Flexible Containment Controller Design for Second Order Multi-agent Systems

Mengyi Jiang, Chuang Gao, Anatolii K. Pogodaev, and Yonghui Yang

***Abstract*—This paper introduces an innovative containment control strategy tailored for second order multi-agent systems. The primary objective is to achieve containment control, which involves the generation of containment reference signals employing a set of dynamic leaders. Subsequently, followers that directly receive these reference signals can effectively trace their generated counterparts, while the remaining followers track some designated signals to ensure the containment control. Additionally, to streamline operations and circumvent the complexities associated with multiple derivative calculations, a command filtered technique is incorporated to simplify the operations of control scheme. The stability of second order multi-agent systems is analyzed based on Lyapunov theory. An inherent advantage of the proposed scheme lies in its flexibility and allowing arbitrary adjustments to the relative positions among followers by manipulating relevant parameters. Through extensive simulations, the efficacy of the designed scheme is validated.**

***Index Terms*—multi-agent systems, tracking control, containment controller, command filtered control.**

I. Introduction

Currently, the study of multi-agent systems (MASs) has garnered considerable attention and made significant progress in both theory and applications [1]− [3]. Two popular multi-agent control problems include formation control and containment control. For formation control, multiple agents can maintain specified formations by adhering to appropriate control protocols [4]− [6]. Additionally, containment control involves designing distributed control protocols to ensure that all agents move within a convex hull, where the convex hull is constructed from a certain number of leaders [7] and [8]. In the study by Zhao et al. [7], an effective control scheme was introduced for spacecraft, so that the containment errors converged into a small area around origin in finite time. In another work [8], a containment controller was developed for nonlinear MASs with nonstrict feedback form. The contributions of the aforementioned works underscore the wide applicability of containment control in collaborative multi-agent tasks. It is worth noting that backstepping design approach was adopted in [7] and [8].

Backstepping serves as an effective methodology for linear and nonlinear systems [9]− [11]. However, the drawback of the backstepping method becomes evident in scenarios of high system order or complex controller structures. The complexity of backstepping method is primarily attributed to the calculation of partial derivatives of virtual control laws, which leads to the potential encounter of the differential explosion problem.

To address the challenges posed by complex calculations and to conserve computing resources, a command filtered technique was developed to simply the design process of backstepping method [12]. Recently, the command filtered technique has been successfully integrated with other techniques [12]− [14].

Examining existing containment control schemes [7] and [8], two issues pertaining to MASs are identified as follows:

(i) The reference signal of follower is determined by the communication topology to limit the flexibility, which leads to cross-collision problems due to specific communication topology.

(ii) Some transmission failure problems are not considered in the aforementioned schemes. In this case, if a follower loses its connection with the leader, it can only track its neighbor follower's trajectory, which may cause a overlap collision problem.

Motivated by these considerations, we focus on a containment control problem for MASs with command filtered technique to simplify the design process and ensure that the tracking errors are uniformly and ultimately bounded (UUB).

To address the limitations of existing approaches, the following contributions are summarized as follows:

For the issue (i), the proposed scheme ensures that a series of tunable signals can be generated for followers. To mitigate problems (ii), a class of estimated tunable signals are devised for followers losing the connection from leaders. The estimated signals are integrated into the containment controller design, and arbitrary adjustments of the follower's trajectory are allowed to reduce the possibility of collisions.

II. Problem Formulation

First of all, we define a second order MASs in the following form [15]:

$$m_k \ddot{p}_k + d_k \dot{p}_k = u_k, k = 1, \cdots, K. \quad (1)$$

where p_k is the position of the kth follower, m_k denotes the mass and d_k represents the damping constant. K denotes the number of follower agents. Define $p_k = x_{k,1}$ and $\dot{p}_k = x_{k,2}$, we have

$$\begin{cases} \dot{x}_{k,1} = x_{k,2}, \\ \dot{x}_{k,2} = \frac{1}{m_k}\left(u_k - d_k x_{k,2}\right), \\ y_k = x_{k,1}, \end{cases} \quad (2)$$

where $x_{k,1} \in \Re$ and $x_{k,2} \in \Re$ represent the states of the kth follower agent. $u_k \in \Re$ is its actual control law.

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$y_k \in \Re$ denotes its output. In this paper, we consider a directed graph communication among agents (2) by following the introduction in [15], where a Laplacian matrix of a directed graph G can be defined by $L = D - A$ with $D = diag(\sum_{j=1}^{k} a_{k,j}) \in \Re^{K\times K}$ and $A = [a_{k,j}] \in \Re^{K\times K}$. The control goal of this paper is to design a controller u_k to realize the containment control of MASs (2) such that the output $x_{k,1}$ can follow a reference signal $y_{d,k}$ when the signal $y_{d,k}$ is active; otherwise, a flexible estimated signal $\hat{y}_{k,0}$ of y_d is generated if $y_{d,k}$ is not available to solve the potential collision problem among followers. Additionally, the tracking error $\zeta_{k,1}$ is UUB and will be defined in next section. Then, we need to select an appropriate reference signal y_d within the convex hull according to **Definition 1** in [16]. For the kth follower, $y_{d,k}$ can be determined from $y_{d,k} = y_d + b_k$ with $b_k \in \Re$ being an adjusted design parameter to generate a suitable signal $\hat{y}_{d,0}$ if $y_{d,k}$ is not available so as to ensure the containment control for all followers from any initial position. In addition, we assume that the minimum number of follower accessing the reference signal $y_{d,k}$ is one. If the kth follower can access the signal from $y_{d,k}$ directly, we define $\delta_k = 1$; otherwise, $\delta_k = 0$. Thus, $\sum_{k=1}^{K} \delta_k > 0$. Furthermore, the reference signal y_d and its 1st-order derivative $\dot{y}_d$ satisfy $|y_d| < Y$, and $|\dot{y}_d| < \bar{Y}$ with $Y > 0$ and $\bar{Y} > 0$ being unknown constants.

III. Containment Controller Design

In this section, we start from the following error transformations:

$$\begin{cases} \zeta_{k,1} = y_k - \delta_k\left(y_d + b_k\right) - \left(1-\delta_k\right)\left(\hat{y}_{k,0} + b_k\right) \\ = \Xi_k + (1-\delta_k)\tilde{y}_{k,0} \\ \xi_{k,1} = \zeta_{k,1} - \vartheta_{k,1} \\ \xi_{k,2} = x_{k,2} - \upsilon_{k,2} \end{cases}, \quad (3)$$

where $\Xi_k = y_k - y_d - b_k$, then we define a vector $\Xi = [\Xi_1, \cdots, \Xi_N]^T$. $\upsilon_{k,2}$ is the output of the following filter:

$$a_{k,2}\dot{\upsilon}_{k,2} + \upsilon_{k,2} = \upsilon_{k,1}, \upsilon_{k,2}\left(0\right) = \upsilon_{k,1}\left(0\right), \quad (4)$$

where $a_{k,2}$ is a positive design parameter and $\upsilon_{k,1}$ is a virtual control law that needs to be designed. $\hat{y}_{k,0}$ denotes an estimate value of y_d, and the estimation error is defined by $\tilde{y}_{k,0} = y_d - \hat{y}_{k,0}$. $\xi_{k,1}$ is called a compensation error, and $\vartheta_{k,1}$ represents a compensating signal which is generated by

$$\dot{\vartheta}_{k,1} = -l_{k,1}\vartheta_{k,1} + \upsilon_{k,2} - \upsilon_{k,1}, \quad \vartheta_{k,1}(0) = 0 \quad (5)$$

where $l_{k,1} > 0$ is a positive design parameter. Next, a Lyapunov candidate function is chosen as follows:

$$V_1 = \sum_{k=1}^{K} \frac{1}{2}\left(\xi_{k,1}^2 + \xi_{k,2}^2 + \frac{c(1-\delta_k)}{f_k}\tilde{y}_{k,0}^2\right), \quad (6)$$

where c and f_k are positive constants. Then, it produces

$$\dot{V}_1 = \sum_{k=1}^{K}\left(\xi_{k,1}\dot{\xi}_{k,1} + \xi_{k,2}\dot{\xi}_{k,2} + \frac{c(1-\delta_k)}{f_k}\tilde{y}_{k,0}\dot{\tilde{y}}_{k,0}\right). \quad (7)$$

Based on $\xi_{k,1} = \zeta_{k,1} - \vartheta_{k,1}$ and $\tilde{y}_{k,0} = y_d - \hat{y}_{k,0}$, it yields

$$\begin{aligned} \dot{V}_1 &= \sum_{k=1}^{K}\left(\xi_{k,1}(\dot{\zeta}_{k,1} - \dot{\vartheta}_{k,1}) + \xi_{k,2}\dot{\xi}_{k,2}\right) \\ &\quad + \sum_{k=1}^{K}\left(\frac{c(1-\delta_k)}{f_k}\tilde{y}_{k,0}(\dot{y}_d - \dot{\hat{y}}_{k,0})\right). \quad (8) \end{aligned}$$

Combining (3) with (8) produces

$$\begin{aligned} \dot{V}_1 &= \sum_{k=1}^{K}\xi_{k,1}\left(\dot{y}_k - \delta_k\dot{y}_d - (1-\delta_k)\dot{\hat{y}}_{k,0}\right) \\ &\quad + \sum_{k=1}^{K}\xi_{k,1}\left(l_{k,1}\vartheta_{k,1} - \upsilon_{k,2} + \upsilon_{k,1}\right) \\ &\quad + \sum_{k=1}^{K}\frac{c(1-\delta_k)}{f_k}\tilde{y}_{k,0}(\dot{y}_d - \dot{\hat{y}}_{k,0}) + \xi_{k,2}\dot{\xi}_{k,2}. \quad (9) \end{aligned}$$

Next, we design $\upsilon_{k,1}$ as

$$\upsilon_{k,1} = \delta_k\dot{y}_d + (1-\delta_k)\dot{\hat{y}}_{k,0} - l_{k,1}\xi_{k,1} - c\psi_k E_k. \quad (10)$$

where $E_k = \varsigma_k - \vartheta_{k,1}\delta_k - \sum_{j=1}^{N} a_{k,j}(\vartheta_{k,1} - \vartheta_{j,1}) - \sum_{j=1}^{N} a_{k,j}(b_k - b_j)$, $\varsigma_k = \sum_{j=1}^{N} a_{k,j}(y_k - y_j) + \delta_k(y_k - y_{d,k})$ and $\psi_k = 1/\Phi_k$ with Φ_k being obtained by defining a vector $\Phi = [\Phi_1, \cdots, \Phi_N]^T = (L + C)^{-1}[1, \cdots, 1]^T$. Then, $\hat{y}_{k,0}$ is generated by

$$\dot{\hat{y}}_{k,0} = -f_k\psi_k E_k - f_k q_k(\hat{y}_{k,0} - w_k), \quad (11)$$

where $q_k > 0$ and $w_k \in R$ are design parameters.

From $\tilde{y}_{k,0} \le |\tilde{y}_{k,0}|$ and $|\dot{y}_d| \le \bar{Y}$, it follows from (9) that

$$\begin{aligned} \dot{V}_1 &\le -\sum_{k=1}^{K} l_{k,1}\xi_{k,1}^2 + \sum_{k=1}^{K}\xi_{k,2}\left(\dot{\xi}_{k,2} + \xi_{k,1}\right) \\ &\quad + \sum_{k=1}^{K}\frac{c(1-\delta_k)}{f_k}\left|\tilde{y}_{k,0}\right|\bar{Y} \\ &\quad + \sum_{k=1}^{K} q_k c(1-\delta_k)\tilde{y}_{k,0}(\hat{y}_{k,0} - b_k) \\ &\quad + \sum_{k=1}^{K}\psi_k E_k c(1-\delta_k)\tilde{y}_{k,0} \\ &\quad - \sum_{k=1}^{K}\vartheta_{k,1}c\psi_k E_k. \quad (12) \end{aligned}$$

For $\sum_{k=1}^{K} q_k c(1-\delta_k)\tilde{y}_{k,0}(\tilde{y}_{k,0} - w_k)$, from $\hat{y}_{k,0} = y_d - \tilde{y}_{k,0}$, we have

$$\begin{aligned} &\sum_{k=1}^{K} q_k c(1-\delta_k)\tilde{y}_{k,0}(y_d - \tilde{y}_{k,0} - w_k) \\ \le\ &\sum_{k=1}^{K}\frac{q_k c(1-\delta_k)}{2}(y_d - w_k)^2 \\ &-\sum_{k=1}^{K}\frac{q_k c(1-\delta_k)}{2}\tilde{y}_{k,0}^2. \quad (13) \end{aligned}$$

For $\sum_{k=1}^{K}\psi_k E_k c(1-\delta_k)\tilde{y}_{k,0} - \sum_{k=1}^{K}\vartheta_{k,1}c\psi_k E_k$ in (12), define a vector as

$$\begin{aligned} \tilde{\Xi} &= \left[\tilde{\Xi}_1, \cdots, \tilde{\Xi}_N\right]^T \\ &= \left[\Xi_1 - \vartheta_{1,1}, \cdots, \Xi_N - \vartheta_{N,1}\right]^T. \quad (14) \end{aligned}$$

From $C = diag\{\delta_1, \cdots, \delta_N\}$ and defining L as

$$\begin{aligned} L &= D - A \\ &= \begin{bmatrix} \sum_{j=1}^{N} a_{1,j} & -a_{1,2} & \cdots & -a_{1,N} \\ -a_{2,1} & \sum_{j=1}^{N} a_{2,j} & \cdots & -a_{2,N} \\ \vdots & \vdots & \ddots & \vdots \\ -a_{N,1} & -a_{N,2} & \cdots & \sum_{j=1}^{N} a_{N,j} \end{bmatrix}, \end{aligned} \quad (15)$$

it gives

$$(L + C)\tilde{\Xi} = E. \quad (16)$$

Then, we obtain

$$\sum_{k=1}^{K} \psi_k E_k c(1-\delta_k)\tilde{y}_{k,0} - \sum_{k=1}^{K} \vartheta_{k,1} c\psi_k E_k = -\frac{c}{2}\tilde{\Xi}^T Q\tilde{\Xi} \leq 0, \quad (17)$$

where $Q = \Theta(L + C) + (L + C)^T\Theta$ and $\Theta = diag\{\psi_1, \cdots, \psi_N\}$. Then Q is positive definite.

Combining (13) with (17) into (12) produces

$$\begin{aligned} \dot{V}_1 \leq & -\sum_{k=1}^{K} l_{k,1}\xi_{k,1}^2 + \sum_{k=1}^{K} \xi_{k,2}\left(\dot{\xi}_{k,2} + \xi_{k,1}\right) \\ & + \sum_{k=1}^{K} \frac{c(1-\delta_k)}{f_k}\left|\tilde{y}_{k,0}\right| \bar{Y} \\ & + \sum_{k=1}^{K} \frac{q_k c(1-\delta_k)}{2}(y_d - w_k)^2 \\ & - \sum_{k=1}^{K} \frac{q_k c(1-\delta_k)}{2}\tilde{y}_{k,0}^2 - \frac{k}{2}\tilde{\Xi}^T\tilde{Q}\tilde{\Xi}. \end{aligned} \quad (18)$$

For $\sum_{k=1}^{K} c(1-\delta_k)\left|\tilde{y}_{k,0}\right|\bar{Y}/f_k$ and $-\sum_{k=1}^{K} 0.5q_k c(1-\delta_k)\tilde{y}_{k,0}^2$, based on $(2\bar{Y} - q_k f_k \tilde{y}_{k,0})^2 \geq 0$, it yields

$$\begin{aligned} & \sum_{k=1}^{K} \frac{c(1-\delta_k)}{f_k}\left|\tilde{y}_{k,0}\right| \bar{Y} - \sum_{k=1}^{K} \frac{q_k c(1-\delta_k)}{2}\tilde{y}_{k,0}^2 \\ \leq & \sum_{k=1}^{K} \frac{c(1-\delta_k)}{f_k q_k}\bar{Y}^2 - \sum_{k=1}^{K} \frac{q_k c(1-\delta_k)}{4}\tilde{y}_{k,0}^2. \end{aligned} \quad (19)$$

Then, one has

$$\begin{aligned} \dot{V}_1 \leq & -\sum_{k=1}^{K} l_{k,1}\xi_{k,1}^2 + \sum_{k=1}^{K} \xi_{k,2}\left(\dot{\xi}_{k,2} + \xi_{k,1}\right) \\ & + \sum_{k=1}^{K} \frac{c(1-\delta_k)}{f_k q_k}\bar{Y}^2 - \sum_{k=1}^{K} \frac{q_k c(1-\delta_k)}{4}\tilde{y}_{k,0}^2 \\ & + \sum_{k=1}^{K} \frac{q_k c(1-\delta_k)}{2}(y_d - w_k)^2. \end{aligned}$$

Define N_1 as

$$\begin{aligned} N_1 = & \sum_{k=1}^{K}\left(\frac{c(1-\delta_k)}{f_k q_k}\bar{Y}^2\right) \\ & + \sum_{k=1}^{K}\left(\frac{q_k c(1-\delta_k)}{2}(Y + |w_k|)^2\right), \end{aligned} \quad (20)$$

and it produces

$$\begin{aligned} \dot{V}_1 \leq & -\sum_{k=1}^{K} l_{k,1}\xi_{k,1}^2 + \sum_{k=1}^{K} \xi_{k,2}\left(\dot{\xi}_{k,2} + \xi_{k,1}\right) \\ & - \sum_{k=1}^{K} \frac{q_k c(1-\delta_k)}{4}\tilde{y}_{k,0}^2 + N_1. \end{aligned} \quad (21)$$

Next, we have

$$\begin{aligned} \dot{V}_1 \leq & -\sum_{k=1}^{K} l_{k,1}\xi_{k,1}^2 - \sum_{k=1}^{K} \frac{q_k c(1-\delta_k)}{4}\tilde{y}_{k,0}^2 + N_1 \\ & + \sum_{k=1}^{K} \xi_{k,2}\left(\frac{u_k}{m_k} - \frac{d_k}{m_k}x_{k,2} - \dot{\upsilon}_{k,2} + \xi_{k,1}\right). \end{aligned} \quad (22)$$

Design an actual control input u_k as

$$u_k = -m_k\left(l_{k,2}\xi_{k,2} - \frac{d_k}{m_k}x_{k,2} - \dot{\upsilon}_{k,2} + \xi_{k,1}\right). \quad (23)$$

Then, we obtain

$$\begin{aligned} \dot{V}_1 \leq & -\sum_{k=1}^{K} l_{k,1}\xi_{k,1}^2 - \sum_{k=1}^{K} l_{k,2}\xi_{k,2}^2 \\ & - \sum_{k=1}^{K} \frac{q_k c(1-\delta_k)}{4}\tilde{y}_{k,0}^2 + N_1. \end{aligned} \quad (24)$$

IV. Stability analysis

First, we define

$$\gamma_1 = \min\left\{2l_{k,1}, 2l_{k,2}, \frac{q_k f_k}{2}\right\}, \quad (25)$$

and it follows from (24) that

$$\dot{V}_1 \leq -\gamma_1 V_1 + N_1. \quad (26)$$

From (26), we obtain

$$V_1(t) \leq V_1(0)e^{-at} + \frac{N_1}{\gamma_1}(1 - e^{-at}), \quad (27)$$

which concludes that V_1 is UUB, it implies that $\xi_{k,1}$, $\xi_{k,2}$ and $\tilde{y}_{k,0}$ are bounded. From $\tilde{y}_{k,0} = y_d - \hat{y}_{k,0}$, we know that y_d is bounded, furthermore, $\hat{y}_{k,0}$ is bounded. Next, define a Lyapunov function as

$$V_2 = \sum_{k=1}^{K} \frac{1}{2}\vartheta_{k,1}^2. \quad (28)$$

Then, it gives

$$\begin{aligned} \dot{V}_2 = & \sum_{k=1}^{K} \vartheta_{k,1}(-l_{k,1}\vartheta_{k,1}^2 + \upsilon_{k,2} - \upsilon_{k,1}) \\ \leq & -\sum_{k=1}^{K} l_{k,1}\vartheta_{k,1}^2 + \sum_{k=1}^{K} \left|\vartheta_{k,1}\right|\left|\upsilon_{k,2} - \upsilon_{k,1}\right|. \end{aligned} \quad (29)$$

From $|\upsilon_{k,2} - \upsilon_{k,1}| \leq \tau_k$ in [12] with τ_k being a positive constant, it yields

$$\dot{V}_2 \leq -\sum_{k=1}^{K}(l_{k,1} - 0.5)\vartheta_{k,1}^2 + \sum_{k=1}^{K} 0.5\tau_k^2. \quad (30)$$

Define

$$\begin{cases} \gamma_2 = 2l_{k,1} - 1, \\ N_2 = \sum_{k=1}^{K} 0.5\tau_k^2. \end{cases} \quad (31)$$

Thus, choosing $l_{k,1} > 0.5$ produces

$$\dot{V}_2 \leq -\gamma_2 V_2 + N_2, \tag{32}$$

which implies that $\vartheta_{k,1}$ are UUB. Then, we have

$$V_2(t) \leq V_2(0)e^{-ct} + \frac{N_2}{\gamma_2}(1 - e^{-ct}). \tag{33}$$

Define

$$\begin{cases} \sigma_1 = V_1(0)e^{-at} + \frac{N1}{\gamma_1}(1 - e^{-at}), \\ \sigma_2 = V_2(0)e^{-ct} + \frac{N_2}{\gamma_2}(1 - e^{-ct}). \end{cases} \tag{34}$$

From (27) and (33), we obtain

$$\frac{1}{2}\xi_{k,1}^2 \leq \sigma_1, \tag{35}$$

then it produces

$$|\xi_{k,1}| \leq \sqrt{2\sigma_1}. \tag{36}$$

From (36) and $\xi_{k,1} = \zeta_{k,1} - \vartheta_{k,1}$, one has

$$|\zeta_{k,1}| - |\vartheta_{k,1}| \leq |\zeta_{k,1} - \vartheta_{k,1}| \leq \sqrt{2\sigma_1}. \tag{37}$$

Based on (36), it shows that

$$|\vartheta_{k,1}| \leq \sqrt{2\sigma_2}. \tag{38}$$

From (37) and (38), it implies that

$$|\zeta_{k,1}| \leq \sqrt{2\sigma_1} + \sqrt{2\sigma_2}. \tag{39}$$

Based on (39), we can see that the tracking errors $\zeta_{k,1}$ are bounded by $\sqrt{2\sigma_1} + \sqrt{2\sigma_2}$ for $t \to \infty$, which illustrates that $\zeta_{k,1}$ is UUB and can converge to an adjusted neighborhood of the origin. Thus, the control goal of this paper has been achieved.

V. Simulation

To realize the containment control of MASs (2), the simulation is carried out for five agents, which include two leaders and three followers. Directed communication graphs are provided in Figures 1 and 2 to demonstrate the effectiveness of the proposed scheme. The dynamic leaders are denoted as $L_1 = \cos(t) + 1$ and $L_2 = \cos(t) - 2.5$, while the followers are labeled as F_1, F_2 and F_3. We consider the following two situations: part 1 covers the initial 15 seconds (depicted in Figure 1), while part 2 represents the scenario after 15 seconds when F_1 and F_3 no longer directly receive the leaders' signals (depicted in Figure 2). The actual control input u_k is given as follows:

$$u_k = -m_k \left(l_{k,2}\vartheta_{k,2} - \frac{d_k}{m_k} x_{k,2} - \dot{\upsilon}_{k,2} + \xi_{k,1} \right), \tag{40}$$

where we set $l_{k,2} = 1$, $m_k = 0.5$ and $d_k = 1$. Then, $\xi_{k,1}$ is defined by

$$\xi_{k,1} = y_k - \delta_k \left(y_d + b_k\right) - \left(1 - \delta_k\right) \left(\hat{y}_{k,0} + b_k\right) - \vartheta_{k,1}, \tag{41}$$

where δ_k is determined as follows:

$$\begin{cases} \text{Case 1 } (t < 15\text{s}): \quad \delta_1 = \delta_2 = \delta_3 = 1, \\ \text{Case 2 } (t \geq 15\text{s}): \quad \delta_1 = \delta_3 = 0 \text{ and } \delta_2 = 1. \end{cases} \tag{42}$$

Next, set a reference signal y_d as $y_d = 0.5r_1 + 0.5r_2$. Then, $\hat{y}_{k,0}$ and $\vartheta_{k,1}$ are given by

$$\begin{cases} \dot{\hat{y}}_{k,0} = -f_k\psi_k E_k - f_k q_k(\hat{y}_{k,0} - w_k), \\ \dot{\vartheta}_{k,1} = -l_{k,1}\vartheta_{k,1} + \upsilon_{k,2} - \upsilon_{k,1}. \end{cases} \tag{43}$$

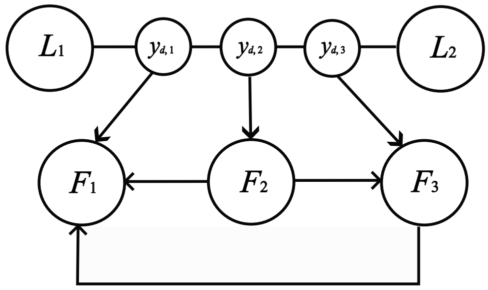

Fig. 1. Communication graph at the beginning 15s.

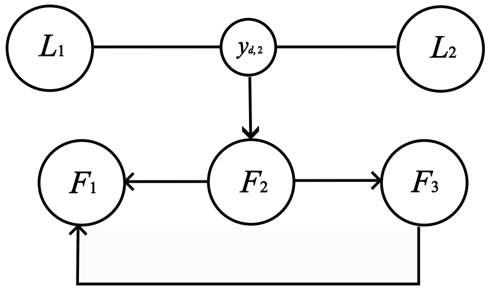

Fig. 2. Communication graph after 15s.

Select $f_k = 1$, $q_k = 1$, $\psi_k = 0.5$ and $l_{k,1} = 1$. For the parameters of $\upsilon_{k,1}$ and $\upsilon_{k,2}$, choose $c = 1$ and $a_{k,2} = 0.1$. Additionally, ς_k can be represented by

$$\begin{cases} \varsigma_1 = 2x_{1,1} - x_{2,1} - x_{3,1} + \delta_1(x_{1,1} - y_{d,1}), \\ \varsigma_2 = \delta_2 \left(x_{2,1} - y_{d,2}\right), \\ \varsigma_3 = (x_{3,1} - x_{2,1}) + \delta_3(x_{3,1} - y_{d,3}). \end{cases} \tag{44}$$

The initial conditions are set as follows: $[x_{1,1}(0),\ x_{2,1}(0),\ x_{3,1}(0)] = [-3,\ 0.8,\ 3]$ and $\hat{y}_{k,0}(0) = x_{k,2}(0) = 0$. The simulation outcomes are depicted in Figures 3–5. Figure 3 illustrates the tracking performances of the proposed containment control. F_1 and F_3 initiate from positions outside the leaders' trajectories, while F_2 starts within the convex hull. Subsequently, F_1 and F_3 rapidly converge into the convex hull. By setting $b_1 = -0.8$, $b_2 = 0$, and $b_3 = 0.8$, the proposed scheme ensures all three followers maintain a close proximity to each other. When irresistible failure occurs at 15 seconds, F_2 is the only follower that can receive the information from the leaders. Figure 4 illustrates that $\hat{y}_{1,0}$ and $\hat{y}_{3,0}$ are accessible and confined. To mitigate potential collision issues, F_1 and F_3 follow the trajectories of $\hat{y}_{1,0}$ and $\hat{y}_{3,0}$ by setting $w_1 = -1.2$ and $w_3 = 0.5$. The control laws for F_1, F_2 and F_3 are provided in Figure 5, and we can see that the control signals are bounded and converge around the origin after the followers enter the convex hull. These results affirm the effectiveness and superiority of the designed controller.

VI. Conclusion

In this paper, an innovative containment control scheme has been designed for second order multi-agent systems. If the reference signals are available, then the followers trace their generated counterparts effectively, while the remaining followers track some estimated signals to ensure the containment control. Additionally, a command filtered technique has been incorporated into the design scheme to streamline operations and circumvent the complexities associated with

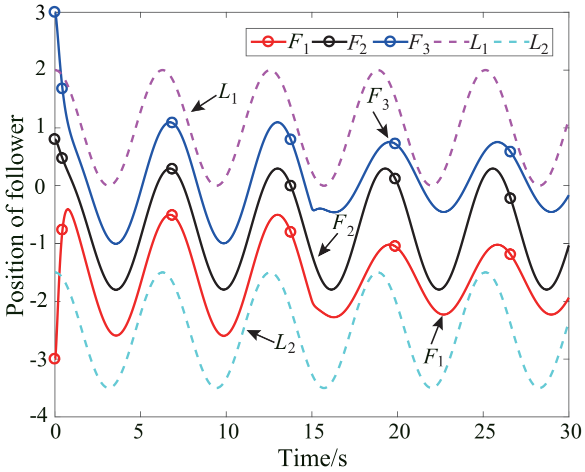

Fig. 3. Control performance of three followers.

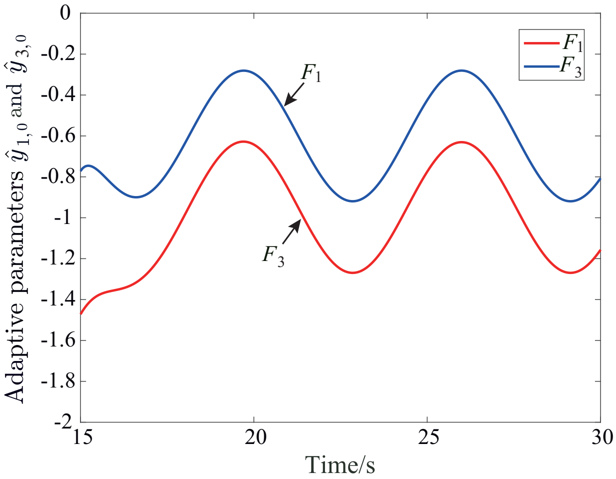

Fig. 4. Curves of $\hat{y}_{1,0}$ and $\hat{y}_{3,0}$.

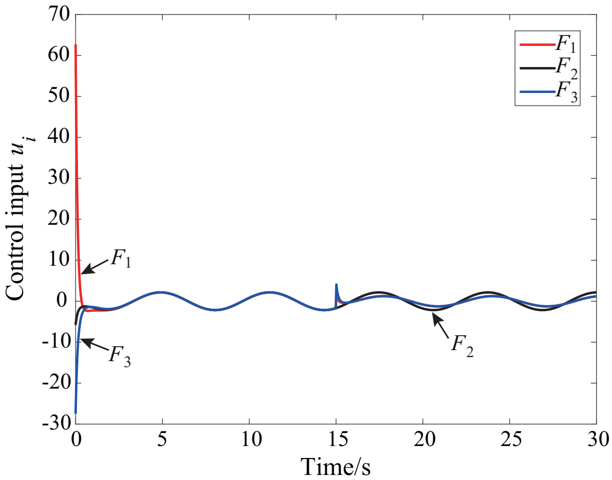

Fig. 5. Actual control laws.

multiple derivative calculation. Through extensive simulations, the efficacy of the designed scheme has been validated.

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