Influence of Harmonic Components in Traction System on the Traction Motor of High-speed Train

Xun Wang, Pingbo Wu*, Sheng Qu, Hao Wu

Abstract—AC-DC-AC traction transmission system provides power for high-speed train and is one of the key technologies for rail transit. The rectifier part adopts a four-quadrant rectifier, which has advantages such as high power factor, low harmonic content, and energy feedback. However, due to the power characteristics of single-phase power supply in electrified railways, there is a double power frequency fluctuation in the DC link voltage of the traction transmission system. In addition, the current output controlled by the inverter contains higher-order harmonics. In order to study the influence of harmonic components on the traction motor of high-speed train, an AC-DC-AC traction system simulation model for high-speed train was established. The results show that after LC filtering, at the condition that the stator current frequency is close to 100Hz, the motor stator current still undergoes distortion, but the effect on torque is not significant. When the LC filter fails, the torque ripple caused by 100Hz is very significant. It is recommended to consider this in the fault detection system of the traction transmission system. During train operation, the torque ripple at frequency of 6-times motor stator current is prominent, and this frequency also exists in the voltage and current of the DC link.

Index Terms—High-speed train, Traction motor, DC link Voltage Fluctuation, Torque vibration

I. INTRODUCTION

The AC-DC-AC traction transmission system [1-3] is widely used in electric traction systems for railway in China, mainly including transformer, single-phase pulse rectifiers, intermediate DC links, three-phase inverters, and traction motors, as shown in Fig. 1. The train receives power from the single-phase 25kV contact network through a pantograph, then depressurizes it through a power frequency transformer and inputs it into a single-phase pulse rectifier. The single-phase pulse rectifier rectifies and outputs stable DC voltage, which is input to the three-phase inverter through the DC link. The inverter generates three-phase AC voltage to drive the three-phase asynchronous traction motor [4-6], thereby achieving power output.

In the AC-DC-AC traction system of high-speed trains, the DC voltage output by a single-phase pulse rectifier contains a pulsating component, which is twice the frequency of the power grid. This component can cause an increase in electromagnetic torque ripple and stator current distortion, known as beat frequency phenomenon [7-10]. When the operating frequency of the motor is close to the beat frequency voltage frequency, the beat frequency phenomenon is the most severe. At this point, a smaller beat frequency voltage would also generate a very large beat frequency current, causing problems such as rapid increase in motor power loss, rapid temperature rise, and severe fluctuations in electromagnetic torque, posing a serious threat to the safe operation of the train [11, 12]. At the same time, the three-phase AC output from the inverter contains a series of high-frequency harmonic components, which interact with the fundamental magnetic flux to form pulsating torque [13, 14]. Through the electromechanical coupling system, it would cause vibration fatigue problems in the mechanical system of high-speed trains.

Wu et al. [15] analyzed the influence of torque pulsation caused by voltage pulsation in the DC-link of traction drive system on motor hanger and studied the structural fatigue damage of the drive system. A method with Voltage Linear Interpolation Compensation for suppressing the beat frequency phenomenon was proposed by Song et al. [16],
which is a Digital OCC Scheme. Ouyang et al. [17] improved compensation performance by using a repetitive DC link voltage predictor, which can significantly suppress steady-state beat current by several times. Diao et al. [18] used a ripple control scheme in vector control systems to reduce torque and current ripple amplitude, which compensates for slip frequency without the need for hardware filters. He et al. [19] used interdisciplinary research to improve switching frequency and mode selection by balancing torque, efficiency, and NVH performance of traction motor.

Double power frequency pulsation in the DC link would have an adverse impact on the control performance of the four-quadrant converter and traction motor [20]. Based on the AC-DC-AC traction system simulation model, the influence of harmonics on traction motor and their mutual influence in traction systems were investigated. The acceleration, uniform speed, and braking processes of train operation were simulated, and the results were analyzed by using short-time Fourier transform in this study.

II. MOTOR VOLTAGE AND TORQUE ANALYSIS UNDER DC LINK VOLTAGE FLUCTUATION

Pulse width modulation technology is used for inverter in traction drive systems. If only basic components are considered, the switch function is represented as [17]

\[
\begin{bmatrix}
S_a \\
S_b \\
S_c
\end{bmatrix} = \begin{bmatrix}
u_a^*(t) \\
u_b^*(t) \\
u_c^*(t)
\end{bmatrix} / U_d = \begin{bmatrix}
U_{lm} \sin(\alpha_0 t) \\
U_{lm} \sin(\alpha_0 t - \frac{2\pi}{3}) \\
U_{lm} \sin(\alpha_0 t + \frac{2\pi}{3})
\end{bmatrix} / U_d
\]

Where, \(u_a^*(t), u_b^*(t), u_c^*(t)\) are output phase voltages, \(U_d\) is DC voltage, \(U_{lm}\) is amplitude of the output voltages, \(\alpha_0\) is angular frequency of output voltages. Assuming there is a fluctuating component in DC voltage, the DC voltage could be expressed as

\[
u_d(t) = U_d + U_r \sin(\omega_r t + \theta_r)
\]

Where, \(U_r, \theta_r\) and \(\omega_r\) are amplitude, phase, and angular frequency, respectively.

The available output voltage is

\[
\begin{bmatrix}
u_a \\
u_b \\
u_c
\end{bmatrix} = \begin{bmatrix}
S_a \\
S_b \\
S_c
\end{bmatrix} \cdot \begin{bmatrix}
u_d^*(t)
\end{bmatrix} / 2
\]

From Eq. 4, it can be seen that the output phase voltage of the inverter consists of three components. The latter two components are formed by modulating the fluctuation component through the inverter. If there are more fluctuation components in the system, there would also be more harmonic frequencies in the output voltage.

Where \(m = 2U_{lm}/U_d\), the output voltage of phase a is represented as

\[
u_a(t) = S_a \cdot u_d^*(t) = \frac{1}{2} \sin(\alpha_0 t) \cdot \left[ U_d + U_r \sin(\omega_r t + \theta_r) \right] = \frac{1}{2} U_{lm} \sin(\alpha_0 t) + m \frac{U}{2} \sin(\omega_r t + \theta_r)
\]

\[
u_b(t) = S_b \cdot u_d^*(t) = \frac{1}{2} \sin(\alpha_0 t - \frac{2\pi}{3}) \cdot \left[ U_d + U_r \sin(\omega_r t + \theta_r) \right] = \frac{1}{2} U_{lm} \sin(\alpha_0 t - \frac{2\pi}{3}) + m \frac{U}{4} \cos(\omega_r t + \theta_r)
\]

\[
u_c(t) = S_c \cdot u_d^*(t) = \frac{1}{2} \sin(\alpha_0 t + \frac{2\pi}{3}) \cdot \left[ U_d + U_r \sin(\omega_r t + \theta_r) \right] = \frac{1}{2} U_{lm} \sin(\alpha_0 t + \frac{2\pi}{3}) - m \frac{U}{4} \cos(\omega_r t + \theta_r)
\]

When the output voltage frequency approaches the fluctuation frequency, the beat frequency (\(\omega_r - \alpha_0\)) could be small, however, it could still form a large beat current, causing torque ripple of the traction motor.

According to PWM rectification theory, ignoring the loss of rectifier power devices and the equivalent impedance of transformer secondary side, the output power is equal to the input power, and the DC side voltage ripple component could be obtained as follows

\[
u_{dc} = \frac{I_s}{2\alpha} C_s \cos(\phi)
\]

Where, \(\alpha\) — Grid side angle frequency, \(\phi\) — Phase angle of grid side current lead voltage, \(I_s\) — Average value of load current. From Eq. 5, it can be found that after PWM rectification, there is a component of twice the grid frequency (\(f_s = 50Hz\) in the DC link, which is 100Hz).

When the frequency of harmonic magnetic flux and harmonic rotor current is different, their interaction would generate vibration harmonic torque. For example, the instantaneous torque value generated by the stator current of the 5th harmonic current and the fundamental magnetic flux in each phase of the motor is

\[
T_{r5-1} = \frac{n_s}{2\pi f_1} \sqrt{S_{15}} \sin(5\omega_1 t - \phi_1) \sqrt{E_s} \sin(\phi_1)
\]

\[
T_{r5-2} = \frac{n_s}{2\pi f_2} \sqrt{S_{15}} \sin \left( (5\omega_1 t - \phi_1) + \frac{2\pi}{3} \right) \sqrt{E_s} \sin \left( \phi_1 - \frac{2\pi}{3} \right)
\]

\[
T_{r5-3} = \frac{n_s}{2\pi f_3} \sqrt{S_{15}} \sin \left( (5\omega_1 t - \phi_1) - \frac{2\pi}{3} \right) \sqrt{E_s} \sin \left( \phi_1 - \frac{4\pi}{3} \right)
\]

Where, \(f_i\) is fundamental voltage frequency of motor stator, \(n_p\) — number of pole pairs.

Using trigonometric relationships:

\[
\sin \alpha \sin \beta = \frac{1}{2} \left[ \cos(\alpha - \beta) - \cos(\alpha + \beta) \right]
\]

the vibration harmonic torque generating by the 5th harmonic current could be obtained by adding the torque of three phases

\[
T_{r5} = \frac{3n_s}{2\pi f_1} I_s E_s \cos(\phi_1 - \phi_2) = \frac{3n_s}{2\pi f_1} I_s E_s \cos(\phi_2 + \pi - \phi_1)
\]

Similarly, the vibration harmonic torque generated by the 7th harmonic current is

Eq. 7 and Eq. 8 indicate that the 5th and 7th stator current harmonics generate a vibration torque of 6 times the stator
voltage fundamental frequency during the operation of the traction motor.

\[ T_{r-1} = \frac{3T_s}{2\pi f_1} \hat{I}_s E_i \cos(6\alpha t - \phi_2) \]  

(8)

In turn, the vibration torque formed by the voltage ripple component of the DC link could be obtained

\[ T_d = \frac{3n_s}{2\pi f_1} I_{ua} E_i \cos(\alpha t - \phi_2) + \frac{3n_s}{2\pi f_1} I_{UB} E_i \cos(\alpha t + \pi - \phi_2) \]  

(9)

From Eq. 5, it can be seen that the DC link contains a pulsating component that is twice the grid frequency, i.e., \( \omega_p = 2\omega_s \).

III. RESULTS AND DISCUSSION

The modeling illustration of the traction system is shown in Fig. 2. Based on the principles of transient current control and direct torque control, models were established for PWM rectifiers and traction inverters [21, 22].

Fig. 2 Block diagram of traction system model for high-speed train.

Transient direct current control could achieve voltage stabilization, voltage regulation, unit power factor, and stable switching for working state in the rectifier link through voltage and current double closed-loop control. Its expression is as follows

\[ \begin{aligned}
I_{ua} &= K_p (U_a - U_i) + K_i \int (U_a - U_i) dt \\
I_{UB} &= I_{Ua} + I_{Ue} \\
I_e &= I_{Us} + I_{Ue} \\
U_a(t) &= u_a(t) - \{\omega L_1 I_{Ua} \cos \omega t + I_e \cos \omega t\} - R_s \sin \omega t \\
\end{aligned} \]  

(10)

Where, \( K_p \) and \( K_i \) are the proportional and integral coefficients of the voltage loop PI controller, respectively. \( K \) is the proportional coefficient of the current inner loop.

Direct torque control uses the space vector analysis method to directly control the stator flux and electromagnetic torque by stator field orientation. In order to achieve feedback control in direct torque control systems, it is necessary to accurately estimate the current stator flux and torque. The estimation of stator magnetic flux of traction motor is as follows

\[ \begin{aligned}
\psi_{mu} &= \int (u_{ma} - i_{ma} R_s) dt \\
\psi_{mb} &= \int (u_{mb} - i_{mb} R_s) dt \\
\end{aligned} \]  

(11)

According to the vector equation of asynchronous motors, the electromagnetic torque can be expressed as

\[ T_e = n_s \psi_s \times \vec{I} \]  

(12)

Observations of electromagnetic torque can be obtained as follows

\[ T_e = n_s (\psi_{ma} i_{mb} - \psi_{mb} i_{ma}) \]  

(13)

Where, \( R_s \) is the stator resistance, \( n_s \) is the number of poles. \( \psi_s \) and \( \vec{I} \) represent the stator flux space vector and stator current space vector under the three-phase shaft system, respectively. \( \psi_{ma} \) and \( \psi_{mb} \) are estimated values, \( i_{ma} \) and \( i_{mb} \) are measured values.

The simulation model of the traction system for high-speed train was established with Simulink module as in Matlab package, and the influence of the DC link voltage fluctuation on the stator current and torque of the traction motor was analyzed. Simulations were conducted at the presence or absence of an LC filter in the DC link at full speed. The motor speed was set to accelerate from 0 to 367 rad/s, run at a

Fig. 3 Time diagram of traction motor stator current at full speed, (a) DC link with LC filter, (b) DC link without LC filter.
constant speed for 10 seconds, and then decelerate.

The time diagram of traction motor stator current at full speed is shown in Fig. 3, and the 100Hz is the pulsating component that occurs after PWM rectification. In the enlarged image of Fig. 3, the phenomenon of current distortion exists in the stator current during the traction and braking stages, when the stator current frequency of the traction motor approaches 100Hz, and the distortion is most severe when the stator current frequency is equal to 100Hz. From Fig. 3, it can be found that regardless of the presence or absence of LC filter in the DC link, when the stator current frequency of the traction motor approaches 100Hz, distortion of the stator current would occur. It indicated that even if the 100Hz component of the DC link is filtered, it can still cause stator current distortion. Comparing Fig. 3(a) and Fig. 3(b), it can be found that when the DC link is equipped with LC filter, the stator current fluctuation is smaller than that without LC filter, indicating the necessity of DC link filtering.

The current time-domain diagram of the traction motor of line test is shown in Fig. 4, where blue represents current and red represents current frequency. This time-domain diagram includes traction and braking conditions, both of which have passed through the current frequency of 100Hz, as shown in the enlarged part of the diagram. It can be observed that when the current frequency of the traction motor approaches 100Hz, i.e. 2f, the current would undergo distortion, which is consistent with the simulation results in Fig. 3. It is indicated that the simulation model could effectively simulate this phenomenon.

The Short-time Fourier transform (STFT) was used in this work to detect changes in the frequency and time domain of the simulation results, and time frequency diagram of stator current of traction motor is shown in Fig. 5. In Fig. 5(b), the stator current has three distinct frequencies that vary with motor speed, namely fundamental frequency, harmonic frequency 5f, and 7f due to PWM inverter. Meanwhile, the stator current has other two harmonic components with the frequencies of f - 2f, and f + 2f, due to DC voltage pulsations. The frequencies of 5f ± 2f, and 7f ± 2f, exist in stator current, but the amplitudes of which are smaller than that of f ± 2f. After the DC link is filtered by LC, in Fig. 5(a), the harmonic components caused by DC voltage fluctuation are not significant in the time frequency, but harmonic components caused by PWM inverter are still prominent.

The time-frequency diagram obtained from the short-time Fourier transform of the traction motor current in the line test is shown in Fig. 6. Several significant frequency bands that vary with speed in the fig. 6 are consistent with the simulation results in Fig. 5(b). The results of the line test were obtained from the power filtering effect of the DC-link in the traction drive system, so prominent frequency bands f - 2f and f + 2f, can be found, which are formed by the modulation of the f and 2f, components.

Fig. 7 shows the time frequency diagram of the torque of traction motor. In the spectrogram of Fig. 7(b), there is a remarkable harmonic component with the frequency of 100Hz, which is caused by DC link voltage fluctuation. In addition, there is also a harmonic component with frequency of 6f that varies with speed, which is caused by PWM inverter, as is mentioned previously. After f is close to 100Hz, the vibration frequency 100Hz is more profound.
Compared with Fig. 7(b), it shows that only the harmonic component with frequency $6f_1$ is particularly prominent, indicating that after LC filtering in the DC link, the impact of 100Hz on the motor torque is significantly reduced. The harmonic components with frequency $6f_1$ and 100Hz would cause vibrations on the vehicle’s mechanical system through electromechanical coupling.

![Fig. 7 Time frequency diagram of the torque of traction motor, (a) DC link with LC filter, (b) DC link without LC filter.](image)

In Fig. 8, time frequency diagram of DC link voltage shows that harmonic component with frequency $6f_1$ exists in DC link voltage, which is caused by PWM inverter. This indicates that the harmonic torque frequency formed by the stator harmonic current and magnetic flux of the motor would affect the DC voltage through the inverter. In addition, two times the power frequency, i.e., 100Hz, exists in the DC link voltage, regardless of whether the LC filter is present or not. Compared with Fig. 8(b), Fig. 8(a) shows that the amplitude of 100Hz is significantly smaller, indicating that the LC filter has a significant filtering effect at 100Hz.

![Fig. 8 Time frequency diagram of DC link voltage of traction system, (a) DC link with LC filter, (b) DC link without LC filter.](image)

The frequency domain diagram of DC link current during a constant speed operation period at $\omega=367\text{rad/s}$ is shown in Fig. 9. Regardless of whether there is an LC filter or not, there is a harmonic frequency of 6 times the motor stator current fundamental frequency in the frequency domain of the DC link current, and the amplitude is basically the same. When the DC link without LC filter, 100Hz is prominent in the spectrum of the DC link current, indicating that the LC filter can effectively filter out the pulsation of 100Hz.

![Fig. 9 Frequency domain diagram of DC link current of traction system.](image)

**IV. CONCLUSIONS**

In the AC-DC-AC traction transmission system, the DC voltage output by the single-phase rectifier has a pulsating voltage with a frequency twice that of the grid side voltage. When the stator current frequency of the traction motor is around 2-times the grid frequency, this pulsating voltage would cause current distortion and threaten the safe and stable operation of the train. In this paper, the simulation model of the traction system for high-speed train was established. The results reproduced the phenomenon of motor stator current distortion caused by pulsating voltage in the DC link, and found that harmonics at twice the grid frequency had a significant impact on motor torque without LC filter in DC link. The fault detection design of high-speed train traction drive system would consider the case of 100Hz filtering failure. In addition, the harmonic torque frequency formed by the stator harmonic current and magnetic flux of the motor would significantly affect the DC link through the inverter.

REFERENCES
