean	1.1451E+03	1.5385E+03	1.3991E+03	1.1184E+03	9.8504E+02	1.1158E+03	1.2778E+03	1.1529E+03	8.8117E+02
	Std	4.8761E+01	2.4469E+01	3.6626E+01	2.9628E+01	3.1615E+01	4.7218E+01	8.6708E+01	6.6966E+01	2.0108E+01
	Best	1.0408E+03	1.4988E+03	1.3096E+03	1.0666E+03	9.2835E+02	1.0277E+03	1.1199E+03	1.0152E+03	8.4417E+02
	Rank	5	9	8	4	2	3	7	6	1
f_9	Mean	1.1306E+04	4.2753E+04	3.3756E+04	1.5588E+04	4.4347E+03	1.0824E+04	2.7277E+04	1.5257E+04	9.0000E+02
	Std	1.7165E+03	2.8769E+03	4.5842E+03	1.6573E+03	1.8544E+03	3.9315E+03	8.1204E+03	5.8482E+03	1.4048E-09
	Best	8.1580E+03	3.5751E+04	2.6162E+04	1.2416E+04	2.1420E+03	5.4412E+03	1.8556E+04	6.5212E+03	9.0000E+02
	Rank	4	9	8	6	2	3	7	5	1
f_{10}	Mean	8.1767E+03	1.5735E+04	1.4423E+04	7.0016E+03	9.0776E+03	8.9368E+03	1.1085E+04	9.0127E+03	5.8956E+03
	Std	9.0689E+02	5.1990E+02	7.3835E+02	5.5640E+02	1.5333E+03	1.2120E+03	1.8869E+03	9.1873E+02	7.6730E+02
	Best	6.4899E+03	1.4470E+04	1.2353E+04	5.7397E+03	6.3127E+03	6.8677E+03	8.4643E+03	6.9564E+03	4.5687E+03
	Rank	3	9	8	2	6	4	7	5	1
f_{11}	Mean	1.3216E+03	2.5242E+04	1.3814E+04	1.6160E+03	5.9318E+03	1.5929E+03	2.2622E+03	2.5089E+03	1.2469E+03
	Std	4.3281E+01	1.5422E+03	2.6395E+03	1.9069E+02	2.3549E+04	1.2967E+02	2.8284E+02	1.1174E+03	7.1623E+01
	Best	1.2454E+03	2.1167E+04	1.0524E+04	1.2847E+03	1.3736E+03	1.3696E+03	1.8100E+03	1.4057E+03	1.1606E+03
	Rank	2	9	8	4	7	3	5	6	1
f_{12}	Mean	3.2828E+06	7.5228E+10	6.0068E+10	1.2852E+07	5.3564E+09	4.0597E+07	6.1935E+08	2.3172E+09	5.8661E+06
	Std	2.2227E+06	1.2593E+10	9.0592E+09	5.1569E+06	5.0139E+09	1.8829E+07	4.0896E+08	2.2557E+09	1.5660E+06
	Best	7.8397E+05	4.1659E+10	4.5134E+10	2.8500E+06	2.2811E+06	1.0729E+07	1.3263E+08	1.8021E+07	2.8203E+06
	Rank	1	9	8	3	7	4	5	6	2
f_{13}	Mean	1.6717E+04	5.1578E+10	3.3738E+10	2.3867E+04	2.5587E+09	4.9948E+04	1.1624E+07	6.0797E+08	3.2844E+03
	Std	1.4338E+04	1.0019E+10	9.9158E+09	5.6970E+03	5.1648E+09	2.6810E+04	2.2066E+07	8.3552E+08	2.4937E+03
	Best	3.3980E+03	2.1570E+10	2.1511E+10	1.4135E+04	5.7644E+04	1.6033E+04	7.5009E+05	8.1532E+04	1.5422E+03
	Rank	2	9	8	3	7	4	5	6	1
f_{14}	Mean	4.4451E+04	1.2824E+08	1.6780E+07	1.1260E+06	1.1522E+07	6.4863E+04	2.2359E+06	8.6612E+05	4.8689E+05
	Std	4.3563E+04	6.6101E+07	1.3291E+07	7.0041E+05	2.0851E+07	6.8839E+04	1.7527E+06	9.8431E+05	2.2200E+05
	Best	2.7026E+03	3.2372E+07	4.6570E+06	1.5424E+05	1.1717E+04	3.7012E+03	3.6234E+05	8.4244E+04	2.1312E+05
	Rank	1	9	8	5	7	2	6	4	3
f_{15}	Mean	1.4073E+04	8.5494E+09	5.9968E+09	6.5493E+03	2.6723E+08	1.3464E+04	1.0796E+06	3.9823E+07	9.7622E+03
	Std	7.4798E+03	1.9975E+09	9.7477E+08	3.6266E+03	7.7381E+08	6.7312E+03	2.1817E+06	1.0632E+08	6.9959E+03
	Best	2.3253E+03	4.0896E+09	3.6205E+09	2.3518E+03	1.3381E+04	3.2012E+03	5.8455E+04	1.6190E+04	1.8390E+03
	Rank	4	9	8	1	7	3	5	6	2
f_{16}	Mean	3.5614E+03	9.9805E+03	5.4697E+03	3.1342E+03	3.7480E+03	3.5985E+03	5.4334E+03	4.2836E+03	2.6799E+03
	Std	4.5212E+02	1.2070E+03	5.2343E+02	3.6326E+02	8.3724E+02	5.2004E+02	4.8142E+02	5.9826E+02	2.9275E+02
	Best	2.6018E+03	7.2830E+03	4.4584E+03	2.4163E+03	2.2826E+03	2.8583E+03	4.6812E+03	3.3728E+03	2.1757E+03

CONTINUED TABLE V

$f(x)$	Index	AEO	BWO	RSO	TS	WSO	FDA	WOA	EWOA	MIWOA
	Rank	3	9	8	2	5	4	7	6	1
f_{17}	Mean	3.3132E+03	8.9360E+03	6.9277E+03	2.9898E+03	3.1285E+03	3.3709E+03	3.8880E+03	3.8706E+03	2.5514E+03
	Std	4.2647E+02	3.1420E+03	1.8335E+03	2.8652E+02	4.6499E+02	3.7523E+02	4.7892E+02	4.7857E+02	3.7641E+02
	Best	2.6996E+03	5.7032E+03	5.6026E+03	2.3337E+03	2.3678E+03	2.1648E+03	3.1990E+03	3.0070E+03	1.8604E+03
	Rank	4	9	8	2	3	5	7	6	1
f_{18}	Mean	2.3764E+05	2.5883E+08	6.6970E+07	2.4261E+06	5.3484E+06	8.7447E+05	1.8022E+07	2.0661E+06	9.0375E+05
	Std	1.9419E+05	1.1138E+08	5.0163E+07	1.3250E+06	1.0490E+07	7.8077E+05	1.0746E+07	3.7973E+06	5.1659E+05
	Best	4.5791E+04	8.7076E+07	2.3901E+07	6.4814E+05	5.2153E+04	1.6532E+05	1.5214E+06	1.3804E+05	2.2163E+05
	Rank	1	9	8	5	6	2	7	4	3
f_{19}	Mean	2.2187E+04	4.7205E+09	6.8875E+09	9.5053E+03	1.0948E+06	1.5937E+04	3.1449E+06	9.9350E+06	1.2080E+04
	Std	1.3154E+04	1.1777E+09	1.9680E+09	3.5259E+03	2.6009E+06	8.8588E+03	2.8951E+06	1.8499E+07	7.2496E+03
	Best	2.3592E+03	1.3484E+09	3.6615E+09	2.8912E+03	4.5532E+03	4.3347E+03	3.7509E+04	2.2946E+03	1.9939E+03
	Rank	4	8	9	1	5	3	6	7	2
f_{20}	Mean	3.4776E+03	4.3335E+03	3.7332E+03	3.0157E+03	3.1058E+03	3.5128E+03	3.7580E+03	3.4906E+03	2.4896E+03
	Std	3.5336E+02	2.0972E+02	3.6487E+02	2.5358E+02	3.8968E+02	3.2296E+02	2.8681E+02	3.9112E+02	2.8203E+02
	Best	2.7857E+03	3.8762E+03	2.9956E+03	2.3950E+03	2.4330E+03	2.8826E+03	3.0103E+03	2.6830E+03	2.0709E+03
	Rank	4	9	7	2	3	6	8	5	1
f_{21}	Mean	2.6526E+03	3.2594E+03	3.0441E+03	2.6089E+03	2.5648E+03	2.5903E+03	2.9049E+03	2.6860E+03	2.3787E+03
	Std	8.6063E+01	5.4525E+01	5.4396E+01	6.3940E+01	5.6112E+01	6.2614E+01	7.7332E+01	6.1615E+01	1.3876E+01
	Best	2.5084E+03	3.0888E+03	2.9356E+03	2.3495E+03	2.4392E+03	2.4285E+03	2.7528E+03	2.5837E+03	2.3641E+03
	Rank	5	9	8	4	2	3	7	6	1
f_{22}	Mean	1.0190E+04	1.7526E+04	1.6909E+04	8.7639E+03	1.0240E+04	1.0264E+04	1.3153E+04	1.0278E+04	7.4456E+03
	Std	9.6435E+02	3.7869E+02	6.1511E+02	2.2624E+03	2.4269E+03	2.3899E+03	9.5926E+02	8.8306E+02	1.0244E+03
	Best	8.2555E+03	1.6835E+04	1.4704E+04	2.3454E+03	2.4508E+03	2.4274E+03	1.1096E+04	8.5083E+03	6.3948E+03
	Rank	3	9	8	2	4	5	7	6	1
f_{23}	Mean	3.1663E+03	4.2706E+03	3.7865E+03	3.2186E+03	3.5226E+03	3.0763E+03	3.6533E+03	3.2448E+03	2.8054E+03
	Std	8.9211E+01	1.0105E+02	8.9077E+01	9.0181E+01	2.1987E+02	9.9752E+01	1.7415E+02	8.4700E+01	2.2990E+01
	Best	2.9954E+03	4.0201E+03	3.6368E+03	3.0603E+03	3.1731E+03	2.9026E+03	3.3527E+03	3.1091E+03	2.7780E+03
	Rank	3	9	8	4	6	2	7	5	1
f_{24}	Mean	3.2448E+03	4.7462E+03	4.2674E+03	3.5265E+03	3.8181E+03	3.2178E+03	3.7555E+03	3.3859E+03	2.9891E+03
	Std	8.7385E+01	2.3096E+02	2.0610E+02	1.2249E+02	1.7174E+02	6.6630E+01	1.5246E+02	1.0653E+02	2.0731E+01
	Best	3.0916E+03	4.3863E+03	4.1011E+03	3.3361E+03	3.5859E+03	3.0544E+03	3.4944E+03	3.1517E+03	2.9527E+03
	Rank	3	9	8	5	7	2	6	4	1
f_{25}	Mean	3.0904E+03	1.5400E+04	1.4271E+04	3.1074E+03	3.1532E+03	3.1865E+03	3.4322E+03	3.4700E+03	3.0651E+03
	Std	2.5736E+01	8.5016E+02	9.3275E+02	2.3196E+01	3.0334E+02	4.6765E+01	1.0259E+02	6.1620E+02	1.9863E+01
	Best	3.0291E+03	1.3585E+04	1.2963E+04	3.0557E+03	2.9634E+03	3.0920E+03	3.2353E+03	2.9699E+03	3.0205E+03
	Rank	2	9	8	3	4	5	6	7	1
f_{26}	Mean	7.6860E+03	1.7260E+04	1.3541E+04	3.1577E+03	1.0255E+04	6.9907E+03	1.2948E+04	8.6034E+03	4.5274E+03
	Std	3.1654E+03	4.4971E+02	7.9784E+02	1.0403E+02	2.8629E+03	8.1998E+02	8.5837E+02	9.5986E+02	2.0790E+02
	Best	2.9072E+03	1.6372E+04	1.2238E+04	3.0181E+03	3.2022E+03	5.1627E+03	1.1394E+04	6.5061E+03	4.2182E+03
	Rank	4	9	8	1	6	3	7	5	2
f_{27}	Mean	3.5567E+03	6.5898E+03	5.4734E+03	3.5102E+03	4.6446E+03	3.6211E+03	4.4369E+03	3.6644E+03	3.4546E+03
	Std	1.1402E+02	4.6375E+02	4.5230E+02	7.8197E+01	5.4864E+02	1.1842E+02	3.9457E+02	1.5839E+02	7.5751E+01
	Best	3.3760E+03	5.6520E+03	4.6130E+03	3.3519E+03	3.9719E+03	3.3872E+03	3.7663E+03	3.4216E+03	3.2967E+03
	Rank	3	9	8	2	7	4	6	5	1
f_{28}	Mean	3.3516E+03	1.3302E+04	7.9046E+03	3.3727E+03	4.5282E+03	3.5422E+03	4.0491E+03	4.6853E+03	3.2977E+03
	Std	3.1478E+01	7.2297E+02	9.1966E+02	2.4130E+01	2.4029E+03	8.2206E+01	2.3444E+02	1.0660E+03	1.5627E+01
	Best	3.3003E+03	1.1803E+04	6.5102E+03	3.3241E+03	3.2710E+03	3.4195E+03	3.7297E+03	3.3164E+03	3.2648E+03
	Rank	2	9	8	3	6	4	5	7	1
f_{29}	Mean	4.8095E+03	5.6168E+04	3.5909E+04	4.1918E+03	5.9817E+03	5.0465E+03	8.2036E+03	5.1409E+03	3.6167E+03
	Std	3.4894E+02	3.1553E+04	4.0832E+04	2.1772E+02	1.4833E+03	3.7802E+02	1.3783E+03	4.9871E+02	1.7896E+02
	Best	4.3011E+03	1.5127E+04	1.1188E+04	3.7823E+03	4.4524E+03	4.3915E+03	6.4161E+03	4.1428E+03	3.3033E+03
	Rank	3	9	8	2	6	4	7	5	1
f_{30}	Mean	2.6336E+06	6.2721E+09	6.5116E+09	1.4813E+06	3.9352E+07	8.0156E+06	1.5578E+08	8.5640E+07	1.1307E+06
	Std	1.8910E+06	1.4266E+09	1.5648E+09	2.8385E+05	4.3441E+07	5.9151E+06	5.9340E+07	1.4294E+08	1.5309E+05
	Best	1.0182E+06	3.2456E+09	4.9261E+09	9.9617E+05	2.3441E+06	1.6709E+06	6.6586E+07	5.6615E+06	8.7006E+05
	Rank	3	8	9	2	5	4	7	6	1
Total Rank		89	259	231	88	139	107	185	166	41
Final Rank		3	9	8	2	5	4	7	6	1

TABLE VI
RESULTS OF COMPARING ALGORITHMS ON THE CEC2017 BENCHMARK FUNCTION (D=100)

$f(x)$	Index	AEO	BWO	RSO	TS	WSO	FDA	WOA	EWOA	MIWOA
f_1	Mean	2.4520E+07	2.6890E+11	2.3860E+11	1.7674E+09	2.9132E+09	2.0982E+10	1.3456E+10	3.0975E+10	4.1727E+04
	Std	1.8722E+07	8.6327E+09	1.2780E+10	2.1621E+08	3.1210E+09	2.8662E+09	3.4978E+09	1.5733E+10	2.5746E+04
	Best	8.4687E+06	2.4581E+11	2.1201E+11	1.3552E+09	5.7005E+05	1.5170E+10	8.0989E+09	1.4448E+10	9.5647E+03
	Rank	2	9	8	3	4	6	5	7	**1**
f_3	Mean	2.0097E+05	5.0718E+05	3.2588E+05	3.0820E+05	4.2985E+05	2.6875E+05	8.3786E+05	7.5815E+05	2.5958E+05
	Std	3.2414E+04	2.2711E+05	1.2239E+04	1.0366E+04	1.4645E+05	3.3914E+04	1.0162E+05	7.0237E+04	4.9743E+04
	Best	1.3694E+05	3.5651E+05	2.9393E+05	2.8521E+05	2.1435E+05	2.0775E+05	6.2892E+05	5.9718E+05	1.7812E+05
	Rank	1	7	5	4	6	3	9	8	2
f_4	Mean	9.1531E+02	1.0836E+05	6.8847E+04	1.0886E+03	1.5333E+03	2.2179E+03	3.9306E+03	4.0139E+03	6.7882E+02
	Std	1.0808E+02	1.0663E+04	9.3134E+03	5.9737E+01	1.8631E+03	3.8628E+02	7.0001E+02	2.1181E+03	2.3967E+01
	Best	7.7880E+02	8.1699E+04	5.4155E+04	9.1077E+02	7.3965E+02	1.5696E+03	2.6410E+03	1.1255E+03	6.2057E+02
	Rank	2	9	8	3	4	5	6	7	1
f_5	Mean	1.3211E+03	2.1487E+03	1.9138E+03	1.4383E+03	1.1064E+03	1.3631E+03	1.6609E+03	1.3925E+03	7.7082E+02
	Std	5.9135E+01	3.5260E+01	7.6341E+01	5.1834E+01	6.5405E+01	9.3062E+01	1.2951E+02	1.3870E+02	4.3166E+01
	Best	1.2099E+03	2.0554E+03	1.7800E+03	1.3302E+03	9.7563E+02	1.1571E+03	1.4172E+03	1.1655E+03	6.8512E+02
	Rank	3	9	8	6	2	4	7	5	1
f_6	Mean	6.6117E+02	7.1574E+02	7.0349E+02	6.5706E+02	6.2828E+02	6.7242E+02	6.9398E+02	6.6952E+02	6.0019E+02
	Std	4.8075E+00	2.4317E+00	3.7152E+00	4.4773E+00	3.3171E+00	5.4478E+00	1.0740E+01	8.0704E+00	6.1347E-02
	Best	6.5017E+02	7.1060E+02	6.9820E+02	6.4636E+02	6.2262E+02	6.6265E+02	6.8116E+02	6.4928E+02	6.0012E+02
	Rank	4	9	8	3	2	6	7	5	1
f_7	Mean	3.0876E+03	3.9330E+03	3.7618E+03	2.5642E+03	2.1571E+03	2.5863E+03	3.4633E+03	2.5862E+03	1.0695E+03
	Std	2.1593E+02	6.3768E+01	9.0274E+01	1.1685E+02	2.1898E+02	2.0712E+02	1.0444E+02	3.6718E+02	5.1278E+01
	Best	2.4907E+03	3.7600E+03	3.6011E+03	2.3039E+03	1.6830E+03	2.0813E+03	3.2420E+03	1.9360E+03	9.7787E+02
	Rank	6	9	8	3	2	5	7	4	1
f_8	Mean	1.7428E+03	2.6365E+03	2.3365E+03	1.8195E+03	1.4451E+03	1.7396E+03	2.1124E+03	1.7941E+03	1.0572E+03
	Std	8.5406E+01	3.9135E+01	6.9907E+01	6.7322E+01	9.1795E+01	9.8295E+01	1.1441E+02	1.6361E+02	3.6178E+01
	Best	1.4846E+03	2.5190E+03	2.1989E+03	1.6632E+03	1.2578E+03	1.5735E+03	1.9045E+03	1.4981E+03	1.0015E+03
	Rank	4	9	8	6	2	3	7	5	1
f_9	Mean	2.4906E+04	8.4417E+04	7.6278E+04	4.7997E+04	1.8048E+04	4.0861E+04	5.7648E+04	3.5606E+04	9.2630E+02
	Std	2.3238E+03	3.8031E+03	9.0371E+03	2.7380E+03	2.3839E+03	7.5876E+03	1.1830E+04	7.4636E+03	2.7196E+01
	Best	2.0292E+04	7.4275E+04	6.3820E+04	3.9799E+04	1.3704E+04	2.5971E+04	4.0270E+04	2.0128E+04	9.0086E+02
	Rank	3	9	8	6	2	5	7	4	1
f_{10}	Mean	1.7317E+04	3.3133E+04	3.2093E+04	1.8465E+04	1.7034E+04	2.2078E+04	2.5053E+04	1.7416E+04	1.3440E+04
	Std	1.6442E+03	6.6671E+02	9.7053E+02	1.1061E+03	2.4097E+03	2.3055E+03	1.7709E+03	1.4214E+03	1.1375E+03
	Best	1.4410E+04	3.1787E+04	2.8513E+04	1.5676E+04	1.2445E+04	1.8675E+04	2.1377E+04	1.4553E+04	1.1330E+04
	Rank	3	9	8	5	2	6	7	4	1
f_{11}	Mean	4.2498E+03	4.6662E+05	1.6472E+05	2.7653E+04	5.1806E+04	3.0542E+04	9.8361E+04	3.8798E+04	6.3449E+03
	Std	6.5777E+02	1.0556E+05	1.1171E+05	5.6772E+03	9.0602E+04	5.3162E+03	4.5660E+04	1.6814E+04	1.6694E+03
	Best	3.0707E+03	2.0118E+05	1.0769E+05	1.7756E+04	4.6074E+03	2.0376E+04	3.6656E+04	1.2210E+04	3.4500E+03
	Rank	1	9	8	3	6	4	7	5	2
f_{12}	Mean	5.3831E+07	2.1130E+11	1.4450E+11	3.1543E+08	2.2691E+09	2.0248E+09	2.8230E+09	1.1720E+10	3.4760E+07
	Std	2.9388E+07	1.3090E+10	1.9076E+10	5.7380E+07	2.7841E+09	7.1879E+08	6.6039E+08	1.0836E+10	1.1157E+07
	Best	2.0210E+07	1.7711E+11	1.1741E+11	2.2134E+08	2.4190E+07	8.3492E+08	1.4693E+09	1.5189E+09	1.2682E+07
	Rank	2	9	8	3	5	4	6	7	1
f_{13}	Mean	1.9086E+04	4.8459E+10	4.3999E+10	8.6904E+05	8.0367E+08	1.0357E+07	2.3382E+07	1.5810E+09	5.0409E+03
	Std	6.9642E+03	2.6030E+09	2.5885E+09	2.1829E+05	1.4993E+09	7.7714E+06	1.6477E+07	1.8188E+09	3.7571E+03
	Best	7.2742E+03	4.0869E+10	3.9980E+10	4.7543E+05	1.4341E+04	2.6567E+06	8.6822E+06	1.0330E+05	1.8938E+03
	Rank	2	9	8	3	6	4	5	7	1
f_{14}	Mean	4.7260E+05	1.0954E+08	2.7752E+07	3.4999E+06	3.6902E+06	1.9863E+06	8.7379E+06	1.2216E+07	2.6597E+06
	Std	2.0570E+05	4.0388E+07	1.1603E+07	1.0791E+06	8.6200E+06	1.0839E+06	4.8958E+06	1.3034E+07	1.4341E+06
	Best	1.7277E+05	6.4206E+07	1.0396E+07	1.5105E+06	6.5075E+04	8.0769E+05	1.1924E+06	5.9703E+05	5.1976E+05
	Rank	1	9	8	4	5	2	6	7	3
f_{15}	Mean	9.9142E+03	2.6348E+10	1.7020E+10	5.2537E+04	9.2360E+07	1.1499E+05	1.2935E+07	9.9299E+08	2.9253E+03
	Std	7.7934E+03	2.1674E+09	1.5051E+09	1.4029E+04	3.5411E+08	6.7008E+04	4.8731E+07	9.2873E+08	1.0065E+03
	Best	3.6263E+03	2.0521E+10	1.3686E+10	2.6465E+04	5.5381E+03	4.4693E+04	4.1161E+05	3.4376E+04	1.9733E+03
	Rank	2	9	8	3	6	4	5	7	1
f_{16}	Mean	6.4294E+03	2.5042E+04	1.9337E+04	5.9433E+03	6.2998E+03	7.0153E+03	1.3179E+04	7.5377E+03	5.2058E+03
	Std	7.8901E+02	1.6639E+03	1.2203E+03	4.6061E+02	8.7740E+02	8.7254E+02	2.0597E+03	8.1950E+02	6.7089E+02
	Best	5.0803E+03	2.1752E+04	1.7616E+04	4.7187E+03	4.8416E+03	5.0465E+03	9.3837E+03	5.4828E+03	3.9250E+03

CONTINUED TABLE VI

$f(x)$	Index	AEO	BWO	RSO	TS	WSO	FDA	WOA	EWOA	MIWOA
	Rank	4	9	8	2	3	5	7	6	1
f_{17}	Mean	5.6527E+03	9.3358E+06	1.9298E+05	5.2117E+03	7.7327E+03	5.7323E+03	8.4066E+03	9.1726E+03	4.3573E+03
	Std	6.0268E+02	5.7857E+06	5.1754E+05	4.9899E+02	2.5382E+03	7.4091E+02	1.4117E+03	4.4433E+03	4.9829E+02
	Best	4.6575E+03	2.3374E+06	3.0863E+04	3.9551E+03	4.3444E+03	4.3914E+03	5.6335E+03	5.6495E+03	3.3616E+03
	Rank	3	9	8	2	5	4	6	7	1
f_{18}	Mean	8.0782E+05	3.2194E+08	2.9785E+07	3.4712E+06	1.7424E+07	2.3606E+06	6.3716E+06	1.1789E+07	4.9625E+06
	Std	3.1116E+05	9.9658E+07	1.7507E+07	9.7625E+05	2.4374E+07	9.5758E+05	2.9317E+06	1.0456E+07	1.5171E+06
	Best	3.4563E+05	1.3406E+08	1.1677E+07	1.0477E+06	3.3486E+05	6.3021E+05	1.6351E+06	1.7370E+06	2.1716E+06
	Rank	1	9	8	3	7	2	5	6	4
f_{19}	Mean	1.0979E+04	2.5048E+10	2.0824E+10	6.5188E+04	3.1479E+08	6.0987E+05	1.8470E+07	8.1752E+08	5.5652E+03
	Std	9.5641E+03	3.7835E+09	2.3046E+09	2.3461E+04	5.4841E+08	4.3445E+05	1.8345E+07	1.1310E+09	3.1216E+03
	Best	2.4270E+03	1.5697E+10	1.4067E+10	2.9916E+04	2.8137E+03	1.6168E+05	2.2873E+06	5.7578E+05	2.1508E+03
	Rank	2	9	8	3	6	4	5	7	1
f_{20}	Mean	5.7501E+03	8.2429E+03	7.4608E+03	5.1557E+03	4.9814E+03	6.0435E+03	6.4955E+03	6.1359E+03	4.3100E+03
	Std	5.4593E+02	2.9725E+02	3.6988E+02	3.8792E+02	7.2851E+02	5.1341E+02	6.0990E+02	5.8301E+02	4.6324E+02
	Best	4.6968E+03	7.6232E+03	6.4453E+03	4.3704E+03	3.9440E+03	5.1531E+03	5.2363E+03	5.2442E+03	3.3947E+03
	Rank	4	9	8	3	2	5	7	6	1
f_{21}	Mean	3.3188E+03	4.9086E+03	4.5255E+03	3.4829E+03	3.2589E+03	3.1817E+03	4.1588E+03	3.4062E+03	2.6004E+03
	Std	1.3857E+02	1.1183E+02	1.8825E+02	9.4380E+01	1.3868E+02	9.2950E+01	1.6457E+02	1.9396E+02	4.9471E+01
	Best	3.0395E+03	4.6229E+03	4.2891E+03	3.2413E+03	3.0769E+03	3.0416E+03	3.8230E+03	3.1208E+03	2.5319E+03
	Rank	4	9	8	6	3	2	7	5	1
f_{22}	Mean	2.0324E+04	3.5699E+04	3.4258E+04	2.2255E+04	2.0360E+04	2.4589E+04	2.8257E+04	2.0481E+04	1.5205E+04
	Std	1.9856E+03	5.7619E+02	1.2185E+03	1.1044E+03	4.1157E+03	2.0650E+03	1.8736E+03	1.3071E+03	1.2137E+03
	Best	1.7225E+04	3.4501E+04	3.1339E+04	1.9865E+04	1.5925E+04	2.0107E+04	2.3537E+04	1.7654E+04	1.3519E+04
	Rank	2	9	8	5	3	6	7	4	1
f_{23}	Mean	3.7180E+03	6.3163E+03	5.8617E+03	3.9219E+03	5.1419E+03	3.8726E+03	4.9216E+03	4.0156E+03	3.0363E+03
	Std	1.4733E+02	1.7008E+02	3.4396E+02	1.1253E+02	4.2123E+02	1.5394E+02	2.5302E+02	1.6329E+02	2.4147E+01
	Best	3.4698E+03	6.0355E+03	5.6307E+03	3.6763E+03	4.2623E+03	3.6269E+03	4.3516E+03	3.7698E+03	2.9856E+03
	Rank	2	9	8	4	7	3	6	5	1
f_{24}	Mean	4.5549E+03	1.0494E+04	8.2028E+03	4.5626E+03	7.2781E+03	4.5231E+03	6.1715E+03	4.7507E+03	3.5295E+03
	Std	2.3577E+02	1.5557E+03	3.1607E+02	1.2806E+02	6.5632E+02	1.7466E+02	4.0606E+02	2.0543E+02	3.9274E+01
	Best	4.1675E+03	8.8338E+03	7.6523E+03	4.2778E+03	5.7689E+03	4.2237E+03	5.2915E+03	4.3223E+03	3.4509E+03
	Rank	3	9	8	4	7	2	6	5	1
f_{25}	Mean	3.5817E+03	2.9120E+04	2.1524E+04	3.8038E+03	3.5212E+03	4.7214E+03	5.1496E+03	5.6345E+03	3.3523E+03
	Std	5.4973E+01	1.3404E+03	2.9050E+03	5.4907E+01	1.6240E+02	2.9429E+02	3.1539E+02	1.9858E+03	3.0580E+01
	Best	3.4733E+03	2.5735E+04	1.8115E+04	3.7236E+03	3.3303E+03	4.2287E+03	4.4640E+03	3.6813E+03	3.2760E+03
	Rank	3	9	8	4	2	5	6	7	1
f_{26}	Mean	2.1448E+04	5.2108E+04	3.5644E+04	7.1219E+03	3.2480E+04	1.9068E+04	3.4015E+04	2.0579E+04	9.0252E+03
	Std	4.1741E+03	1.4786E+03	3.2095E+03	4.9104E+03	6.5706E+03	1.7338E+03	3.0900E+03	2.5753E+03	7.3345E+02
	Best	4.8336E+03	4.8851E+04	2.9274E+04	4.6385E+03	2.1282E+04	1.5659E+04	2.5375E+04	1.5998E+04	7.7471E+03
	Rank	5	9	8	1	6	3	7	4	2
f_{27}	Mean	3.8202E+03	1.3490E+04	8.5641E+03	3.7447E+03	6.6806E+03	4.1786E+03	5.7029E+03	3.9219E+03	3.5013E+03
	Std	1.6554E+02	1.0414E+03	1.0627E+03	9.8414E+01	1.5368E+03	1.9780E+02	6.9320E+02	2.1870E+02	4.3765E+01
	Best	3.5556E+03	1.1134E+04	7.0144E+03	3.5649E+03	4.4473E+03	3.8590E+03	4.6578E+03	3.5883E+03	3.3994E+03
	Rank	3	9	8	2	7	5	6	4	1
f_{28}	Mean	3.7013E+03	2.8423E+04	2.2112E+04	3.8531E+03	7.4582E+03	6.1577E+03	6.4462E+03	1.0054E+04	3.4071E+03
	Std	6.3577E+01	9.6943E+02	2.5994E+03	7.4565E+01	6.3739E+03	5.8454E+02	6.6736E+02	3.6116E+03	2.5316E+01
	Best	3.5634E+03	2.6510E+04	1.6851E+04	3.7063E+03	3.5149E+03	4.8977E+03	5.3787E+03	4.1067E+03	3.3711E+03
	Rank	2	9	8	3	6	4	5	7	1
f_{29}	Mean	7.6833E+03	6.8253E+05	1.8448E+05	7.4546E+03	7.7392E+03	9.0883E+03	1.6013E+04	1.0278E+04	5.7308E+03
	Std	6.0057E+02	3.6078E+05	2.0407E+05	4.9459E+02	1.1658E+03	8.4336E+02	2.0237E+03	2.1872E+03	5.2132E+02
	Best	6.4994E+03	6.5335E+04	5.9046E+04	6.6848E+03	6.3697E+03	7.6484E+03	1.1966E+04	7.1472E+03	4.3585E+03
	Rank	3	9	8	2	4	5	7	6	1
f_{30}	Mean	1.9978E+06	4.5700E+10	3.1900E+10	2.8878E+06	1.3010E+09	2.1313E+07	6.7383E+08	1.1691E+09	4.7526E+04
	Std	2.3640E+06	3.6416E+09	4.0757E+09	9.0766E+05	2.0821E+09	1.0245E+07	3.4610E+08	8.8148E+08	1.8997E+04
	Best	9.6462E+04	3.1647E+10	2.3099E+10	1.4119E+06	3.1959E+06	7.6080E+06	1.8980E+08	6.6662E+06	1.2224E+04
	Rank	2	9	8	3	7	4	5	6	1
Total Rank		79	259	229	102	129	120	183	167	37
Final Rank		2	9	8	3	5	4	7	6	1

TABLE VII
PERFORMANCE OF COMPARISON ALGORITHMS ON CEC2017

Name	D=50 Mean Best/Second/Worst	D=50 Std Best/Second/Worst	D=50 Best Best/Second/Worst	D=100 Mean Best/Second/Worst	D=100 Std Best/Second/Worst	D=100 Best Best/Second/Worst
AEO	4/6/0	4/2/3	5/4/0	4/10/0	3/7/0	2/12/0
BWO	0/0/27	3/2/9	0/0/26	0/0/28	4/4/10	0/0/28
RSO	0/0/2	0/2/6	0/0/3	0/0/0	0/3/4	0/0/0
TS	3/8/0	6/11/0	2/6/0	1/4/0	3/9/0	1/2/0
WSO	0/6/0	0/1/5	1/11/0	0/8/0	0/0/7	2/13/0
FDA	0/5/0	0/4/0	0/5/0	0/4/0	0/2/0	0/0/0
WOA	0/0/0	0/0/3	0/0/0	0/0/1	0/0/3	0/0/1
EWOA	0/0/0	0/1/3	0/2/0	0/0/0	0/0/5	0/0/0
EIWOA	22/4/0	16/6/0	21/1/0	24/3/0	19/4/0	24/2/0

(Comparative results under different indicators)

TABLE VIII
MEAN RANKS OF DIFFERENT ALGORITHMS ON DIFFERENT INDICATORS OBTAINED BY FRIEDMAN TEST ON CEC2017

Name	D=50 Mean	D=50 Std	D=50 Best	D=100 Mean	D=100 Std	D=100 Best
AEO	3.1333	4.1333	3.3333	2.8000	3.5667	2.9667
BWO	8.8000	6.1667	8.7667	8.8000	6.0333	8.8000
RSO	7.8667	6.4333	7.9000	7.8000	6.5333	7.8333
TS	3.1000	2.8000	3.6667	3.5667	2.9333	4.2667
WSO	4.8000	6.5333	3.5667	4.4667	6.6333	3.0333
FDA	3.7333	4.4333	4.1667	4.1667	4.6333	4.9000
WOA	6.3333	6.1667	6.8667	6.2667	6.1667	6.7000
EWOA	5.7000	6.3333	4.8333	5.7333	6.6333	4.9667
EIWOA	1.5333	2.0000	1.9000	1.4000	1.8667	1.5333
p-value	1.3028e-36	8.4193e-18	2.6655e-34	3.0632e-36	5.5232e-20	7.9822e-37

(Mean ranks)

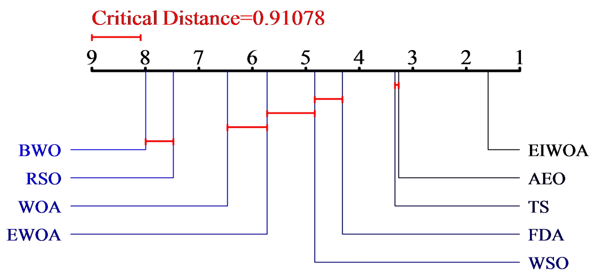


Fig.8 Critical difference in Friedman's ranking

TABLE IX
RESULTS OF WILCOXON SIGNED RANK TEST (MEAN)

EIWOA vs	50 *p*-Value	50 *R+*	50 *R-*	50 +/=/-	100 *p*-Value	100 *R+*	100 *R-*	100 +/=/-
AEO	1.03E-03	340.10	124.90	24/0/5	4.60E-04	375.97	89.03	29/0/0
BWO	1.73E-06	465.00	0.00	29/0/0	1.73E-06	465.0	0.0	29/0/0
RSO	1.86E-06	461.93	3.07	29/0/0	2.24E-06	459.31	5.69	29/0/0
TS	2.85E-02	356.24	108.76	24/3/2	4.88E-04	409.90	55.10	26/0/3
WSO	3.45E-02	387.55	77.45	25/2/2	1.02E-02	409.90	55.10	27/2/0
FDA	2.41E-02	387.76	77.24	24/2/3	1.77E-02	421.00	44.00	26/2/1
WOA	2.29E-06	460.03	4.97	29/0/0	9.34E-04	454.17	10.83	28/0/1
EWOA	2.45E-02	434.66	30.34	27/2/0	1.21E-04	449.34	15.66	28/0/1
Mean Value	1.41E-02	411.66	53.34	26.38/0.73/1.5	3.74E-03	430.57	34.43	27.75/0.5/0.75

(Dimension)

between the two algorithms. In Table IX, all p-Value values are less than 0.05, this shows that the statistical results are significant. The results of Wilcoxon signed rank test show that EIWOA achieves a large score victory for both 50-and 100-dimensional test functions. For the second best AEO algorithm, the advantages of EIWOA are also obvious. Therefore, in a statistical sense, the performance of EIWOA is significant better to comparison algorithms.

F. Convergence comparisons on the selected functions

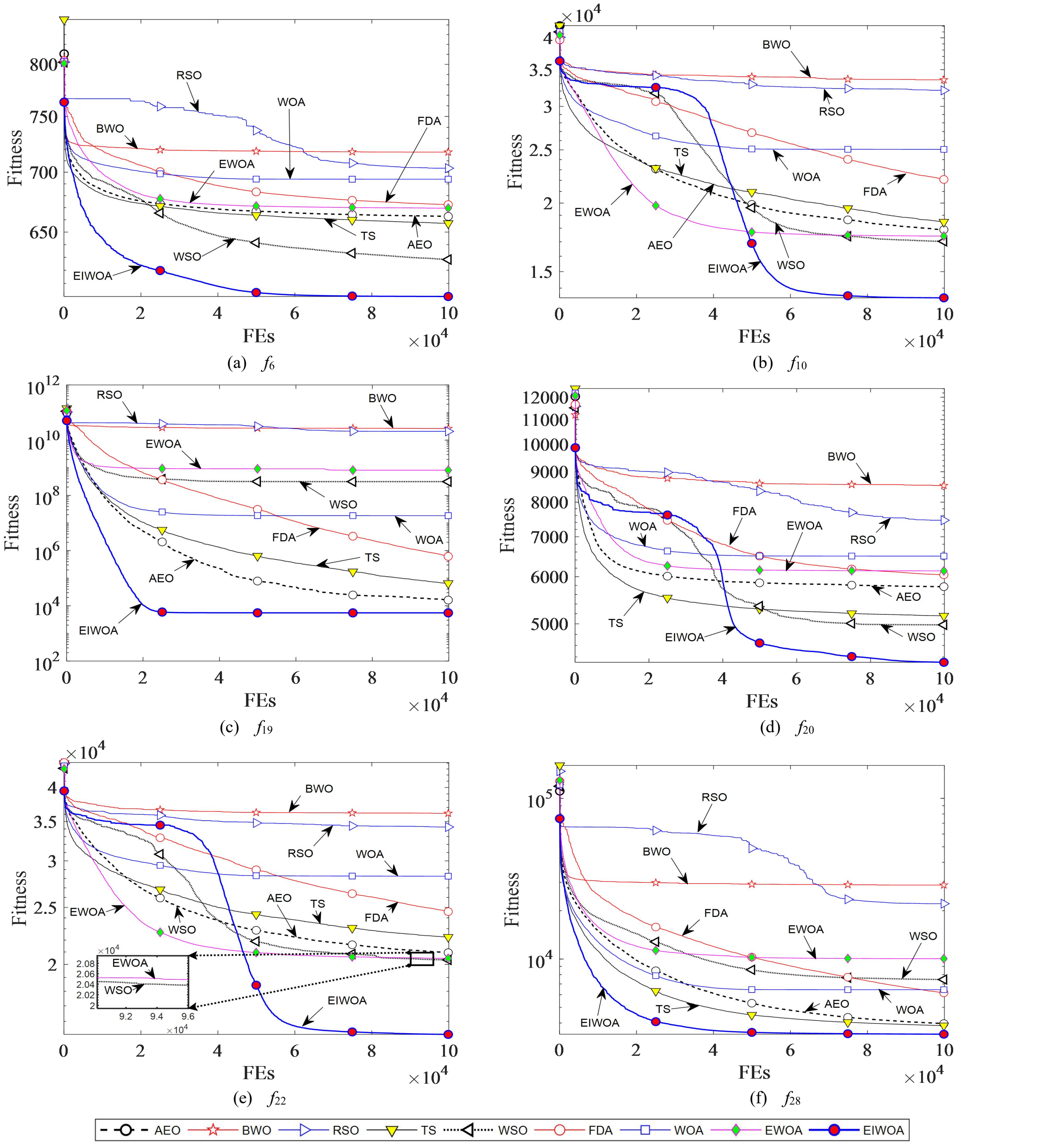
Convergence curves are an intuitive way to observe the convergence speed and accuracy of the algorithms. We plotted the convergence curves of the comparison algorithms for six functions selected from the test functions(D=100), as shown in Fig.9. From Fig.9, it can be seen that for the majority of functions, EIWOA has obvious advantages in convergence speed and convergence accuracy. Its search efficiency is significantly improved over both WOA and EWOA algorithms. This indicates that the improvement of EIWOA algorithm is successful.

V. APPLICATION OF EIWOA IN ENGINEERING PROBLEM

The use of the EIWOA algorithm to solve real-world engineering optimization problems was the original intent of the algorithm's design [40, 41]. In this section, two engineering problems, i.e., speed reducer design problem, wireless sensor network (WSN) coverage optimization problem, and two feature selection problems in data mining are selected to validate the ability of the EIWOA algorithm to solve engineering problems.

A. Speed reducer design problem

The design of reducers is an optimization problem in the field of mechanical engineering, which involves a set of constraints such as gear bending stress, tooth surface stress, axial deflection of shafts and axial stress as shown in Fig.10. The goal of this problem is to minimize the weight of the reducer[42]. The mathematical model of this problem is presented in the literature [43].



（g）Curve Legend

Fig.9. Convergence curve of comparison algorithms under different functions (D=100)

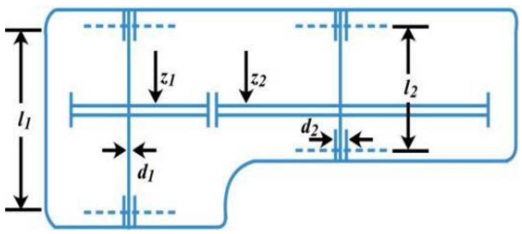


Fig. 10. Speed reducer design problem

TABLE XI
THE DATASETS FOR FEATURE SELECTION PROBLEM

NO.	Datasets	Instances	Features	Classes
1	Musk1	476	166	2
2	Libras	309	90	3

B. WSN coverage optimization problem

WSN is a network of many wireless sensors that can accomplish data collection and transmission tasks through information interaction between nodes. How to arrange the location of sensor nodes so as to arrive at the maximum spatial coverage is an important issue in WSN research. A WSN network has as few as dozens of nodes or as many as hundreds of nodes, and determining the optimal location of each sensor node to reach the maximum network coverage is a challenging optimization problem.

A mathematical model of 2D wireless sensor network coverage is given in literature [44] and a schematic of the 2D WSN coverage optimization process is given in Fig. 11. In this paper, the parameters of the 2D WSN are set as follows: regional edge length L = 50, number of nodes V=40, perception radius Rs = 5, communication radius Rc = 10, perception error Pe = 0.01 and discrete granularity dg=1.

C. Feature selection problems

Dealing with optimization problems with discrete variables is also an important application of meta- heuristic algorithms. Feature selection problem is typical discrete variable optimization problem in data mining. Two feature selection problems in UCI data set are used to verify the performance of EWOA. The information of the two data datasets used for feature selection is described in Table XI.

The two problems are classification problems. This paper uses k-nearest neighbor (k-NN) method (K=5) to build the classification models [45]. The U-shaped transfer function [18] is used to map the continuous search space to binary space. The U-shaped transfer is shown in Eq. (19).

$$u(x) = 1.0 \times \left| x^{1.5} \right| \tag{19}$$

The binary conversion result is calculated by Eq. (20).

$$b = \begin{cases} 1, u(x) \ge rand(0,1) \\ 0, u(x) < rand(0,1) \end{cases} \tag{20}$$

Where 1 means that the feature is selected whereas 0 means that the feature is not selected.

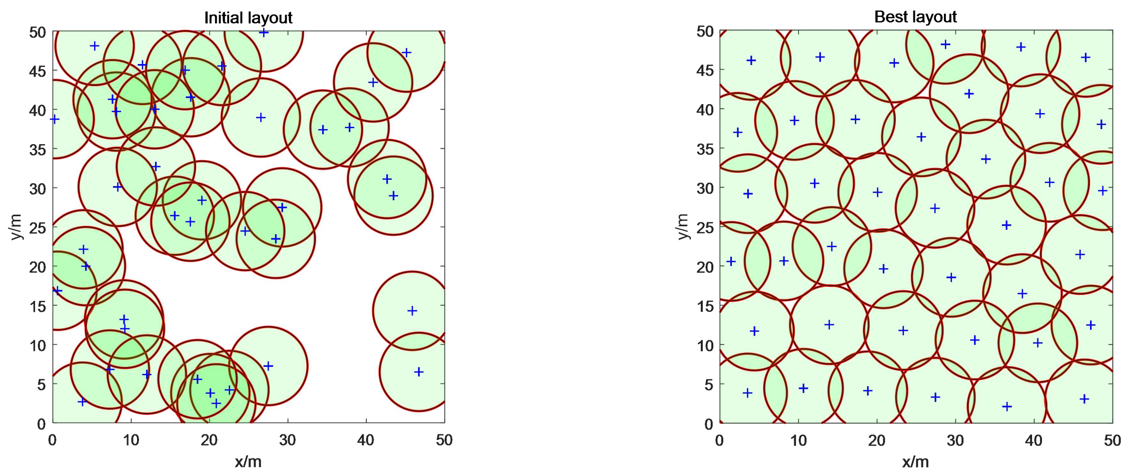
The fitness value is defined as Eq.(21).

$$fitness = 0.95 \times \frac{1}{ACC_{cv5}} + 0.05 \times \frac{n_{sel}}{N_{tot}} \tag{21}$$

Where n_{sel} is the number of selected features, N_{tot} is the total number of features. ACC_{cv5} is the accuracy of 5 fold cross-validation of the classification model.

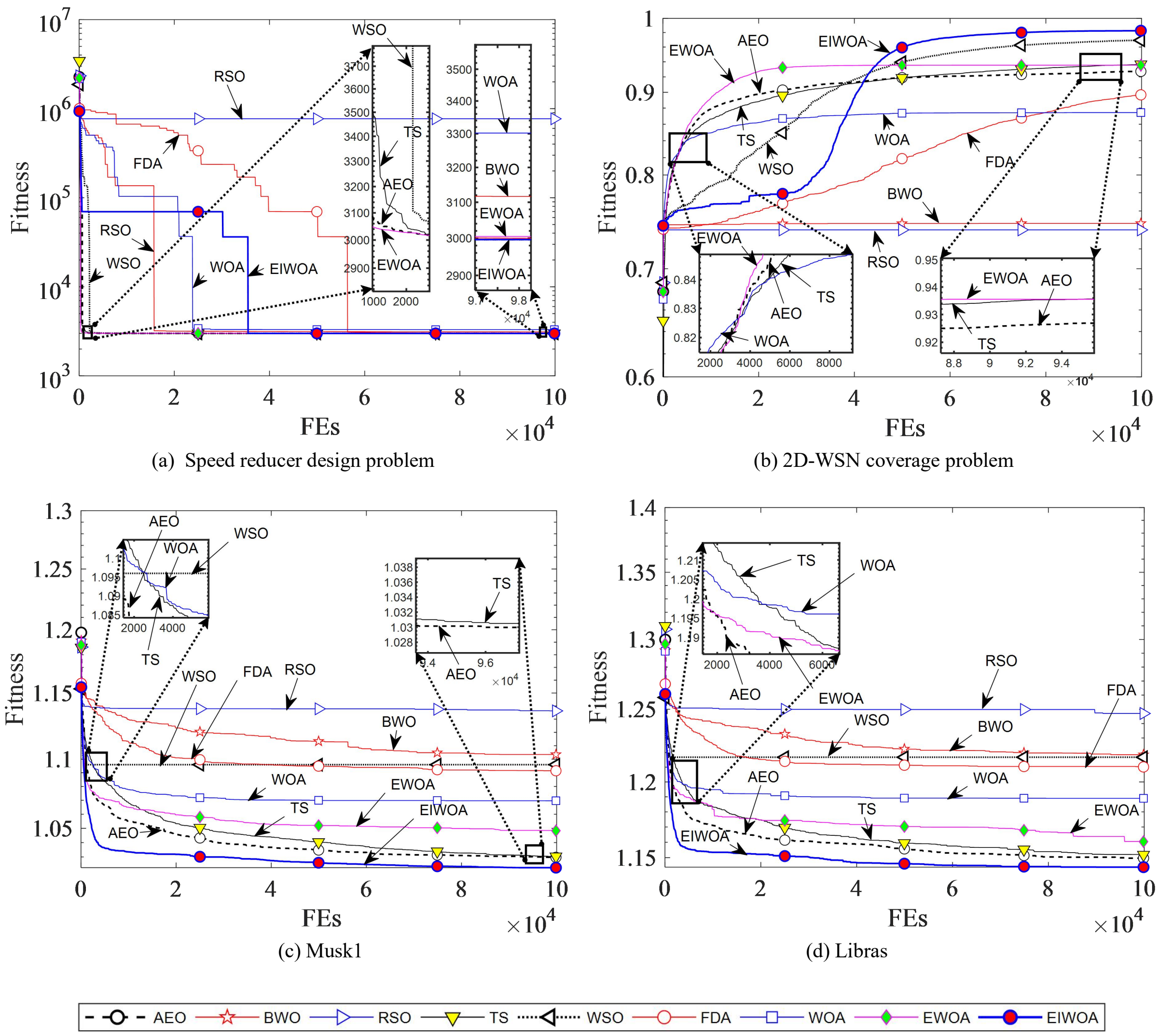
The results of comparison algorithms (The population size is 50, and the maximum number of evaluations are 1×10^5) over 30 runs on these four real world problems are shown in Table XII.

As can be seen from Table XII, for all the real world engineering problems, the EIWOA algorithm achieves the best performance. For the other algorithms, only AEO, TS22 and WSO are able to achieve satisfactory results on the speed reducer design problem. The comparison algorithm performs inferior to the EIWOA algorithm in handling all other problems. The average convergence curves of each algorithm on the four problems are shown in Fig.12. As can be seen from Fig.12, the advantages of the EIWOA algorithm are obvious both in terms of convergence speed and convergence accuracy.



(a) Sensors initial layout (b) Sensors best layout

Fig. 11 Coverage optimization of 2D wireless sensor network



(a) Speed reducer design problem (b) 2D-WSN coverage problem

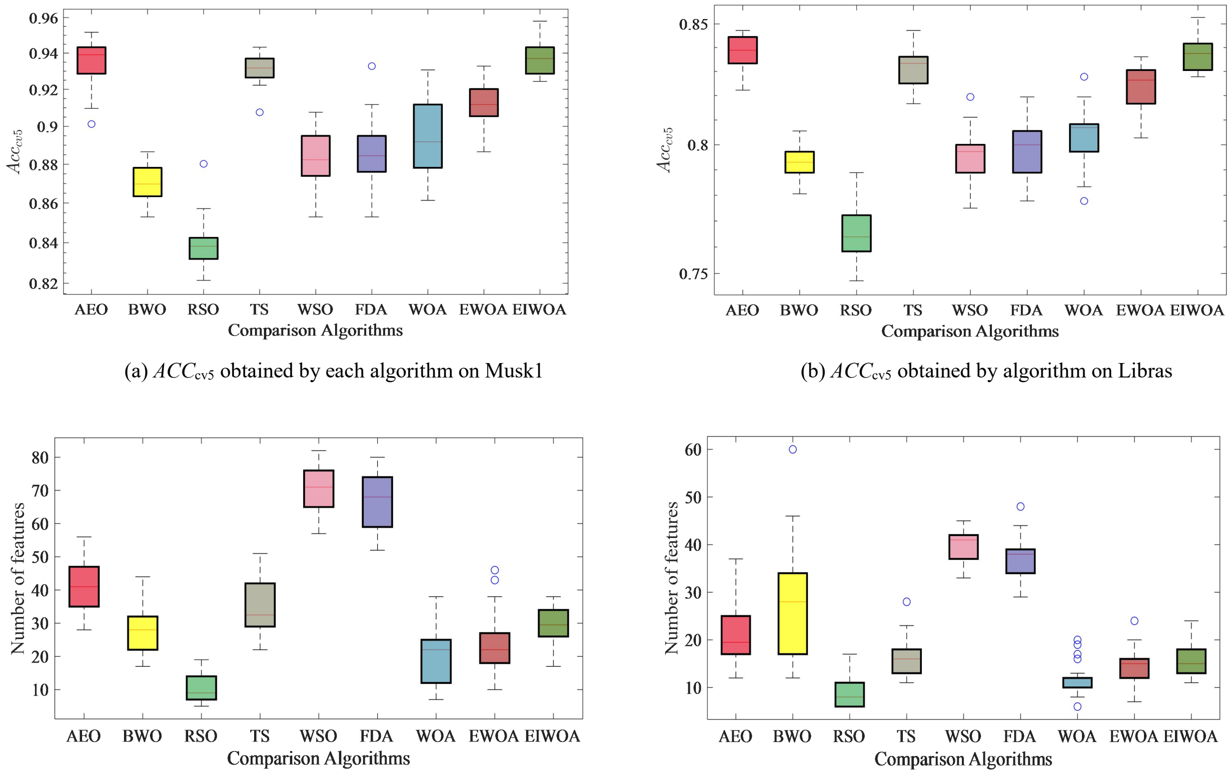
(c) Musk1 (d) Libras

（e）Curve Legend

Fig.12. Convergence curve of comparison algorithms on engineering problems

TABLE XII
RESULTS OF THE FOUR ENGINEERING PROBLEMS ACHIEVED BY ALL COMPARISON ALGORITHMS

Problem	Index	AEO	BWO	RSO	TS22	WSO	FDA	WOA	EWOA	EIWOA
Speed Reducer Design	Mean	2.9963E+03	3.1193E+03	7.7025E+05	2.9963E+03	2.9963E+03	2.9995E+03	3.3020E+03	3.0048E+03	2.9963E+03
	Std	4.9318E-04	3.9338E+01	4.2979E+05	4.6910E-07	2.0942E-12	3.7515E+00	5.7657E+02	1.6173E+01	1.3876E-12
	Best	2.9963E+03	3.0133E+03	4.2631E+03	2.9963E+03	2.9963E+03	2.9964E+03	3.0015E+03	2.9963E+03	2.9963E+03
	Rank	1	4	6	1	1	2	5	3	1
2D-WSN	Mean	0.9327	0.7471	0.7398	0.9366	0.9693	0.8971	0.8744	0.9355	0.9826
	Std	1.19E-02	1.20E-02	1.36E-02	4.73E-03	5.85E-03	1.40E-02	1.53E-02	1.51E-02	4.28E-03
	Best	0.9516	0.7762	0.7651	0.9469	0.9781	0.9143	0.9027	0.9550	0.9885
	Rank	5	8	9	3	2	6	7	4	1
Musk1	Mean	1.0285E+00	1.0997E+00	1.1363E+00	1.0304E+00	1.0959E+00	1.0915E+00	1.0696E+00	1.0483E+00	1.0226E+00
	Std	1.3589E-02	1.0351E-02	1.5149E-02	7.7225E-03	1.7924E-02	1.9916E-02	1.9643E-02	1.0889E-02	1.0440E-02
	Best	1.0088E+00	1.0821E+00	1.0844E+00	1.0181E+00	1.0666E+00	1.0347E+00	1.0301E+00	1.0251E+00	1.0010E+00
	Rank	2	8	9	3	7	6	5	4	1
Libras	Mean	1.1461E+00	1.2132E+00	1.2471E+00	1.1519E+00	1.2168E+00	1.2102E+00	1.1890E+00	1.1604E+00	1.1439E+00
	Std	1.2000E-02	1.3273E-02	1.7243E-02	9.4344E-03	1.3761E-02	1.5406E-02	1.6112E-02	1.1895E-02	9.1934E-03
	Best	1.1319E+00	1.1888E+00	1.2087E+00	1.1330E+00	1.1804E+00	1.1810E+00	1.1543E+00	1.143434847	1.125673181
	Rank	2	7	9	3	8	6	5	4	1
Total Rank		10	27	33	10	18	20	22	15	4
Final Rank		2	8	9	2	5	6	7	4	1



(a) ACC_{cv5} obtained by each algorithm on Musk1

(b) ACC_{cv5} obtained by algorithm on Libras

(c) Number of features selected by each algorithm on Musk1

(d) Number of features selected by each algorithm on Libras

Fig.13. Cross validation accuracy(ACCcv5) and number of selected features of each algorithm on feature selection problems

In order to observe cross validation accuracy (ACC_{cv5}) and number of features selected by each algorithm on feature selection problems, we draw boxplots, as shown in Fig.13. From Fig. 13(a) and (b), it can be seen that among the comparison algorithms, the AEO and EIWOA obtain better 5-fold cross-validation accuracy of the *k*-NN model on the two feature selection problems than other algorithms. From Fig. 13(c) and (d), it can be seen that the number of features selected by EIWOA is generally smaller than that of the AEO, which is favorable to the simplification of the model structure. Thus in summary, the EIWOA algorithm can handle the feature selection problem well.

VI. Conclusions and Prospects

In this paper, an improved WOA algorithm, i.e., EIWOA, is proposed to solve the problems of low accuracy, slow convergence and tendency to local optimization of WOA. The results of the qualitative analysis of EIWOA and WOA showed that the new global search mechanism, new encircling prey strategy based on differential evolution and sine-cosine search strategy improved the global search efficiency of WOA, the new Lévy flight-based spiral update position strategy enhanced the ability of whales to search unknown regions, the new balancing factor could better balance the exploration and exploitation, and the DOL-based whale-fall strategy gave the algorithm the ability to jump out of the local optimum. The proposed EIWOA was fully compared with WOA, EWOA and recently developed meta-heuristic algorithms, i.e., AEO, BWO, RSO, TS, WSO and FDA. The numerical results and convergence curves on CEC2017 benchmark functions show that the comprehensive performance of EIWOA is

In conclusion, the improved strategies, i.e., the new global search mechanism and encircling prey strategy, the lévy flight-based spiral updating position strategy, the balancing factor with fluctuation decay properties, and a dynamic opposite learning-based whale-fall strategy, are satisfactory and competitive in improving the performance of EIWOA. Experimental results on engineering problem problems show that the comprehensive performance of EIWOA is significantly better than other similar algorithms, and shows good performance in solving optimization problems with continuous or discrete variables.

significantly better than those of comparison algorithms. Friedman and Wilcoxon signed rank test provided further statistical evidence. Comparison results on real world problems showed that EIWOA has obvious advantages in dealing with the optimization problems with continuous or discrete variables. For continuous optimization problems, EIWOA had fast convergence and good accuracy. For discrete optimization problems, EIWOA was able to build classification models with good performance using fewer features. Therefore, the proposed EIWOA has good application prospects.

In the future, we will strengthen the theoretical analysis of the EIWOA algorithm, such as convergence proof. The improvement strategies proposed in this paper will provide inspiration for other meta-inspired algorithms, and we will develop more efficient variants of WOA.

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