Stability Analysis of Dynamic Competition-Cooperation Model with Economic Factors for Transportation Systems

Shih-Ching Lo and Ching-Fen Lin

***Abstract*—Dynamic competition-cooperation models are employed to scrutinize the interplay among products, markets, or services, either in terms of competition or cooperation. Additionally, they serve to predict the behavior of the entities under examination, such as passenger volumes within transportation systems. Utilizing steady-state and asymptotic analyses, researchers delve into the long-term dynamics of these entities. While steady-state analysis delves into equilibrium conditions, asymptotic analysis scrutinizes system behavior as time extends toward infinity. However, the influence of socio-economic factors, which are external, cannot be disregarded in dynamic competition-cooperation models concerning passenger volumes in transportation systems. These factors typically encompass population dynamics, gross domestic product (GDP), oil prices, unemployment rates, consumer price indices, among others. The primary aim of this study is to assess the stability and asymptotic behavior of dynamic competition-cooperation models in light of economic factors. The investigation reveals that the solutions derived from these models do not exhibit periodic behavior. Rather, the system tends to converge or diverge. Notably, when convergence transpires, it does not converge towards a fixed point but towards a function of the economic factors. This underscores the profound influence of prevailing economic conditions on the behavior of transportation systems.**

***Index Terms*—**dynamic competition-cooperation model, steady state, stability analysis, economic factors.

I. Introduction

Various modes of transportation, including high-speed rail, highway coaches, airlines, and conventional rail, often cater to similar travel routes. It is imperative to grasp their individual standings within the transportation landscape to facilitate efficient coordination towards optimizing system performance. Thus, employing dynamic cooperative-competitive models becomes essential for forecasting and analyzing passenger volumes within interlinked transportation systems, services, or tourism sectors in Taiwan. Examples include the tourist influx to the three southern offshore islands [1], domestic air travel and high-speed rail [2, 3], vehicle ownership statistics encompassing cars and motorcycles [4], and the turnover rate of service areas along Freeway No. 3 [5].

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The dynamic cooperative-competitive model finds its roots in the population growth model, with foundational contributions from Fourt and Woodlock [6], Mansfield [7], and Bass [8]. Initially, these scholars embarked on a comprehensive theoretical journey, focusing on the univariate analysis of the life cycle of individual products. Subsequently, the Bass model underwent expansions to accommodate various complexities. Bemmaor and Lee [9], along with Karmeshu and Goswami [10], tackled the challenge posed by heterogeneous agents within the market. Additionally, Bass et al. [11] introduced a versatile intervention function, enabling the incorporation of marketing-mix variables, managerial controls, external contingencies, incentives, policy measures, and more into the modeling framework.

While the Bass model [8, 12] remains widely utilized for analyzing competition between two species or products, it inherently lacks consideration for market competition. Fisher and Pry [13] proposed a simple substitution model, predicated on the notion that newer technologies would supplant established ones, yet it too overlooks competition dynamics. Norton and Bass [14], recognizing this gap, integrated the Bass model with the Fisher and Pry model, demonstrating the substitution effect and enabling the forecasting of new technology diffusion.

In addition to these models, the mathematical Lotka-Volterra (LV) model has seen extensive use in exploring the diffusion phenomenon and reciprocal competition between two species [15-19]. Recognized as a two-species biological model, the LV model is commonly referred to as the predator-prey model. Moreover, its application extends to scenarios involving correlated populations [20-22].

Furthermore, equations of the system are not restricted to two. A technology systematic model considering three interacting technologies has been introduced by Jackson [23] and Meadows [24]. Three case sets of general application of the system dynamics model focusing on the transition from asymptotic to cyclic behavior of the technology system have been considered. Pretorius et al. [25] proposed a three-technology system and simulated three competing technologies that interact. Chang et al. [26] analyzed the competition among three companies with adjusted data from a Korean government-affiliated institute, the Korea Information Society Development Institute (KISDI). The performances of the LV and the extended Bass models are discussed in the study. The results show that the goodness of fit of the three-species LV model in the case of competition among three companies is better than that of the extended Bass model.

Primarily, forecasting the diffusion of new technologies and predicting passenger volumes within transportation systems predominantly rely on numerical methods. While analytical discussions on global stability, solution existence, and asymptotic analysis of dynamic competition-cooperation models are prevalent [27, 28], models incorporating economic factors have received scant attention. Typically, social-economic factors such as population dynamics, gross domestic product (GDP), oil prices, unemployment rates, and consumer price indices are crucial considerations. Given the inherent relationship between transportation systems and economic factors, it becomes imperative to analyze the stability and asymptotic behavior of dynamic competition-cooperation models incorporating economic variables. This study thus aims to fill this gap by delving into the stability and asymptotic analysis of such models in the context of transportation systems.

II. Dynamic Competition-Cooperation Model

In this section, we introduce the modeling of dynamic competition-cooperation firstly. Then, the applications of the dynamic competition-cooperation models in transportation related research are presented.

A. Modeling Dynamic Competition-Cooperation

Dynamic competition-cooperation model is extended from the population model. The simplest population model considered the growth rate of population is proportional to the population. If $N(t)$ is the function of population and t is time. Let α be the proportional constant. Then, we have $dN(t)/dt = \alpha N(t)$. The model can be solved analytically if the initial condition is given. The solution is $N(t) = N_0 e^{\alpha(t-t_0)}$, where t_0 is the initial time and N_0 is the initial population. According to empirical studies, the model can only to apply to short-term forecasting because when t increases N increases exponentially. The result is unreasonable and unrealistic. In real world, the growth of population is also proportional to the capacity of the system, that is, $dN(t)/dt \propto (M - N(t))$, where M is a constant, which presents the capacity of the system. Combine the capacity restriction to the previous model, the analytical model is obtained by separation of variables. It is in the logistic form, which is given by

$$N(t) = M \Big/ \left[1 + \left(\frac{M - N_0}{N_0} \right) e^{-\alpha M t} \right]. \tag{1}$$

Under the close system assumption, Eq. (1) can be represented by a general form, which is $dN(t)/dt = F(N, t)$. $F(N, t)$ is the growth function. If two species of population are considered, the model is consisted by two equations with interaction terms, such as $dN_1(t)/dt = F_1(N_1, N_2, t)$ and $dN_2(t)/dt = F_2(N_1, N_2, t)$, where N_1 and N_2 are two species, F_1 and F_2 are growth functions of N_1 and N_2, respectively. The most famous model in this form is the Lotka-Volterra (LV) model [29-31], the model is given by

$$dN_1(t)/dt = a_1 N_1(t) - b_1 N_1^2(t) - c_1 N_1(t) N_2(t) \tag{2}$$

$$dN_2(t)/dt = a_2 N_2(t) - b_2 N_2^2(t) - c_2 N_1(t) N_2(t) \tag{3}$$

where a_1, a_2, b_1, b_2 c_1 and c_2 are coefficients which are calibrated by empirical data. For biology studies, a_1, and a_2, are the growth rates of species 1 and 2. b_1 and b_2 are related to carrying capacities of the populations. For competitive market studies, a_i is the logistic parameters for product i when it is living alone. b_i is the limitation parameter of the niche capacity related to the niche size. c_i is with the other product i. The multi-mode form of LV model is illustrated in Table 1 for the case of two species. Although there are five types of modes, note that there are two possible predator-prey interactions (depending on which species is the predator or prey) in the predator-prey mode and two possible states (depending on which species is the stronger of the two) in the amensalism model.

Table 1 The relationship according to the signs of c_1 and c_2

coefficient c_1	c_2	Type	Explanation
+	+	Pure competition	Both species suffer from each other’s existence.
–	+	Predator-prey	One of them serves as direct food (N_2) for the other (N_1).
–	–	Mutualism	It is the case of symbiosis or a win-win situation.
– /+	0	Amensalism	One (N_1) suffers from the existence of the other (N_2), who is impervious to what is happening.
0	0	Neutralism	There is no interaction.

Since there are still three social-economic variables must be compared, the model is generalized as follows.

$$dN_1(t)/dt = a_1 N_1(t) - b_1 N_1^2(t) - c_1 N_1(t) N_2(t) + d_1 E(t), \tag{4}$$

$$dN_2(t)/dt = a_2 N_2(t) - b_2 N_2^2(t) - c_2 N_1(t) N_2(t) + d_2 E(t), \tag{5}$$

where $E(t)$ denotes the variables of economic factor. d_i is the coefficient of that. If $d_i > 0$, the economic factor influences the change of N_i positively, and vice versa. To ensure the systematic equations can be solved, the function form of $E(t)$ is suggested to choose as simple as possible.

B. Applications for Transportation Related Studies

In this section, we review three applications of the dynamic competition-cooperation model in transportation and tourism studies to highlight the model's advantages and underscore the necessity of considering economic factors.

The first application involves analyzing the cooperation and competition dynamics among the three central service areas of Freeway No.3 and forecasting their turnover using the competition-cooperation model [5]. The study aimed to elucidate the relationships among the Xihu service area, Qingshui service area, and Nantou service area, and predict their turnover utilizing a dynamic competition-cooperation model. The model, initially developed on a basic framework without economic factors, was calibrated using time series data on turnover spanning from 2008 to 2013. Furthermore, data from 2014 were employed to validate the model's performance.

The findings can be summarized as follows: the Xihu service area and Nantou service area were identified as having subpar reputations, while the Qingshui and Xihu service areas were noted to suffer from inadequate infrastructure and capacity. The analysis revealed a competitive relationship between the Xihu service area and

Qingshui service area, potentially diverting tourists from the Nantou service area. Only the Nantou service area appeared capable of meeting travelers' needs in terms of infrastructure and capacity. Despite the absence of economic factors, the dynamic competition-cooperation model exhibited remarkable accuracy in predicting turnover, with a mean absolute percentage error (MAPE) of less than 10%, signifying its precise forecasting capabilities.

In the second example, the dynamic competition-cooperation model was utilized to scrutinize passenger volumes for Taiwan High-Speed Rail (THSR) and domestic airlines within Taiwan [2], spanning from 2007 to 2010. The study sought to assess the forecasting accuracy of the dynamic competition-cooperation model with and without the inclusion of economic factors.

The findings unveiled that incorporating economic variables, particularly oil prices, resulted in a reduced MAPE compared to the model devoid of economic considerations. This suggests that integrating economic factors led to more precise forecasts of passenger volumes for THSR and domestic airlines. The forecasts rooted in economic factors exhibited closer alignment with actual data, indicating enhanced predictive capabilities. Surprisingly, the study uncovered a symbiotic relationship in passenger volumes between THSR and domestic airlines, contrary to conventional expectations. This unexpected outcome was ascribed to the incorporation of economic factors, notably oil prices. The study discerned that higher oil prices spurred an increase in passenger volumes for both THSR and airlines. Elevated oil prices not only inflate the operational costs for airlines but also influence overall transportation demand.

The surge in oil prices tends to influence transportation preferences, with individuals gravitating towards more fuel-efficient modes such as public transportation like THSR. Hence, despite the potential dampening effect on airline demand, higher oil prices propel an uptick in passenger volumes for both THSR and airlines. This observation underscores the intricate interplay between economic factors, mode choice, and transportation dynamics.

Ultimately, the study underscores the imperative of integrating economic variables into dynamic competition-cooperation models for transportation systems. Economic factors, such as oil prices, wield substantial influence over passenger demand and mode selection, thereby shaping the landscape of competition and cooperation among different transportation modes.

Table 2 MAPE of the forecasting results of THSR and domestic airlines.

Model	Mode	MAPE (%)
without oil price	airline	58.91
	THSR	16.17
with oil price	airline	39.84
	THSR	3.47

In the third example, the dynamic competition-cooperation model was deployed to examine tourist volumes on three southern offshore islands in Taiwan: Ludao (LD), Lanyu (LY), and Little Liuqiu (LLQ) [1], spanning from 2008 to 2014. Table 3 provides a comparative analysis of the forecasting outcomes for the three islands with and without the integration of economic factors, specifically the unemployment rate.

The study discerned that the model lacking economic considerations yielded inaccurate forecasts. In stark contrast, the inclusion of the unemployment rate significantly reduced the MAPE compared to the model devoid of this economic factor.

According to the model's analysis, Little Liuqiu and Lanyu exhibited a neutralistic relationship, while both Ludao-Lanyu and Ludao-Little Liuqiu were characterized by a predator-prey dynamic, with Ludao assumed the role of the predator owing to its superior resources, amenities, and transportation infrastructure.

However, given the geographical proximity of Lanyu and Ludao, both falling under the governance of Taitung County, there arises an opportunity to transform their predator-prey relationship into a cooperative one. Such a transition could prove advantageous in bolstering offshore island tourism in Taitung County.

Table 3 MAPE of the forecasting results of the visitor volume of the three southern offshore islands.

Model	island	MAPE (%)
without unemployment rate	LLQ	110.74
	LD	59.89
	LY	147.88
with unemployment rate	LLQ	58.75
	LD	21.38
	LY	38.99

III. Stability Analysis of the Model without Economic Factors

Stationary points denote stable states where the system's behavior remains consistent and unaffected by time. Conversely, unstable cases arise when the system's equations become unsolvable, indicating inherent instability within the system. In the realm of dynamic competition-cooperation models for transportation systems, only stable and periodic scenarios prove beneficial for forecasting and analysis.

Stability implies that the system tends to converge towards a stationary point, ensuring a stable equilibrium within the transportation market. Periodicity, on the other hand, refers to the oscillations observed in the variables described by the system equations. These oscillations manifest when the system, although not in a steady state, exhibits regular patterns over time.

Whether the systematic equations converge to a stationary point or exhibit periodic behavior, it provides valuable insights for planners and operators of transportation systems. Understanding the system's status enables stakeholders to make informed decisions and implement strategies to enhance service quality and efficiency.

In dynamic systems, the behavior of the system near a stationary point can be determined by the eigenvalues of the Jacobian matrix at the stationary point. The Jacobian matrix of Eqs (2) and (3) is

$$\mathbf{J}=\begin{bmatrix} a_1-2b_1N_1-c_1N_2 & -c_1N_1 \\ -c_2N_2 & a_2-2b_2N_2-c_2N_1 \end{bmatrix}. \tag{6}$$

The sign of the real parts of the eigenvalues of the Jacobian matrix will determine the behaviors of surrounding points. The stationary points can be characterized as follow:

1. If both the real parts of eigenvalues are negative, it is stable and

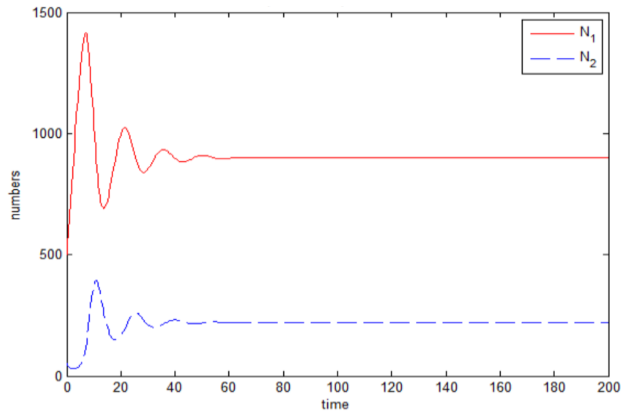
$\left((a_2b_1 - a_1c_2)/(b_1b_2 - c_1c_2),\quad (a_1b_{21} - a_2c_1)/(b_1b_2 - c_1c_2)\right)$ and $(0,\quad 0)$ are the stationary points.

2. If one of the real parts of eigenvalues is positive, it is unstable.
3. If both the real parts of eigenvalues are zero, it is a center.

Three sets of coefficients are given to show the three cases numerically. The systematic model is coupled by Eqs (2) and (3) and the coefficients and eigenvalues are given in Table 4. The initial values and the value of the coefficients are chosen to show the trajectories clearly, which do not have physical meanings. λ_1 and λ_2 are eigenvalues of the Jacobian matrix. Figures 1 to 3 illustrate the three cases. For all cases, the initial N_1 is 500 and N_2 is 50. In the figures, "+" denotes the points of initial conditions and "o" denotes the stationary point. (900, 220) is the stationary point of the stable case, (900, 400) is the center of the periodic case. According to the c_1 and c_2, N_1 is the prey and N_2 is the predator in the system.

Table 4 The coefficients and eigenvalues of stable, unstable and periodic cases.

coefficient / cases	a_1	a_2	b_1	b_2	c_1	c_2	eigenvalue	
							λ_1	λ_2
stable	0.4	-0.9	0.0002	0.0	0.001	-0.001	-0.09	-0.09
unstable	0.4	-0.9	0.001	0.0	0.001	-0.001	-1.26	0.36
periodic	0.4	-0.9	0.0	0.0	0.001	-0.001	0.0	0.0



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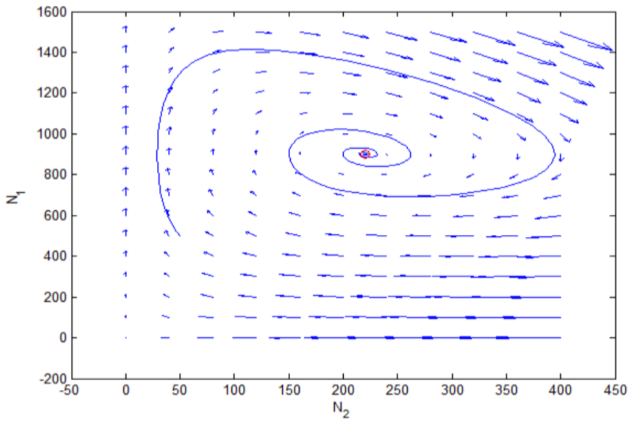

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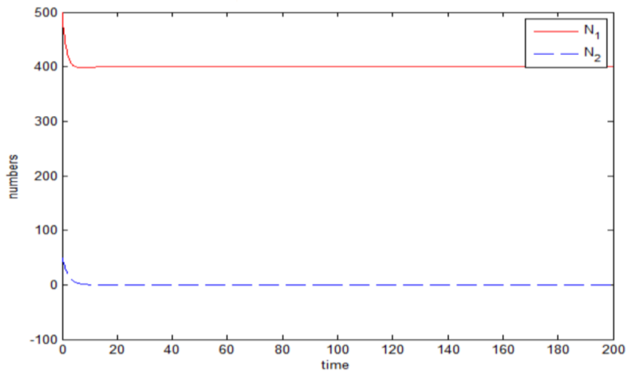
Figure 1 Stable case: (a) The numbers of N_1 and N_2.vary with time; (b) Phase plane and the vector field of N_1 and N_2.



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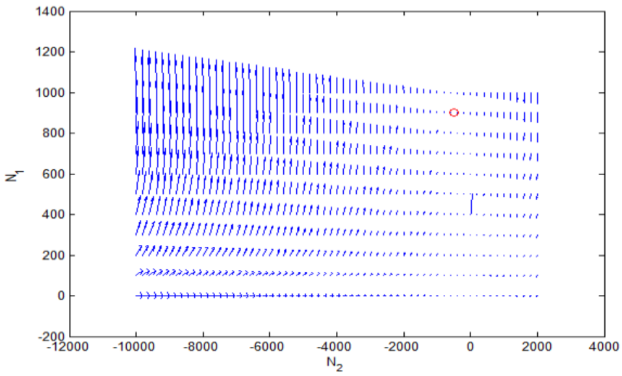

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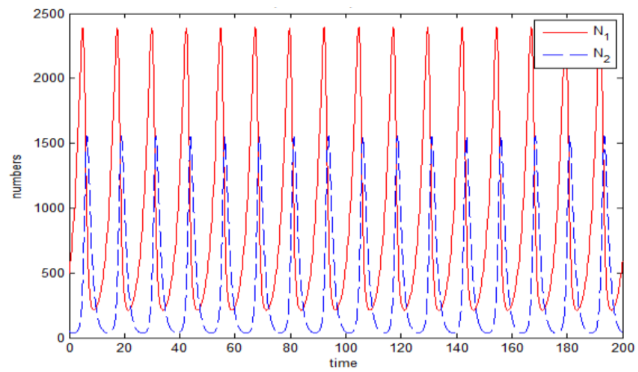
Figure 2 Unstable case: (a) The numbers of N_1 and N_2.vary with time; (b) Phase plane and the vector field of N_1 and N_2.



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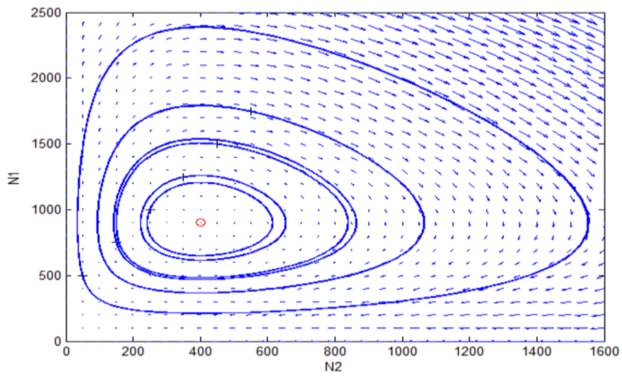


(b)

Figure 3 Periodic case: (a) The numbers of N_1 and N_2.vary with time; (b) Phase plane and the vector field of N_1 and N_2, which shows the periodic results of different initial conditions and the stationary point.

From Fig. 1 (a), the numbers of N_1 and N_2 oscillate with time and become stable in the stable case. Figure 1 (b) shows the convergent trajectory to the stationary point and the vector field, which illustrates the direction and the intensity of the convergence. The divergent case is shown in Fig. 2. The system cannot converge to the theoretical stationary point. Figure 3 is the periodic case. Different initial conditions are given and each one of them derived the solution to an orbit around the center.

IV. Stability Analysis of the Model with Economic Factors

Passenger and cargo volumes are generally subject to a myriad of factors, including income levels, pricing dynamics, health crises such as diseases, political climates, economic conditions, weather patterns, holiday seasons, governmental regulations, foreign exchange rates, and advancements in transportation technology. For instance, events like the SARS outbreak from 2002 to 2003 and the financial crisis of 2007 to 2008 significantly impacted the global economy, leading to a decline in consumer wealth and subsequently affecting transportation demand during these periods.

Given the profound influence of socio-economic factors on transportation demand, it is imperative to incorporate economic considerations when applying the dynamic competition-cooperation model to forecast and analyze passenger volumes within transportation systems. Based on insights gleaned from previous studies, we propose a structured modeling procedure for this purpose.

1. Calibrate the coefficients of the model and examine the significance of each coefficient.
2. Omit the variable whose coefficient is insignificant and check the R-squared of each equation is acceptable or not.
3. If the R-squared after omitting the variable becomes much worse than it before omitting the variable, the variable should be remained in the systematic equations.
4. Repeat Step 1 to 3 until all coefficients are significant and the R-squared of all equations are acceptable.

Through the modeling procedure, we will have a concise and significant model. The economic factors might not always be significant and could be omitted.

GDP, population, oil prices, and unemployment rate are selected as independent variables in modeling transportation demand. GDP represents the total monetary value of all finished goods and services produced within a country during a specified period, commonly viewed as an indicator of a nation's standard of living. A rise in population often correlates with increased transportation demand. Oil prices significantly affect the operational expenses of transportation systems, with potential cost transfers to consumers. Additionally, the unemployment rate, reflecting economic conditions, inversely impacts transportation demand, serving as a gauge of the overall economic environment.

The basic dynamic competition-cooperation model has been extended to encompass phenomena such as diffusion [32-35], delays [36], and random environmental factors [37-39]. Despite these extensions, the core structure of these models retains the dependency on the studied species in their right-hand side equations.

Subsequently, numerical examples are provided to elucidate the stability and asymptotic behavior. To facilitate solvability of Equations (4) and (5), it is recommended that the economic factor function remain linear. The systematic equations are rewritten as

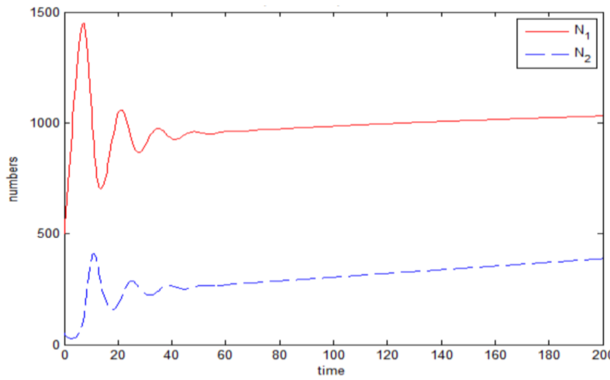
$$dN_1(t)/dt = a_1N_1(t)+b_1N_1^2(t)+c_1N_1(t)N_2(t)+d_1(mt+n), \quad (7)$$

$$dN_2(t)/dt = a_2N_2(t)+b_2N_2^2(t)+c_2N_1(t)N_2(t)+d_2(mt+n), \quad (8)$$

where m and n are coefficients. t denotes time. From Eqs (7) and (8), the Jacobian matrix of Eqs (7) and (8) is the same as Eq. (6) because the parts of the economic factor are independent to N_1 and N_2. Since the unstable case diverges, we discuss the stable and periodic case only. Table 4 gives the coefficients of the model with the economic factor effect. The value of a_1, b_1 and c_1 of case 1 in Table 5 are equal to the stable case in Table 3 and a_2, b_2 and c_2 of case 2 in Table 5 are equal to the periodic case in Table 3. Figure 4 illustrates the numbers of N_1 and N_2 varying with time for the two cases. Figures 5 and 6 are the trajectories of solution for the two cases at given time.

Table 5 The coefficients of cases for the model with the economic factor.

coefficient / cases	a_1	a_2	b_1	b_2	c_1	c_2
case 1 (stable case)	0.4	-0.9	0.0002	0.0	0.001	-0.001
case 2 (periodic case)	0.4	-0.9	0.0	0.0	0.001	-0.001
coefficient / cases	d_1	d_2	m	n		
case 1 (stable case)	0.2	-0.05	5	2		
case 2 (periodic case)	0.2	-0.05	5	2		



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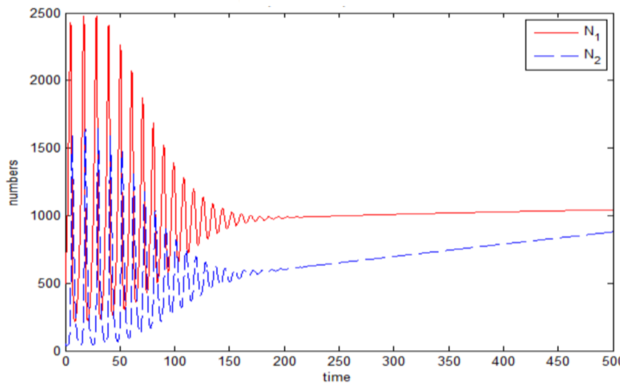

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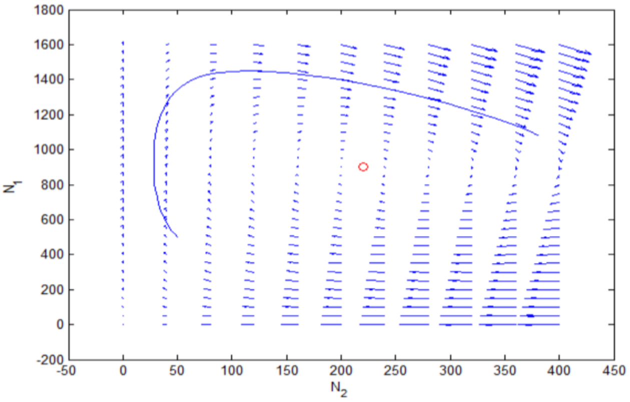
Figure 4 The numbers of N_1 and N_2.vary with time (a) case 1 (stable case); (b) case 2 (periodic case).



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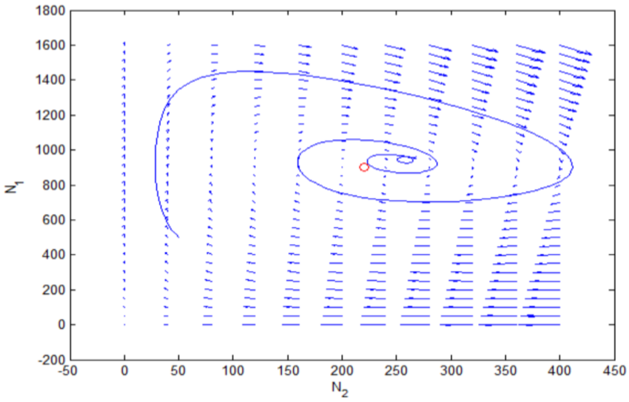

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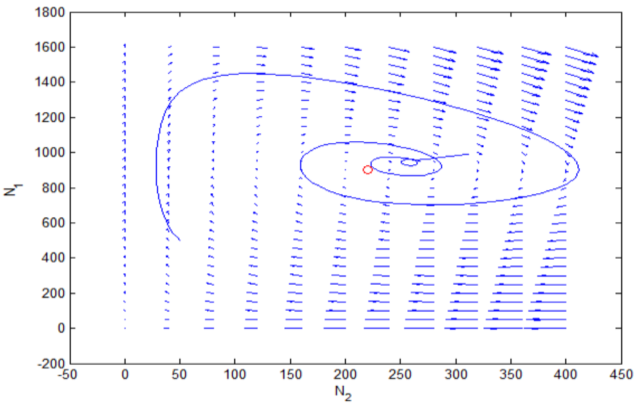

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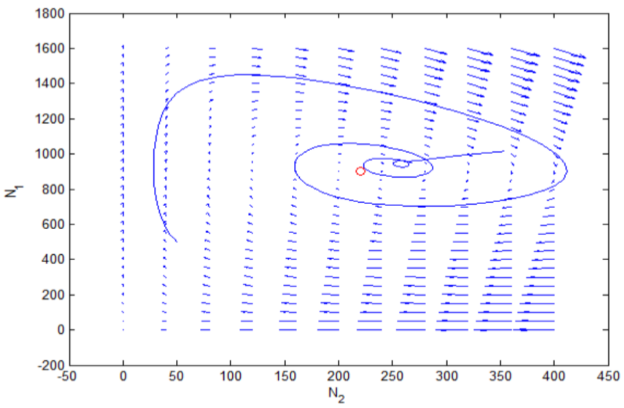
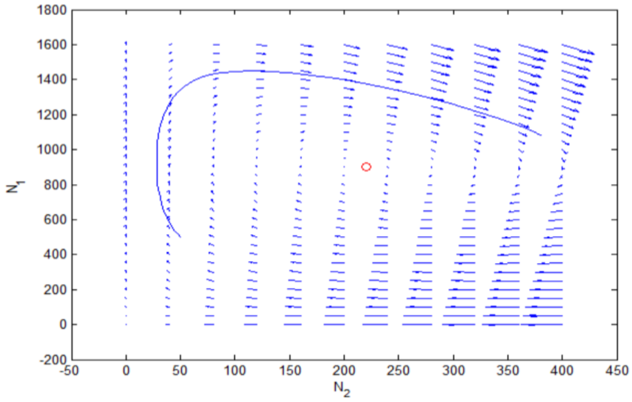


(d)

Figure 5 Trajectory of solution for case 1 (stable case) on phase plane and its vector field with different time (t): (a) t = 10; (b) t = 60; (c) t = 110; (d) t = 160.



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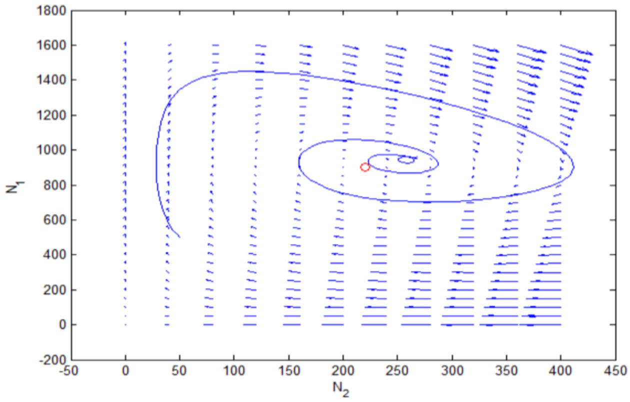

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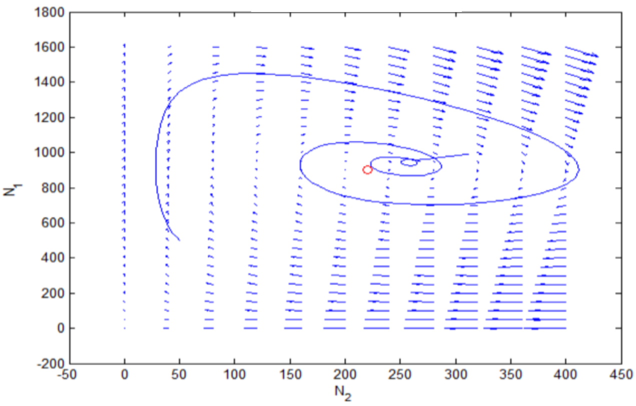

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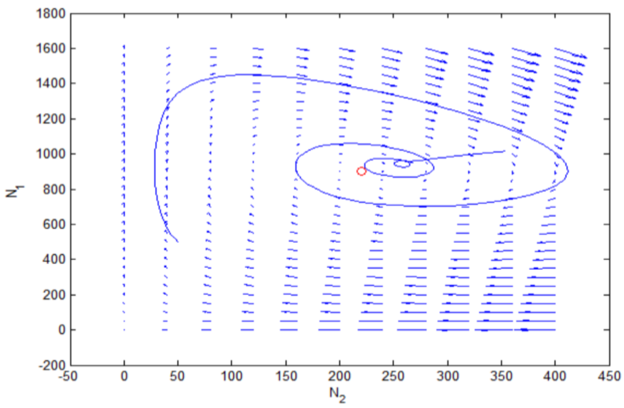


(d)

Figure 6 Trajectory of solution for case 2 (periodic case) on phase plane and its vector field with different time (t): (a) t = 10; (b) t = 60; (c) t = 110; (d) t = 160.

To make the discussion clearly, we denote the $a_i N_i(t)+b_i N_i^2(t)+c_i N_i(t) N_i(t)$ part of Eqs (7) and (8) as the competition-cooperation part (CC part) and $d_i(mt+n)$ part of Eqs (7) and (8) as the economic factor part (EF part). In stable case, the CC part dominates the behavior of the model and the solution will move close to the stationary point gradually. This phenomenon can be observed in Fig. 4 (a) when time is smaller than 50 and can be observed in Figs 5 (a) and (b). Unlike the case without economic factors, the solution of the stable case will not converge to the stationary point, which is discussed in Sec. 3. When the trajectory of solution is close to the stationary point means the CC part is close to zero, the model is dominated by the EF part. Figures 5 (c) and (d) present the behavior well. Thus, the stable case of model with economic factors will not converge to a stationary point but will converge to the function of the economic factor.

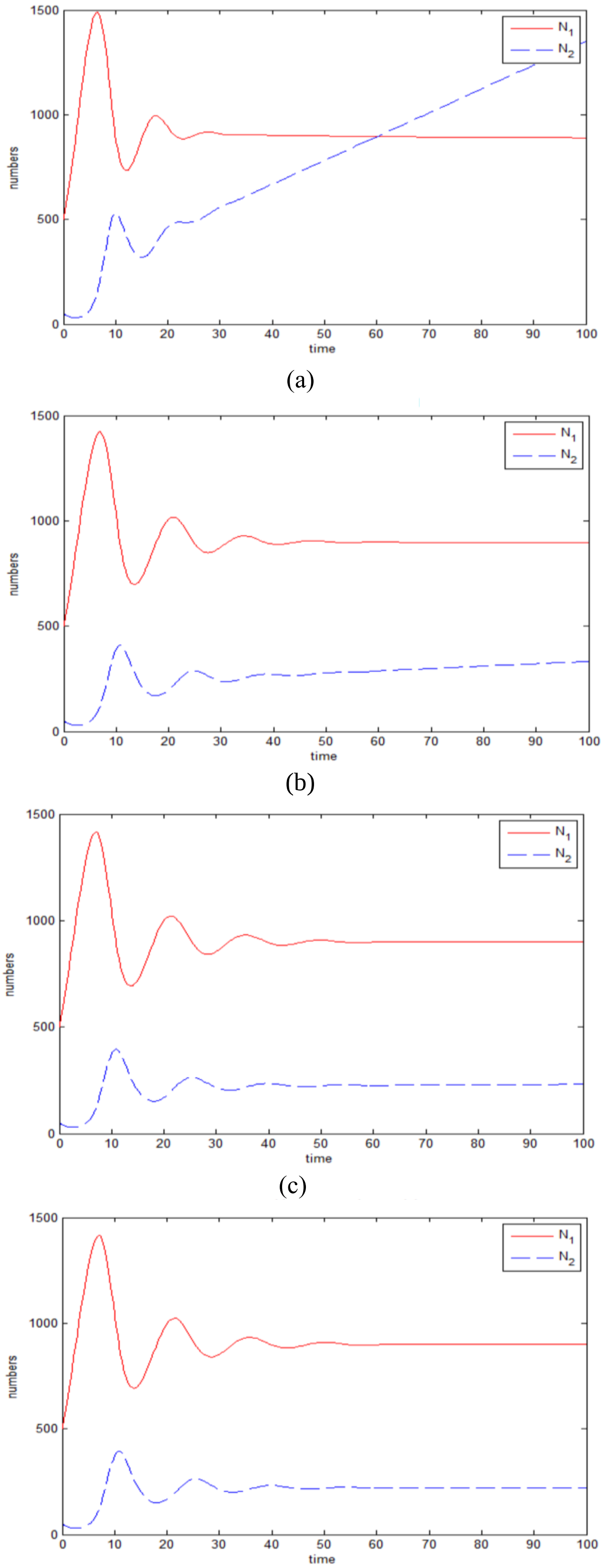
In the periodic case, we can find the solution still converges to the function of the economic factor in long-term trend from Figs 4(b) and 6. From Fig. 4(b), the solutions of N_1 and N_2 oscillate larger than the solutions of the stable case. The EF part of the periodic case takes a longer time to take over the system than that of the stable case. Therefore, there is no periodic case when an economic factor is considered in the dynamic competition-cooperation model. The economic factor plays a role like the perturbation to the periodic orbit and push the solution close to the center (stationary point).

Next, the magnitude of the EF part to the model is discussed in stable and periodic cases. Table 6 provides the coefficients of the economic factor, as well as the initial values of both the EF and CC parts of the model. By comparing the magnitudes of these parts, insights into their relative influence on the system dynamics can be gained. The other coefficients are the same as Table 5. Initial values are presented to compare the magnitude of both parts because the magnitude of the CC part decreases with time and the magnitude of the EF part increases with time. In addition, the initial point of the system is set to be (500, 50).

Table 6 The coefficients and the initial values for stable and periodic cases.

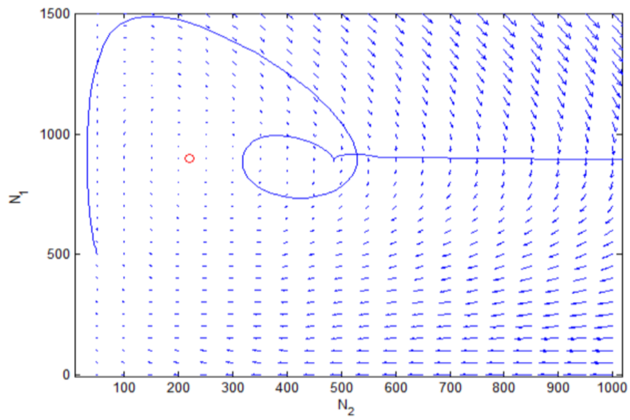
	Case 3 (stable case)					
	d_1	d_2	initial value of the EF part		initial value of the CC part	
			dN_1	dN_2	dN_1	dN_2
case 3-1	2	0.05	14	0.35	125	-20
case 3-2	0.2	0.005	1.4	0.035	125	-20
case 3-3	0.02	0.0005	0.14	0.0035	125	-20
case 3-4	0.002	0.00005	0.014	0.00035	125	-20
	Case 4 (periodic case)					
	d_1	d_2	initial value of the EF part		initial value of the CC part	
			dN_1	dN_2	dN_1	dN_2
case 4-1	2	0.05	14	0.35	175	-20
case 4-2	0.2	0.005	1.4	0.035	175	-20
case 4-3	0.02	0.0005	0.14	0.0035	175	-20
case 4-4	0.002	0.00005	0.014	0.00035	175	-20

Figures 7 and 8 depict the results of the stable case, while Figures 9 and 10 show the results of the periodic case. By examining these figures, it becomes apparent how the EF part influences the behavior of the model in both scenarios. In both the stable and periodic cases, if the magnitude of the EF part is larger, the solution tends to converge to the EF part faster. The economic factor enhances the convergence of the dynamic competition-cooperation model, improving the accuracy of forecasts.

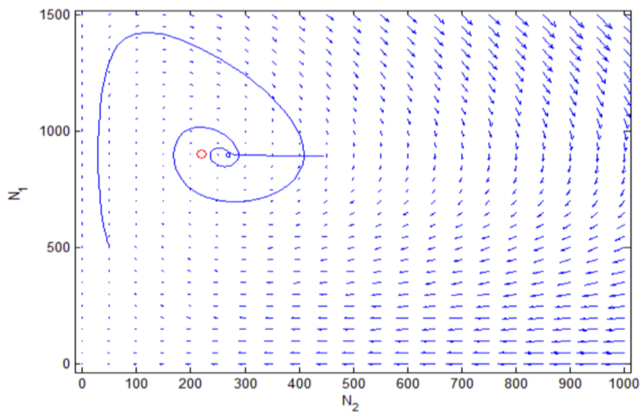


(d)

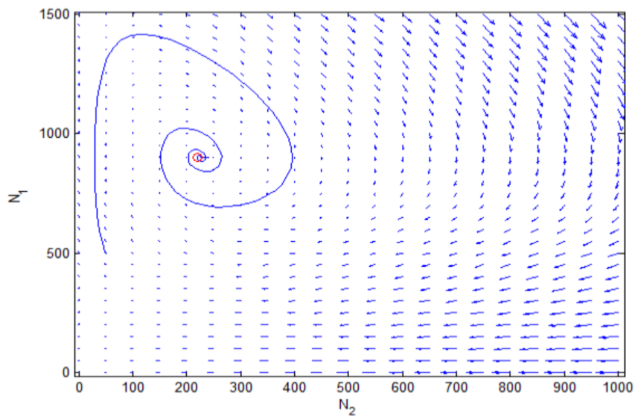
Figure 7 The numbers of N_1 and N_2.vary with time for case 3 with different d_i, i=1, 2: (a) case 3-1; (b) case 3-2; (c) case 3-3; (d) case 3-4.



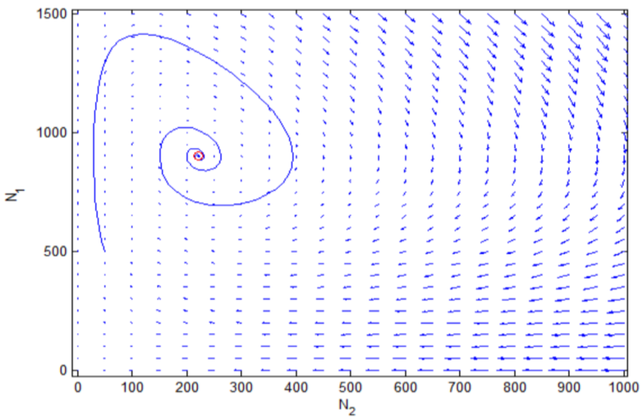
(a)



(b)

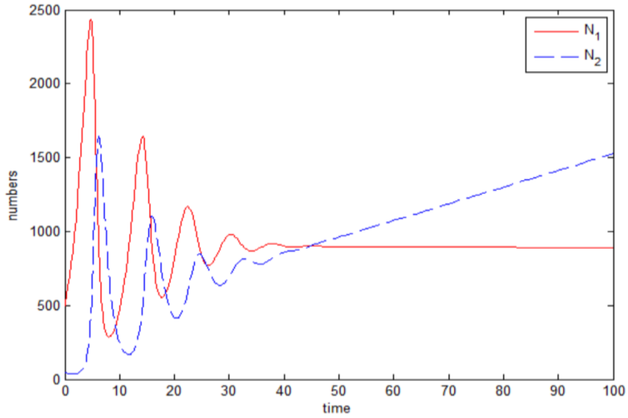


(c)

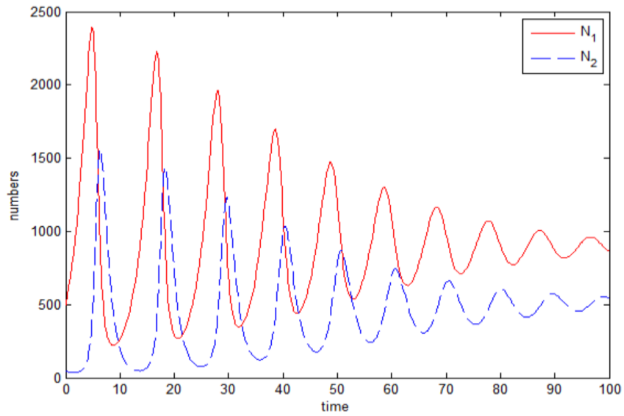


(d)

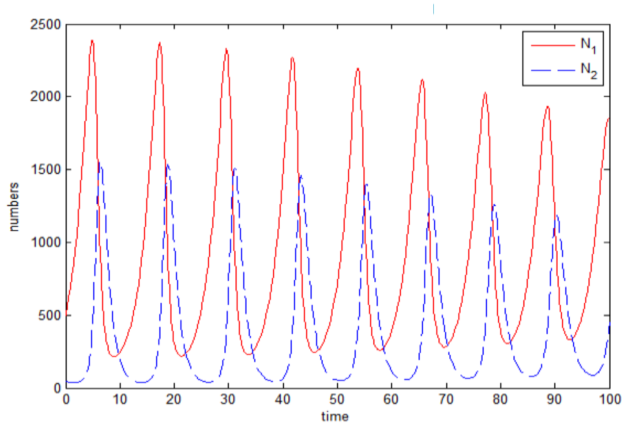
Figure 8 Trajectory of solution for case 3 on phase plane and its vector field with different d_i, i=1, 2: (a) case 3-1; (b) case 3-2; (c) case 3-3; (d) case 3-4.



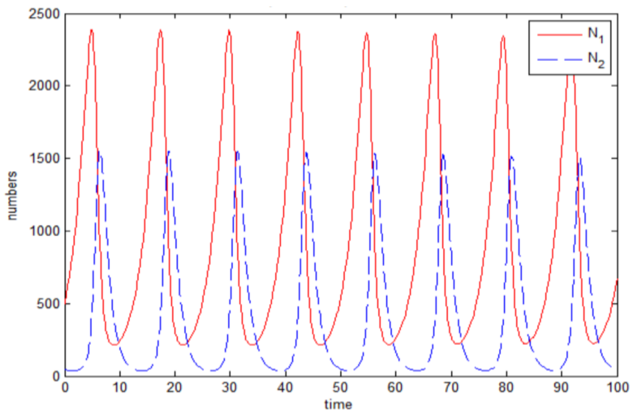
(a)



(b)



(c)



(d)

Figure 9 The numbers of N_1 and N_2.vary with time for case 4 with different d_i, i=1, 2: (a) case 4-1; (b) case 4-2; (c) case 4-3; (d) case 4-4.

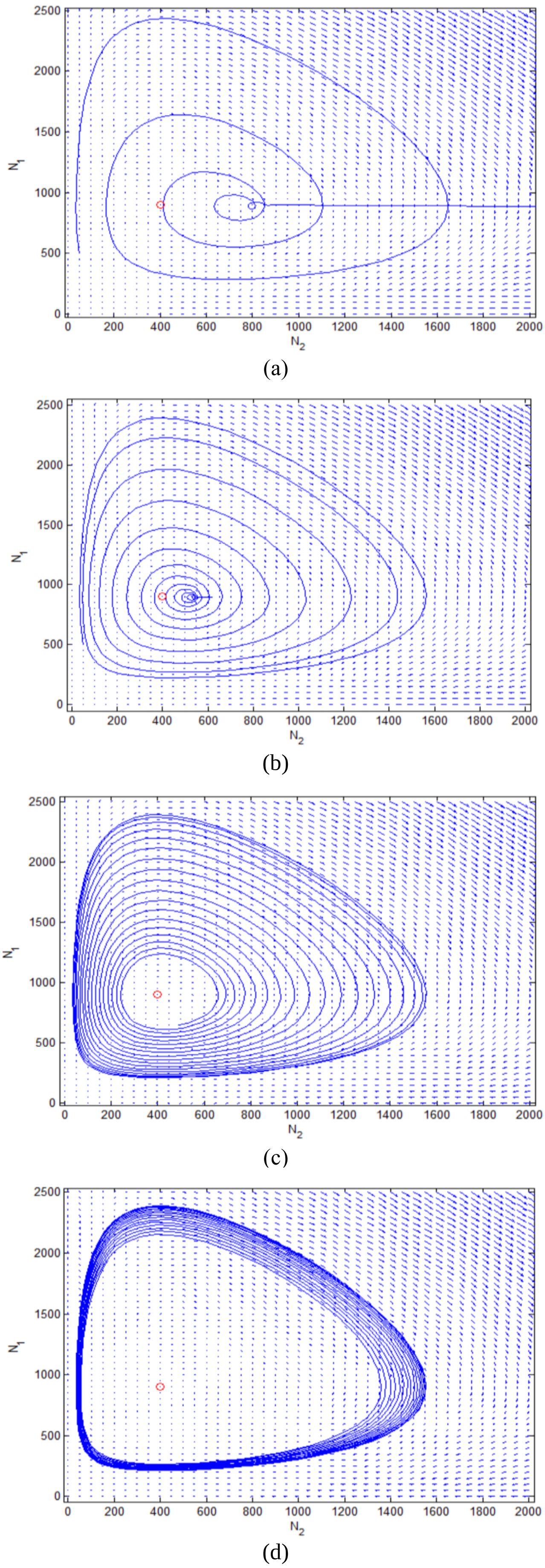


Figure 10 Trajectory of solution for case 4 on phase plane and its vector field with different d_i, i=1, 2: (a) case 4-1; (b) case 4-2; (c) case 4-3; (d) case 4-4.

However, if the results resemble those shown in Figures 7(a) and 9(a), where the solution primarily converges to the EF part, it suggests that developing a forecasting model based solely on the economic factor might be more appropriate. These observations underscore the importance of the economic factor in shaping the behavior of the dynamic competition-cooperation model. Depending on the relative magnitudes of the EF and CC parts, the model's convergence and accuracy may vary. Understanding these dynamics can inform decisions regarding the development of forecasting models and the incorporation of economic factors in transportation planning and management.

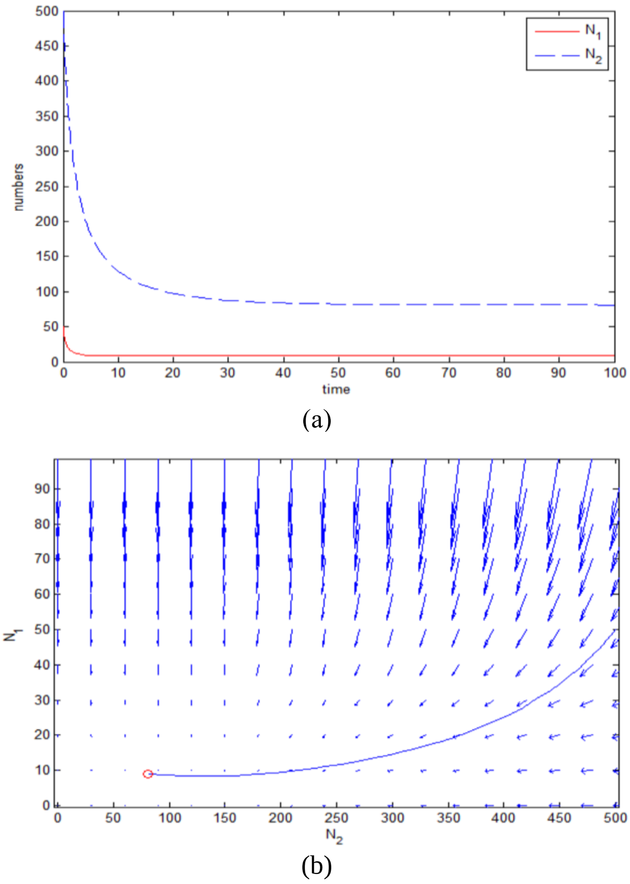
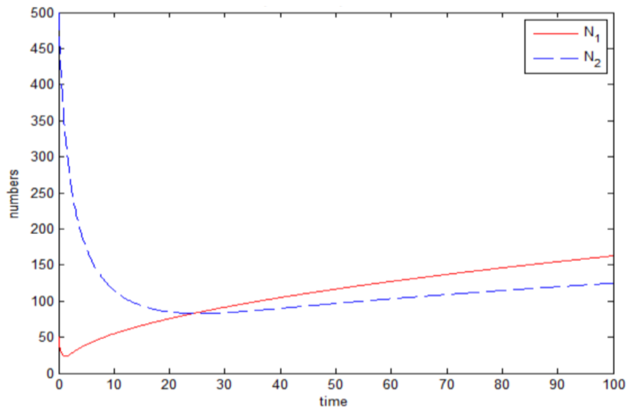


Figure 11 Competitive type model: (a) The numbers of N_1 and N_2 .vary with time; (b) Phase plane and the vector field of N_1 and N_2.

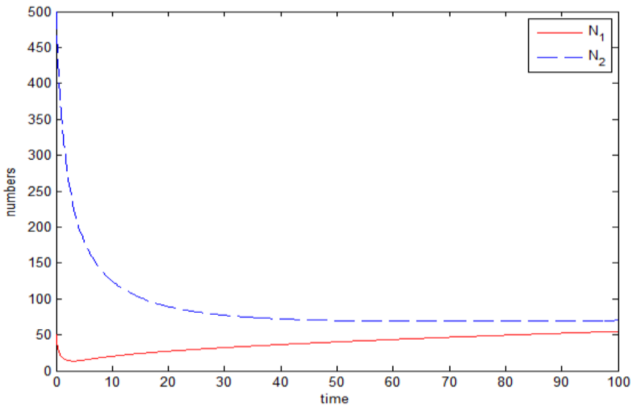
Table 7 The coefficients and eigenvalues of the competitive type model.

coefficient	a_1	a_2	b_1	b_2	c_1	c_2	eigenvalue	
							λ_1	λ_2
value	0.4	0.09	-0.04	-0.001	-0.0005	-0.001	-0.36	-0.08

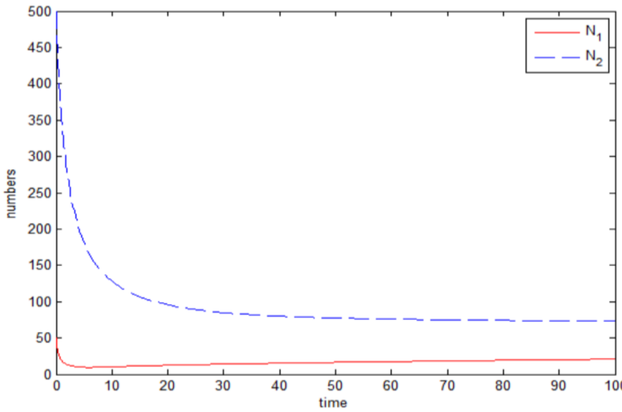
All discussions above are based on the predator-prey type model because c_1 is positive and c_2 is negative, which means N_1 is the predator and N_2 is the prey. A competitive type model is employed to discuss, and the coefficients are given in Table 7. According to the Jacobian analysis, the system is stable because the two eigenvalues of the Jacobian matrix are negative. The system converges to (8.99, 81.01) and the initial condition is (50, 500). Figure 11 illustrates the numbers of N_1 and N_2 and their phase plane. Figures 12 and 13 are the results with the economic factor. d_1 and d_2 are given the same as Table 6.



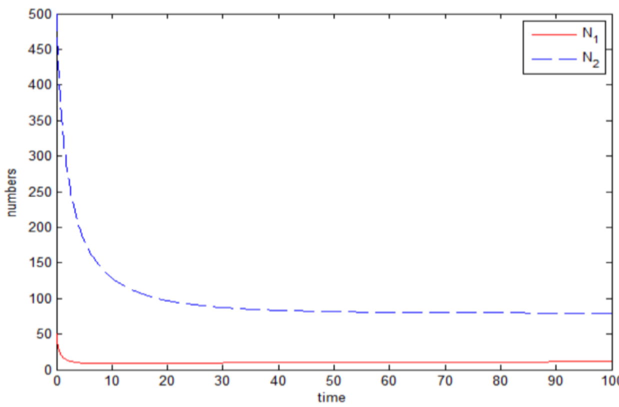
(a)



(b)

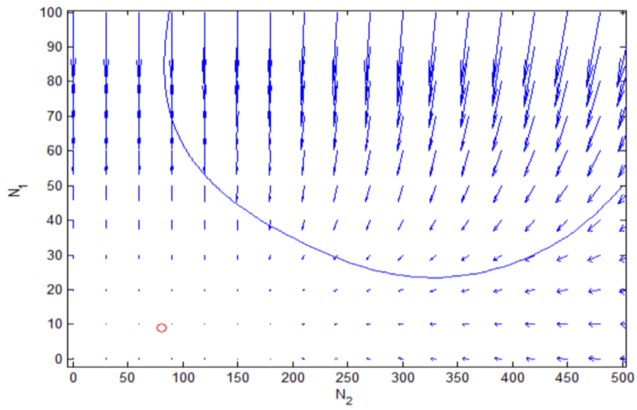


(c)

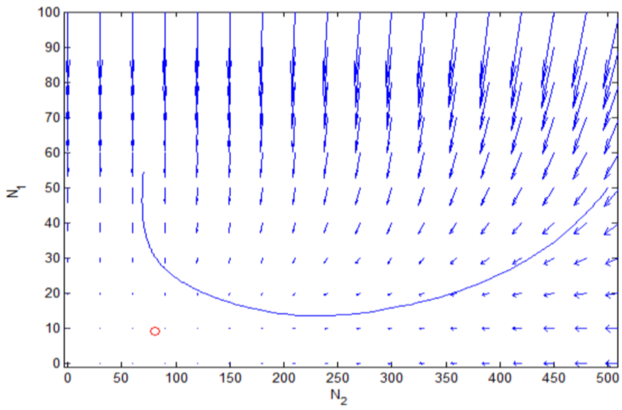


(d)

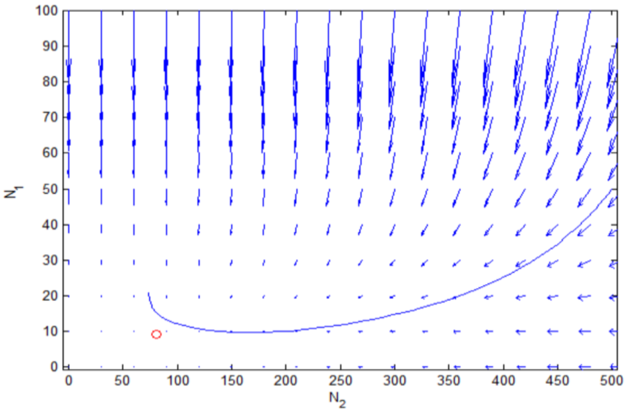
Figure 12 The numbers of N_1 and N_2 .vary with time for the competitive type model with different d_i, i=1, 2: (a) d_1 =2, d_2 =0.05; (b) d_1 =0.2, d_2 =0.005; (c) d_1 =0.02, d_2 =0.0005; (d) d_1 =0.002, d_2 =0.00005.



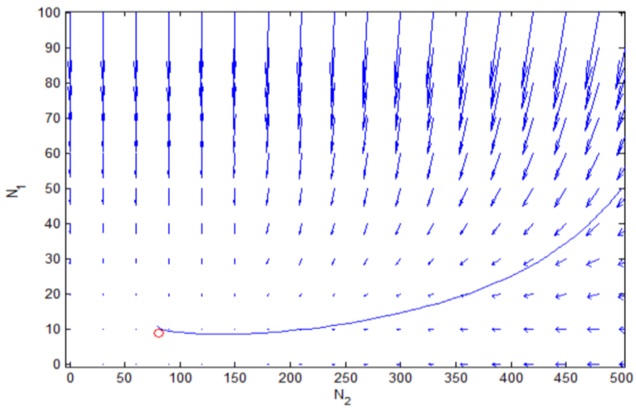
(a)



(b)



(c)



(d)

Figure 13 Trajectory of solution for the competitive type model with different d_i, i=1, 2: (a) d_1 =2, d_2 =0.05; (b) d_1 =0.2, d_2 =0.005; (c) d_1 =0.02, d_2 =0.0005; (d) d_1 =0.002, d_2 =0.00005.

V. Conclusion and Perspectives

Forecasting passenger or visitor volumes in transportation systems necessitates accounting for the influence of socio-economic factors. However, traditional Jacobian analysis falls short when economic factors are incorporated into the model. In this study, we complement the Jacobian analysis with numerical examples to explore the stability and asymptotic behavior of the competition-cooperation model enriched with economic factors. Our findings can be summarized as follows:

1. In unstable models, solutions diverge over time, leading to increasingly unpredictable dynamics that may fail to converge to a steady state.
2. In stable models, solutions converge towards the economic factor function in the long term. This underscores the significant role of economic factors in shaping the model's behavior over time, influencing the convergence pattern.
3. Similarly, in periodic models characterized by oscillations, solutions still converge to the economic factor function over the long term. This indicates the enduring influence of economic factors even in dynamic and fluctuating scenarios.

Both convergent and periodic models with economic factors exhibit stability, ensuring bounded trajectories over time. Initially, competition-cooperation dynamics dominate, leading trajectories towards a stationary or center point. As time progresses, economic factors gradually exert their influence, guiding trajectories towards the economic factor function. This highlights the pivotal role of economic factors in shaping model behavior over time. By integrating economic factors, the model yields more accurate forecasts, reflecting real-world dynamics and enhancing decision-making in transportation planning and management.

In real-world transportation systems, competition and cooperation often involve multiple entities. For instance, in Taiwan's transportation landscape, high-speed rail, freeway coaches, and rail services in the western corridor may compete or cooperate. Analyzing systems with three or more entities introduces complexity and dynamics surpassing two-species models. These systems may exhibit emergent behaviors, nonlinear interactions, and intricate feedback loops, necessitating advanced modeling techniques.

While this study delves into the stability and asymptotic behavior of two-species models, extending these analyses to multispecies systems presents a challenging yet promising research direction. Understanding the stability and convergence properties of such models is crucial for predicting system behavior and guiding decision-making in transportation planning and management. The stability and asymptotic analyses of systems involving three or more entities warrant further investigation.

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