TOPSIS Techniques on Picture Fuzzy Soft Sets

Salsabeela V and Sunil Jacob John

***Abstract*—TOPSIS (Technique for Order of Preference by Similarity to Ideal Solution) is one among the most useful Multi-Criteria Decision Making (MCDM) methods available to apply in real-world problems. Picture Fuzzy Soft Sets (PFS$_f$S) generalize fuzzy soft sets and are more flexible and efficient in comparison with other fuzzy soft set models that are currently in use. Making use of these advantages of PFS$_f$S, a TOPSIS model under a Picture Fuzzy Soft environment is proposed. Besides giving the procedure of TOPSIS applied to multiple criteria group decision-making problems, an illustrative numerical example is also provided. A comparative study is also used to prove the advantages as well as the effectiveness of the developed approach over existing techniques.**

***Index Terms*—MCDM problems, Picture Fuzzy Sets, Picture Fuzzy Soft Sets, Soft Sets.**

I. Introduction

IN real-world practices, We often come across tasks and activities that necessitate the implementation of processes for making decisions. Decision-making can be viewed as a problem-solving activity that concludes in an optimal, or at least appropriate, solution. Typically, decision-making is an intellectual procedure involving a number of intellectual and rational processes that result in the choice of an adequate substitute from such a group of possible alternative solutions in a decision-making scenario. TOPSIS or The Technique for Order of Preference by Similarity to Ideal Solution is an effective method for solving real-world Multiple-Criteria Decision Making (MCDM) challenges. Ching-Lai Hwang and Yoon [1] developed such a technique in 1981, with improvements made by Yoon in 1987. TOPSIS grades choices to determine the best compromise solution to an ideal solution. TOPSIS utilizes the notion of distance measures to classify and then choose numerous externally defined alternative options. This approach takes into account the distances of both positive ideal solution (PIS) and negative ideal solution (NIS) at the same time, and an The priority of their choice is determined by their closeness. To decide the better choice, they suggested that, the shortest distance taken from the positive ideal solution and the longest distance taken from the negative ideal solution are used to rate the system ([2], [3], [4]). The PIS targets to maximise benefits whereas minimising costs, but the NIS targets to maximise costs whereas minimize benefits. Up to this point, TOPSIS has been absolutely researched by pioneers and professionals and has been effectively implemented in a diverse selection of decision-making situations.

Even though the philosophies commenced by Zadeh [5] proved like a revolution in the ecosphere of dynamic mathematicians, it always faced the problem of assigning unique membership values. As an answer to this Molodtsov [6] suggested soft sets, a creative strategy to demonstrating uncertainty that was free of this difficulty. A soft set is a parameterized family of subsets of a crisp universal set. This theory can be effectively utilized in a wide spectrum of areas comprising game theory, operations research, and Reimann integration. Further results about soft sets can be seen in [7]. Maji et al. [8] were the first to define fuzzy soft sets and the operations associated with them. Chen et al [9] prolonged the TOPSIS approach for MCDM in a fuzzy context. Boran et al. [10] used intuitionistic fuzzy sets to create a TOPSIS technique for solving MCDM problems. TOPSIS was generalized by Chi and Liu [11] to Interval Neutrosophic Sets (INSs) and numerous attribute judgment problems with unknown attribute weights and INSs. TOPSIS on soft set theory was used by M.Eraslan [12] to create a decision-making device. According to the findings of the study [13], between 2000 and 2015, 49 scholars developed the TOPSIS methodology, and about 56 scholars suggested or added several alterations to the problem-solving method based on TOPSIS. The decision-making process under the technique of TOPSIS in the fuzzy soft setting was established by Selim Eraslan and Faruk Karaaslan. There is a research [14] which describes the Pythagorean fuzzy soft (PFS) TOPSIS method and the Pythagorean fuzzy soft VIKOR method in the context of MCGDM problems. Then, TOPSIS Model strategy for tackling MCGDM problems with Pythagorean m-polar Fuzzy Soft Sets was proposed by Riaz Khalid et al.[15]. Boran et.al [16] suggested an extended TOPSIS method for the multiple attribute decision-making problems based on interval neutrosophic sets. At last Salsabeela and John put ward TOPSIS techniques in the context of Fermatean Fuzzy Soft atmosphere [17]. Riaz et.al [18] developed TOPSIS model based on a Spherical fuzzy soft environment. Dong Qiu et.al [19] proposes the optimization problems as continuous decision problems and introduces a new method of using TOPSIS model in fuzzy decision-making to solve the interval-valued optimization problems. Xu et al. [20] establishes an extended TODIM method to comprehensively mirror the psychological state of decision-makers. Yang et al [15] defined picture fuzzy soft set (PFS$_f$S). Even though several studies addressed Picture Fuzzy Soft (FS) sets [21] have been suggested to define different modifications of TOPSIS and then applied in various fields of application, MCDM problems received very little attention.

Further results related with decision-making techniques on picture fuzzy soft sets are discussed in [22]. Here an extension of TOPSIS method for dealing with group decision problems in a Picture Fuzzy Soft atmosphere is proposed and with an illustrative example related to a real-life situation. The following is how the study is organized.

Section II collects fundamental results related to fuzzy sets, Picture Fuzzy Sets, Soft Sets, and Picture Fuzzy Soft

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Sets. The third section is dedicated to the TOPSIS method and it outlines a step-by-step algorithm for the conventional TOPSIS method. Section 4 provides the concept to demonstrate the MCGDM technique with PFS$_f$S set utilizing PFS$_f$S matrices. Then, to show how the suggested solution is effective, an illustrative example is provided in section V, for selecting the best suitable location for the construction of a college campus in the city. Section VI includes a comparative analysis that demonstrates the superior performance of the recommended algorithm. Finally, section VII gives the conclusions.

II. PRELIMINARIES

Definition 1. [5] Consider Σ as a universe of discourse. A fuzzy set over Σ is described by a membership.

$$\Xi : \Sigma \to [0,1]$$

For $\Upsilon \in \Sigma$; the membership value $\Xi(\Upsilon)$ represents the degree to fuzzy set Ξ. As a result a fuzzy set Ξ over Σ can always be signified as, $\Xi = \{\Xi(\Upsilon)/\Upsilon : \Upsilon \in \Sigma, \Xi(\Upsilon) \in [0,1]\}$. $F(\Upsilon)$ denotes the set of all fuzzy sets over Σ.

Definition 2. [14] A Picture Fuzzy Set (PFS), $\wp$ on a universe of discourse Σ is an object in the form of,

$$\wp = \{(\Upsilon,\ \Gamma_{\wp}(\Upsilon), \Upsilon_{\wp}(\Upsilon), \Delta_{\wp}(\Upsilon)) | \Upsilon \in \Sigma\}$$

in which, $\Gamma_{\wp}(\Upsilon) \in [0,1]$ is termed the "positive membership degree of Υ in Σ", $\Upsilon_{\wp}(\Upsilon) \in [0,1]$ is termed the "neutral membership degree of Υ in Σ" and $\Delta_{\wp}(\Upsilon) \in [0,1]$ is termed the "negative membership degree of Υ in Σ" and where $\Gamma_{\wp}$, $\Upsilon_{\wp}$ and $\Delta_{\wp}$ satisfy the given criteria,

$$\forall \Upsilon \in \Sigma,\ \Gamma_{\wp}(\Upsilon) + \Upsilon_{\wp}(\Upsilon) + \Delta_{\wp}(\Upsilon) \leq 1.$$

Then for $\Upsilon \in \Sigma$, $\pi_{\wp}(\Upsilon) = 1 - (\Gamma_{\wp}(\Upsilon) + \Upsilon_{\wp}(\Upsilon) + \Delta_{\wp}(\Upsilon))$ is known as the refusal-membership degree of Υ in Σ. For simpleness, we call $\wp(\Gamma_{\wp}(\Upsilon), \Upsilon_{\wp}(\Upsilon), \Delta_{\wp}(\Upsilon))$ a picture fuzzy number (PFN) denoted by $k = \wp(\Gamma_k, \Upsilon_k, \Delta_k)$ where $\Gamma_k, \Upsilon_e, \Delta_k \in [0,1]$,
$\pi_k = 1 - (\Gamma_k + \Upsilon_k + \Delta_k)$, and $\Gamma_k + \Upsilon_k + \Delta_k \leq 1$.

Definition 3. [7] Consider $K = \{k_1, k_2, k_3, \cdots k_n\}$ is the set of parameters and $\Lambda \subseteq K$. A pair $(\complement, \Lambda)$ is said to be a soft set over the universe Σ, in which $\complement$ is a mapping defined by

$$\complement : \Lambda \to P(\Sigma)$$

In the simplest terms, a soft set is a parameterized family of subsets of the universe Σ.

Definition 4. Denote Σ as the universal of discourse, K be a collection that includes every parameter, and $\Im \subseteq K$. A pair $\langle \complement, \Im \rangle$ is a picture fuzzy soft set (PFS$_f$S) over the universe Σ, where $\complement$ is the mapping determined by $\complement : \Im \to PFS(\Sigma)$. $\complement(k)$ can be described as a picture fuzzy soft set for any parameter $k \in K$, in the following way

$$\complement(k) = \{(\Upsilon, \Gamma_{\complement(k)}(\Upsilon), \Upsilon_{\complement(k)}(\Upsilon), \Delta_{\complement(k)}(\Upsilon)) | \Upsilon \in \Sigma\}$$

$\Gamma_{\complement(k)}(\Upsilon)$ denotes the positive membership degree, $\Upsilon_{\complement(k)}(\Upsilon)$ the corresponding neutral membership degree, and $\Delta_{\complement(k)}(\Upsilon)$ represents the negative membership degree function, provided that $\Gamma_{\complement(k)}(\Upsilon) + \Upsilon_{\complement(k)}(\Upsilon) + \Delta_{\complement(e)}(\Upsilon) \leq 1$. Here $\complement(k)$ becomes a Pythagorean fuzzy set and $\langle \complement, \Im \rangle$ becomes an Intuitionistic fuzzy soft set for any parameter $k \in \Im$ and any $\Upsilon \in \Sigma$, $\Upsilon_{\complement(k)}(\Upsilon) = 0$, then it is valid for all $k \in \Im$. In this case, $PFS_fS(\Sigma)$ symbolizes the set of all picture fuzzy soft sets over Σ.

TABLE I
TABULAR REPRESENTATION OF PFS$_f$S

Σ	g_1	g_2	g_3	g_3
γ_1	(0.4,0.1,0.1)	(0.4,0.2,0.3)	(0.8,0.1,0)	(0.4,0.3,0.1)
γ_2	(0.3,0.4,0.1)	(0.3,0.2,0.4)	(0.4,0.2,0.1)	(0.2,0.3,0.1)
γ_3	(0.5,0.3,0.1)	(0.5,0.2,0.1)	(0.4,0,0.5)	(0.4,0.2,0.2)
γ_4	(0.4,0.3,0.2)	(0.3,0.4,0.2)	(0.3,0.3,0.2)	(0.3,0.4,0.2)

Example 1.
Nowadays Recruitment must be viewed as a component of a larger engine that will propel the company forward. One of the necessary qualities of a recruiter is the ability to see the big picture in order to ensure long-term benefits to the company. The managing system of an international company is decided to make recruitment for different posts. Some considerations must be made to ensure that hiring is in line with the company's strategies. When it comes to talent acquisition, the following qualities of a recruiter are more or less non-negotiable. Let $\Sigma = \{\gamma_1, \gamma_2, \gamma_3, \gamma_4\}$ be the decision makers, $G = \{g_1, g_2, g_3, g_4\}$ be the set of all parameters, where
g_1 = Good communication skills
g_2 = Foresight
g_3 = Marketing skills
g_4 = Bonus tip: Be a Team worker
A decision maker assesses the various options based on the preceding criteria and the outcome are described using Picture Fuzzy Soft numbers, as shown in Table I.

III. TOPSIS METHOD

The TOPSIS or Technique for Order of Preference by Similarity to Ideal Solution has been considered as a great MCDM procedure that has been employed satisfactorily to solve a variety of selection problems. Typically, the TOPSIS algorithm procedure starts with the formation of a decision matrix containing every alternative [10]. Following that, we perform matrix normalization and compute positive and negative ideal solutions. Finally, the alternatives are ranked according to the corresponding closeness coefficients. For more information on TOPSIS, see the previous studies [16]. The notion of a traditional TOPSIS method is illustrated in the steps below.
Note that $\mathscr{I}_n = \{1, 2, 3, \cdots, n\}$ for every $n \in N$.

Step i. Defining problem.
Consider $\aleph = \{\hat{\aleph}_p; (p \in \mathscr{I}_n)\}$ as a collection of decision makers; $\{\eth_i; (i \in \mathscr{I}_m)\}$ is the set of alternatives and $\{\wp_j; (j \in \mathscr{I}_n)\}$ denotes the group of parameters;

Step ii. Creating decision matrix $\hat{\aleph}$.

$$\hat{\aleph} = \begin{array}{c} \\ Q_1 \\ Q_2 \\ \vdots \\ Q_i \\ \vdots \\ Q_m \end{array} \begin{array}{c} \begin{array}{cccc} P_1 & P_2 & \cdots & P_n \end{array} \\ \begin{pmatrix} \hat{d}_{11} & \hat{d}_{12} & \dots & \hat{d}_{1n} \\ \hat{d}_{21} & \hat{d}_{22} & \dots & \hat{d}_{2n} \\ \vdots & \vdots & \vdots & \\ \hat{d}_{i1} & \hat{d}_{i2} & \dots & \hat{d}_{in} \\ \vdots & \vdots & \vdots & \\ \hat{d}_{m1} & \hat{d}_{m2} & \dots & \hat{d}_{mn} \end{pmatrix} \end{array} = [\hat{d}_{ij}]_{m\times n}$$

Step iii. Formulating standard (normalized) decision matrix, $\Box$.
The given standardized formula can be used to find out normalization of values, $\Box_{ij} = \frac{\hat{d}_{ij}}{\sqrt{\sum_{k=1}^{m} \hat{d}_{kj}^2}}$,$\forall \hat{d}_{ij} \neq 0$ and for all $i \in \mathscr{I}_m$, and for all $j \in \mathscr{I}_n$. From this point, the normalized decision matrix could be constructed as

$$\Box = \begin{pmatrix} \Box_{11} & \Box_{12} & \cdots & \Box_{1n} \\ \Box_{21} & \Box r_{22} & \cdots & \Box_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ \Box_{i1} & \Box_{i2} & \cdots & \Box_{in} \\ \vdots & \vdots & \vdots & \vdots \\ \Box_{m1} & \Box_{m2} & \cdots & \Box_{mn} \end{pmatrix} = [\Box_{ij}]_{m\times n}$$

Step iv. Constructing weighted normalized decision matrix ∇.
The weighted normalised decision matrix is described as follows: $\nabla = [\nabla_{ij}]_{m\times n} = [\daleth_j \Box_{ij}]_{m\times n}$; $i \in \mathscr{I}_m$, in which $\daleth_j = \frac{\hbar_j}{\sum_{j=1}^{n} \hbar_j}$, so that $\sum_{j=1}^{n} \daleth_j = 1$ and $\hbar_j$ is the original weight given to the j^{th} criteria $\wp_j$.

$$\nabla = \begin{pmatrix} \nabla_{11} & \nabla_{12} & \cdots & \nabla_{1n} \\ \nabla_{21} & \nabla_{22} & \cdots & \nabla_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ \nabla_{i1} & \nabla_{i2} & \cdots & \nabla_{in} \\ \vdots & \vdots & \vdots & \vdots \\ \nabla_{m1} & \nabla_{m2} & \cdots & \nabla_{mn} \end{pmatrix} = [\nabla_{ij}]_{m\times n}$$

Step v. Calculating positive (S^+) and negative (S^-) ideal solutions.
We are calculating S^+ and S^- by referring above matrix ∇. We can use the formula given to accomplish this.

$$S^+ = \{\nabla_1^+ \cdots \nabla_j^+ \cdots, \nabla_n^+\} = \{(\max_i \nabla_{ij}, \min_i \nabla_{ij}), i \in \mathscr{I}_m\}$$

$$S^- = \{\nabla_1^-, \cdots, \nabla_j^- \cdots, \nabla_n^-\} = \{(\min_i \nabla_{ij}, \max_i \nabla_{ij}), i \in \mathscr{I}_m\}$$

Step vi. Finding separation measures from positive S^+ and S^-.
The Euclidean distance formula is used here to calculate the PIS and NIS separation measurements for each alternative.

$$\Re_i^+ = \sqrt{\sum_{j=1}^{n} (\nabla_{ij} - \nabla_j^+)^2}, \forall i \in \mathscr{I}_m. \quad (1)$$

$$\Re_i^- = \sqrt{\sum_{j=1}^{n} (\nabla_{ij} - \nabla_j^-)^2}, \forall i \in \mathscr{I}_m. \quad (2)$$

Step vii. Calculating closeness coefficient of each alternative with ideal solution.
The relative proximity of each option to an optimal solution can be calculated as φ_i+.

$$\varphi_i^+ = \frac{\Re_i^-}{(\Re_i^+ + \Re_i^-)}, 0 \leq \S_i^+ \leq 1, \forall i \in \mathscr{I}_m \quad (3)$$

Step viii. Ranking your preferences.
Alternatives must be sorted out in order to receive preferential order.

IV. MULTIPLE CRITERIA GROUP DECISION MAKING WITH THE PFS$_f$S LINGUISTIC TOPSIS METHOD

The statistical summary of the suggested model for MCGDM in the PFS$_f$S environment will be presented here. The proposed PFS$_f$S TOPSIS is an extension of Eraslan and Karaaslan's fuzzy soft TOPSIS [12]. In this segment, we will look at how PFS$_f$S-sets can be combined with TOPSIS in Multiple Criteria Decision Making Criteria (MCGDM). The following process clearly show the procedures for determining the required approach:

step i. Defining problem.
Consider $\sqsupset = \{\sqsupset_i, i \in \mathscr{I}_m.\}$ as a group of decision makers, $\mho = \{\mho_i, i \in \mathscr{I}_m\}$ represents the set of alternatives and $\daleth = \{\daleth_i, i \in \mathscr{I}_m\}$ denotes the family of parameters.

step ii. Creating weighted parameter matrix.
Create a weighted parameter matrix Z using the linguistic terms listed in Table 1.

$$Z = [z_{ij}]_{n\times m} \begin{pmatrix} z_{11} & z_{12} & \cdots & z_{1m} \\ z_{21} & z_{22} & \cdots & z_{2m} \\ \vdots & \vdots & \vdots & \vdots \\ z_{i1} & z_{i2} & \cdots & z_{im} \\ \vdots & \vdots & \vdots & \vdots \\ z_{n1} & z_{n2} & \cdots & z_{nm} \end{pmatrix}$$

in which z_{ij} gives the weight given by decision maker $\sqsupset_i$ to the alternative $\mho_i$.

step iii. Formulating normalised weighted matrix.

$$\hat{Z} = [\hat{z}_{ij}]_{n\times m} \begin{pmatrix} \hat{z}_{11} & \hat{z}_{12} & \cdots & \hat{z}_{1m} \\ \hat{z}_{21} & \hat{z}_{22} & \cdots & \hat{z}_{2m} \\ \vdots & \vdots & \vdots & \vdots \\ \hat{z}_{i1} & \hat{z}_{i2} & \cdots & \hat{z}_{im} \\ \vdots & \vdots & \vdots & \vdots \\ \hat{z}_{n1} & \hat{z}_{n2} & \cdots & \hat{z}_{nm} \end{pmatrix}$$

where $\hat{z}_{ij} = \frac{zij}{\sqrt{\sum_{i=1}^{n} z_{ij}^2}}$ and find out the weighted matrix $\varpi = (\Omega_1, \Omega_2 \cdots, \Omega_m)$ where $\Omega_i = \frac{z_i}{\sum_{i=1}^{n} z_i}$ and $z_j = \frac{\sum_{i=1}^{n} \hat{z}_{ij}}{n}$

step iv. Creating the PFS$_f$S decision matrix.
Picture Fuzzy Soft (PFS$_f$S) decision matrix can be created

as

$$X_i = [\hat{x}_{rs}]_{l\times m} \begin{pmatrix} \hat{x}_{11} & \hat{x}_{12} & \cdots & \hat{x}_{1m} \\ x_{21} & \hat{x}_{22} & \cdots & \hat{x}_{2m} \\ \vdots & \vdots & \vdots & \vdots \\ \hat{x}_{i1} & \hat{x}_{i2} & \cdots & \hat{x}_{im} \\ \vdots & \vdots & \vdots & \vdots \\ \hat{x}_{l1} & \hat{x}_{l2} & \cdots & \hat{x}_{lm} \end{pmatrix}$$

in which $\hat{x}_{rs}$ is a PFS$_f$S element, for $r\ th$ decision maker so that M_r assign $r\ th$ alternative with respect to the $s\ th$ criteria. Then obtain the aggregating matrix $\tilde{\mathrm{X}} = \frac{X_1+X_2+\cdots X_n}{n}$ = $[\ddot{\mathrm{x}}_{rs}]_{l\times m}$.

step v. Finding the weighted PFS decision matrix.
The weighted PFS$_f$S decision matrix can be created as per following.

$$\hat{\tau} = [\hat{\tau}_{rs}]_{l\times m} \begin{pmatrix} \hat{\tau}_{11} & \hat{\tau}_{12} & \cdots & \hat{\tau}_{1m} \\ \hat{\tau}_{21} & \hat{\tau}_{22} & \cdots & \hat{\tau}_{2m} \\ \vdots & \vdots & \vdots & \vdots \\ \hat{\tau}_{i1} & \hat{\tau}_{i2} & \cdots & \hat{\tau}_{im} \\ \vdots & \vdots & \vdots & \vdots \\ \hat{\tau}_{l1} & \hat{\tau}_{l2} & \cdots & \hat{\tau}_{lm} \end{pmatrix}$$

where $\hat{\tau}_{rs} = \Omega_s \times\ \ddot{\mathrm{x}}_{rs}$.

step vi. Calculating PFS-valued positive ideal solution $(PFS_v - PIS)$ and PFS-valued negative ideal solution $(PFS_v - NIS)$.

$$PFS_v - PIS = \{\hat{\tau}_1^+, \hat{\tau}_2^+, \cdots, \hat{\tau}_l^+\} = \{(\vee_s \hat{\tau}_{rs}, \wedge_s \hat{\tau}_{rs}, \wedge_s \hat{\tau}_{rs}) : s \in \mathscr{I}_m\}$$

and

$$PFS_v - NIS = \{\hat{\tau}_1^-, \hat{\tau}_2^-, \cdots, \hat{\tau}_l^-\} = \{(\wedge_s \hat{\tau}_{rs}, \vee_s \hat{\tau}_{rs}, \vee_s \hat{\tau}_{rs}) : s \in \mathscr{I}_m\}$$

Here $\vee$ re[presents PFS union and $\wedge$ represents PFS intersection.

step vii. Computing Euclidean distances.
Compute PFS-Euclidean distances for alternatives from $FFS_v - PIS$ and $FFS_v - NIS$, by referring

$$\S_i^+ = \left\{ \frac{1}{2mn} \sum_{j=1}^{n} \sum_{i=1}^{m} \left[\left(\mu_{B(\daleth_j)}(\Upsilon_i) - \mu_{B^+(\daleth_j)}(\Upsilon_i)\right)^2 + \left(\nu_{B(\daleth_j)}(\Upsilon_i) - \nu_{B^+(\daleth_j)}(\Upsilon_i)\right)^2 + \left(\psi_{B(\daleth_j)}(\Upsilon_i) - \psi_{B^+(\daleth_j)}(\Upsilon_i)\right)^2 \right] \right\}^{\frac{1}{2}} \quad (4)$$

$$\S_i^- = \left\{ \frac{1}{2mn} \sum_{j=1}^{m} \sum_{i=1}^{n} \left[\left(\mu_{B(\daleth_j)}(\Upsilon_i) - \mu_{B^-(\daleth_j)}(\Upsilon_i)\right)^2 + \left(\nu_{B(\daleth_j)}(\Upsilon_i) - \nu_{B^-(\daleth_j)}(\Upsilon_i)\right)^2 + \left(\psi_{B(\daleth_j)}(\Upsilon_i) - \psi_{B^-(\daleth_j)}(\Upsilon_i)\right)^2 \right] \right\}^{\frac{1}{2}} \quad (5)$$

step viii.
The relative closeness of every choices to an optimal solution is given by,

$$\varphi(\hat{\tau}_j) = \frac{\S_j^-}{\S_j^+ + \S_j^-} \in [0,1] \quad (6)$$

step ix. Ranking preferential order of alternatives
Here, we can sort order of alternatives in order to the value of $R^*(\hat{\tau}_j)$.

The step by step process of the decision-making process is shown in the figure below.

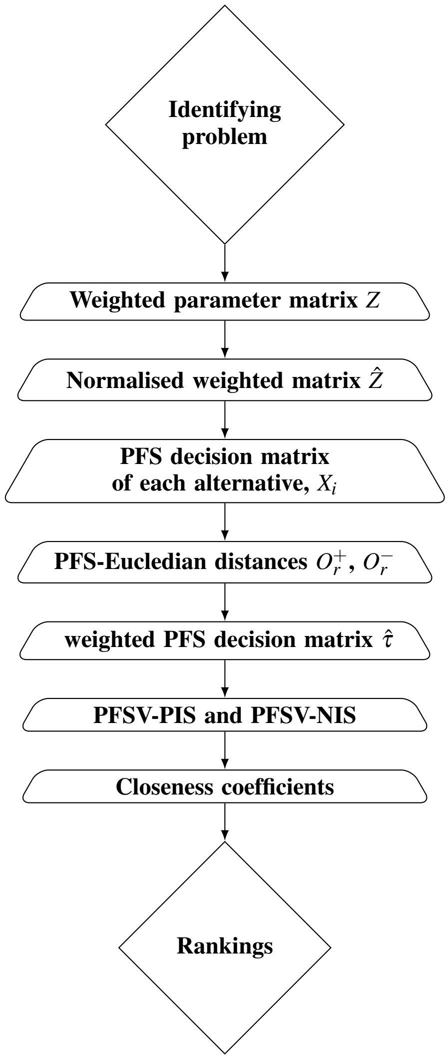

Fig 1. Flow Chart of Decision Making Process

V. NUMERICAL EXAMPLE

This section tries to develop an illustrative implementation for a group decision-making technique based on the Picture Fuzzy Soft Set theory using TOPSIS. Now, Employing the algorithm of the newly suggested approach, we can solve the following problem step by step, as shown below. For problem solving, we employ the linguistic terms listed below.

The Nation is dedicated to exploring the effect of education on the younger citizens of the country and utilising the country's biggest educational network. To meet the academic requirements of the ever-growing settlements, they decided to establish a college campus with the goal of providing high-quality education and to provide students with an inspiring and comfortable environment. Following the city visit and preevaluation, five alternative locations $(\mho_1, \mho_2, \mho_3, \mho_4, \mho_5)$ continued to be for additional consideration in order to select the The most appropriate spot in the city for the establishment of a college campus. To address this decision making issue, the educational institute's owners form a committee of four decision makers comprised of legal advisors business manager and census specialists to assess available locations based on four criteria:

- T_1 : Legally permissible and academically appropriate.

TABLE II
LINGUISTIC VARIABLES FOR EVALUATING ALTERNATIVES

Extremely Super (*ES*)	0.29
Super (*F*)	0.18
Good (*G*)	0.31
Normal (*N*)	0.22

- T_2 : Students will be able to see and use it if it is visible and easily accessible.
- T_3 : Construction costs
- T_4 : Area population and literacy rate

Step i. Consider $\{\sqsupset = \sqsupset_1, \sqsupset_2, \sqsupset_3, \sqsupset_4\}$ is a group of legal advisors/Decision makers, $\mho = \{\mho_1, \mho_2, \mho_3, \mho_4, \mho_5\}$ is a set of alternative locations and $T = \{T_1, T_2, T_3, T_4\}$ is a collection of four criterias.

Step ii. Create the weighted parameter matrix using the information in Table 1.

$$Z = [z_{ij}]_{4\times4} = \begin{pmatrix} ES & S & G & ES \\ S & ES & G & N \\ S & N & N & G \\ N & G & N & S \end{pmatrix}$$
$$= \begin{pmatrix} 0.31 & 0.31 & 0.29 & 0.29 \\ 0.22 & 0.18 & 0.31 & 0.22 \\ 0.29 & 0.31 & 0.18 & 0.18 \\ 0.18 & 0.29 & 0.18 & 0.29 \end{pmatrix}$$

Step iii. Construct the normalized weighted matrix as

$$\hat{Z} = [\hat{z}_{ij}]_{4\times4} = \begin{pmatrix} 0.61 & 0.55 & 0.59 & 0.59 \\ 0.43 & 0.32 & 0.63 & 0.45 \\ 0.57 & 0.55 & 0.37 & 0.37 \\ 0.35 & 0.52 & 0.37 & 0.59 \end{pmatrix}$$

As a result, the weighted vector is ϖ = (0.08, 0.30, 0.31, 0.31).

Step iv. Assume that four decision makers create the PFS$_f$S matrix shown below, where alternatives are expressed row-wise and parameters are expressed column-wise.

$X_1 =$

$$\begin{pmatrix} (0.5,0.2,0.1) & (0.8,0.1,0) & (0.3,0.1,0.2) & (0.5,0.3,0.1) \\ (0.4,0.3,0.1) & (0.6,0.1,0.1) & (0.6,0.2,0.1) & (0.5,0.3,0) \\ (0.7,0.1,0.1) & (0.7,0.1,0) & (0.4,0.4,0.1) & (0.6,0.2,0.1) \\ (0.6,0.1,0.1) & (0.3,0.2,0.1) & (0.5,0.2,0.1) & (0.5,0.3,0.1) \end{pmatrix}$$

$X_2 =$

$$\begin{pmatrix} (0.2,0.3,0.1) & (0.4,0.2,0.2) & (0.2,0.3,0.1) & (0.4,0.3,0.1) \\ (0.5,0.1,0.2) & (0.4,0,0.5) & (0.4,0.2,0.1) & (0.8,0.1,0) \\ (0.6,0.1,0.1) & (0.5,0.2,0.1) & (0.3,0.2,0.4) & (0.4,0.2,0.3) \\ (0.8,0.1,0) & (0.5,0.3,0.1) & (0.3,0.4,0.1) & (0.4,0.1,0.1) \end{pmatrix}$$

$X_3 =$

$$\begin{pmatrix} (0.3,0.4,0.2) & (0.3,0.2,0.2) & (0.6,0.2,0.1) & (0.3,0,0.6) \\ (0.3,0.3,0.2) & (0.5,0.2,0.1) & (0.3,0.4,0.1) & (0.4,0.1,0.1) \\ (0.3,0.4,0.2) & (0.3,0.2,0.2) & (0.6,0.2,0.1) & (0.3,0,0.6) \\ (0.4,0.3,0.2) & (0.3,0.3,0.2) & (0.5,0.2,0.1) & (0.3,0.4,0.1) \end{pmatrix}$$

$X_4 =$

$$\begin{pmatrix} (0.3,0.3,0.1) & (0.4,0.2,0.2) & (0.5,0.2,0.2) & (0.3,0.3,0.2) \\ (0.1,0.6,0.2) & (0.2,0.4,0.3) & (0.1,0.7,0.1) & (0.5,0.1,0.3) \\ (0.1,0.5,0.1) & (0.1,0.4,0.3) & (0.1,0.5,0.3) & (0.1,0.5,0.1) \\ (0.8,0.1,0) & (0.3,0,0.5) & (0.2,0.3,0.4) & (0.5,0,0.3) \end{pmatrix}$$

The average decision matrix can be obtained as $\mathfrak{W} =$

$$\begin{pmatrix} (0.33,0.30,0.13) & (0.48,0.18,0.15) & (0.40,0.20,0.15) & (0.38,0.23,0.25) \\ (0.33,0.33,0.18) & (0.43,0.18,0.25) & (0.35,0.38,0.1) & (0.55,0.15,0.1) \\ (0.43,0.28,0.18) & (0.40,0.23,0.15) & (0.35,0.33,0.23) & (0.35,0.23,0.28) \\ (0.65,0.15,0.08) & (0.35,0.20,0.23) & (0.38,0.28,0.18) & (0.43,0.20,0.15) \end{pmatrix}$$

Step v.
The weighted PFS$_f$S decision matrix is $\hat{\tau} = [\hat{\tau}_{rs}]_{5\times4} =$

TABLE III
DISTANCE MEASURES AND CLOSENESS COEFFICIENT

Alternative ($\mho_i$)	O_i^+	O_i^-	R_i*
$\mho_1$	0.0271	0.0345	0.5601
$\mho_2$	0.0853	0.1297	0.6033
$\mho_3$	0.0313	0.0556	0.6398
$\mho_4$	0.0330	0.0336	0.5045

$$\begin{pmatrix} (0.03,0.02,0.01) & (0.14,0.05,0.05) & (0.12,0.06,0.05) & (0.12,0.07,0.08) \\ (0.03,0.03,0.01) & (0.13,0.05,0.08) & (0.11,0.12,0.03) & (0.17,0.47,0.03) \\ (0.03,0.02,0.01) & (0.12,0.07,0.05) & (0.11,0.10,0.07) & (0.11,0.07,0.09) \\ (0.05,0.01,0.006) & (0.16,0.06,0.07) & (0.12,0.09,0.06) & (0.13,0.06,0.05) \end{pmatrix}$$

Step vi. PFS-Valued Positive Ideal Solution $(PFS_v - PIS)$ and PFS-Valued Negative Ideal Solution $(FFS_v - NIS)$ are found here,

$FFS_v - PIS$=$\{\hat{\tau}_1^+, \hat{\tau}_2^+, \hat{\tau}_3^+, \hat{\tau}_4^+, \hat{\tau}_5^+\}$
$= \{(0.14,0.02,0.01),(0.17,0.03,0.01),(0.12,0.02,0.01),$
$(0.16,0.01,0.006)\}$
$FFS_v - NIS$=$\{\hat{\tau}_1^-, \hat{\tau}_2^-, \hat{\tau}_3^-, \hat{\tau}_4^-, \hat{\tau}_5^-\}$
$= \{(0.03,0.07,0.08),(0.03,0.47,0.08),(0.03,0.10,0.09),$
$(0.05,0.09,0.07)\}$

Step vii. Table II shows the PFS-Eucledean distances of alternatives from $PFS_v - PIS$ and $PFS_v - NIS$, as well as the closeness coefficients.
Step viii. As a result, the preference order of the alternatives is.

$$\mho_3 > \mho_2 > \mho_1 > \mho_4$$

VI. COMPARISON ANALYSIS AND DISCUSSION

The present section compares the proposed TOPSIS method in the framework of PFS$_f$S to the existing MCDM approaches.[Section V-Para 1]

The previously described Algorithm is used in the q-ROFSS-based TOPSIS for alternative classification, and its findings are contrasted with various other methods for decision-making, displayed in Table III, which summarises the most effective alternative. We recognize that ξ_2 is the most suitable alternative for the recommended Algorithm. Applying this fictitious example with other methods for decision-making reveals that the optimal alternative for each decision-making methodology is ξ_2. It can be viewed that the recommended algorithm's optimum solution is unchanged when compared to the algorithm [23] described in "Applications of generalised picture fuzzy soft set in concept selection," which yields an optimal design idea. In the article [24], "Multi-valued picture fuzzy soft sets and their applications in group decision-making problems", ξ_2 is selected as the optimal one while solving multi-attribute group decision-making problems in a multi-valued picture fuzzy soft surroundings. The article [11] named "Neutrosophic N-Soft Sets with TOPSIS method for Multiple Attribute Decision Making" also selects ξ_2 as the optimal alternative. ξ_2 is also selected as the optimal one in the development of a robust VIKOR method for PFS$_f$S in the article [25], "Picture Fuzzy Soft Robust VIKOR Method and its Applications". The paper titled [26], "Spherical Fuzzy Soft Topology and Its Application in Group Decision-Making Problems," also recommends ξ_2 as the most appropriate one.
All approaches produce a ranking list. When we compare our ranking to the solutions offered through the available literature, we find small variations in the conclusions. Nonetheless, option with the greatest ranking by TOPSIS is the most effective in regards to ranking the criteria being used which doesn't always mean that it is very near to the ideal solution. Since it is founded on fundamentally supported distance measures, our suggested approach is both accurate and reliable.

VII. CONCLUSION

TOPSIS can be included in the category of the most well-known MCDM techniques in the world. The process of making

TABLE IV
ALGORITHM 1-BASED COMPARISON ANALYSIS OF FINALIZED RANKING WITH EXISTING TECHNIQUES

Methods	Ranking Orders of Alternatives	The Optimal Alternative
Algorithm 1 (Proposed)	$\xi_2 > \xi_3 > \xi_1 > \xi_4$	ξ_2
Algorithm [23]	$\xi_2 > \xi_3 > \xi_1 > \xi_4$	ξ_2
Algorithm [24]	$\xi_2 > \xi_3 > \xi_1 > \xi_4$	ξ_2
Algorithm [11]	$\xi_2 > \xi_3 > \xi_1 > \xi_4$	ξ_2
Algorithm [25]	$\xi_2 > \xi_3 > \xi_1 > \xi_4$	ξ_2
Algorithm [26]	$\xi_2 > \xi_3 > \xi_1 > \xi_4$	ξ_2

decisions involves phases like recognizing a decision, collecting information, and assessing potential alternatives. A step-by-step procedure for making choices will assist you in making more informed, deliberate choices by organising specific information and defining alternatives. As a consequence, DM has a major effect on real-world circumstances. The given method is a An integrated investigation of TOPSIS Techniques centred around an innovative definition, which, in comparison to other methods, creates a common foundation for mathematical issues by incorporating parametrization tools all through the investigation.

This work provides a group decision-making technique centred on multiple criteria utilizing TOPSIS techniques under Picture Fuzzy Soft settings followed by a flowchart. A comparison analysis using some already developed MCDM methods is used in order to show the selection of the optimal alternative. Picture fuzzy soft sets can be conceived as a broader version of fuzzy soft sets. In some cases, this novel soft set framework appears more achievable and exact than earlier soft set foundations. Finally, we use hypothetical evidence to determine the An appropriate spot in a city for the establishment of an educational institution. The principles presented in this article can potentially be extended to incorporate a TOPSIS Procedure for PFSf$_f$S sets utilizing similarity measures. Picture Fuzzy Soft TOPSIS framework may be used in scientific investigations for a broad range of multifunctional decision-making processes.

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