An EPQ-Type Inventory Model Incorporating Deterioration, Defective Products, Quadratic Demand Rate, Linear Holding Cost Rate of Non-Defective Products, and Carbon Emission

Antonius Tri Wahyudi, Dharma Lesmono, Jonathan Hoseana

***Abstract*—The challenges faced by companies to operate in an environmentally-friendly manner have prompted researchers to incorporate into inventory models factors measuring the companies' environment-friendliness, such as the companies' carbon emission. In this paper, we propose a single-product EPQ-type inventory model that takes into account the product's deterioration, the presence of defective products, a quadratic time-dependent demand rate, a linear time-dependent holding cost of non-defective products, as well as carbon emission. We construct the company's expected total cost and expected total carbon emission as functions of the existing parameters. Through a numerical example, we demonstrate the computation of the inventory cycle length that minimises the expected total cost, and of the corresponding expected total carbon emission. Finally, we conduct a sensitivity analysis, which reveals that the expected total cost depends most sensitively on the demand rate's constant term and the per-unit production cost, whereas the expected total carbon emission depends most sensitively on the demand rate's constant term, the per-unit average electricity consumption for production, and the standard carbon emission for electricity generation.**

***Index Terms*—EPQ, deterioration, defective product, quadratic demand, linear holding cost, carbon emission**

I. Introduction

THE dawn of the 21st century, driven by rapid modernisation, has brought about major impacts on various aspects of life, including the industrial sector. Amidst the existing competitions, companies that could operate with minimal, manageable inventory tend to generate higher profits than those that are unable to manage their inventory effectively [32]. Indeed, effective inventory management leads to an appropriate pricing of the items to be produced, as it directly impacts the flow of production materials within the company.

In 1918, Taft [30] constructed a model to minimise the production of items while maximising profits, known as the EPQ (Economic Product Quantity) model. As time progresses, Taft's EPQ model has undergone significant development with the consideration of numerous factors, leading to the birth of more complex and realistic EPQ-type inventory models. Examples of such models, which take into account, among others, environmental factors, have been proposed by Daryanto and Wee [8], [9]. In 2018, the two authors proposed two EPQ-type models that take into account the respective companies' carbon emission, the second model being obtained from the first model by the incorporation of backorders [9]. In the subsequent year, the same authors proposed a more sophisticated EPQ-type model that incorporates simultaneously the product's deterioration, the presence of defective products, and carbon emission [8].

Despite the sophistication it already exhibits, the model in [8] possesses elements that are open for improvement. First, the model assumes the product's demand rate to be constant [8]. Such an assumption may not be realistic, as the demand rate is a parameter whose exact value is, in reality, largely uncertain. Accordingly, it is unsurprising that numerous past inventory models already utilised more complex forms of demand rate. For instance, Goswami and Chaudhuri [11], as well as Chakrabarti and Chaudhuri [3], developed inventory models in which the demand rate is assumed to depend linearly on time. Alternative forms of time-dependent demand rate utilised in recent studies include quadratic [1], [6], [7], [25], [26], [27], power-law [24], exponential [21], ramp-type [2], [19], and stochastic-type [4]. Of particular note is the applicability of the quadratic demand rate across various industrial sectors, including the production of seasonal goods, clothing materials, automobiles, and mobile phones [6].

In addition, the model in [8] assumes that the holding cost rates, for both the non-defective and defective products, are constant. In reality, as noted by Malumfashi, et al. [16], there are numerous instances in which holding cost rates vary significantly over time, such as due to rental costs that typically escalate over time. Accordingly, past inventory models have used more complex forms of holding cost rate, such as linear [6], [16].

In the present paper, as a development of the model proposed by Daryanto and Wee in 2019 [8], we construct a more realistic EPQ-type inventory model, which takes into account simultaneously carbon emission costs, item deterioration, and the presence of defective products, whereby the demand rate is assumed to depend quadratically on time, while the holding cost rate of non-defective products is assumed to depend linearly on time. At the same time, we assume that the holding cost rate of defective products remains constant. Our objective, as in the case of [8], is to minimise the company's expected total cost, and calculate

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the expected total carbon emission at the state where the expected total cost is minimum.

We organise our work as follows. In the upcoming section II, we first construct our model itself, detailing the underlying assumptions as well as the employed mathematical notation. Subsequently, in section III, we exemplify numerically the application of our model to compute the company's minimum expected total cost, as well as the inventory cycle length and the number of products to be produced that must be set in order to achieve this minimum. Additionally, we compute the company's expected total carbon emission at the minimum state. In section IV, we analyse the sensitivity of both the company's minimum expected total cost and the company's expected total carbon emission at the minimum state, with respect to each of the model's parameters. Finally, in section V, we state our conclusions and describe avenues for further investigation.

II. Model construction

Let us begin by constructing our inventory model itself. For this purpose, we assume the following.

1. The company produces only one type of product.
2. Both non-defective and defective products deteriorate at the same constant rate.
3. The demand rate of non-defective products depends quadratically on time, while defective products experience no demand.
4. The holding cost rate of non-defective products depends linearly on time, while the holding cost rate of defective products is constant.
5. The production rate is constant, and is greater than the demand rate, ensuring the absence of product shortages.
6. The company conducts a comprehensive quality inspection, and detected defective products are stored until the end of the production-consumption period.
7. The per-year disposal cost is a function of the per-ton-of-waste disposal cost, the per-unit-of-product average weight of solid waste produced, and the total production [9], [18], [29].
8. Carbon emission originate only from production and inventory holding.
9. The per-unit production carbon emission cost is the product of the per-unit average electricity consumption for production, the standard carbon emission from electricity generation, and the carbon tax rate [17], [34].
10. The per-unit inventory carbon emission cost is the product of the per-unit-of-product storage space occupancy, the per-warehouse-space-unit average electricity consumption, the standard carbon emission from electricity generation, and the carbon tax rate [13], [31].

The notation used for the time-dependent variables and parameters in our model is summarised in Tables I and II, respectively.

As in [8], we aim to model the dependence of the inventory level of non-defective products $I_p(t)$ and of defective products $I_{pd}(t)$ on $t \in [0, T]$ as depicted in Figure 1, where a period $[0, T]$ consists of a production-consumption period $[0, T_1]$ followed by a consumption period $[T_1, T_1 + T_2]$, so that $T = T_1 + T_2$. The inventory level of non-defective

TABLE I
Time-dependent variables involved in our model.

Variable	Description	Unit
$D(t)$	Demand rate of non-defective products, depending quadratically on time $t \in [0, \infty)$	unit/year
$I_p(t)$	Inventory level of non-defective products at time $t \in [0, T]$	unit
$I_{p_1}(t_1)$	Inventory level of non-defective products at time $t_1 \in [0, T_1]$	unit
$I_{p_2}(t_2)$	Inventory level of non-defective products at time $t_2 \in [0, T_2]$	unit
$I_{pd}(t)$	Inventory level of defective products at time $t \in [0, T_1]$	unit
$c_{h_1}(t)$	Per-unit holding cost rate of non-defective products, depending linearly on time $t \in [0, \infty)$	\$/(unit · year)

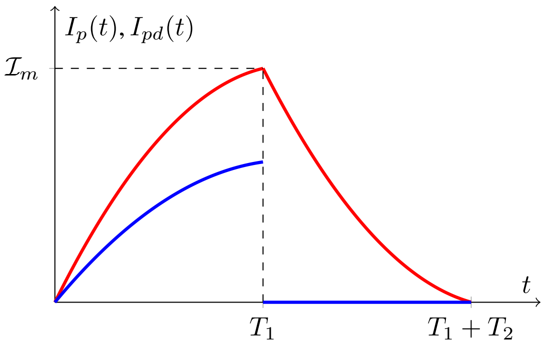

Fig. 1. Inventory level of non-defective products $I_p(t)$ (red) and of defective products $I_{pd}(t)$ (blue) at time $t \in [0, T]$, where $T = T_1 + T_2$.

products thus achieves its maximum $\mathcal{I}_m$ at $t = T_1$. At this point, the company discards all defective products, so that no defective products remain in storage during the consumption period.

Let us now begin the construction of our model. First, the total production can be formulated as

$$Q_0 = PT_1. \tag{1}$$

Next, let us express the demand rate of non-defective products, which is assumed to depend quadratically on time, at $t_1 \in [0, T_1]$ and $t_2 \in [0, T_2]$ as $D(t_1) = a + bt_1 + ct_1^2$ and $D(t_2) = a + bt_2 + ct_2^2$, respectively. Therefore, the inventory level of non-defective products $I_{p_1}(t_1)$ at time $t_1 \in [0, T_1]$ satisfies the ordinary differential equation

$$\frac{\mathrm{d}I_{p_1}(t_1)}{\mathrm{d}t_1} = (1 - u) P - D(t_1) - \theta I_{p_1}(t_1),$$

i.e.,

$$I'_{p_1}(t_1) + \theta I_{p_1}(t_1) = (1 - u) P - \left(a + bt_1 + ct_1^2\right), \tag{2}$$

while the inventory level of non-defective products $I_{p_2}(t_2)$ at time $t_2 \in [0, T_2]$ satisfies the ordinary differential equation

$$\frac{\mathrm{d}I_{p_2}(t_2)}{\mathrm{d}t_2} = -D(t_2) - \theta I_{p_2}(t_2),$$

i.e.,

$$I'_{p_2}(t_2) + \theta I_{p_2}(t_2) = -\left(a + bt_2 + ct_2^2\right). \tag{3}$$

TABLE II
PARAMETERS INVOLVED IN OUR MODEL.

Parameter	Description	Unit
P	Production rate	unit/year
a	Constant term of $D(t)$	unit/year
b	Coefficient of t in $D(t)$	unit/year^2
c	Coefficient of t^2 in $D(t)$	unit/year^3
u	Per-cycle proportion of defective products	-
$\mathrm{E}[u]$	Expected value of u	-
θ	Deterioration rate	year^{-1}
$\mathcal{I}_p$	Per-cycle inventory level of non-defective products	unit
$\mathcal{I}_m$	Maximum inventory level of non-defective products	unit
$\mathcal{I}_{pd}$	Per-cycle inventory level of defective products	unit
$\mathcal{I}_d$	Per-cycle inventory level of deteriorated products	unit
T	Cycle length	year
T_1	Length of the production-consumption period	year
T_2	Length of the consumption period	year
T^*	Optimal cycle length	year
T_1^*	Optimal length of the production-consumption period	year
T_2^*	Optimal length of the consumption period	year
Q	Total production of non-defective products	unit
Q_0	Total production	unit
c_s	Per-cycle set-up cost	\$/cycle
c_p	Per-unit production cost	\$/unit
c_{pe}	Per-unit production carbon emission cost	\$/unit
c_i	Per-cycle fixed quality inspection cost	\$/cycle
c_u	Unit inspection cost	\$/unit
A	Constant term of $c_{h_1}(t)$	\$/(unit · year)
B	Coefficient of t in $c_{h_1}(t)$	\$/(unit · year^2)
$\bar{c}_{h_1}$	Average value of $c_{h_1}(t)$ for $t \in [0, T]$	\$/(unit · year)
c_{h_2}	Per-unit holding cost rate of defective products	\$/(unit · year)
c_{he}	Per-unit inventory emission cost	\$/(unit · year)
c_d	Per-unit deterioration cost	\$/unit
c_w	Per-ton-of-waste disposal cost	\$/ton
e_p	Per-unit average electricity consumption for production	kWh/unit
e_w	Per-warehouse-space-unit average electricity consumption	kWh/(m^3 · year)
v	Per-unit-of-product storage space occupancy	m^3/unit
$\overline{w}$	Per-unit-of-product average weight of solid waste produced	ton/unit
C_{TX}	Carbon tax rate	\$/ton of CO_2
E_g	Standard carbon emission from electricity generation	ton of CO_2/kWh
C_1	Per-year set-up cost	\$/year
C_2	Per-year production cost	\$/year
C_3	Per-year inspection quality cost	\$/year
C_4	Per-year holding cost	\$/year
C_5	Per-year deterioration cost	\$/year
C_6	Per-year disposal cost	\$/year
ETC	Per-year expected total cost	\$/year
ETE	Per-year expected total carbon emission	ton of CO_2/year

Together with the boundary conditions

$$I_{p_1}(0) = 0, \quad I_{p_1}(t_1) = I_{p_2}(0) = \mathcal{I}_m, \quad \text{and} \quad I_{p_2}(t_2) = 0,$$

which are apparent from Figure 1, the equations (2) and (3) can be solved to obtain that

$$I_{p_1}(t_1) = \frac{(1-u)P - a}{\theta} - \frac{b(\theta t_1 - 1)}{\theta^2} - \frac{c\left(\theta^2 t_1^2 - 2\theta t_1 + 2\right)}{\theta^3} + \mathrm{e}^{-\theta t_1}\left[-\frac{(1-u)P - a}{\theta} - \frac{b}{\theta^2} + \frac{2c}{\theta^3}\right] \quad (4)$$

and

$$I_{p_2}(t_2) = -\frac{a}{\theta} - \frac{b(\theta t_2 - 1)}{\theta^2} - \frac{c\left(\theta^2 t_2^2 - 2\theta t_2 + 2\right)}{\theta^3} + \mathrm{e}^{\theta(T_2 - t_2)}\left[\frac{a}{\theta} + \frac{b(\theta T_2 - 1)}{\theta^2} + \frac{c\left(\theta^2 T_2^2 - 2\theta T_2 + 2\right)}{\theta^3}\right]. \quad (5)$$

Substituting (4) and (5) into the boundary condition

$$I_{p_1}(t_1) = I_{p_2}(0)$$

gives

$$\frac{(1-u)P - a}{\theta} - \frac{b(\theta T_1 - 1)}{\theta^2} - \frac{c\left(\theta^2 T_1^2 - 2\theta T_1 + 2\right)}{\theta^3} + \mathrm{e}^{-\theta T_1}\left[-\frac{(1-u)P - a}{\theta} - \frac{b}{\theta^2} + \frac{2c}{\theta^3}\right] = -\frac{a}{\theta} + \frac{b}{\theta^2} - \frac{2c}{\theta^3} + \mathrm{e}^{\theta T_2}\left[\frac{a}{\theta} + \frac{b(\theta T_2 - 1)}{\theta^2} + \frac{c\left(\theta^2 T_2^2 - 2\theta T_2 + 2\right)}{\theta^3}\right]. \quad (6)$$

Replacing $\mathrm{e}^{-\theta T_1}$ with its second-order Maclaurin approximant $1 - \theta T_1 + \theta^2 T_1^2/2$ and carrying out algebraic simplifications, one obtains from (6) the following quadratic equation in T_1:

$$[a\theta - P\theta(1-u) - b]\,T_1^2 + [2P(1-u) - 2a]\,T_1 + \kappa = 0,$$

where

$$\kappa = \frac{2a}{\theta} - \frac{2b}{\theta^2} + \frac{4c}{\theta^3} - 2\mathrm{e}^{\theta T_2}\left[\frac{a}{\theta} + \frac{b(\theta T_2 - 1)}{\theta^2} + \frac{c(\theta^2 T_2^2 - 2\theta T_2 + 2)}{\theta^3}\right]. \quad (7)$$

Letting

$$\Gamma = [P(1-u) - a]^2 - [a\theta - P\theta(1-u) - b]\,\kappa,$$

the quadratic formula then gives

$$T_1 = \frac{-P(1-u) + a \pm \sqrt{\Gamma}}{a\theta - P\theta(1-u) - b}, \quad (8)$$

and so

$$T = T_1 + T_2 = \frac{-P(1-u) + a \pm \sqrt{\Gamma}}{a\theta - P\theta(1-u) - b} + T_2. \quad (9)$$

Therefore, the per-cycle inventory level of non-defective products is given by

$$
\begin{aligned}
\mathcal{I}_p &= \frac{1}{T}\left[\int_0^{T_1} I_{p_1}(t_1)\,\mathrm{d}t_1 + \int_0^{T_2} I_{p_2}(t_2)\,\mathrm{d}t_2\right] \\
&= \frac{1}{T}\left[\frac{T_1}{\theta}\left((1-u)P-a\right) - \frac{b}{\theta^2}\left(\frac{\theta T_1^2}{2}-T_1\right)\right. \\
&\quad - \frac{c}{\theta^3}\left(\frac{\theta^2 T_1^3}{3}-\theta T_1^2+2T_1\right) \\
&\quad + \left(\frac{1-\mathrm{e}^{-\theta T_1}}{\theta}\right)\left(\frac{a-P(1-u)}{\theta}-\frac{b}{\theta^2}+\frac{2c}{\theta^3}\right) \\
&\quad - \frac{aT_2}{\theta} - \frac{b}{\theta^2}\left(\frac{\theta T_2^2}{2}-T_2\right) - \frac{c}{\theta^3}\left(\frac{\theta^2 T_2^3}{3}-\theta T_2^2+2T_2\right) \\
&\quad + \left(\frac{\mathrm{e}^{\theta T_2}-1}{\theta}\right)\left(\frac{a}{\theta}+\frac{b(\theta T_2-1)}{\theta^2}\right. \\
&\quad \left.\left. + \frac{c\left(\theta^2 T_2^2-2\theta T_2+2\right)}{\theta^3}\right)\right],
\end{aligned} \tag{10}
$$

where we have used (4) and (5) and calculated the definite integrals.

On the other hand, since defective products experience no demand, the inventory level of defective products $I_{pd}(t_1)$ at $t_1 \in [0, T_1]$ satisfies the ordinary differential equation

$$
\frac{\mathrm{d}I_{pd}(t_1)}{\mathrm{d}t_1} = uP - \theta I_{pd}(t_1),
$$

i.e.,

$$
I'_{pd}(t_1) + \theta I_{pd}(t_1) = uP. \tag{11}
$$

Using the boundary condition $I_{pd}(0) = 0$, one solves (11) to obtain

$$
I_{pd}(t_1) = \frac{uP}{\theta}\left(1-\mathrm{e}^{-\theta t_1}\right).
$$

Therefore, the inventory level of defective products per cycle can be expressed as

$$
\begin{aligned}
\mathcal{I}_{pd} &= \frac{1}{T}\int_0^{T_1} I_{pd}(t_1)\,\mathrm{d}t_1 \\
&= \frac{1}{T}\left[\frac{uPT_1}{\theta} + \frac{uP}{\theta^2}(\mathrm{e}^{-\theta T_1}-1)\right].
\end{aligned} \tag{12}
$$

Next, the per-cycle inventory level of deteriorated products is given by

$$
\begin{aligned}
\mathcal{I}_d &= \left[(1-u)PT_1 - \int_0^{T_1+T_2}\left(a+bt+ct^2\right)\mathrm{d}t\right] \\
&\quad + \left[\frac{uPT_1}{\theta} + \frac{uP}{\theta^2}\left(\mathrm{e}^{-\theta T_1}-1\right)\right]\theta \\
&= PT_1 - aT - \frac{bT^2}{2} - \frac{cT^3}{3} + \frac{uP}{\theta}\left(\mathrm{e}^{-\theta T_1}-1\right).
\end{aligned} \tag{13}
$$

On the other hand, the per-unit holding cost rate of non-defective products, which we assume to depend linearly on time, can be expressed as

$$
c_{h_1}(t) = A + Bt, \tag{14}
$$

so that its average value for $t \in [0, T]$ is given by

$$
\bar{c}_{h_1} = \frac{1}{T-0}\int_0^T c_{h_1}(t)\,\mathrm{d}t = A + \frac{B}{2}T. \tag{15}
$$

Finally, the company's expected total cost is the sum of the per-year set-up cost, the per-year production cost, the per-year inspection quality cost, the per-year holding cost, the per-year deterioration cost, and the per-year disposal cost:

$$
\begin{aligned}
\mathrm{ETC} &= C_1 + C_2 + C_3 + C_4 + C_5 + C_6 \\
&= \frac{c_s}{T} + \frac{c_p + c_{pe}}{T}\cdot PT_1 + \frac{c_i + c_u PT_1}{T} \\
&\quad + (\bar{c}_{h_1} + c_{he})\cdot\mathcal{I}_p + (c_{h_2} + c_{he})\cdot\mathcal{I}_{pd} + \frac{c_d}{T}\cdot\mathcal{I}_d \\
&\quad + \frac{c_w \overline{w} PT_1}{T} \\
&= \frac{c_s}{T} + \frac{c_p + c_{pe}}{T}PT_1 + \frac{c_i + c_u PT_1}{T} \\
&\quad + \frac{A + (B/2)T + c_{he}}{T}\left[\frac{T_1}{\theta}\left[(1-\mathrm{E}[u])P - a\right]\right. \\
&\quad - \frac{b}{\theta^2}\left(\frac{\theta T_1^2}{2}-T_1\right) - \frac{c}{\theta^3} \\
&\quad + \frac{c_w \overline{w} PT_1}{T}\left(\frac{\theta^2 T_1^3}{3}-\theta T_1^2+2T_1\right) \\
&\quad + \left(\frac{1-\mathrm{e}^{-\theta T_1}}{\theta}\right)\left(\frac{a-P(1-\mathrm{E}[u])}{\theta}-\frac{b}{\theta^2}+\frac{2c}{\theta^3}\right) \\
&\quad - \frac{aT_2}{\theta} - \frac{b}{\theta^2}\left(\frac{\theta T_2^2}{2}-T_2\right) + \frac{c_w \overline{w} PT_1}{T} \\
&\quad - \frac{c}{\theta^3}\left(\frac{\theta^2 T_2^3}{3}-\theta T_2^2+2T_2\right) \\
&\quad + \left(\frac{\mathrm{e}^{\theta T_2}-1}{\theta}\right)\left(\frac{a}{\theta}+\frac{b(\theta T_2-1)}{\theta^2}\right. \\
&\quad \left.\left. + \frac{c\left(\theta^2 T_2^2-2\theta T_2+2\right)}{\theta^3}\right)\right] + \frac{c_{h_2}+c_{he}}{T}\left[\frac{\mathrm{E}[u]PT_1}{\theta}\right. \\
&\quad \left. + \frac{\mathrm{E}[u]P}{\theta^2}\left(\mathrm{e}^{-\theta T_1}-1\right)\right] + \frac{c_d}{T}\left[PT_1 - aT - \frac{bT^2}{2}\right. \\
&\quad \left. - \frac{cT^3}{3} + \frac{\mathrm{E}[u]P}{\theta}\left(\mathrm{e}^{-\theta T_1}-1\right)\right] + \frac{c_w \overline{w} PT_1}{T}.
\end{aligned} \tag{16}
$$

For the company to achieve the maximum profit, this expected total cost is to be minimised. In the subsequent section, we shall carry out the minimisation numerically, by substituting (8) and (9) into (16), and thus treating ETC as a single-variable function of the consumption period length T_2. This means that, for a specific set of parameter values, we shall determine the solution T_2^* of the stationarity equation $\partial\mathrm{ETC}/\partial T_2 = 0$ at which the graph of $\mathrm{ETC}(T_2)$ versus T_2 is convex: $\partial^2\mathrm{ETC}/\partial T_2^2 > 0$. After obtaining the optimal consumption period length T_2^* and the minimum expected total cost $\mathrm{ETC}(T_2^*)$, we shall calculate the expected total carbon emission at this minimum state. Using the equations (10) and (12), the expected total carbon emission can be formulated as

$$
\begin{aligned}
\mathrm{ETE} &= \frac{e_p E_g}{T}PT_1 + (ve_w E_g)\cdot\mathcal{I}_p + (ve_w E_g)\cdot\mathcal{I}_{pd} \\
&= \frac{e_p E_g}{T}PT_1 + \frac{ve_w E_g}{T}\left[\frac{T_1}{\theta}\left((1-\mathrm{E}[u])P - a\right)\right. \\
&\quad - \frac{b}{\theta^2}\left(\frac{\theta T_1^2}{2}-T_1\right) - \frac{c}{\theta^3}\left(\frac{\theta^2 T_1^3}{3}-\theta T_1^2+2T_1\right) \\
&\quad + \left(\frac{1-\mathrm{e}^{-\theta T_1}}{\theta}\right)\left(\frac{a-P(1-\mathrm{E}[u])}{\theta}-\frac{b}{\theta^2}+\frac{2c}{\theta^3}\right)
\end{aligned}
$$

TABLE III
PARAMETER VALUES USED IN OUR NUMERICAL EXAMPLE.

Parameter	Value	Unit
P	100	unit/year
a	45	unit/year
b	3	unit/year^2
c	-0.05	unit/year^3
c_s	20	\$/cycle
c_p	7	\$/unit
c_i	10	\$/cycle
c_u	0.1	\$/unit
A	2.5	\$/(unit · year)
B	0.01	\$/(unit · year^2)
c_{h_2}	0.5	\$/(unit · year)
c_d	2	\$/unit
c_w	0.5	\$/ton
$\overline{w}$	0.02	ton/unit
θ	0.1	/year
v	1.7	m^3/unit
C_{TX}	75	\$/ton of CO_2
e_p	80	kWh/unit
e_w	8	kWh/(m^3 · year)
E_g	0.5×10^{-3}	ton CO_2/kWh
$\mathrm{E}[u]$	0.02	-

$$- \frac{aT_2}{\theta} - \frac{b}{\theta^2}\left(\frac{\theta T_2^2}{2} - T_2\right) - \frac{c}{\theta^3}\left(\frac{\theta^2 T_2^3}{3} - \theta T_2^2 + 2T_2\right) + \left(\frac{\mathrm{e}^{\theta T_2}-1}{\theta}\right)\left(\frac{a}{\theta} + \frac{b(\theta T_2 - 1)}{\theta^2} + \frac{c\left(\theta^2 T_2^2 - 2\theta T_2 + 2\right)}{\theta^3}\right)\Bigg] + \frac{v e_w E_g}{T}\left[\frac{\mathrm{E}[u] P T_1}{\theta} + \frac{\mathrm{E}[u] P}{\theta^2}\left(\mathrm{e}^{-\theta T_1} - 1\right)\right]. \quad (17)$$

III. NUMERICAL EXAMPLE

For a concrete illustration, let us now apply our constructed model to a specific numerical scenario. For this purpose, let us employ the set of parameter values presented in Table III, which originates from a study conducted by Taleizadeh, et al. [31] in an Iranian petrochemical factory. Following our last two assumptions in section II, to compute the per-unit production carbon emission cost, we shall employ the formula in [17], [34]:

$$c_{pe} = e_p \cdot E_g \cdot C_{TX}. \quad (18)$$

whereas to compute the per-unit inventory emission cost, we shall employ the formula in [13], [31]:

$$c_{he} = v \cdot e_w \cdot E_g \cdot C_{TX}. \quad (19)$$

Using the parameter values in Table III and the formulae (18) and (19), we carry out the following procedure.

Step 1 Substitute the equations (8) and (9) into the equation (16), where the value of κ is calculated using the equation (7). Since (8) potentially gives two different possibilities for T_1, this step potentially produces two different triples $(T_1(T_2), T_2, T(T_2))$, which lead to two different expressions for $\mathrm{ETC}(T_2)$.

Step 2 Substitute the parameter values in Table III into each of the obtained expressions for $\mathrm{ETC}(T_2)$.

Step 3 For each of the obtained expressions for $\mathrm{ETC}(T_2)$, determine the concavity of the graph $\mathrm{ETC}(T_2)$ versus T_2 (e.g., by direct graphing). Eliminate, if any, expressions for $\mathrm{ETC}(T_2)$ for which the graph $\mathrm{ETC}(T_2)$ versus T_2 is concave. Perform the subsequent steps only for each of the expressions for $\mathrm{ETC}(T_2)$ that are not eliminated.

Step 4 Solve the stationarity equation $\partial\mathrm{ETC}/\partial T_2 = 0$ to find the optimal consumption period length T_2^*.

Step 5 Substitute T_2^* into the equations (7), (8), and (9) to obtain the optimal length of the production-consumption period T_1^* and the optimal cycle length T^*.

Step 6 Substitute T_1^* into the equation (1) to obtain the number $Q_0(T_1^*)$ of products to be produced in order to minimise the expected total cost.

Step 7 Substitute T_2^* into the corresponding expression for $\mathrm{ETC}(T_2^*)$ to obtain the corresponding minimum expected total cost $\mathrm{ETC}(T_2^*)$.

Step 8 Substitute the parameter values in Table III, the value of T_2^*, as well as the corresponding values of T_1^* and T^*, into the equation (17), to obtain the expected total carbon emission $\mathrm{ETE}(T_2^*)$ at the state where the expected total cost is minimum.

For the parameter values in Table III, there exists only a single triple $(T_1(T_2), T_2, T(T_2))$ for which the graph $\mathrm{ETC}(T_2)$ versus T_2 is convex, leading to only a single expression $\mathrm{ETC}(T_2)$ that possesses a minimum. At its minimum state, the production-consumption period length, the consumption period length, and the cycle length are, respectively, $T_1^* \approx 0.3371$ years, $T_2^* \approx 0.3747$ years, and $T^* \approx 0.7118$ years. Moreover, the minimum expected total cost $\mathrm{ETC}(T_2^*)$ amounts to \$548.1643 per year, the optimal number of products to be produced is 34 units, and the expected total carbon emission at this minimum state is 1.9549 tons of CO_2 per year.

IV. SENSITIVITY ANALYSIS

Let us now analyse the sensitivity of the expected total cost ETC and the expected total carbon emission ETE at the minimum state obtained in section III, with respect of each of the model's parameters. To analyse the sensitivity of ETC with respect to a parameter, we calculate the percentage of variation

$$\frac{\mathrm{ETC}_{\mathrm{new}} - \mathrm{ETC}_{\mathrm{old}}}{\mathrm{ETC}_{\mathrm{old}}} \times 100\%,$$

where $\mathrm{ETC}_{\mathrm{old}} = 548.1643$ is the minimum expected total cost obtained in section III, while $\mathrm{ETC}_{\mathrm{new}}$ is the value of ETC after the value of the corresponding parameter is changed. Similarly, to analyse the sensitivity of ETE with respect to a parameter, we calculate the percentage of variation

$$\frac{\mathrm{ETE}_{\mathrm{new}} - \mathrm{ETE}_{\mathrm{old}}}{\mathrm{ETE}_{\mathrm{old}}} \times 100\%,$$

where $\mathrm{ETE}_{\mathrm{old}} = 1.9549$ is the expected total carbon emission at the minimum state obtained in section III, while $\mathrm{ETE}_{\mathrm{new}}$ after the value of the corresponding parameter is changed. Clearly, sensitive dependences of ETC and ETE on a parameter is signified by large percentages of variation. By performing computations with percentages of changes in parameter values of -20%, -10%, 0, 10%, and 20%, we obtain the results presented in Tables IV and V.

Based on these results, we conclude that, in the vicinity of the minimum state obtained in section III, ETC is a monotonically increasing function of each parameter. Furthermore, a and c_p are the parameters upon which ETC depends most sensitively (indicated by yellow highlights), while c is the parameter upon which ETC depends least sensitively. On the other hand, in the vicinity of the minimum state obtained in section III, ETE is a monotonically increasing function of P, a, b, c_s, c_i, θ, v, e_p, e_w, e_w, E_g, and $\mathrm{E}[u]$, but is a monotonically decreasing function of c, c_p, c_u, A, B, c_{h_2}, c_d, c_w, $\overline{w}$, and C_{TX}. Furthermore, a, e_p, and E_g are the parameters upon which ETE depends most sensitively (indicated by green highlights), while c, B, c_{h_2}, c_w, $\overline{w}$ are the parameters upon which ETE depends least sensitively.

V. Conclusions and Future Research

Developing the model proposed in 2019 by Daryanto and Wee [8], we have constructed an EPQ-type inventory model for a deteriorating product, which takes into account both the presence of defective products and the company's carbon emission, in which the demand rate is assumed to depend quadratically on time, while the holding cost of non-defective products is assumed to depend linearly on time. Using the parameter values originating from [31], we have conducted numerical computations, which reveal that the company's expected total cost achieves its minimum in the case of the production-consumption period length, the consumption period length, and the cycle length are set to be 0.3371 years, 0.3747 years, and 0.7118 years, respectively, and the quantity of products to be produced is set to be 34 units. At this minimum state, the minimum expected total cost itself amounts to \$548.1643 per year, while the corresponding expected total carbon emission amounts to 1.9549 tons of CO_2 per year. Furthermore, we have carried out a sensitivity analysis, which leads to the conclusion that, at the above minimum state, the expected total cost depends most sensitively on the demand rate's constant term and the per-unit production cost, while the expected total carbon emission depends most sensitively on the demand rate's constant term, the per-unit average electricity consumption for production, and the standard carbon emission from electricity generation.

Certainly, our model remains open for further development. First, since one our model's assumptions is made to ensure the absence of shortages, for an advancement, one could replace this assumption with a specified scheme for managing shortages [3], [7], [10], [25], [33]. In addition, one could consider multiple different products [5], thereby extending the model from single-item to multi-item. Furthermore, there are also opportunities for modification concerning the functional forms of the product's demand and holding cost rates. One could employ, for instance, an exponential time-dependent demand rate [16], [23], [26], a stock-dependent demand rate [12], a quadratic time-dependent holding cost rate [15], or a stock-dependent holding cost rate [20]. The need for warm-up production runs, product recovery, rework, and/or discounts could also be considered [10], [14], [22], [35].

TABLE IV
The results of our sensitivity analysis (part 1 of 2).

Parameter (initial value)	Percentage of change in parameter value	Value				Variation (%)	
		T_1^*	T_2^*	ETC	ETE	ETC	ETE
P (100)	$+20\%$	0.2632	0.4029	553.7413	1.9611	1.0174	0.3199
	$+10\%$	0.2956	0.3906	551.2083	1.9582	0.5553	0.1727
	0	0.3371	0.3747	548.1643	1.9549	0	0
	-10%	0.3924	0.3536	544.4357	1.9509	-0.6802	-0.2046
	-20%	0.4696	0.3241	539.7591	1.9461	-1.5333	-0.4487
a (45)	$+20\%$	0.4052	0.3112	640.7219	2.3226	16.8850	18.8107
	$+10\%$	0.3699	0.3421	594.6946	2.1390	8.4884	9.4180
	0	0.3371	0.3747	548.1643	1.9549	0	0
	-10%	0.3062	0.4097	501.1300	1.7703	-8.5803	-9.4437
	-20%	0.2765	0.4477	453.5816	1.5851	-17.2544	-18.9139
b (3)	$+20\%$	0.3340	0.3698	549.0965	1.9580	0.1701	0.1587
	$+10\%$	0.3356	0.3722	548.6319	1.9564	0.0853	0.0797
	0	0.3371	0.3747	548.1643	1.9549	0	0
	-10%	0.3388	0.3772	547.6936	1.9533	-0.0859	-0.0804
	-20%	0.3404	0.3798	547.2197	1.9517	-0.1723	-0.1616
c (-0.05)	$+20\%$	0.3371	0.3747	548.1644	1.9549	0.0000	-0.0006
	$+10\%$	0.3371	0.3747	548.1644	1.9549	0.0000	-0.0003
	0	0.3371	0.3747	548.1643	1.9549	0	0
	-10%	0.3371	0.3747	548.1643	1.9549	-0.0000	0.0003
	-20%	0.3371	0.3747	548.1642	1.9549	-0.0000	0.0006
c_s (20)	$+20\%$	0.3594	0.3980	553.6091	1.9625	0.9933	0.3900
	$+10\%$	0.3485	0.3865	550.9289	1.9587	0.5043	0.1981
	0	0.3371	0.3747	548.1643	1.9549	0	0
	-10%	0.3255	0.3624	545.3066	1.9509	-0.5213	-0.2048
	-20%	0.3134	0.3497	542.3458	1.9467	-1.0615	-0.4170
c_p (7)	$+20\%$	0.3291	0.3663	614.4464	1.9521	12.0916	-0.1408
	$+10\%$	0.3331	0.3704	581.3112	1.9535	6.0469	-0.0716
	0	0.3371	0.3747	548.1643	1.9549	0	0
	-10%	0.3414	0.3791	515.0052	1.9563	-6.0491	0.0743
	-20%	0.3458	0.3838	481.8334	1.9578	-12.1005	0.1514
c_i (10)	$+20\%$	0.3485	0.3865	550.9289	1.9587	0.5043	0.1981
	$+10\%$	0.3428	0.3807	549.5577	1.9568	0.2542	0.0998
	0	0.3371	0.3747	548.1643	1.9549	0	0
	-10%	0.3313	0.3686	546.7477	1.9529	-0.2584	-0.1015
	-20%	0.3255	0.3624	545.3066	1.9509	-0.5213	-0.2048
c_u (0.1)	$+20\%$	0.3370	0.3746	549.1115	1.9548	0.1728	-0.0021
	$+10\%$	0.3371	0.3746	548.6379	1.9548	0.0864	-0.0010
	0	0.3371	0.3747	548.1643	1.9549	0	0
	-10%	0.3372	0.3748	547.6907	1.9549	-0.0864	0.0010
	-20%	0.3373	0.3748	547.2171	1.9549	-0.1728	0.0021
A (2.5)	$+20\%$	0.3205	0.3572	552.4194	1.9492	0.7762	-0.2920
	$+10\%$	0.3285	0.3656	550.3182	1.9519	0.3929	-0.1514
	0	0.3371	0.3747	548.1643	1.9549	0	0
	-10%	0.3465	0.3845	545.9537	1.9581	-0.4033	0.1637
	-20%	0.3567	0.3951	543.6815	1.9615	-0.8178	0.3414
B (0.01)	$+20\%$	0.3371	0.3747	548.1705	1.9548	0.0011	-0.0009
	$+10\%$	0.3371	0.3747	548.1674	1.9549	0.0006	-0.0004
	0	0.3371	0.3747	548.1643	1.9549	0	0
	-10%	0.3372	0.3747	548.1612	1.9549	-0.0006	0.0004
	-20%	0.3372	0.3748	548.1581	1.9549	-0.0011	0.0009
c_{h_2} (0.5)	$+20\%$	0.3371	0.3746	548.1801	1.9548	0.0029	-0.0012
	$+10\%$	0.3371	0.3747	548.1722	1.9549	0.0014	-0.0006
	0	0.3371	0.3747	548.1643	1.9549	0	0
	-10%	0.3372	0.3747	548.1564	1.9549	-0.0014	0.0006
	-20%	0.3372	0.3748	548.1485	1.9549	-0.0029	0.0012
c_d (2)	$+20\%$	0.3365	0.3740	548.3124	1.9546	0.0270	-0.0112
	$+10\%$	0.3368	0.3744	548.2384	1.9548	0.0135	-0.0056
	0	0.3371	0.3747	548.1643	1.9549	0	0
	-10%	0.3375	0.3750	548.0902	1.9550	-0.0135	0.0056
	-20%	0.3378	0.3754	548.0159	1.9551	-0.0271	0.0112
c_w (0.5)	$+20\%$	0.3371	0.3747	548.2590	1.9549	0.0173	-0.0002
	$+10\%$	0.3371	0.3747	548.2117	1.9549	0.0086	-0.0001
	0	0.3371	0.3747	548.1643	1.9549	0	0
	-10%	0.3371	0.3747	548.1169	1.9549	-0.0086	0.0001
	-20%	0.3372	0.3747	548.0696	1.9549	-0.0173	0.0002
$\overline{w}$ (0.02)	$+20\%$	0.3371	0.3747	548.2590	1.9549	0.0173	-0.0002
	$+10\%$	0.3371	0.3747	548.2117	1.9549	0.0086	-0.0001
	0	0.3371	0.3747	548.1643	1.9549	0	0
	-10%	0.3371	0.3747	548.1169	1.9549	-0.0086	0.0001
	-20%	0.3372	0.3747	548.0696	1.9549	-0.0173	0.0002

TABLE V
THE RESULTS OF OUR SENSITIVITY ANALYSIS (PART 2 OF 2).

Parameter (initial value)	Percentage of change in parameter value	Value				Variation (%)	
		T_1^*	T_2^*	ETC	ETE	ETC	ETE
θ (0.1)	+20%	0.3295	0.3642	550.3311	1.9589	0.3953	0.2065
	+10%	0.3333	0.3694	549.2542	1.9569	0.1988	0.1041
	0	0.3371	0.3747	548.1643	1.9549	0	0
	−10%	0.3412	0.3803	547.0611	1.9528	−0.2013	−0.1058
	−20%	0.3453	0.3861	545.9441	1.9507	−0.4050	−0.2133
v (1.7)	+20%	0.3335	0.3709	549.0656	1.9656	0.1644	0.5471
	+10%	0.3353	0.3728	548.6162	1.9602	0.0824	0.2749
	0	0.3371	0.3747	548.1643	1.9549	0	0
	−10%	0.3390	0.3767	547.7100	1.9494	−0.0829	−0.2778
	−20%	0.3409	0.3787	547.2532	1.9439	−0.1662	−0.5584
C_{TX} (75)	+20%	0.3301	0.3673	577.4689	1.9524	5.3460	−0.1240
	+10%	0.3335	0.3709	562.8212	1.9536	2.6738	−0.0630
	0	0.3371	0.3747	548.1643	1.9549	0	0
	−10%	0.3408	0.3786	533.4981	1.9561	−2.6755	0.0650
	−20%	0.3447	0.3826	518.8222	1.9574	−5.3528	0.1321
e_p (80)	+20%	0.3336	0.3710	576.5767	2.3324	5.1832	19.3144
	+10%	0.3354	0.3728	562.3716	2.1437	2.5918	9.6584
	0	0.3371	0.3747	548.1643	1.9549	0	0
	−10%	0.3389	0.3766	533.9548	1.7660	−2.5922	−9.6610
	−20%	0.3408	0.3785	519.7430	1.5771	−5.1848	−19.3247
e_w (8)	+20%	0.3335	0.3709	549.0656	1.9656	0.1644	0.5471
	+10%	0.3353	0.3728	548.6162	1.9602	0.0824	0.2749
	0	0.3371	0.3747	548.1643	1.9549	0	0
	−10%	0.3390	0.3767	547.7100	1.9494	−0.0829	−0.2778
	−20%	0.3409	0.3787	547.2532	1.9439	−0.1662	−0.5584
E_g (0.5×10^{-3})	+20%	0.3301	0.3673	577.4689	2.3429	5.3460	19.8512
	+10%	0.3335	0.3709	562.8212	2.1490	2.6738	9.9308
	0	0.3371	0.3747	548.1643	1.9549	0	0
	−10%	0.3408	0.3786	533.4981	1.7605	−2.6755	−9.9415
	−20%	0.3447	0.3826	518.8222	1.5660	−5.3528	−19.8943
$E[u]$ (0.02)	+20%	0.3387	0.3735	550.0286	1.9626	0.3401	0.3935
	+10%	0.3379	0.3741	549.0945	1.9587	0.1697	0.1963
	0	0.3371	0.3747	548.1643	1.9549	0	0
	−10%	0.3364	0.3753	547.2379	1.9510	−0.1690	−0.1955
	−20%	0.3356	0.3759	546.3154	1.9472	−0.3373	−0.3902

Finally, the constant deterioration assumption could be made more realistic by assuming that the deterioration follows a certain probability distribution, such as the Weibull distribution [19], [26], [28], [33].

REFERENCES

[1] M. Balasubramanian and R. Kamali, "An optimal green inventory model for a deteriorating product with non-linear quadratic demand and permissible payment systems," IAENG International Journal of Applied Mathematics, vol. 55, no. 2, pp278-284, 2025.

[2] M. Balasubramanian and R. Kamali, "The mathematical modelling of multi-stage inventory supply chain to determine product demand with ramp-type," IAENG International Journal of Applied Mathematics, vol. 55, no. 4, pp835-841, 2025.

[3] T. Chakrabarti and K. S. Chaudhuri, "An EOQ model for deteriorating items with a linear trend in demand and shortages in all cycles," International Journal of Production Economics, vol. 49, no. 3, pp205-213, 1997.

[4] D. Castellano and C. H. Glock, "Economic production quantity for a decaying item with stochastic demand and positive lead time," International Journal of Production Economics, vol. 267, 109094, 2024.

[5] S. W. Chiu, C. T. Tseng, M. F. Wu, and P. C. Sung, "Multi-item EPQ model with scrap, rework and multi-delivery using common cycle policy," Journal of Applied Research and Technology, vol. 12, no. 3, pp615-622, 2014.

[6] S. Dari and B. Sani, "An EPQ model for delayed deteriorating items with quadratic demand and linear holding cost," OPSEARCH, vol. 57, pp46-72, 2019.

[7] S. Dari and B. Sani, "An EPQ model for delayed deteriorating items with quadratic demand and shortages," Asian Journal of Mathematics and Computer Research, vol. 22, no. 2, pp87-103, 2017.

[8] Y. Daryanto and H. M. Wee, "Low carbon economic production quantity model for imperfect quality deteriorating items," International Journal of Industrial Engineering and Engineering Management, vol. 2, no. 1, pp1-8, 2019.

[9] Y. Daryanto and H. M. Wee, "Sustainable economic production models: An approach toward a cleaner production," Journal of Advanced Management Science, vol. 6, no. 4, pp206-212, 2018.

[10] S. Ganesan and R. Uthayakumar, "EPQ models for an imperfect manufacturing system considering warm-up production run, shortages during hybrid maintenance period and partial backordering," Advances in Industrial and Manufacturing Engineering, vol. 1, 100005, 2020.

[11] A. Goswami and K. S. Chaudhuri, "An EOQ model for deteriorating items with a linear trend in demand," International Journal of Production Economics, vol. 42, no. 3, pp1105-1110, 1991.

[12] M. A. Halim, A. Paul, M. Mahmoud, B. Alshahrani, A. Y. M. Alazzawi, and G. M. Ismail, "An overtime production inventory model for deteriorating item with nonlinear price and stock dependent demand," Alexandria Engineering Journal, vol. 60, no. 3, pp2779-2786, 2021.

[13] M. Hariga, R. As'ad, and A. Shamayleh, "Integrated economic and environmental models for a multi stage cold supply chain under carbon tax regulation," Journal of Cleaner Production, vol. 166, pp1357-1371, 2017.

[14] M. Y. Jaber and J. Peltokorpi, "Economic order/production quantity (EOQ/EPQ) models with product recovery: A review of mathematical modeling (1967-2022)," Applied Mathematical Modelling, vol. 129, pp655-672, 2024.

[15] P. Kumar, "Inventory optimization model for quadratic increasing holding cost and linearly increasing deterministic demand," International Journal of Recent Technology and Engineering, vol. 7, no. 6, pp1999-2004, 2019.

[16] M. L. Malumfashi, M. T. Ismail, and M. K. M. Ali, "An EPQ model for delayed deteriorating items with two-phase production period, exponential demand rate and linear holding cost," Bulletin of the Malaysian Mathematical Sciences Society, vol. 45, no. Suppl. 1, pp395-424, 2022.

[17] B. Marchi, S. Zanoni, L. E. Zavanella, and M. Y. Jaber, "Supply chain models with greenhouse gases emmssions, energy usage, imperfect process under different coordination decisions," International Journal of Production Economics, vol. 211, pp145-153, 2019.

[18] M. C. Monte, E. Fuente, A. Blanco, and C. Negro, "Waste management from pulp and paper production in the European Union," Waste Management, vol. 29, no. 1, pp293-308, 2009.

[19] S. Pal, G. S. Mahapatra, and G. P. Samanta, "An EPQ model of ramp type demand with Weibull deterioration under inflation and finite horizon in crisp and fuzzy environment," International Journal of Production Economics, vol. 156, pp159-166, 2014.

[20] V. Pando, L. A. San-José, J. Sicilia, and D. Alcaide-López-de-Pablo, "An inventory model with price- and stock-dependent demand and time- and stock quantity-dependent holding cost under profitability maximization," Computers and Operations Research, vol. 164, 106520, 2024.

[21] K. Pathak, A. S. Yadav, and P. Agarwal, "An inventory model for deteriorating goods with exponential demand, variable holding costs, and partial backlogging across two-warehouses in an inflationary environment," IAENG International Journal of Applied Mathematics, vol. 54, no. 5, pp950-960, 2024.

[22] S. Priyan, P. Mala, and M. Palanivel. "A cleaner EPQ inventory model involving synchronous and asynchronous rework process with green technology investment," Cleaner Logistics and Supply Chain, vol. 4, 100056, 2022.

[23] R. Raj, N. K. Kaliraman, S. Chandra, and H. Chaudhary, "A production inventory model with exponential demand rate and reverse logistics," International Journal of Industrial Engineering Computations, vol. 5, no. 4, pp521-542, 2014.

[24] L. A. San-José, J. Sicilia, L. E. Cárdenas-Barrón, and M. Gonzáles-de-la-Rosa, "A sustainable inventory model for deteriorating items with power demand and full backlogging under a carbon emission tax," International Journal of Production Economics, vol. 264, 109098, 2024.

[25] D. Sani. "An EPQ model for delayed deteriorating items with reliability consideration, quadratic demand and shortages," Pakistan Journal of Statistics and Operation Research, vol. 19, no. 3, pp475-489, 2023.

[26] R. I. P. Setiawan, J. D. Lesmono, and T. Limansyah, "Inventory control problems with exponential and quadratic demand considering Weibull deterioration," Journal of Physics: Conference Series, vol. 1821, no. 1, 012057, 2021.

[27] N. H. Shah, M. Y. Jani, and U. Chaudhari, "EPQ model for deteriorating items with quadratic demand, dynamic production rate and production time dependent selling price: A risk analysis approach,"

International Journal of Supply Chain Management, vol. 7, no. 4, pp377-385, 2018.
[28] S. Shekhawat, H. Rathore, and K. Sharma, "Economic production quantity model for deteriorating items with Weibull deterioration rate over the finite time horizon," International Journal of Applied and Computational Mathematics, vol. 7, 56, 2019.
[29] V. R. Soleymanfar, A. A. Taleizadeh, and N. P. Zia, "A sustainable lot-sizing model with partial backordering," International Journal of Advanced Operations Management, vol. 7, no. 2, pp157-172, 2015.
[30] E. W. Taft, "The most economical production lot," The Iron Age, vol. 101, no. 18, pp1410-1412, 1918.
[31] A. A. Taleizadeh, V. R. Soleymanfar, and K. Govindan, "Sustainable economic production quantity models for inventory systems with shortage," Journal of Cleaner Production, vol. 174, pp1011-1021, 2018.
[32] R. J. Tersine, Principles of Inventory and Materials Management, 4th edition, Prentice Hall, New Jersey, 1994.
[33] G. Viji and K. Karthikeyan, "An economic production quantity model for three levels of production with Weibull distribution deterioration and shortage," Ain Shams Engineering Journal, vol. 9, no. 4, pp1481-1487, 2018.
[34] I. D. Wangsa, "Greenhouse gas penalty and incentive policies for a joint economic lot size model with industrial and transport emissions," International Journal of Industrial Engineering Computations, vol. 8, no. 4, pp453-480, 2017.
[35] Y. W. Zhou, J. Chen, Y. Wu, and W. Zhou, "EPQ models for item with imperfect quality and one-time-only discount," Applied Mathematical Modelling, vol. 39, no. 3-4, pp1000-1018, 2015.