Event-Triggered Prescribed Performance Control for Switched Uncertain Nonlinear Systems

Yu Luo, Nannan Zhao*, Xinyu Ouyang, and Decai Liu

Abstract—**An event-triggered prescribed performance control (PPC) method is presented for switched uncertain nonlinear systems. Firstly, in order to ensure the comprehensive performance of the system, an effective error transformation function was introduced in the design process of the controller. Then, considering the mismatch behavior caused by the interaction between system switching and event-triggered sampling, a new switching event-triggered mechanism and adaptive control law were designed. Without strict assumptions about the event-triggered control of switching system, the mismatch problem between subsystems and corresponding controllers was effectively solved, and Zeno behavior was avoided. Finally, the effectiveness of this method is verified by the tracking control simulation of a single connecting-rod mechanical arm.**

Index Terms—**switched nonlinear systems, prescribed performance control (PPC), event-triggered control (ETC), mismatch behavior, adaptive control, mechanical arm**

I. Introduction

In recent years, adaptive control combining backstepping technology is one of the commonly used methods in nonlinear system control. However, for switching nonlinear systems, the combination of adaptive control and backstepping control is not easy due to the fact that coordinate transformations of different subsystems often lead to different state spaces. The existing solutions, such as the common Lyapunov function method used in [1][2][3], all require the presence of stable control, losing the flexibility of switching control. Therefore, multiple Lyapunov function methods can be used to control switched nonlinear systems [4][5].

In addition, the PPC can ensure both the transient performance and steady-state performance of the system, so it has been extensively studied and applied [6][7][8]. With the deepening of research, the application of ETC for switched systems has attracted people's attention. However, the interaction between switched system and triggers may cause mismatch problems between subsystems and corresponding controllers, most existing results make strict assumptions to avoid this issue. For example, in [9][10][11], switching is only allowed at the triggering moment. In [12][13][14][15][16][17], the switching of controller/filter parameters always matches the subsystem models. These assumptions greatly simplify the design process, but in reality, it's difficult to satisfy them. Therefore, it is necessary to design a controller suitable for general situations to solve the mismatch problem caused by switched systems. In [14][18][19][20], time-triggered control scheme [21][22][27] has been used to solve it. However, this method requires a known maximum sampling interval to ensure the maximum mismatch period. In fact, it is difficult to know the maximum sampling interval in advance. So additional processing or some conservative estimates are needed to be carried out in the event-triggered mechanism, For example, incorporating the time term into the trigger condition to generate the maximum sampling interval [14][20]. In [19], the maximum mismatch period is directly assumed. Due to hardware limitations, Zeno behavior may occur in event-triggered control. For periodic ETC systems [9][10][12][14][18] or discrete time systems [13][14][15], Zeno behavior can be avoided by using the sampling period as the minimum triggering event. For systems with continuous event-triggered control, existing methods include adding a small constant to the threshold function [23][24], or allowing a small steady-state error [16][25], or using dynamic threshold functions [19][26]. But when using the switching event-triggered mechanism, they are not applicable due to the possible coupling information between the switching and the triggers.

Based on the previous discussion, a new switching event-triggered controller is designed to solve the tracking problem of switched nonlinear systems. The event-triggered controller includes switching event-triggered mechanism and adaptive control laws related to the mode, which effectively solves the incompatible problem in the system. The comprehensive performance of the system is guaranteed by adopting PPC. The method was finally validated to be effective through simulation.

Manuscript received September 9, 2023; revised March 3, 2024.

Yu Luo is a postgraduate student of School of Electronic and Information Engineering, Liaoning University of Science and Technology, Anshan, Liaoning, 114051, China.(e-mail: 635728568@qq.com)

Nannan Zhao is a professor of School of Electronic and Information Engineering, Liaoning University of Science and Technology, Anshan, Liaoning, 114051, China.(Corresponding author, e-mail: 723306003@qq.com)

Xinyu Ouyang is a professor of School of Electronic and Information Engineering, Liaoning University of Science and Technology, Anshan, Liaoning, 114051, China.(Corresponding author, e-mail: 13392862@qq.com)

Decai Liu is a postgraduate student of School of Electronic and Information Engineering, Liaoning University of Science and Technology, Anshan, Liaoning, 114051, China.(e-mail: 1938356908@qq.com)

II. Problem Description

A. System descriptions and assumptions

Consider the expression for a class of stochastic nonlinear systems as follows

$$\begin{cases} \dot{x}_i = x_{i+1} + f_i\left(x_1, \ldots, x_i\right), \quad i = 1, \ldots, n-1 \\ \dot{x}_n = m_\sigma u + \vartheta_\sigma^{\mathrm{T}} \varphi_\sigma(x) + f_{\sigma,n}(x) \\ y = x_1 \end{cases} \tag{1}$$

where $x = [x_1, \ldots, x_n]^T \in R_n$ with initial value $x(t_0) = x_0$, $u \in R$ and $y \in R$ are the states, input and output of the system, respectively. m denotes the number of subsystems, $\sigma : R_+ \to M = \{1, 2, \ldots, m\}$ represents the function of switching signal, $\sigma(t) = p \in M$, $t \in [\pi_s, \pi_{s+1}]$ represents the p-th subsystem. The known functions $\varphi_p : R^n \to R^l$, $f_i : R^i \to R$ and $f_{p,n} : R^n \to R$ are smooth, the vector $\vartheta_p : R^n \to R^l$ is unknown, and $m_p \neq 0$. Assuming that there is no state jump at the switching moment, the interval between any two switches should be $\pi_d > 0$, that is, $\pi_{s+1} - \pi_s \geq \pi_d$, it indicates that there are no cumulative switching points. For subsequent analysis, several assumptions [1][4][23] need to be made as follows.

Assumption 1. The sign of m_p is known and there exist positive constants m_0 and $\bar{m}$ such that $m_0 \leq |m_p| \leq \bar{m}$.

Assumption 2. There exists $\vartheta_0 > 0$ such that $||\vartheta_p|| \leq \vartheta_0$ with $\forall p \in M$.

Assumption 3. The expected trajectory $y_d(t)$ and its $(n+1)$ order derivative are known continuous bounded functions.

To facilitate the design of control schemes, all states and switching signals can be measured. The control signal associated with the switch is transmitted through event-triggered. The zero-order synchronization refresh in the actuator. That is to say, for $\forall t \in [t_k, t_{k+1})$, the actual input is as follows

$$u(t) = w_{\sigma(t_k)}(t_k) \tag{2}$$

Here $w_{\sigma(t)}(t)$ denotes the control law, $\{t_k\}_{k \in Z_+}$ is the sampling time series. At this point, system switching and event triggering may result in a mismatch between the controlled object and its corresponding controller. Because the controlling signal is transmitted at the triggered time, and the system can be switched at any time.

Remark 1. Avoiding Zeno behavior means that it can only be triggered a limited number of times within any limited time [28][29]. Therefore, as long as the minimum triggering time is positive, i.e., $f_{k \in Z_+}\{t_{k+1} - t_k\} > 0$, Zeno behavior can be avoided. However when triggering condition is related to the switch, this goal is difficult to achieve due to the discontinuity caused by switches. The switches may trigger additional events. It may result in an infinitesimal distance between two triggers. Therefore, a new event-triggered mechanism must be designed to settle the Zeno behavior.

III. Controller Design

A. PPC

In the design of the controller below, the backstepping method will be adopted. First, let's define the virtual control signals as follows

$$\alpha_1 = -f_1 + \frac{z_1\dot{\theta}_1}{\theta_1} - \frac{k_1}{\phi_1}\eta_1 \tag{3}$$

$$\begin{aligned} \alpha_i =& -k_i z_i - f_i - z_{i-1} \\ &+ \sum_{j=1}^{i-1}\left[\frac{\partial\alpha_{i-1}}{\partial x_j}(x_{j+1}+f_j) + \frac{\partial\alpha_{i-1}}{\partial y_{\mathrm{d}}^{(j-1)}} y_{\mathrm{d}}^{(j)}\right] \\ &+ \sum_{j=0}^{i-1}\frac{\partial\alpha_{i-1}}{\theta_1^{(j)}}\theta^{(j+1)} \\ \alpha_n =& -k_n z_n - f_n - z_{n-1} \\ &+ \sum_{j=1}^{n-1}\left[\frac{\partial\alpha_{n-1}}{\partial x_j}(x_{j+1}+f_j) + \frac{\partial\alpha_{n-1}}{\partial y_{\mathrm{d}}^{(j-1)}} y_{\mathrm{d}}^{(j)}\right] \\ &+ \sum_{j=0}^{n-1}\frac{\partial\alpha_{n-1}}{\theta_1^{(j)}}\theta^{(j+1)} \end{aligned} \tag{4}$$

The expressions for the state errors are given as follows

$$\begin{aligned} z_1 =& x_1 - y_d \\ z_i =& x_i - y_d^{i-1} - \alpha_{i-1}, \quad i = 2, \dots n \end{aligned} \tag{5}$$

Define the performance function of PPC as follows

$$\theta_1 = (\theta_0 - \theta_\infty)e^{-lt} + \theta_\infty \tag{6}$$

Where θ_0, θ_∞ and l are predefined positive constants, θ_∞ represents the upper limit of steady-state error. Define error transformation as follows

$$\eta_1 = \ln\left(\frac{\theta_1 + z_1}{\theta_1 - z_1}\right) \tag{7}$$

Its derivative over time is as follows

$$\dot{\eta}_1 = \phi_1\left(x_2 + f_1 - \dot{y}_d - \frac{z_1\dot{\theta}_1}{\theta_1}\right) \tag{8}$$

where $\phi_1 = 2\theta_1/(\theta_1^2 - z_1^2)$.

Remark 2. The hyperbolic tangent function $tanh^{-1}(x) = \frac{1}{2}\ln\frac{1+x}{1-x}$ will be used to construct η_1. If $\eta_1^2(t)$ is bounded, then $|z_1(t)| < \theta_1$ holds.

B. Event-triggered controller design

1) Virtual control design:

step 1: Select the Lyapunov function as follows

$$V_1 = \frac{1}{2}\eta_1^2 \tag{9}$$

By combining equations (1) and (5), $\dot{V}_1$ can be get

$$\dot{V}_1 = \eta_1\phi_1(\alpha_1 + f_1 - \frac{z_1\dot{\theta}_1}{\theta_1}) + \eta_1\phi_1 z \tag{10}$$

Substituting $\alpha_1 = -f_1 + \frac{z_1\dot{\theta}_1}{\theta_1} - \frac{k_1}{\phi_1}\eta_1$ yields

$$\dot{V}_1 = -k_1\eta_1^2 + \eta_1\phi_1 z_2 \tag{11}$$

step 2: Similarly, according to $z_2 = x_2 - \alpha_1 - \dot{y}_d$, select the Lyapunov function as follows

$$V_2 = V_1 + \frac{1}{2}z_2^2 \tag{12}$$

Combining equations (1), (5) and (10), the time-derivative of $V_2(t)$ can be obtained

$$\dot{V}_2 = -k_1\eta_1^2 + z_2(\eta_1\phi_1 + z_3 + \alpha_2 + f_2 - \dot{\alpha}_1) \tag{13}$$

Substituting $\alpha_2 = -k_2 z_2 - f_2 - \eta_1\phi_1 + \frac{\partial\alpha_1}{\partial x_1}(x_2 + f_1) + \frac{\partial\alpha_1}{\partial y_d}\dot{y}_d + \frac{\partial\alpha_1}{\partial\theta_1}\dot{\theta}_1 + \frac{\partial\alpha_1}{\partial\dot{\theta}_1}\ddot{\theta}_1$ yields

$$\dot{V}_2 = -k_1\eta_1^2 - k_2 z_2^2 + z_2 z_3 \tag{14}$$

where $\dot{\alpha}_1 = \frac{\partial\alpha_1}{\partial x_1}(x_2 + f_1) + \frac{\partial\alpha_1}{\partial y_d}\dot{y}_d + \frac{\partial\alpha_1}{\partial\theta_1}\dot{\theta}_1 + \frac{\partial\alpha_1}{\partial\dot{\theta}_1}\ddot{\theta}_1$.

step i: Similar to step 2, for each step i, $3 \leq i \leq n-1$, consider the i-th subsystem in equation (1) and coordinate transformation (5), it has

$$\dot{z}_i = x_{i+1} + f_i - \dot{\alpha}_{i-1} - \dot{y}_d \tag{15}$$

where $\dot{\alpha}_{i-1} = \sum_{j=1}^{i-1}\left[\frac{\partial\alpha_{i-1}}{\partial x_j}(x_{j+1}+f_j) + \frac{\partial\alpha_{i-1}}{\partial y_{\mathrm{d}}^{(j-1)}}y_{\mathrm{d}}^{(j)}\right] + \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\theta_1^{(j)}}\theta_1^{(j+1)}$.

Construct a Lyapunov function as follows

$$V_i = V_{i-1} + \tfrac{1}{2}z_i^2$$

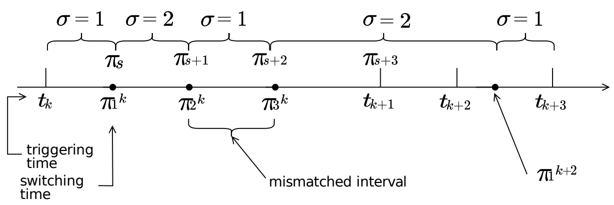

Fig. 1. Example of switching and triggering time positions (in the case of two subsystems)

Similar to the previous two steps, there has

$$\dot{V}_i = -k_1\eta_1^2 - \sum_{j=2}^{i-1} k_j z_j^2 + z_i(z_{i-1} + \alpha_i + f_i - \dot{\alpha}_{i-1}) + z_i z_{i-1} \tag{16}$$

Substituting α_i in equation (4), it can be obtained

$$\dot{V}_i \le -k_1\eta_1^2 - \sum_{j=2}^{i} k_j z_j^2 + z_i z_{i-1} \tag{17}$$

2) Event-triggered control design:

Next is the final recursive step, define $\lambda_p = 1/m_p$, $\forall p \in M$. Define $\hat{\lambda}_p$ and $\hat{\vartheta}_p$ as the estimated values of λ_p and ϑ_p. Define their estimation errors as $\tilde{\lambda}_p = \lambda_p - \hat{\lambda}_p$ and $\tilde{\vartheta}_p = \vartheta_p - \hat{\vartheta}_p$.

According to the explanation in Section 2, the relationship between the switches and triggered time is shown in Figure 1, where $\pi_1^k = \pi_s$ and $\pi_2^k = \pi_{s+1}$ denote the first and second switching respectively, and so on. Next, the switching event-triggered mechanism is designed as follows

$$\begin{aligned} t_{k+1} =& inf\left\{t \in R_+ : \left|e_{\sigma(t)}\right| \ge \Gamma_w\right\} \\ \Gamma_w =& \epsilon + \Delta_1 \left|w_{\sigma(t_k)} t_k\right| + \Delta_2 \left|B_w\right| \\ B_w(t) =& \begin{cases} w_{\sigma(\pi_1^k)}\left(\pi_1^k\right) - w_{\sigma(t_k)}\left(\pi_1^k\right), & t \in \left[\pi_1^k, \bar{T}_J^k\right) \\ 0, & \text{otherwise} \end{cases} \\ \bar{T}_J^k =& \begin{cases} \pi_2^k, & N_k > 1 \\ t_{k+1}, & N_k = 1 \end{cases} \end{aligned} \tag{18}$$

where $\epsilon > 0$, $0 < \Delta_1 < 1$, $\Delta_2 \ge 1$, and $e_{\sigma(t)}(t)$ denotes switching sampling error, and N_k denotes switching frequency on the (t_k, t_{k+1}) interval, equation (18) represents $B_w \ne 0$ only holds on the first mismatched interval, such as $\left[\pi_1^k, \pi_2^k\right)$ or $\left[\pi_1^{k+2}, t_{k+3}\right)$, and on the interval without switch, $B_w = 0$, such as interval $[t_{k+1}, t_{k+2})$.

The designed adaptive control law $(p \in M)$ is as follows

$$\begin{aligned} w_p =& -\bar{\Delta} s_p \left[\hat{\lambda}_p \phi_p \tanh\left(\hat{\lambda}_p z_n \phi_p/\xi\right) + \frac{\epsilon \tanh\left(\epsilon z_n/\xi\right) + \Delta_2 B_w \tanh\left(z_n B_w/\xi\right)}{1 - \Delta_1}\right] \\ \dot{\hat{\vartheta}}_p =& \mathrm{Proj}\left\{r\left(z_n \varphi_p - \delta \hat{\vartheta}_p\right)\right\}, \left\|\hat{\vartheta}_p\left(t_0\right)\right\| \le \vartheta_0 \\ \dot{\hat{\lambda}}_p =& \mathrm{Proj}\left\{h s_p z_n \phi_p - \rho \hat{\lambda}_p\right\}, \left|\hat{\lambda}_p\left(t_0\right)\right| \le 1/m_0 \end{aligned} \tag{19}$$

where $\phi_p = z_{n-1} + k_n z_n + f_{p,n} + \bar{\vartheta}_p^T - y_d^{(n)} - \dot{\alpha}_{n-1}$, $\bar{\Delta} = 1 + \Delta_1$. ξ, r, δ and ρ are all greater than 0. $Proj\left(\cdot\right)$ represents projection operator [29]. f_i, $f_{p,i}$, s_p, φ_p and $y_d^{(i)}$ are all continuous and differentiable, and there will be no state jumps or buildups during switches. Assuming the maximum interval is $[t_0, T_{max})$, the projection operator ensures $\left\|\hat{\vartheta}_p\right\| \le \vartheta_0 + \varepsilon_1 = \bar{\vartheta}$, $\left|\hat{\lambda}_p\right| \le 1/m_0 + \varepsilon_2 = 1/\tilde{m}$ in interval $[t_0, T_{max})$, where ε_1, ε_2 are two arbitrary normal numbers.

step n: When the system is in the matching interval (with no switch in the interval), it is known that $B_w = 0$. Assuming that $\sigma(t) = \sigma(t_k) = p$, it has

$$|e_p(t)| = |w_p(t_k) - w_p(t)| \le \epsilon + \Delta_1 |w_p(t_k)| \tag{20}$$

For equation (20), there exist $|\zeta_1(t)| \le 1$, $|\zeta_2(t)| \le 1$, so that $e_p(t) = \epsilon\zeta_1(t) + \Delta_1 w_p(t_k)\zeta_2(t)$, then

$$w_p(t_k) = \frac{w_p(t)}{1 - \zeta_2(t)\Delta_1} + \frac{\zeta_1(t)\epsilon}{1 - \zeta_2(t)\Delta_1} \tag{21}$$

According to $m_p s_p = m_p sgn(m_p) = |m_p| > 0$, it is determined that $z_n m_p w_p \le 0$, so

$$z_n m_p w_p(t_k) \le \frac{z_n m_p w_p(t)}{1 + \Delta_1} + \frac{|z_n|\,|m_p|\,\epsilon}{1 - \Delta_1} \tag{22}$$

The system is in the matching range, and at this point $B_w = 0$, $\bar{\Delta} = 1 + \Delta_1$. By combining equation (19), it can be obtained that

$$z_n m_p w_p\left(t_k\right) \le \frac{\epsilon\,|z_n|\,|m_p|}{1 - \Delta_1} - z_n\,|m_p|\,\hat{\lambda}_p \phi_p \tanh\left(\frac{\hat{\lambda}_p z_n \phi_p}{\xi}\right) - \frac{z_n\,|m_p|\,\epsilon \tanh\left(\epsilon z_n/\xi\right)}{1 - \Delta_1} \tag{23}$$

Structure the following Lyapunov function:

$$\overline{V}_p = V_{n-1} + \frac{1}{2}z_n^2 + \frac{1}{2r}\tilde{\vartheta}_p^T\tilde{\vartheta}_p + \frac{|m_p|}{2c}\tilde{\lambda}_p^2, \quad \forall p \in M \tag{24}$$

Taking its time-derivative yields

$$\dot{\overline{V}}_p = \dot{V}_{n-1} + z_n\dot{z}_n - \frac{1}{r}\tilde{\vartheta}_p^T\dot{\hat{\vartheta}}_p - \frac{|m_p|}{c}\tilde{\lambda}_p\dot{\hat{\lambda}}_p \tag{25}$$

The function $tanh(\cdot)$ satisfies the following inequality [31]:

$$\rho \le |\rho| \le \rho tanh\left(\rho/\zeta\right) + 0.2785\zeta, \quad \zeta > 0 \tag{26}$$

Combinating equations (1), (5) and (26), there has

$$\begin{aligned} \dot{\overline{V}}_p =& -k_1\eta_1^2 - \sum_{j=2}^{n-1} k_j z_j + z_{n-1}z_n \\ &+ z_n(\vartheta_p^T\varphi_p + f_{p,n} - \dot{\alpha}_{n-1} - y_d^{(n)}) \\ &- z_n m_p \hat{\lambda}_p \phi_p \\ &+ \frac{2 - \Delta_1}{1 - \Delta_1}|m_p| 0.2785\xi - \frac{1}{r}\tilde{\vartheta}_p^T\dot{\hat{\vartheta}}_p - \frac{|m_p|}{c}\tilde{\lambda}_p\dot{\hat{\lambda}}_p \end{aligned} \tag{27}$$

The projection operator $Proj\left(\cdot\right)$ have the following properties [30]

$$-\tilde{\lambda}_p Proj\left\{c s_p z_n \phi_p - \rho\hat{\lambda}_p\right\} \le -\tilde{\lambda}_p\left(c s_p z_n \phi_p - \rho\hat{\lambda}_p\right) \tag{28a}$$

$$-\tilde{\vartheta}_p^T Proj\left\{r(z_n\varphi_p - \delta\hat{\vartheta}_p)\right\} \le -\tilde{\vartheta}_p^T r(z_n\varphi_p - \delta\hat{\vartheta}_p) \tag{28b}$$

Based on the above properties and ϕ_p, $\tilde{\lambda}_p = \lambda_p - \hat{\lambda}_p$ and $\tilde{\vartheta}_p = \vartheta_p - \hat{\vartheta}_p$, it can be obtained that

$$\begin{aligned}\dot{\overline{V}}_p \leq & -k_1\eta_1^2 - \sum_{j=2}^{n} k_j z_j^2 + \frac{2-\Delta_1}{1-\Delta_1}|m_p|0.2785\xi \\ & + \frac{\rho|m_p|}{c}\tilde{\lambda}_p\hat{\lambda}_p + \delta\tilde{\vartheta}_p^T\hat{\vartheta}_p\end{aligned} \tag{29}$$

and $\tilde{\lambda}_p\hat{\lambda}_p \leq 1/2\lambda_p^2 - 1/2\tilde{\lambda}_p^2$,$\tilde{\vartheta}_p^T\hat{\vartheta}_p \leq 1/2\vartheta_p^T\vartheta_p - 1/2\tilde{\vartheta}_p^T\tilde{\vartheta}_p$, then it can be obtained that

$$\begin{aligned}\dot{\overline{V}}_p \leq & -k_1\eta_1^2 - \sum_{j=2}^{n} k_j z_j^2 - \rho\frac{|m_p|}{2c}\tilde{\lambda}_p^2 - r\delta\frac{1}{2r}\tilde{\vartheta}_p^T\vartheta_p \\ & + \frac{\rho}{2c|m_p|} + \frac{\delta}{2}\vartheta_p^T\vartheta_p + \frac{2-\Delta_1}{1-\Delta_1}|m_p|0.2785\xi\end{aligned} \tag{30}$$

so

$$\dot{\overline{V}}_p \leq -\gamma_0\overline{V}_p + \gamma_c \tag{31}$$

where $\gamma_0 = min_{i\in\{1,2,\cdots n\}}\{2k_i, r\delta, \rho\}$, $\gamma_c = \frac{\rho}{2c|m_p|} + \frac{\delta}{2}\vartheta_p^T\vartheta_p + \frac{2-\Delta_1}{1-\Delta_1}|m_p|0.2785\xi$.

When the system is in the type-I mismatch interval (there is a switch in the system), and π_1^k is the first switch after t_k, assuming $\sigma(t_k) = p$ and $\sigma(t) = q$ $(q \neq p)$, $t \in (\pi_1^k, t_{k+1})$, and at this time there has $B_w = w_q(\pi_1^k) - w_p(\pi_1^k)$.

Through the event-triggered mechanism, it can be inferred that

$$\begin{aligned}|e_q(t)| \leq & \epsilon + \Delta_1 |w_p(t_k)| \\ & + \Delta_2 B_w(t)\end{aligned} \tag{32}$$

For equation (32), $|\zeta_3(t)| \leq 1$, $|\zeta_4(t)| \leq 1$, $|\zeta_5(t)| \leq 1$ cause $e_q(t) = \epsilon\zeta_3(t) + \zeta_4(t)\Delta_1 w_p(t_k) + \zeta_5\Delta_2 B_w$, there has

$$w_p(t_k) = \frac{w_q(t) + \epsilon\zeta_3(t) + \Delta_2\zeta_5(t)B_w(t)}{1 - \Delta_1\zeta_4(t)} \tag{33}$$

then

$$\begin{aligned}z_n m_q w_p(t_k) \leq & \frac{z_n m_q w_q(t)}{1+\Delta_1} + \frac{\epsilon|z_n||m_q|}{1-\Delta_1} \\ & + \frac{\Delta_2|m_q||z_n B_w|}{1-\Delta_1} \\ \leq & -z_n m_q\hat{\lambda}_q\phi_q \\ & + \frac{2-\Delta_1+\Delta_2}{1-\Delta_1}|m_q|0.2785\xi.\end{aligned} \tag{34}$$

Similar to the matching interval, it can be obtained

$$\dot{\overline{V}}_q \leq -\gamma_0\overline{V}_q + \gamma_c \tag{35}$$

where $\gamma_0 = min_{i\in\{1,2,\cdots n\}}\{2k_i, r\delta, \rho\}$, $\gamma_c = \frac{\rho}{2c|m_q|} + \frac{\delta}{2}\vartheta_q^T\vartheta_q + \frac{2-\Delta_1+\Delta_2}{1-\Delta_1}0.2785\xi$.

When the system is in the type-II mismatch interval (there are two switches in the system), due to $B_w \neq 0$ only holds on the first mismatched interval, and $B_w = 0$ on subsequent mismatched intervals, so the derivation process is the same as on type-I mismatched intervals.

When switch occurs in the system at the time of event-triggered, only one switch moment π_s needs to analyze the relationship between $\overline{V}_p$ and $\overline{V}_q$, and define $\left|\hat{\lambda}_p\right| \leq 1/\tilde{m} = d$, $|\lambda_p| \leq 1/m_0 \leq 1/\tilde{m} < d$. It can be obtained that

$$|m_q|\tilde{\lambda}_q^2 \leq |m_p|\tilde{\lambda}_p^2 + 4|m_q|d^2 \tag{36a}$$

$$\tilde{\vartheta}_q^T\tilde{\vartheta}_q \leq 4\bar{\vartheta}^2 + \tilde{\vartheta}_p^T\tilde{\vartheta}_p \tag{36b}$$

so

$$\begin{aligned}\overline{V}_q - \overline{V}_p = & \frac{1}{2r}\tilde{\vartheta}_q^T\tilde{\vartheta}_q + \frac{|m_q|}{2c}\tilde{\lambda}_q^2 - \frac{1}{2r}\tilde{\vartheta}_p^T\tilde{\vartheta}_p - \frac{|m_p|}{2c}\tilde{\lambda}_p^2 \\ \leq & \frac{2\bar{\vartheta}^2}{r} + \frac{2\bar{m}}{c\tilde{m}^2}\end{aligned} \tag{37}$$

In summary, it can be concluded that

$$\dot{\overline{V}}_p \leq \gamma_0\overline{V}_p + \gamma_c, \quad \forall p \in M \tag{38}$$

$$\overline{V}_q - \overline{V}_p \leq \frac{2\bar{\vartheta}^2}{r} + \frac{2\bar{m}}{c\tilde{m}^2} = \kappa \tag{39}$$

where $\gamma_0 = min_{i\in\{1,2,\cdots n\}}\{2k_i, r\delta, \rho\}$, $\gamma_c = \frac{\rho}{2c|m_p|} + \frac{\delta}{2}\vartheta_p^T\vartheta_p + \frac{2-\Delta_1}{1-\Delta_1}|m_p|0.2785\xi$.

3) Stability analysis:

Theorem 1: Considering the switching system (1) that satisfies Assumptions 1-3, by introducing event-triggered mechanism (18)-(19), and selecting appropriate design parameters for (2) and switching signals with π_d, the conclusions below hold:

(1) All closed-loop signals are bounded, and the output error can meet the preset requirement;

(2) The interval time between event-triggered satisfies $t_{k+1} - t_k \geq t^*$, indicating no Zeno behavior.

Proof:

(1) From the previous derivation, it can be seen that the time-derivatives of all selected Lyapunov functions have the form $\dot{V}(t) \leq -\gamma_0 V(t) + \gamma_c$. Take piecewise continuous differentiable function $V(t)$ and define it as

$$V(t) = \overline{V}_p, \quad p \in M, \forall t \in [\pi_s, \pi_{s+1}) \tag{40}$$

According to equations (38), (39), it has

$$\begin{aligned}V(t) \leq & V(\pi_s)e^{-\gamma_0(t-\pi_s)} + \frac{\gamma_c}{\gamma_0}(1 - e^{-\gamma_0(t-\pi_s)}) \\ V(\pi_s) \leq & V(\pi_s^-) + \kappa\end{aligned} \tag{41}$$

So, for $\forall t \in [t_0, T_{max})$, there has

$$\begin{aligned}V(t) \leq & V(t_0)e^{-\gamma_0(t-t_0)} + \frac{\gamma_c}{\gamma_0}(1 - e^{-\gamma_0(t-t_0)}) \\ & + \sum_{j=1}^{N_\sigma} e^{-\gamma_0(t-\pi_j)}\kappa\end{aligned} \tag{42}$$

Assuming that N_σ is switching frequency on $(t, t_0]$, according to the dwell time $\pi_{s+1} - \pi_s \geq \pi_d$, $\forall s \in Z_+$, it has $t - \pi_j \geq (N_\sigma - j)\pi_d$ and $N_\sigma(t, t_0) \leq 1 + (t - t_0)/\pi_d$, so there has

$$\begin{aligned}\sum_{j=1}^{N_\sigma} e^{-\gamma_0(t-\pi_j)}\kappa \leq & \sum_{j=1}^{N_\sigma} e^{-\gamma_0(N_\sigma-j)\pi_\mathrm{d}}\kappa = \sum_{j=0}^{N_\sigma-1} e^{-\gamma_0\pi_\mathrm{d}j}\kappa \\ = & \frac{1 - e^{-\gamma_0\pi_\mathrm{d}N_\sigma}}{1 - e^{-\gamma_0\pi_\mathrm{d}}}\kappa \\ \leq & \frac{1 - e^{-\gamma_0(t-t_0+\pi_\mathrm{d})}}{1 - e^{-\gamma_0\pi_\mathrm{d}}}\kappa\end{aligned} \tag{43}$$

Substituting equation (43) into equation (42) yields

$$\begin{aligned}V(t) \leq & e^{-\gamma_0(t-t_0)}V(t_0) + \frac{\gamma_c}{\gamma_0}\left(1 - e^{-\gamma_0(t-t_0)}\right) \\ & + \frac{1 - e^{-\gamma_0(t-t_0+\pi_\mathrm{d})}}{1 - e^{-\gamma_0\pi_\mathrm{d}}}\kappa\end{aligned} \tag{44}$$

Assumption 1 and the adaptive laws (19) based on projection operators have been proven to be bounded for all $p \in M$,

ϑ_p, $\hat{\vartheta}_p$, $\tilde{\vartheta}_p$, λ_p, $\hat{\lambda}_p$, $\tilde{\lambda}_p$. From the equation (41), it can be seen that $V(t)$ is bounded. For $\forall t \in [t_0, T_{Max})$, according to (24), z_1, ..., z_n are bounded. Since y_d is bounded, it ensures that x_1 and $f_1(x_1)$ are also bounded. By selecting appropriate parameters, the tracking error can converge to the preset range. According to Assumption 3 and expression of α_i, x_i, $f_i(x_1, ..., x_n)$, $f_{p,n}(x)$, $\varphi_p(x)$ are bounded. According to equation (19), the input $u(t)$ is bounded. So all signals of the system (1) are bounded.

(2) When the system is in the matching interval, for any $t \in [t_k, t_{k+1})$, assuming $\sigma_t = p$, as can be seen from equation (18), $|e_p(t_k)| = |w_p(t_k) - w_p(t_k)| = 0$, $\left|e_p(t_{k+1}^-)\right| = \left|w_p(t_k) - w_p(t_{k+1}^-)\right| \leq \epsilon + \Delta_1 |w_p(t_k)|$, it can be obtained that

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{dt}}|e_p(t)| =& \frac{e_p(t)}{|e_p(t)|}\dot{e}_p(t) \leq |\dot{e}_p(t)| = |\dot{w}_p(t)| \\ \leq& \bar{\Delta}\left|\frac{\mathrm{d}}{\mathrm{dt}}\left[\hat{\lambda}_p\phi_p \tanh\left(\hat{\lambda}_p z_n \phi_p/\xi\right)\right]\right| \\ &+ \frac{\bar{\Delta}\epsilon}{1-\Delta_1}\left|\frac{\mathrm{d}}{\mathrm{dt}}\tanh\left(\epsilon Z_n/\xi\right)\right| \end{aligned} \tag{45}$$

Since the signals of all closed-loop system are bounded, all signals on the right side of equation (45) is also bounded, that is to say, there exists a constant T_1^* that makes $\frac{d}{d(t)}|e_p(t)| \leq T_1^*$, applying the mid-value theorem yields

$$\begin{aligned} \left|e_p(t_{k+1}^-)\right| - |e_p(t_k)| =& \epsilon + \Delta_1 |w_p(t_k)| \leq T_1^*(t_{k+1} - t_k) \\ t_{k+1} - t_k \geq& \frac{\epsilon + \Delta_1 |w_p(t_k)|}{T_1^*} \geq \frac{\epsilon}{T_1^*} \end{aligned} \tag{46}$$

When the system is in the type-I mismatch interval, the proof process within $t \in [t_k, \pi_1^k)$ is the same as the matching interval. For $t \in [\pi_1^k, t_{k+1})$, assume $\sigma(t_k) = p$, $\sigma(t) = q$ $(q \neq p)$, $t \in (\pi_1^k, t_{k+1})$, and at this point there is $B_w = w_q(\pi_1^k) - w_p(\pi_1^k)$, so it has

$$\begin{aligned} \left|e_q(\pi_1^k)\right| \leq& |w_p(t_k) - w_p(\pi_1^k)| + |w_p(\pi_1^k) - w_q(\pi_1^k)| \\ <& \epsilon + \Delta_1 |w_p(t_k)| + \Delta_2 |B_w(\pi_1^k)| \end{aligned} \tag{47}$$

It can be obtained from the same mismatched interval as follows

$$\begin{aligned} |\dot{e}_q(t)| \leq& \bar{\Delta}\left|\frac{\mathrm{d}}{\mathrm{dt}}\left[\hat{\lambda}_q\phi_q \tanh\left(\hat{\lambda}_q z_n \phi_q/\xi\right)\right]\right| \\ &+ \frac{\epsilon\bar{\Delta}}{1-\Delta_1}\left|\frac{\mathrm{d}}{\mathrm{dt}}\tanh\left(\epsilon z_n/\xi\right)\right| \\ &+ \frac{\bar{\Delta}\Delta_2|B_w|}{1-\Delta_1}\left|\frac{\mathrm{d}}{\mathrm{dt}}\tanh\left(tanh(z_n B_w)/\xi\right)\right| \end{aligned} \tag{48}$$

Due to all closed-loop signals and initial conditions are bounded, there exists an independent constant T_2^* such that $\frac{d}{d(t)}|e_q(t)| \leq T_2^*$. According to the equation (18), it can be obtained $|e_p(t_k)| = |w_p(t_k) - w_p(t_k)| = 0$, $\left|e_q(t_{k+1}^-)\right| = \left|w_p(t_k) - w_q(t_{k+1}^-)\right| \leq \epsilon + \Delta_1 |w_p(t_k)| + \Delta_2 |B_w|$, therefore the total error increment is $\left|e_q(t_{k+1}^-)\right| - |e_p(t_k)| - \Delta_2 |B_w| = \epsilon + \Delta_1 |w_p(t_k)| = Inc(t_k, \pi_1^k) + Inc(\pi_1^k, t_{k+1})$. $Inc(\cdot)$ is the corresponding interval error increment, and it can be obtained through the mean value theorem that

$$\begin{aligned} \pi_1^k - t_k \geq& \frac{Inc(t_k, \pi_1^k)}{T_1^*} \\ t_{k+1}^- - \pi_1^k \geq& \frac{Inc(\pi_1^k, t_{k+1})}{T_2^*} \end{aligned} \tag{49}$$

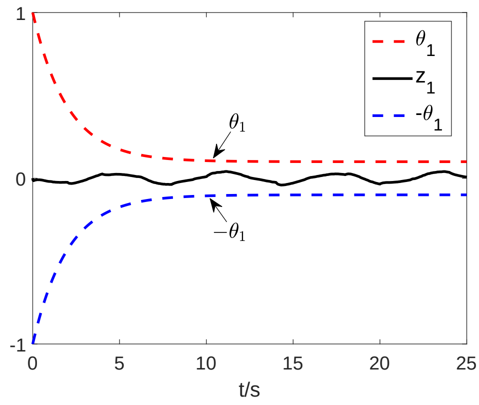

Fig. 2. Tracking error z_1 under the prescribed performance constraint θ_1

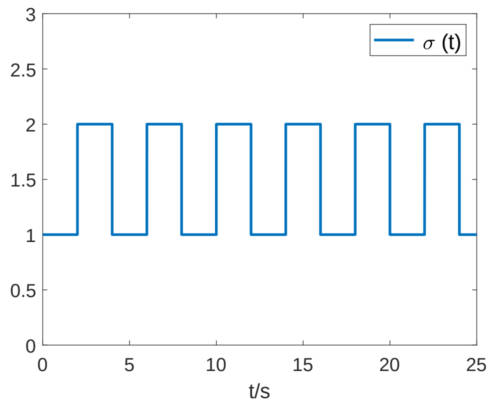

Fig. 3. The switching signal

so it has

$$t_{k+1} - t_k \geq \frac{\epsilon + \Delta_1 |w_p(t_k)|}{max\left\{T_1^*, T_1^*\right\}} \geq \frac{\epsilon}{max\left\{T_1^*, T_2^*\right\}} \tag{50}$$

When there are multiple switches in the system, the following inequality can be obtained

$$t_{k+1} - t_k \geq N_k \pi_d \tag{51}$$

Based on the above, the minimum triggering time is as follows

$$\pi^* = min\left\{\frac{\epsilon}{max\left\{T_1^*, T_2^*\right\}}, \pi_d\right\} \tag{52}$$

Thus, it proves that using event triggered control (18) will not result in Zeno behavior. ■

IV. SIMULATION

The dynamic equation of a single connecting-rod mechanical arm system based on network control [32] is considered as follows

$$J\ddot{q} + B\dot{q} + MgL\sin(q) = K_\tau u \tag{53}$$

Where q denotes the angle of rigid connecting rod, u, J and B denote the control input, the moment of inertia and the unknown damping coefficient respectively. M, L, K_τ and g represent the mass, the length, the torque and gravitational acceleration, respectively.

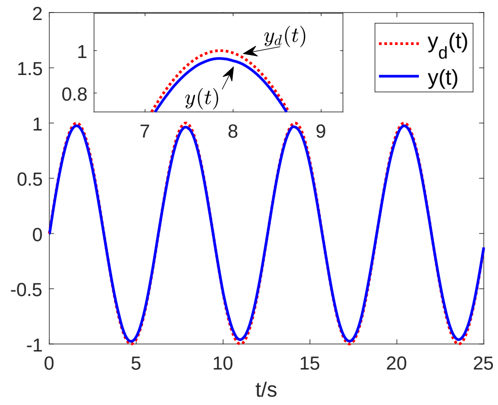

Fig. 4. Actual output and desired output

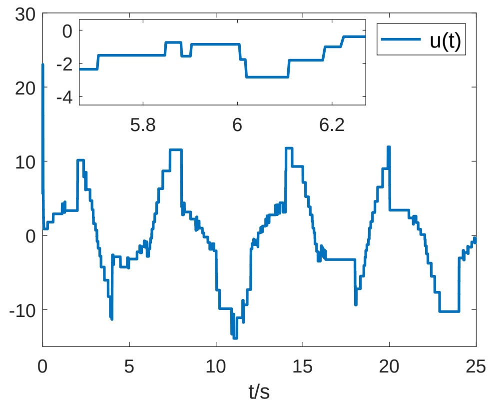

Fig. 5. Event-triggered control signal

Here, it is assumed that the robotic arm moves the same specified loads one after another on a specified trajectory, then (53) can be rewritten as follows

$$J_\sigma \ddot{q} + B\dot{q} + M_\sigma g L_\sigma \sin(q) = K_\tau u \tag{54}$$

where $\sigma \in \{1, 2\}$ denotes the signal of switching. J_σ, M_σ and L_σ all are unknown constant parameters. Let $x_1 = q$, let $x_2 = \dot{q}$, then (54) can be transformed into

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = \dfrac{K_\tau}{J_\sigma} u - \dfrac{B}{J_\sigma} x_2 - \dfrac{M_\sigma L_\sigma g}{J_\sigma} sin(x_1) \\ y = x_1 \end{cases} \tag{55}$$

Assuming $y_d(t) = sin(t)$, $J_1 = 1kgm^2$, $J_2 = 2kgm^2$, $B = 1kgm^2 s^{-1}$, $K_\tau = 2$, $M_1 g L_1 = 10kgm^2 s^{-2}$, $M_2 g L_2 = 30kgm^2 s^{-2}$. The design parameters are $\theta_0 = 1$, $\theta_\infty = 0.3$, $l = 0.5$, $k_1 = 8$, $k_2 = 4$, $\epsilon = 0.5$, $\Delta_1 = 0.2$, $r = 200$, $\Delta_2 = 0.2$, $\delta = 0.2$, $\rho = 0.5$, $c = 20$, $\xi = 0.05$. Taking $x_1(0) = x_2(0) = 0$, $\hat{\vartheta}_1(0) = [-0.5, 5]^T$, $\hat{\vartheta}_2(0) = [-1, -10]^T$, $\hat{\lambda}_1(0) = \hat{\lambda}_2(0) = 1$. The tracking error under PPC is shown in Figure 2. Choose $\tau_d = 2s$, then the switching signal is shown in Figure 3. By using the event-triggered control (18)-(19), Figure 4 shows that the system output can be well tracked. The estimation values of the parameters are shown in Figures 6-8. Through Figure 9, it is noted that $B_w = 0$ on some mismatched intervals, indicating that the event-triggered mechanism (18) in this example operates in a mixed manner. The above simulation results also verify the absence of Zeno behavior.

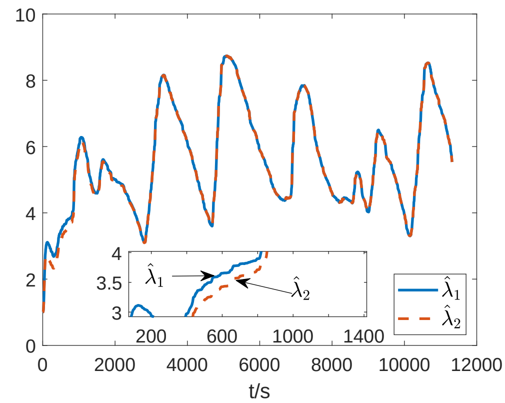

Fig. 6. The estimated value of λ_1 and λ_2

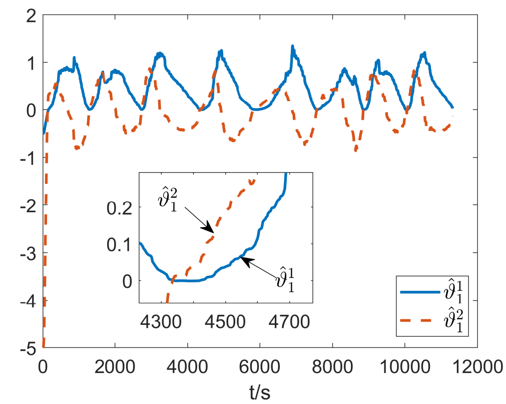

Fig. 7. The estimated value of ϑ_1^1, ϑ_1^2

V. CONCLUSION

This paper studies the event-triggered PPC problem for a class of switched uncertain nonlinear systems. Firstly, an improved PPC method is proposed. Then, an effective switching event-triggered mechanism is given, which effectively solves the mismatch problem between subsystems and corresponding controllers. This method no longer requires strict assumptions about ETC of switching systems in some literatures, so it is more intuitive and effective than existing methods. This paper also designs a mixed threshold function to avoid Zeno behavior, and designs an adaptive control law to compensate for sampling errors, thereby ensuring the convergence of tracking errors.

REFERENCES

[1] M. L. Chiang and L. C. Fu, "Adaptive Stabilization of a Class of Uncertain Switched Nonlinear Systems with Backstepping Control," *Automatica*, vol. 50, no. 8, pp. 2128-2135, 2014.

[2] Z. Liu, B. Chen, and C. Lin, "Adaptive Neural Backstepping for a Class of Switched Nonlinear System without Strict-Feedback Form," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 47, no. 7, pp. 1315-1320, 2016.

[3] F. Mazenc, L. Burlion, and M. Malisoff, "Backstepping Design for Output Feedback Stabilization for a Class of Uncertain Systems," *Systems & Control Letters*, vol. 123, pp. 134-143, 2019.

[4] G. Lai, Z. Liu, Y. Zhang, C. P. Chen, and S. Xie, "Adaptive Backstepping-Based Tracking Control of a Class of Uncertain Switched Nonlinear Systems," *Automatica*, vol. 91, pp. 301-310, 2018.

[5] L. Long and J. Zhao, "Switched-Observer-Based Adaptive Neural Control of MIMO Switched Nonlinear Systems with Unknown Control

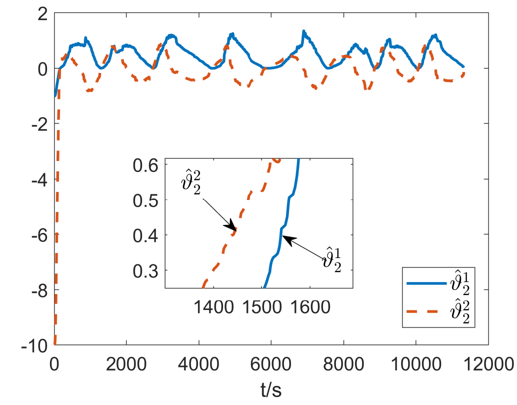

Fig. 8. The estimated value of ϑ_2^1, ϑ_2^2

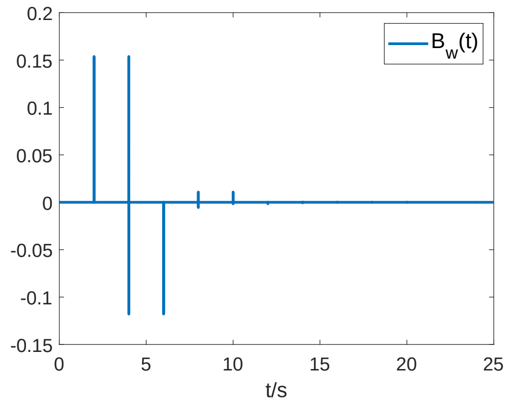

Fig. 9. Hybrid behavior in threshold function

Gains," *IEEE Transactions on Neural Networks and Learning Systems*, vol. 28, no. 7, pp. 1696-1709, 2016.

[6] X. Liu, H. Wang, C. Gao, and M. Chen, "Adaptive Fuzzy Funnel Control for a Class of Strict Feedback Nonlinear Systems," *Neurocomputing*, vol. 241, pp. 71-80, 2017.

[7] C. P. Bechlioulis and G. A. Rovithakis, "A Low-Complexity Global Approximation-Free Control Scheme with Prescribed Performance for Unknown Pure Feedback Systems," *Automatica*, vol. 50, no. 4, pp. 1217-1226, 2014.

[8] A. Theodorakopoulos and G. A. Rovithakis, "Guaranteeing Preselected Tracking Quality for Uncertain Strict-Feedback Systems with Deadzone Input Nonlinearity and Disturbances via Low-Complexity Control," *Automatica*, vol. 54, pp. 135-145, 2015.

[9] L. Peng and Z. Ai, "Event-Triggered Stabilisation for Continuous-Time Switched Nonlinear Systems Under Switching Constraints," *IET Control Theory & Applications*, vol. 13, no. 8, pp. 1182-1186, 2019.

[10] W. Xiang and T. T. Johnson, "Event-Triggered Control for Continuous-Time Switched Linear Systems," *IET Control Theory & Applications*, vol. 11, no. 11, pp. 1694-1703, 2017.

[11] X. Wang, Y. Wang, and W. Hou, "Non-Fragile Reliable Passive Control for Switched Systems Using an Event-Triggered Scheme.," *IAENG International Journal of Applied Mathematics*, vol. 51, no. 2, pp. 431-438 2021.

[12] H. Yan, H. Zhang, X. Zhan, Y. Wang, S. Chen, and F. Yang, "Event-Triggered Sliding Mode Control of Switched Neural Networks with Mode-Dependent Average Dwell Time," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 51, no. 2, pp. 1233-1243, 2019.

[13] X. Liu, X. Su, P. Shi, S. K. Nguang, and C. Shen, "Fault Detection Filtering for Nonlinear Switched Systems Via Event-Triggered Communication Approach," *Automatica*, vol. 101, pp. 365-376, 2019.

[14] X. Xiao, J. H. Park, and L. Zhou, "Event-Triggered H_∞ Filtering of Discrete-Time Switched Linear Systems," *ISA Transactions*, vol. 77, pp. 112-121, 2018.

[15] X. Su, X. Liu, P. Shi, and Y.-D. Song, "Sliding Mode Control of Hybrid Switched Systems via an Event-Triggered Mechanism," *Automatica*, vol. 90, pp. 294-303, 2018.

[16] Y. Qi and M. Cao, "Finite-Time Boundedness and Stabilisation of Switched Linear Systems Using Event-Triggered Controllers," *IET Control Theory & Applications*, vol. 11, no. 18, pp. 3240-3248, 2017.

[17] Y. Qi, P. Zeng, and W. Bao, "Event-Triggered and Self-Triggered $H\infty$ Control of Uncertain Switched Linear Systems," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 50, no. 4, pp. 1442-1454, 2018.

[18] H. Ren, G. Zong, and T. Li, "Event-Triggered Finite-Time Control for Networked Switched Linear Systems with Asynchronous Switching," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 48, no. 11, pp. 1874-1884, 2018.

[19] T.-F. Li, J. Fu, F. Deng, and T. Chai, "Stabilization of Switched Linear Neutral Systems: An Event-Triggered Sampling Control Scheme," *IEEE Transactions on Automatic Control*, vol. 63, no. 10, pp. 3537-3544, 2018.

[20] C. Li and J. Lian, "Event-Triggered Feedback Stabilization of Switched Linear Systems Using Dynamic Quantized Input," *Nonlinear Analysis: Hybrid Systems*, vol. 31, pp. 292-301, 2019.

[21] M. Wakaiki and Y. Yamamoto, "Stability Analysis of Sampled-Data Switched Systems with Quantization," *Automatica*, vol. 69, pp. 157-168, 2016.

[22] D. Liberzon, "Finite Data-Rate Feedback Stabilization of Switched and Hybrid Linear Systems," *Automatica*, vol. 50, no. 2, pp. 409–420, 2014.

[23] L. Xing, C. Wen, Z. Liu, H. Su, and J. Cai, "Event-Triggered Adaptive Control for a Class of Uncertain Nonlinear Systems," *IEEE Transactions on Automatic Control*, vol. 62, no. 4, pp. 2071-2076, 2016.

[24] Y. Huang and Y. Liu, "Switching Event-Triggered Control for a Class of Uncertain Nonlinear Systems," *Automatica*, vol. 108, pp. 108471, 2019.

[25] Y.-X. Li and G.-H. Yang, "Model-Based Adaptive Event-Triggered Control of Strict-Feedback Nonlinear Systems," *IEEE Transactions on Neural Networks and Learning Systems*, vol. 29, no. 4, pp. 1033-1045, 2017.

[26] Y.-F. Gao, X.-M. Sun, C. Wen, and W. Wang, "Event-Triggered Control for Stochastic Nonlinear Systems," *Automatica*, vol. 95, pp. 534-538, 2018.

[27] J. Lian, C. Li, and B. Xia, "Sampled-Data Control of Switched Linear Systems with Application to an F-18 Aircraft," *IEEE Transactions on Industrial Electronics*, vol. 64, no. 2, pp. 1332-1340, 2016.

[28] T. Liu and Z.-P. Jiang, "A Small-Gain Approach to Robust Event-Triggered Control of Nonlinear Systems," *IEEE Transactions on Automatic Control*, vol. 60, no. 8, pp. 2072-2085, 2015.

[29] F. Gao, X. Zhu, J. Huang, and X. Wen, "Finite-Time State Feedback Stabilization for a Class of Uncertain High-Order Nonholonomic Feedforward Systems," *Engineering Letters*, vol. 27, no. 1, pp. 108-113, 2019.

[30] M. Krstic, P. V. Kokotovic, and I. Kanellakopoulos, *Nonlinear and Adaptive Control Design*. John Wiley & Sons, Inc., 1995.

[31] M. M. Polycarpou and P. A. Ioannou, "A Robust Adaptive Nonlinear Control Design," in *1993 American Control Conference*, pp. 1365-1369, IEEE, 1993.

[32] M. W. Spong, S. Hutchinson, and M. Vidyasagar, *Robot Modeling and Control*. John Wiley & Sons, 2020.