Optimization of Urban Rail Multi-composition Operation Plan Based on Improved Space-time Network

Ruifa Niu, Changfeng Zhu, Yunqi Fu, Jie Wang, Linna Cheng, Rongjie Kuang

Abstract—**Due to accelerating urbanization, there is a growing imbalance in passenger demand in terms of time and space distribution along the lines. To address this issue, urban rail transit lines have started implementing multi-routing and multi-composition operational plans to better manage complex passenger flows and reduce operating costs. Based on existing research, a space-time network diagram of multi-composition trains was designed using the physical network diagram as a reference. This optimizes transfer behavior within the network diagram and increases the space-time state of the transfer nodes. An optimization model for the multi-composition operational plan was constructed using composition and network constraints. The dual objective was to minimize enterprise costs and passenger travel time. Using the Beijing Changping Line as an example, the target cost reduction was 6.99% when the Lagrangian relaxation algorithm was employed. This suggests that operating multi-component trains can be more beneficial for enterprises and passengers. Finally, analysis of the network, composition types and routes showed that multi-component trains can reduce the infeasibility of passenger flow under line network conditions and increase the feasibility of passenger travel.**

Index Terms—**Space-time networks; Multi-composition operation plan; Transfer arc optimization; Lagrangian relaxation algorithm**

I. INTRODUCTION

THE development of three-dimensional transport integration has reached a new stage. Concurrently, urbanization is persisting in its development of metropolitan areas and city clusters; however, the issue of disparate regional development within these clusters continues to exert an influence on the process. The interaction of passenger and train flow is constrained by weak transport links between central and peripheral areas. This phenomenon is observable both within and outside the cluster. It is evident that inadequate transport timeliness and insufficient transport capacity have a considerable impact. These factors have been demonstrated to exert a profound influence on the integrated and synergistic development of metropolitan areas and city clusters. In the context of the uneven distribution of passenger flows over time and space, traditional single-route, single-composition train plans are no longer adequate to meet the operational needs of urban rail transport enterprises. Multi-composition trains have been demonstrated to be an effective solution to the problem of time imbalance, thereby facilitating more efficient passenger flow management during periods of peak demand. Consequently, certain urban rail transport lines have initiated the utilization of multi-composition trains, with the objective of optimizing the provision of services to meet the multifaceted demands of passengers and concomitantly reducing operating costs. In light of the prevailing demand, the development of a scientific and rational multi-route and multi-composition train operation plan has emerged as a research problem of theoretical and practical significance.

In the process of train operation organization, running trains with different compositions according to the characteristics of passenger flow can reduce operating costs. This can be done without reducing operating costs. Using different train compositions on metro lines is an effective way to solve the problem of mismatch between transport capacity and passenger demand. Using flexible train compositions saves enterprises money and solves the problem of spatial-temporal imbalance of passenger flow. Yang [1] by analyzed the operation mode of crossing-line trains with multi-composition, based on the crossing-line passenger flow distribution model which analyzed the passenger travelling behavior, and taking into consideration of the constraints such as the interchanging The multi-objective planning model with the frequency of different types of trains and the location of the return station as the decision variables was established by considering the constraints such as the passing capacity of the station and the full load rate of the train. Li [2] by analyzing the urban rail transit which can operate multiple full lines or short distance service routes, and each service route can use multi-composition trains, and taking the constraints such as the return station of the service routes, the train composition and the frequency of the service routes, etc. as the constraints. A

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bi-objective model for minimizing passenger travel time and optimal corporate benefit is established, and a local search algorithm is applied to solve the large-scale instances by decomposing the objective into two sub-problems. Li [3] in response to the problem of insufficient passenger demand of the urban rail transit during off-peak hours, a new train composition mode is proposed, which takes into account the train concatenation/deconfiguration at the intermediate stops with a flexible train composition. To minimize the total passenger waiting time and train operating cost, constraints such as maximum line throughput capacity, passenger flow demand and the number of trains in the train composition are considered. It is verified that the optimization of the composition pattern can better match the passenger demand and capacity, and reduce the operating cost Zhu [4] to better achieve the goal of energy saving and carbon reduction in urban rail transit, a train plan with flexible train schedule (TPFTC) is proposed, and a multi-objective planning model is established under the constraints of the maximum full train capacity and the number of operating trains. An Improved Quantum Genetic Algorithm (IQGA) is designed to solve the mathematical model. Feng [5] in order to satisfy the problem of limited transportation capacity and passenger demand in a high-speed railway network, by optimizing the composition operations at the originating stations, adjusting the train compositions, scheduling additional trains from the capacity pools, and optimizing the seat assignments between ODs with the objective of minimizing the operating costs and the penalties for unfulfilled passengers, consideration is made of the following Various constraints such as flow conservation, loading capacity, flexible composition, etc., a mixed-integer linear programming model is developed and a Lagrangian relaxation algorithm combining parallel computation and sub-gradient method is designed. Feng [6] multi-crossing mode, multi-composition mode as well as fast and slow train modes are considered in the train service design problem. The coupling of the interchanging and multi-scheduling modes into an integer linear programming model is used to integrate the train service route design with the passenger flow allocation problem. A method using the alternating direction multiplier method (ADMM) is designed to solve the model solution. Zhong [7] considering the dynamic and distributional characteristics of passenger flow demand in urban rail transit, a train schedule optimization method that responds to the demand fluctuation and improves the energy efficiency is designed under the flexible train composition mode with the objective of minimizing passenger waiting time and energy consumption, and a hybrid integer nonlinear programming (MINLP) model. An improved Variable Neighborhood Search (VNS) heuristic algorithm is used to solve the high-quality solution. Zhou [8] for the collaborative optimization of train schedule and vehicle scheduling problems on two-way urban rail transit lines. Passengers' spatial and temporal demands are met by splitting and merging multiple modular train units in the yard. The problem is mathematically modelled under one arrival time window, and the proposed model considers two realistic FIFO rules to accurately capture passenger transport in a supersaturated urban rail transit system. Zhao [9] considering the difficulty of the traditional fixed-format train operation mode to meet the diversity of passenger demand, the different adoptions of different train composition modes in urban rail transit can reduce the passenger waiting number and improve the vehicle occupancy rate. Based on the constraints such as turnback constraints, composition constraints, travelling intervals and other constraints. Mixed integer nonlinear programming problem is proposed and solved using existing optimization solvers such as CPLEX. Wang [10] optimization of train composition and train schedules to meet actual passenger demand, a multiple-format train departure plan is used with the objective of minimizing the number of stranded passengers and the number of trains through schedule-related constraints and constraints related to passenger assignment. Mapping constraints are incorporated in the model to match each train service with the corresponding train sequence at different stations to ensure correct passenger allocation and a mixed integer linear programming (MILP) model is proposed.

The optimization model construction of the running plan is based on the solution quality of the algorithm solution, but the traditional heuristic algorithms differ among themselves in the solution speed and the quality of the solution, and the selection of the initial solution may lead to the problem of falling into the local optimum due to the convergence of the initial solution is too fast, so the use of improved heuristic algorithms can be optimal to find the results of the model of the problem. Zhan [11] in order to optimize the frequency of the through express trains and the local trains, with the maximizing passenger travel time savings and average train load factor utilization, a non-dominated sorting genetic algorithm is designed to perform a comprehensive comparative analysis with the full-stopping schedules under the single line operation mode and the through operation mode. Lin [12] by converting passenger travel time into cost through the value coefficient of the Passengers' non-working time, and taking into account the constraints on the capacity matching degree of the different routes, an improved reconciliation search algorithm to solve the problem, and the effectiveness of the method is verified experimentally. Tian [13] in order to reduce the carbon emission and improve the transport efficiency, a multi-objective optimization model for railway heavy-duty trains is established, and a forward learning pigeon flock heuristic optimization algorithm based on an agent-assisted model is proposed by taking into account the joint constraints between the carbon emission and transport efficiency objectives. This approach uses an agent model to compute candidate solutions and a forward learning strategy to enhance the learning capability of non-dominated solutions thereby reducing the time cost. Zhou [14] to address the phenomenon of tidal traffic and to satisfy the imbalance of passenger flow in space and time. better matching of passenger demand, a mixed integer linear programming (MILP) model is proposed and a custom heuristic algorithm based on variable Neighborhood search (VNS) is designed to generate high-quality solutions quickly. Cadarso [15] by the existence of railway vehicle allocation and train route planning problems in railway vehicle operation, a new approach is proposed to obtain better, more robust cycle of railway vehicle units and since the use of Benders decomposition is applicable to the integrated model, an improved heuristic algorithm for Benders decomposition is designed. Based on the experiments, it is proved that the improved algorithm can obtain a more robust and efficient solution. Tian [16] summarizes the characteristics of the through operation of urban railways and suburban railways. In order to reduce the total passenger travelling time and increase the revenue of urban rail transit operating companies. A multi-objective mixed integer nonlinear planning model is designed and then solved by using the Gurobi solver. Dong [17] summarizes the mathematical characteristics of train cycle operation, analyses the impact of the number and location of turnback lines at the turnback station on the train timetable and vehicle turnover plan, and designs an algorithmic solution algorithm integrating the depth, breadth and efficiency of the iterative exploration.

In recent years, scholars with the depth of research gradually combined with space-time networks to portray the dynamic behavior of passengers and trains, which can better portray the process of passengers' travel behavior. Regarding the research of space-time network, White [18] firstly introduced the theory of space-time network into the problem of traffic flow allocation, and constructed a linear model of empty train allocation based on space-time network; Wang [19] for the railway passenger scheduling problem with meal time constraints, through the construction of the time-space-state network, they put forward a kind of improved algorithm based on the Lagrange relaxation algorithm, and the effective lower bound obtained by the Lagrange relaxation algorithm is different from that obtained by solving the set-coverage model. The effective lower bound obtained by the Lagrangian relaxation algorithm is combined with solving the ensemble coverage model to obtain a high-quality upper bound solution. Krygsman [20] by analyzing the networkability elements of public transport, it is proposed that the inbound and outbound passenger flow factors in the public transport network are the weakest links in the public network, which determines the availability and convenience of the public transport. Optimization of entry and exit plans has the potential to significantly reduce public transport travel times and take measures to effectively lower costs. Cadarso [21] based on the complexity of the urban rail network system, and at the same time there is a problem of cross-interference when optimizing timetables and vehicle scheduling. The need to deal with interference on passenger demand is considered. A two-step approach combining an integrated optimization model with a model about passenger behavior is proposed, and the proposed method is able to find a solution with a good balance between various management objectives within a few minutes. Pan [22] to optimize train timetabling and vehicle cycle planning, an integer linear programming model based on a space-time network and taking into account the flexible detrainment activities is proposed in order to better match the passenger demand and transport capacity provided, and to further improve the flexibility of urban rail transit operations and meet dynamic passenger demand. Zheng [23] a matrix-based ALNS heuristic algorithm is proposed to construct a space-time network model with the objective of minimizing the total dwell time of cargo aircraft goods by introducing a capacity matrix .Bao [24] connecting, de-coupling and switching technology optimization is caried out based on the virtual composition technology at intermediate transfer stations through the , unlinking and switching techniques to optimize the train composition pattern and multiple service routes. A path-based spatial-temporal network approach is used to construct the metro network, taking into account constraints such as train service routes and the number of train services and linking and unlinking operations, as well as capacity constraints of the storage lines, with the objective of minimizing passenger waiting time and operating costs. A novel rolling stock cycle model based on passenger demand train schedules and rolling stock cycle planning is designed and solved by applying the adaptive simulated annealing (ASA) algorithm. Liao [25] describe the integrated timetable and vehicle scheduling problem by constructing a hybrid space-time network, and formulate an integer planning model to maximize the overall transport performance. Wang [26] develop an integrated timetable and vehicle scheduling model to maximize the overall transport performance for the railroad passenger scheduling problem with the constraints of dining and service time. time constraints on the railway passenger scheduling problem, proposed an improved algorithm based on the Lagrangian relaxation algorithm by constructing a time-space-state network, and obtained a high-quality upper bound solution by combining the effective lower bounds obtained by the Lagrangian relaxation algorithm with solving the ensemble coverage model. Santos [27] in order to evaluate the impacts of vehicle ranges and different types of chargers in the optimal design of an inter-city SAEV transport systems, a space-time-energy-flow based integer planning model was developed to support the design of regional SAEV systems by optimizing the Vehicle and charging facilities while considering the overall operation of the trains; Yao [28] by introducing passenger route while considering the impacts of travelling time and fares in the Passengers' utility in order to move out the Passengers' path choices on a railway network. A multi-commodity network flow model based on space-time networks was constructed. Qu [29] designed a space-time network computational framework for mesoscopic traffic simulation in large-scale networks. Through the spatial and temporal attributes of events, different types of simulation events are obtained to be mapped to independent logical processes, and multi-threaded logical computation tasks are implemented on a multi-core shared memory architecture.

Based on this, the space-time network method is added to the urban rail transit multi-unit train departure plan. Considering that the passengers generally choose to take the train in the travel process of the transfer time is relatively short, and the Passengers' transfer behavior is not taken into account, so this paper combines the Passengers' transfer behavior, so the space-time network in the distribution of passenger flow when more reasonable. It helps to better portray the dynamic process of passenger travelling, so as to construct a more realistic space-time network.

II. SPACE-TIME NETWORK CONSTRUCTION

The physical network is two-dimensional, portraying the spatial location of stations and lines in the composition, but lacks the ability to describe passenger travel time. To describe both the spatial and temporal aspects of the composition of passengers and trains, the two-dimensional space network is used as the basis and an additional time axis is added to expand the network in the temporal dimension, forming a space-time network (G). This network depicts the situation of trains passing through various stations at different times. The schematic diagram of the space-time network is shown in Fig.1.

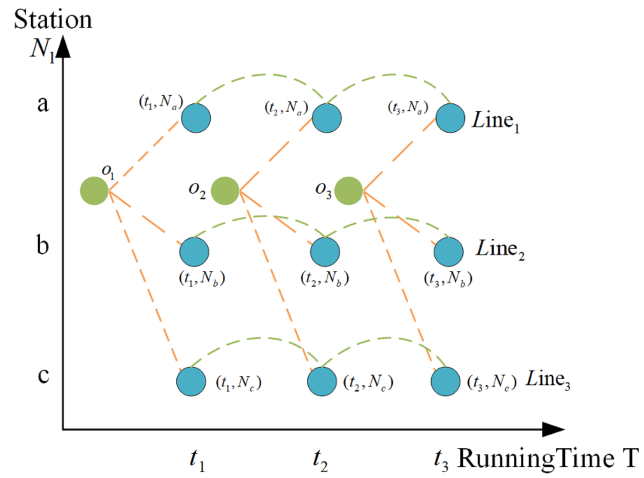


Fig.1 Schematic diagram of space-time network G

A. Problem Description

Consider that in a directed metro network, in multi-composition mode, the network consists of a set of independent metro network lines denoted by $L=\{l\,|\,l=1,2,\cdots,l_{\max}\}$. The set of stations along the line l_i is denoted by N_l , while the set of all stations involved in the whole network is denoted by N_L and the set denoted by $N_L=\bigcup_{l\in L} N_l=\{i\,|\,i=1,2,\cdots,i_{\max}\}$ .The set of travelling paths denoted E which $E=\{k\,|\,k\ =1,2,\ldots,K\}$ between N_L .There is passenger demand between N_L the two locations. between N_L and denoted by N_L^{OD} .The set of trains between denoted by $F=\{f\,|\,f\ =1,2,\ldots,f\}$, There are many transfer stations in the railway network, and passengers can transfer between different lines through the transfer channel to the transfer stations. In this case, there may be a non-empty route between two station sets N_l and $N_{l'}(l,l'\in L, l\neq l')$ in the network. To avoid confusion in the modelling process, denoted $N_l\cap N_{l'}=\varnothing$, the same station in the transfer is therefore replaced with the same number l' ($l,l'\in L$). Based on the generated and associated with the network train schedules, passengers prioritize routes with the shortest travel times to meet their travel demand. The network needs to be reconstructed to clearly portray the passenger transfer process. Number2,5 is transferring stations between lines l_{ac} , l_{bd} , which are actually the same station. Considering that the traditional transfer design considers the transfer of nmber2,5 as the exchange of passengers at the same level. In reality, however, passengers transfer at different platform levels and the purpose of the trip is heterogeneous. The number of virtual nodes has been increased from two to four. The node now also takes into account passengers walking and changing at the same time. To depict the platform levels of transfer time and the rationality of travel behaviors, we need to consider the time dimension in the timetable of each train in the network. Embed train running and passenger behavior into a space-time network to portray train and passenger travel process. The design of traditional transfer nodes and improved transfer nodes is shown in Fig.2 and 3.

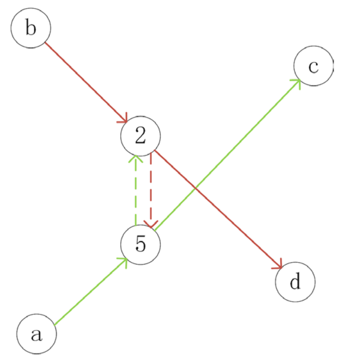


Fig.2 Traditional transfer nodes of passengers

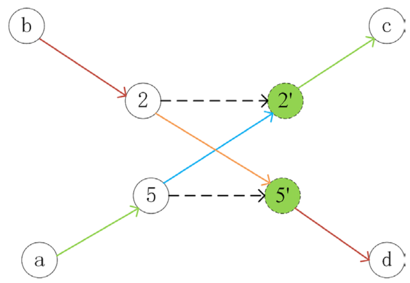


Fig.3 Design improvement of transfer nodes of passengers

B. Space-time-state network construction

The description of passenger behavior is ignored in the physical network, and the heterogeneity of passengers causes differences in selection behavior. The two-dimensional space-time network need to be transformed into a space-time network in three-dimensional coordinates, with the added axes indicating the range of passenger states and the other two coordinates indicating the space-time positions. Each space-time node in the space-time coordinate system portrays the specific position or state of the train or passenger at the corresponding time.

In the space-time network there exist several space-time arcs and space-time nodes. The space-time nodes represent the specific position of the train or passenger at the corresponding time, and the space-time arcs represent the physical connections between the nodes of the network [30] . In the process of constructing a space-time network, time as a whole is discretized into a number of small-time segments and it is assumed that the network state does not change significantly during this period. Time needs to be discretized into a number of small time slots δ , in which the network state is assumed not to change δ . Use time intervals with lengths to discretize the time axis into different time ranges. The time horizon can be discretized into a set of time segments, denoted $T=\{u\,|\,u=t_0,t_0+\delta,t_0+2\delta,\cdots,t_0+M\delta\}$, where, t_0 and $t_0+\delta$ are the left and right endpoints, respectively, of the time horizon under consideration. A schematic diagram of the physical and space-time networks is shown in Fig.4 and 5.

In accordance with the characteristics of the space-time network, the space-time starting point is characterized as the condition of a physical node at a specific time period., represented by a two-element array (i, t_0). At the same time, having the presence of a link (i, j) in the physical network (G, N_L), a passenger or train enters at t_0 the link (i, j) and leaves at t_1 the link (i, j), and the time of departure from this link should be just enough to satisfy the time (t_0, t_1). Space-time arc In this case, the two space-time vertices, (i, t_0) (j, t_1) are connected by a space-time arc, which is indicated by (i, j, t_0, t_1) for and $t_p = t_1 - t_0$ for passenger travel time. Since the transfer arc and the passenger waiting arc are also part of the space-time arc, they are constructed with the same consideration as the running arc. The flowchart of space-time-state network construction is shown in Fig. 6.

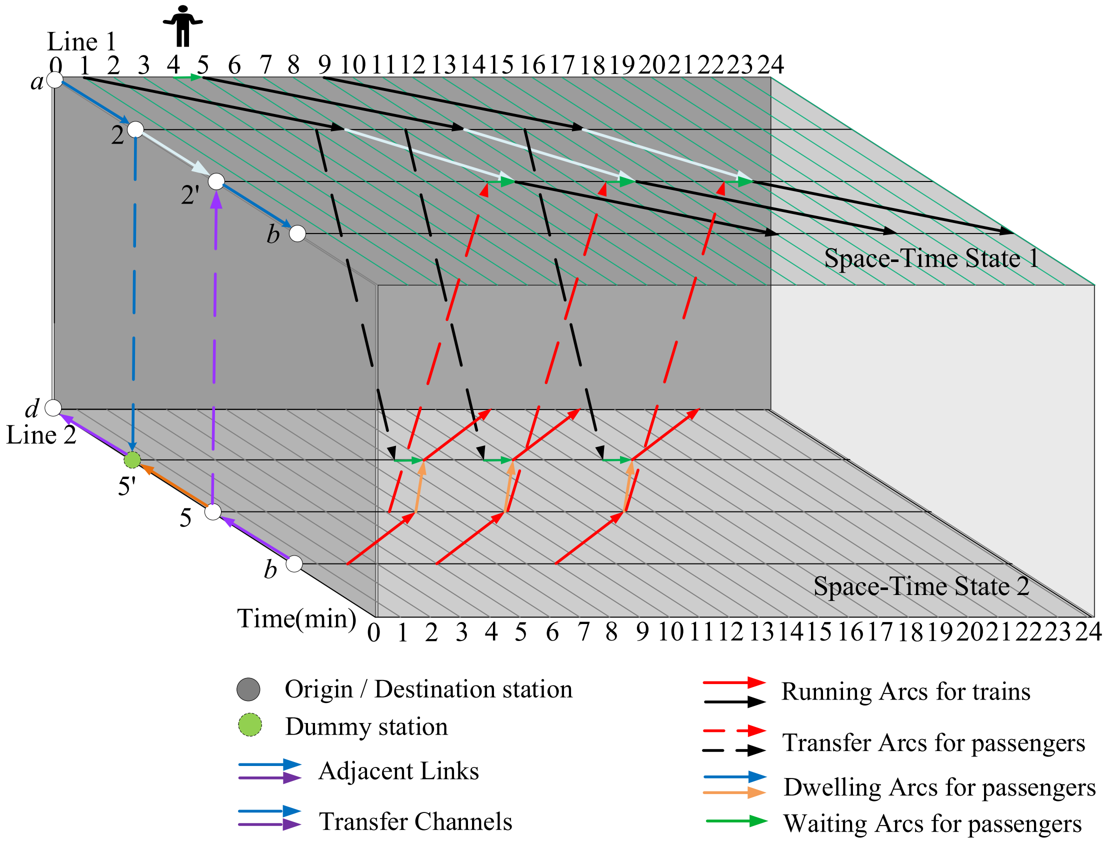


Fig.4 Schematic diagram of the space-time network

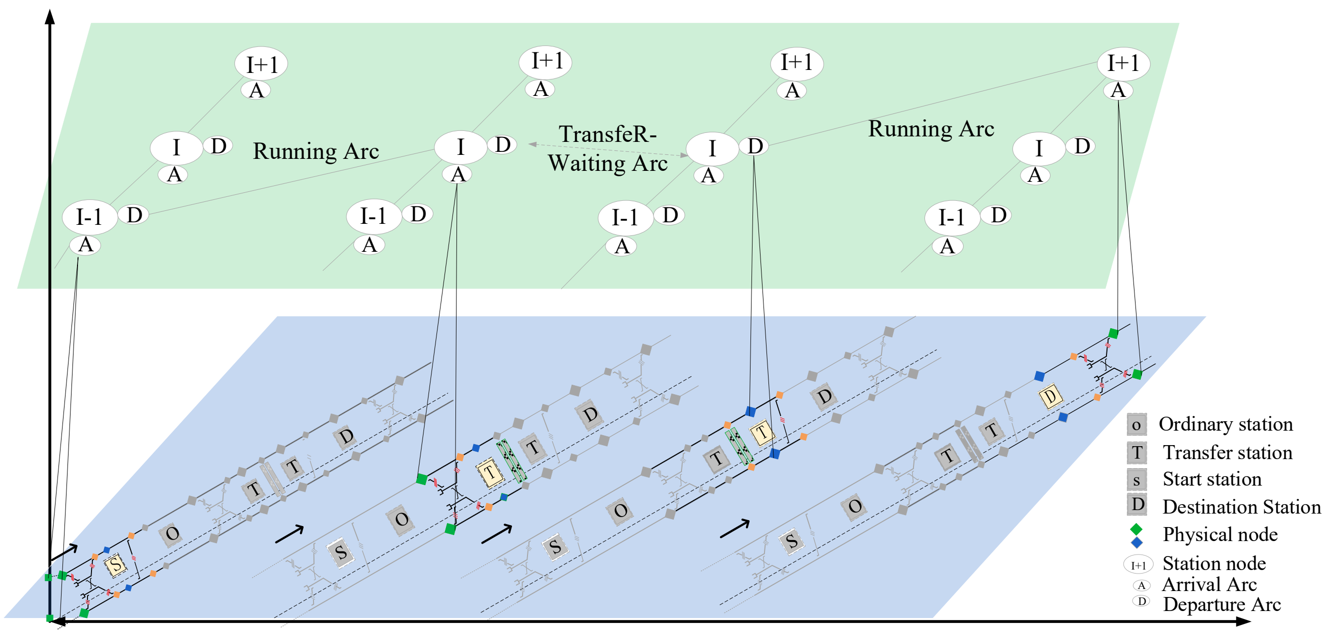


Fig.5 Physical space-time network diagram

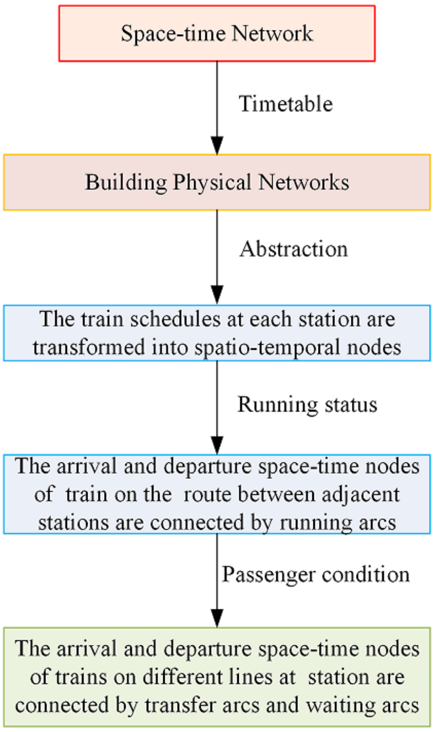


Fig.6 Flowchart for the construction of space-time-state network

The impact of train factors on the network varies according to the timetable during construction of the network. However, the development of the timetable must take into account the prevailing circumstances pertaining to passenger flow. In the peak period, passengers will adopt rational behavior by transferring at the most efficient time and completing the shortest number of transfers possible. During peak periods, there is a special phenomenon of passenger flow transfer at some of the transfer stations. At the peak of the route, there is congestion, and passengers who want to reach their destinations may increase the transfer time at the expense of multiple transfer stations or choose non-optimal line trains for interchanging. A schematic diagram of the special transfer behavior of passengers in the space-time-state network is shown in Fig.7.

III. Model construction

A. Model assumptions

(1) At each station on the line, trains arrive on a schedule-determined basis. To avoid potential conflicts, the physical link at each station in the space-time network can be occupied by at most one train at any one time.

(2) Each train is considered to be a point in the network, where the length of the train is not taken into account. The underline of each route is used independently, and only one composition type of train can be used on a route, but trains of different composition types can be used on different routes.

(3) Trains of different transfer stations and compositions have equal running times when passing through the same intervals on the line and equal stopping times at the same stations. stopping times are equal.

(4) All transfer passengers will be required to walk from the Spur Line compositions to the Connector Line compositions (i.e. transfer between compositions), while only one transfer will take place at each station (i.e. there will be no secondary waiting involved).

(5) There are no two transfer stations at the same station, i.e. consecutive transfer arc for passengers cannot occur.

(6) The stopping plan of the train is selected for the armistice set S_{stop} .The collection of stopping patterns S_{stop} is shown in Table.1.

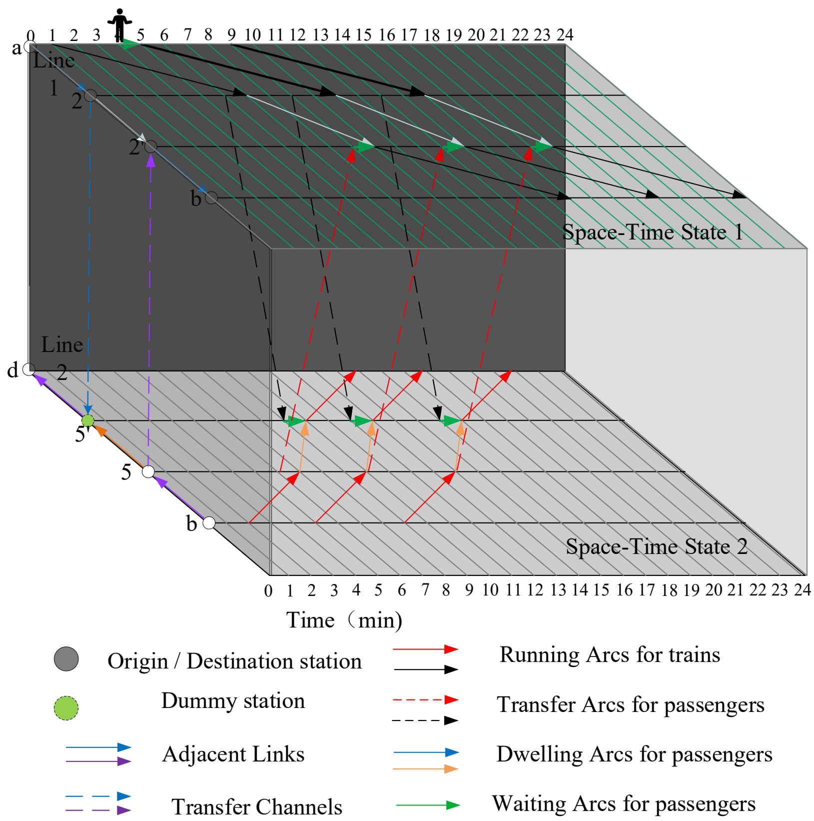


Fig. 7 Schematic diagram of space-time-state network

TABLE I
THE COLLECTION S_{stop} OF STOPPING PATTERNS

Mode of stop	Classification of stops	Schematic diagram
Trains stop at the major station	Access to terminals without transfer stations	●─○─●─○─●
	Go through the transfer to the terminal	●─○─⊘─○─⊘
Trains stop at every station	Access to terminals without transfer stations	●─●─●─●─●
	Go through the transfer to the terminal	●─●─⊘─⊘─⊘
OD Direct	Direct trains from the starting point to the destination	●─○─○─○─●
Train stop at crossing stations	Access to terminals without transfer stations	●─●─○─●─●
	Go through the transfer to the terminal	●─●─⊘─○─⊘

B. Parameters and Variables

Define all the relevant variables and parameters to be used below, based on the formulas in the modelling research problem for the operation plan. Variables and parameters involved in the formulations is shown in Table 2.

C. Enterprise costs

For a transport company operating an urban rail line, costs mainly fall into two categories: fixed and variable. Fixed costs include the cost of purchasing all the trains on the line and their maintenance, while variable costs include operating costs and employee costs.

(1) Vehicle cost C_{g} , Day vehicle cost C_{r_g}

$$C_{\text{g}} = \sum_{h \in H} c_0 N_h b_h \tag{1}$$

$$C_{\text{r}_{\text{g}}} = \frac{C_g}{365 D_g} \tag{2}$$

Where, the number of trains in the composition h N_h ; the number of trains in the composition h b_h .

(2) Vehicle maintenance and servicing

As the average annual overhaul and maintenance cost of the rolling stock is a percentage of the train's acquisition cost. Daily maintenance costs $C_{r\text{j}}$

$$C_{r\text{j}} = \frac{C_{\text{g}}}{365} \alpha_b \tag{3}$$

Where, the coefficient of the share of maintenance costs of the Vehicle α_b .

(3) Train operation C_{y}

$$C_{\text{y}} = L_z c_{\text{my}} \tag{4}$$

Where, kilometers travel by the vehicle L_z .

(4) Employee C_{yg}

$$C_{\text{yg}} = \sum_{h \in H} c_{\text{hy}} N_h \tag{5}$$

In summary, the operating costs of a business C_{q} are made up of four components.

$$\min C_{\text{q}} = C_{\text{r}_{\text{g}}} + C_{\text{rj}} + C_{\text{y}} + C_{\text{yg}} \tag{6}$$

D. Passenger costs

The focus of the study is passenger travel time, which is the main factor influencing passenger travel costs. The main reference object is transport for passengers. In the process of travelling, passenger travel costs consist of three factors: travel time cost, passenger fare cost and passenger travelling running time. Time and cost processing is considered as a broad travel time. As travel time and train costs are weakly correlated, passengers are significantly sensitive to both. The introduction of cost conversion weight coefficients converts the cost of travelling into time.. Then the generalized travel cost $C_{N_L}^k$ of the travel path k between N_L .

$$C_{N_L}^k = C_{N_L}^{k,w} + oc_l + t_l i_l + \alpha^{\text{h}} \tag{7}$$

Where, N_L travel time between paths k $C_{N_L}^{k,w}$; cost shift weighting factor o reflecting differences in passenger sensitivity to travel time and cost; passenger fare cost between N_L c_l ; unit time of operation between N_L t_l ; number of stations between N_L i_l ; and transfer time for passengers choosing to transfer α^{h} .

The Transfer Time α^{h} consists of two parts, including the Transfer Walking Time o and the Transfer Waiting Time τ .

$$\alpha^{\text{h}} = o_{rs} + \tau \tag{8}$$

Where, α^{h} is influenced by the Passengers' transfer walking distance and the Passengers' own factors.

$$o_{rs} = \frac{L_{rs}}{\text{h}} \tag{9}$$

Where, h Representing the average step length, due to the differences in individual step lengths of passengers, the statistical average of is used as the step length , L_{rs} represents the distance from the starting point of the transfer to the target of the transfer in the station of the passenger. τ The waiting time is the average of the statistical data.

Passenger flow allocation based on a space-time network essentially involves dynamically loading passenger flow. In accordance with the principle of Wardrop network equilibrium, passengers will opt for the space-time route with the lowest integrated impedance for travel. The integrated impedance of each space-time path that has been selected is not larger than that of any of the space-time paths that have not been selected. All the multi-path passenger flow allocation models are actually based on the stochastic or deterministic cost theory. rs Selection of N_L cost of path between k impedance c_k^{rs} .

$$c_k^{rs} = c_{\min}^{rs}(1+\sigma), \quad \forall rs, k \in K \tag{10}$$

Where, the cost of the shortest path $c_{\min}^{rs}$, the path extensibility factor σ .

In the urban rail transit space-time state network, each OD to rs between the existence of a number of space-time paths, the time period between the OD of the passenger demand is fixed and known, in the network rs

TABLE II
VARIABLES AND PARAMETERS INVOLVED IN THE FORMULATIONS

notation	define
L	A composition of lines of an urban railway system. l
N_L	Collection of Urban Railway Line Stations , i, j, L_i
K	The set of paths of the line N_L , k
D	Train direction of travel collection. d
R	The set of routes of the line, r
E	The set of intervals of the line,
H	The set of composition of the line. h
F	N_L collection of trains running. f
Q_L	Aggregation of Passenger Traffic in Urban Railway Systems. q_{ij}
rs	Q On the line N_L passengers OD assemble
W	A collection of OD arrays in the space-time network, (i, j)
T	Passengers in the time network OD meet at the starting and stopping time of the walk. (u, v)
B	Collection of arrays of path travelling times (i, j, u, v) in the space-time network
X	$x_{ijuv}^{rs,t}$ set
Y	y_{jivu} set
A	Arc assembly in the metro network. a
c_0	Unit vehicle costs
D_g	Service life of the Vehicle
c_{my}	Cost per vehicle kilometers
c_{hy}	Employee unit costs for codification composition h
m_l	Full turnaround time for route r
g_l	Full turnaround distance for route r
α_{rh}	0-1 parameters, route r covered stations , N_L $\alpha_{rh} = 1$
β_{re}	0-1 parameters, route r covered stations , $\beta_{re} = 1$
γ_{rd}	0-1 parameter, when a train travelling in the direction d on the crossing r turns back at the station L_i . $\gamma_{rd} = 1$
P_{ed}	Cross-sectional passenger flow between zones e in the direction of operation d
p_h	Composition h train capacity
δ	Line capacity surpluses. %
φ	Minimum number of pairs of trains for each traffic
ε_{dh}	0-1 parameter, trains of the composition h in the direction of operation d can turn back. $\varepsilon_{dh} = 1$
s_d	Station L_i turnback capacity to the direction of travel d
π_t	Number of available underframes for trains in the composition h
q_{ij}	Number of passengers travelling to station j from station i . $q_{ij} \in Q$
θ_{ij}	Stations i for travelling to Station j, starting from $\theta_{ij} \in D$
t_f	0-1 variable, the train stops at l between N_L , the $t_f = 1$
u_{rh}	Integer variable, number of train in the required composition h on the crossing r
x_{rh}	0-1 variable, trains of the composition h are used on the route r , and $x_{rh} = 1$
y_{ltf}	0-1 variable, crossing r on the use of composition h of running trains, f $y_{rhf} = 1$
k_a^i	Continuous variable, number of passengers travelling from the arc a to the station node i in network G.
w_n^i	Continuous variable, waiting time at station node i for passengers travelling to station node in network G arc a
$x_{ijuv}^{rs,t}$	Continuous variable, OD passenger travelling flow from station node i to node j in network G. $x_{ijuv}^{rs,t} = 1$
y_{ijuv}	0-1 variable, OD passenger trips in network G select this trip arc, $y_{ijuv} = 1$

set β_{rs} . The network reaches equilibrium when c_k^{rs} and β_{rs} satisfy the relation.

$$\beta_{rs} - c_k^{rs} \begin{cases} = 0 & x_{jivu}^{rs,t} > 0, \\ \leq 0 & x_{jivu}^{rs,t} = 0, \end{cases} \quad \forall rs, k \in K \tag{11}$$

Similar transport network, in a space-time urban rail network, the passenger demand between OD, Q_L^{rs} is equal to the total of all space-time road flows k between OD

$$Q_L^{rs} = \sum_{k \in K} x_{jivu}^{rs,t}, k \in K \tag{12}$$

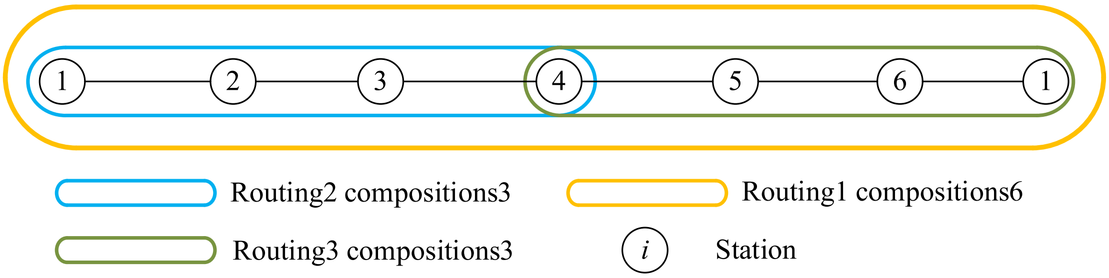


Fig. 8 Schematic diagram of multi-composition and intersection design

Passenger travel costs $C_{N_L}^{k,w}$

$$C_{N_L}^{k,w} = \sum_{l\in L} R_{L}^{rs} t_f \upsilon Q_{L}^{rs} \tag{13}$$

Where, υ is the operating line frequency coefficient.

In summary, the combined travelling costs for passengers $C_{N_L}^{k}$

$$\min C_{N_L}^{k} = C_{N_L}^{k,w} + oc_l + t_l i_l + \alpha^{h} \tag{14}$$

E. restrictive conditions

(1) Composition and route constraints

At least one route R exists between each station N_L and between the intervals E . Schematic diagram of multi-composition and intersection design is shown in Fig. 8.

$$\sum_{r\in R}\sum_{h\in H} \alpha_{rh} x_{rh} \ge 1 \quad \forall h\in H, N_L \tag{15}$$

$$\sum_{r\in R}\sum_{h\in H} \beta_{re} x_{rh} \ge 1 \quad \forall e\in E \tag{16}$$

(2) Cross-sectional passenger flow constraints

The sum of the train conveyance capacities of the intervals is greater than or equal to the maximum intervals cross-section passenger flow demand in the up and down directions of the intervals. The operating organization takes special circumstances into account when formulating the plan, and generally sets a certain capacity redundancy factor δ to constrain the cross-section passenger flow of the interval E .

$$(1-\delta)\sum_{r\in R}\sum_{h\in H}\sum_{f\in F} \beta_{re} p_h y_{rhf} \ge \max_{d\in D}\{p_{ed}\}, \forall e\in E \tag{17}$$

(3) Operational organizational constraints

Restrictions on the number of pairs of trains and types of train composition running on the line in the route

(a) Trains may operate up to one composition type on any transfer r .

$$\sum_{h\in H} x_{rh} \le 1 \quad \forall r\in R \tag{18}$$

(b) The use of any route r on the line L with a number of train pairs of the route of are not less than φ .

$$\sum_{h\in H}\sum_{f\in F} y_{rhf} \ge \varphi \sum_{h\in H} x_{rh} \quad \forall r\in R \tag{19}$$

(c) The use of composition h trains in route r between the lines L ,are not exceeding the maximum $|F|$ number of pairs of trains.

$$\sum_{f\in F} y_{rhf} \le |F| x_{rh} \quad \forall r\in R, \forall h\in H \tag{20}$$

(4) Turnback station capacity constraints

Turnback station setting and interval capacity constraints considering the fact that in stations, urban railways are limited by their own factors to set up turnback stations in some transfer stations and departure stations, such as L_i , L_i there is an upper limit to the number of trains that can be folded back in each direction of operation d . s_d . Due to the existence of cross-interference and other operational impacts of bi-directional turnback stations, a constraint is introduced to limit the total turnback capacity of the bi-directional turnback station to not exceeding an upper limit s_i^{max} in both directions.

(a) Turnback operations are carried out for trains that meet the turnback direction restrictions and train composition restrictions of L_i at the station.

$$\sum_{r\in R}\sum_{f\in F} \gamma_{rd} y_{rhf} \le \varepsilon_{dh} s_d, \forall d\in D, h\in H, L_i\in N_L \tag{21}$$

(b) In the direction d of operation , the number of trains turned back at station L_i does not exceed the turning capacity s_d of this station in the direction d .

$$\sum_{r\in R}\sum_{h\in H}\sum_{f\in F} \gamma_{rd} y_{rhf} \le s_d, \forall L_i\in N_L, d\in D, h\in H \tag{22}$$

(c) The total bi-directional turnback capacity of each station does not exceed the total feasible turnback capacity s_v^{max} of that station.

$$\sum_{r\in R}\sum_{h\in H}\sum_{f\in F}\sum_{d\in D} \gamma_{lvd} y_{ltf} \le s_i^{max}, \quad \forall i\in N_L, r\in R, h\in H, d\in D \tag{23}$$

(d) TE minimum tracking interval between two neighboring trains in the same direction on a zone limit the number of passing trains on each zone to no more than the capacity number capacity b_e^{max} of the zone and no less than the minimum number of pairs b_e^{min} of trains required.

$$b_e^{min} \le \sum_{r\in R}\sum_{h\in H}\sum_{f\in F} \beta_{re} y_{rhf} \le b_e^{max} \quad \forall e\in E \tag{24}$$

(5) Train underframe restraint

The number of underframes for trains of different composition types f must not exceed the number of underframes available for trains of different compositions on the line.

(a) The number of underframes required for the composition t on each l .The number of underframes is equal to the full turnaround time m_l for that crossing compared with the headway for that crossing υ .

$$\sum_{f\in F} \frac{m_l}{\upsilon} \le u_{rh} \quad \forall r\in R, \forall h\in H \tag{25}$$

$$\sum_{f\in F} \frac{m_l}{\upsilon} + 1 \ge u_{rh} \quad \forall r\in R, \forall h\in H \tag{26}$$

(b) The number of underframes of the composition type h necessary for the train plan does not exceed the number of underframes of the composition type available on the line.

$$\sum_{r\in R} u_{rh} \le \pi_h \quad \forall h\in H \tag{27}$$

(6) Space-time arc constraints

(a) The number of passengers on each operating arc is not greater than the capacity of the arc.

$$\sum_{i\in N_1} k_a^i \le \sum_{h\in H}\sum_{f\in F} r_h y_{rhf} \quad \forall a\in A \tag{28}$$

(b) Commutation arc constraints

$$0 \le \alpha^{\mathrm{h}} \le D_W \tag{29}$$

Where, the transfer time does not exceed the maximum transfer time of D_W , with a general maximum limit of two minutes.

(c) Passenger flow constraints

The station has zero flow of passengers travelling on reverse trains from node i to station node j .

$$k_a^i y_{jivu} = 0 \quad \forall a\in A \tag{30}$$

The space-time network is constructed as follows. The trajectory of each train consists of a series of space-time arcs, which can only be defined in one direction according to the traversal direction. As a result, there is no ring in the space-time network on each underground line. To constitute the space-time trajectory of the train, a set of space-time arcs satisfying the constraints mentioned above is generated..

$$\sum_{(i,j,u,v)\in B_l} y_{ijuv} - \sum_{(i,j,u,v)\in B_l} y_{jiuv} = \begin{cases} 1, \text{if } i = O_l^{'}, u = t_l^O \\ -1, \text{if } i = D_l^{'}, u = t_l^D \\ 0, \quad \text{otherwise} \end{cases} \tag{31}$$

Where, $\left(O_l^{'}, t_l^O\right)$ denotes the start of the station and $\left(D_l^{'}, t_l^D\right)$ denotes the end of the station.

(7) Nodal passenger flow conservation constraint

Conservation of passenger flows entering and leaving at the station at each station node.

$$\sum_{(i,j,u,v)\in B_l} x_{ijuv}^{rs,t} - \sum_{(j,i,v,u)\in B_l} x_{jivu}^{rs,t} = \begin{cases} q_{ij}, \text{if } i = O_l^{'}, u = t_l^O \\ -q_{ij}, \text{if } i = D_l^{'}, u = t_l^D \\ 0, \quad \text{otherwise} \end{cases} \tag{32}$$

(8) 0-1 constraints

$$u_{rh} \in Z^+ \quad \forall r\in R, \forall h\in H \tag{33}$$

$$x_{rh} \in \{0,1\} \quad \forall r\in R, \forall h\in H \tag{34}$$

$$y_{rhf} \in \{0,1\} \quad \forall r\in R, \forall h\in H, \forall f\in F \tag{35}$$

$$y_{ijuv} \in \{0,1\}, (i,j,u,v)\in B_l \tag{36}$$

IV. Model algorithm

A. Introduction to the algorithm

The models that have been proposed are complex ones, given the size of the space-time network and the complexity of the objectives or constraints. They take into account a large number of decision variables. In this case, commercial optimization software is usually ineffective at solving the problem. The next step is to suggest an efficient heuristic algorithm based on Lagrange relaxation (La), which can be used to find a near-optimal solution and greatly reduce the computational intensity.

B. model decomposition

The problem is first decomposed into one part representing the cost to the firm P1 and another part representing the cost of passenger travel P2.

$$\begin{array}{l} \text{Model⊠P1:} \begin{cases} \text{Objective function (6)} \\ s.\,t.\ (15) - (27), (33) - (35). \end{cases} \\ \text{Model⊠P2:} \begin{cases} \text{Objective function (14)} \\ s.\,t.\ (28) - (32), (36). \end{cases} \end{array} \tag{37}$$

Among them, the enterprise cost contains variables that are one-dimensional and can be solved in various ways. However, the enterprise cost variable involves a four-dimensional array of variables $x_{ijuv}^{rs,t}$, which is more difficult to solve. Therefore, the passenger travel cost P2 will be decomposed by dualizing the coupling constraints generated by the inequality, and the problem of the decomposition model P 2 is designed separately $x_{ijuv}^{rs,t}$ as the objective, and the change of the function objective is represented by the transformation of $x_{ijuv}^{rs,t}$. After the decomposition of P2, the problem model is decomposed in a simplified way, i.e., the composite multidimensional variables are decomposed into multiple multidimensional variables to achieve the decomposition of the model to achieve the dual-objective optimization solution with P1. A series of non-negative Lagrange multipliers $\lambda_{ijuv}^{rs,t}$,which are composed into an overall vector, are introduced, and then these constraints are dynamized into the objective function.

$$\min: \text{P2:} \begin{cases} \min(X,Y,\lambda) = \sum_{(rs,t)\in W\times T} x_{ijuv}^{rs,t} + \\ \sum_{(rs,t)\in W\times T} \sum_{(rs,t)\in W\times T} \left\lfloor \lambda_{ijuv}^{rs,t} (x_{ijuv}^{rs,t} - y_{ijuv}^{rs,t}) \right\rfloor \\ s.t.(28) - (32) \end{cases} \tag{38}$$

After recombining all the variables in the relaxation model, the objective function of model P2 can be further reconstructed.

$$\begin{aligned} L(X,Y,\lambda) = & \sum_{(rs,t)\in W\times T} x_{ijuv}^{rs,t} + \sum_{(rs,t)\in W\times T} \\ & \sum_{(i,j,u,v)\in B_L} \lambda_{ijuv}^{rs,t} x_{ijuv}^{rs,t} - \sum_{(rs,t)\in W\times T} \sum_{(i,j,u,v)\in B_L} \lambda_{ijuv}^{rs,t} y_{ijuv} \end{aligned} \tag{39}$$

Model P2 is decomposed into the following two subproblems, one of which involves only the decision vector X and the other is related to the decision vector Y . For convenience, these two subproblems are referred to as Model P2-X and Model P2-Y in the following discussion.

$$\text{Model P2-X:} \begin{cases} \min: L_X(X,\lambda) = \sum_{(rs,t)\in W\times T'} (x_{rstv'}^{rs,t} \\ + \sum_{(i,j,u,v)\in B_L^+} \lambda_{ijuv}^{rs,t} x_{ijuv}^{rs,t}) \\ s.t.(28),(29),(32) \end{cases} \tag{40}$$

$$\text{Model P2-Y:} \begin{cases} \min: L_Y(Y,\lambda) = -\sum_{(rs,t)\in W\times T'} \\ \sum_{(i,j,u,v)\in B_L^+} \lambda_{ijuv}^{rs,t} y_{ijuv} \\ s.t.(30),(31),(36) \end{cases} \tag{41}$$

Basically, for any given Lagrange multiplier, the best objective function for the relaxation model is the lower limit of the original problem. So, during the search process, the lower and upper bounds are basically worked on bit by bit to get a solution that's pretty close to optimal. To solve the dualized model, the input data required consists of the initial space-time network and the initial La-multipliers. The calibration algorithm can find any given shortest space-time path in the space-time network. This process continues until the termination condition is satisfied, at which point the best solution is output as an approximate optimal solution to the model.

C. Algorithmic steps

Step1: Initialize the population i $\{i \mid i = 1, 2, \ldots, N\}$ N is the population size, relevant parameters and Lagrange multipliers $\lambda_{ijuv}^{rs,t}$;

Step2: Loosen the coupling constraint relaxation into a penalty term to be added to the objective function for solving the relaxation problem, and construct a new relaxation function.

Step3: Lagrange multipliers update mutation;

The mutation operator adopts a new type of Cauchy mutation, which is introduced to increase the diversity of the population and avoid local optimal solutions. When the closer the position distance between the particle and the current optimal particle is, it indicates that the similarity between the populations is greater, the individual farthest from the current optimal particle is subjected to the Cauchy mutation, if the fitness value of the mutated particle is greater than the original fitness, the mutated particle replaces the original particle, otherwise it remains unchanged. Probability density function of one-dimensional Cauchy distribution.

$$d(x) = \frac{1}{\pi} \bullet \frac{t}{t^2 + x^2}, -\infty < x < +\infty \tag{42}$$

Step4: A loop iteration is performed and when the loop ends, a relaxation solution and its corresponding objective value can be computed. For the spacetime one come pair need to solve the spacetime shortest path problem, where the weights of the spacetime arcs are Lagrange multipliers;

Step5: Determine if it is a feasible solution, if not, go to, if it is Step4, go to Step8;

Step6: Integrate $x_{ijuv}^{rs,t}$ and perform Cauchy's mutation on the solution that is farther away from the current optimal solution, compare the mutated solution with the original solution, keep the better solution, add 1 to the iteration number, and return to step Step4; compute the value of the objective function of P2, and bring it into P1 for computation;

Step7: Judge whether the P1 solution satisfies the requirement, if so, go to step Step8, otherwise go to Step6;

Step8 : End of algorithm. The total flow is shown in Fig.9.

V. CASE STUDY

A. Line Introduction

The Beijing Metro Changping Line starts from Changping Xishankou Station in the north and ends at Xi'erqi Station in the south, connecting Changping District with the central city of Beijing, with a total length of 31.9 km, and 12 stations numbered from Changping Xishankou Station to Xi'erqi Station, respectively No. 1 to No. 12. There are two transfer stations on the line, namely, Zhuxinzhuang Station and Xi'erqi Station, where Zhuxinzhuang Station realizes the same-composition transfer with Subway Line 8, and Xi'erqi Station is connected with Subway Line 13. There are five stations on the Changping Line that have the conditions for switching back, namely: Changping Xishankou Station, Changping Dongguan Station, Nanshao Station, Shahe Higher Education Park Station and Xi'erqi Station. The line is planned to use two types of trains on the line, including 3-unit trains and 6-unit trains. The route map of Changping Line. Peak passenger flow data chart, flat peak passenger flow data chart, Different predicted passenger flows OD Data. are shown in Fig 10 and 11 and 12. The key parameters are shown in Table 3.

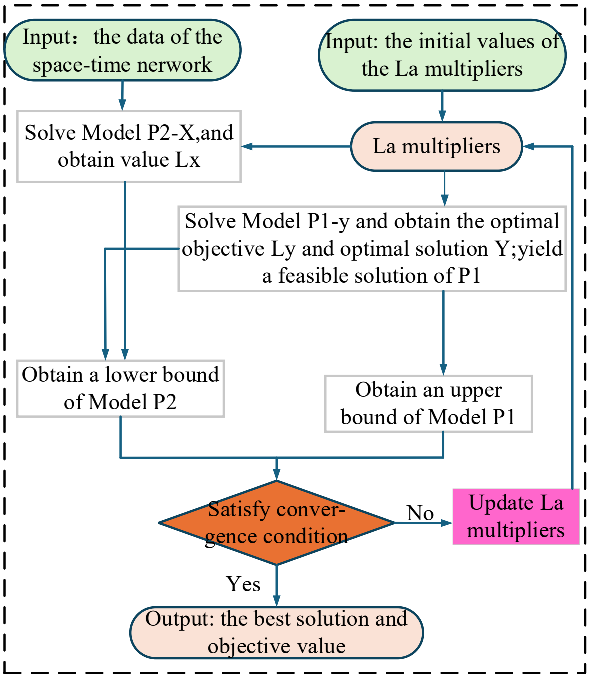


Fig.9 Algorithm flow chart

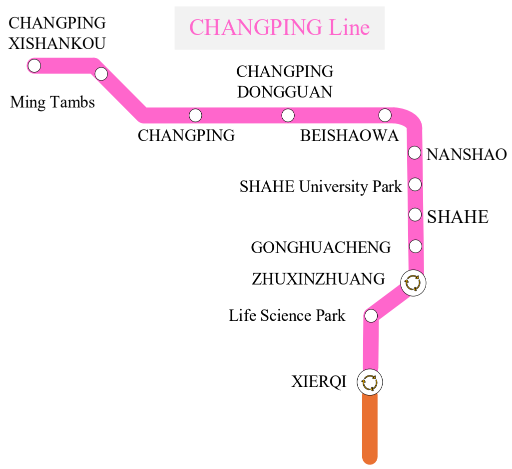


Fig.10 The route map of Changping Line

B. Analysis of results

The Python compilation environment is used to build the proposed algorithm and then solve the model. Considering the high complexity of the model, lower population sizes and maximum number of evolutionary generations may not result in a satisfactory solution, or even in a situation where there is no feasible solution. The iterative convergence curve is shown in Fig.13. The pareto solution is shown in Fig14. A comparison of the optimization plans is shown in Fig.15.

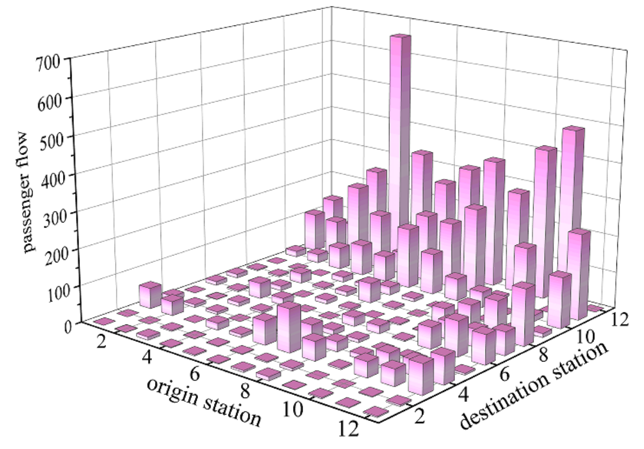
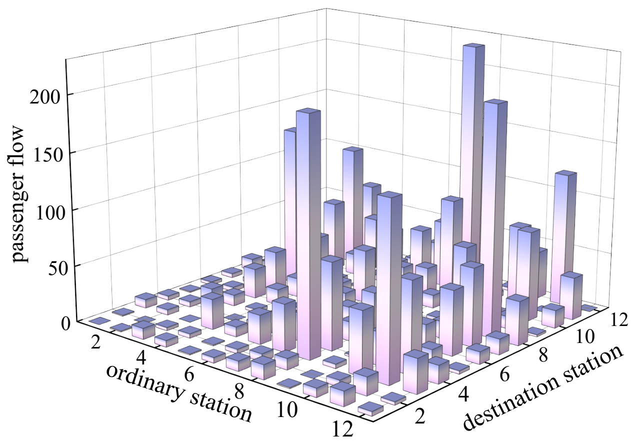


Fig.11 Peak passenger flow data chart



Fi.12 flat peak passenger flow data chart

TABLE III
KEY PARAMETERS

Parameter type	Parameter name	Parameter value
Line parameters	Stop time	0.5min
	Train Tracking Time	3min
	Maximum headway of transfer stations	15min
	Minimum headway of transfer stations	3min
	full load factor	0.8
	Train capacity	1860
Train parameters	Average train operating cost (CNY/train)	4000
	Train mileage cost (CNY/Km)	30
	Useful life of train underframe (years)	30
algorithmic parameter	evolutionary algebra	100
	σ	0.8
	η	0.88
	λ	0.2

As can be seen in Fig. 13, and 14, under the condition of algorithm convergence, there are 19 Pareto solutions. Taking one of these optimized plans compared to the original train operation plan, the optimized plan uses more three-composition trains and fewer six-composition trains. This provides flexibility in controlling high passenger flows in certain sections during peak periods through the use of smaller composition trains. It also better addresses time imbalances in passenger flows. Thus, although the total number of trains in the optimized train operation plan increases, the total number of trains used decreases. Consequently, the total number of trains used in the optimized train operation plan increases, but the total number of trains used decreases. At the same time, as can be seen from Fig. 15, The original train operation plan has an objective function value of 261,407.2, while the optimized plan has a value of 243,141.9. This reduces cost consumption by 6.99%, increasing the enterprise's interests. To a certain extent, using multiple-crewed trains reduces the enterprise's costs and passengers' travel time. However, it may also result in an increase in the number of trains during the initial period of operation. Changes in the initial stage of operation may reduce corporate benefits, but in the long term, they can optimize performance.

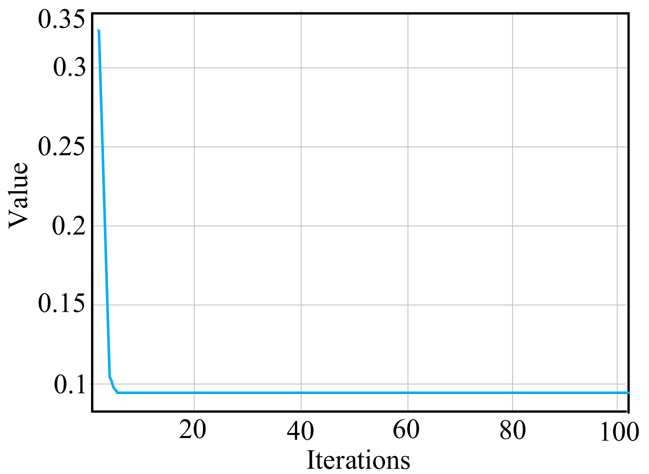


Fig.13 Line diagram and P2X-Y convergence curve for the small-scale case

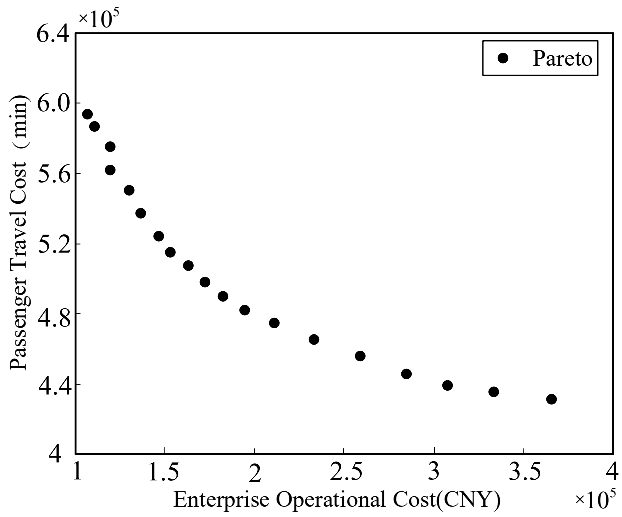


Fig.14 Iterative convergence curve

C. 5.3 Networkability analysis

One line is a tough way to accurately present the characteristics of the metro network, but the space-time network has significant advantages when it comes to characterizing some of these characteristics. A small-scale instance can be used to exhibit the network attributes in a road network, with certain components streamlined. The line network in the network incorporates four metro lines with four transfer stations, 12 intervals and 12 stations. We treat the different directions of a metro line separately, with each direction having its own unique characteristics. The ease with which lines can be changed by passengers is mainly looked at, as is how the number of passengers unable to board or disembark the train can be reduced.. A small-scale case circuit diagram and P2X-Y convergence curve are shown in Fig.16 and 17. Passenger flow variations are shown in Table 3. up and down line departure time optimization is shown in Fig.18.

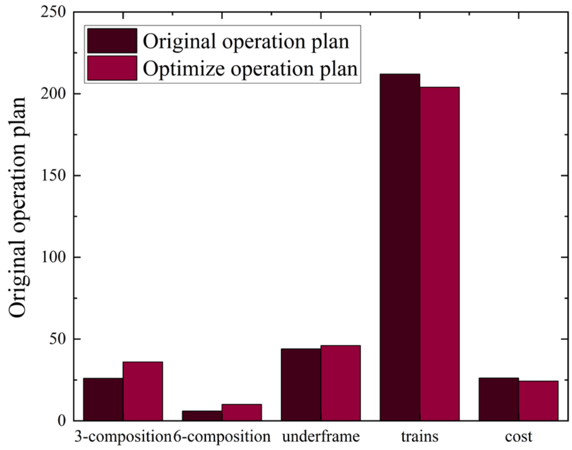


Fig.15 Comparison of optimization options

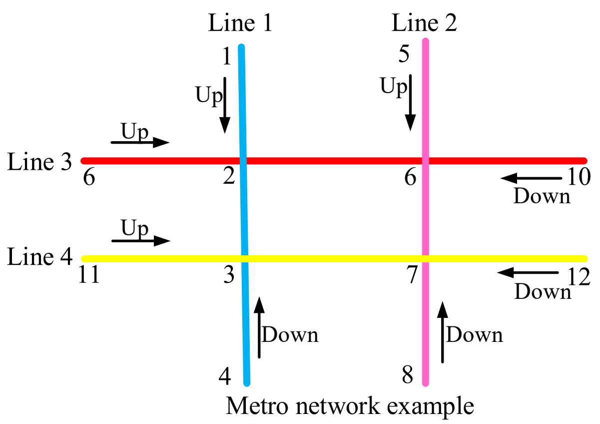


Fig.16 A small-scale case circuit diagram

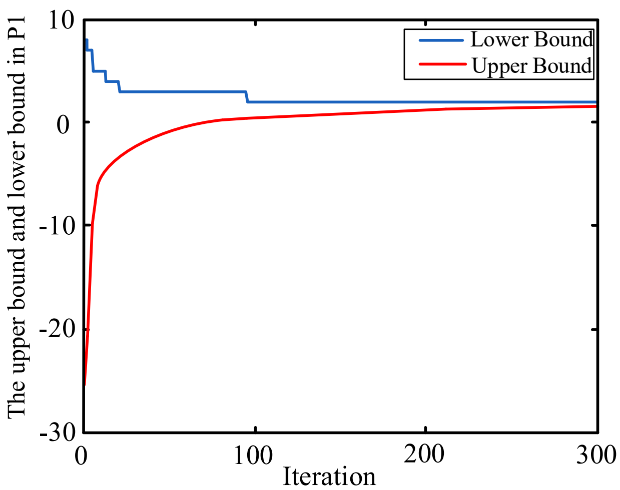


Fig.17 P2X-Y convergence curve for the small-scale case

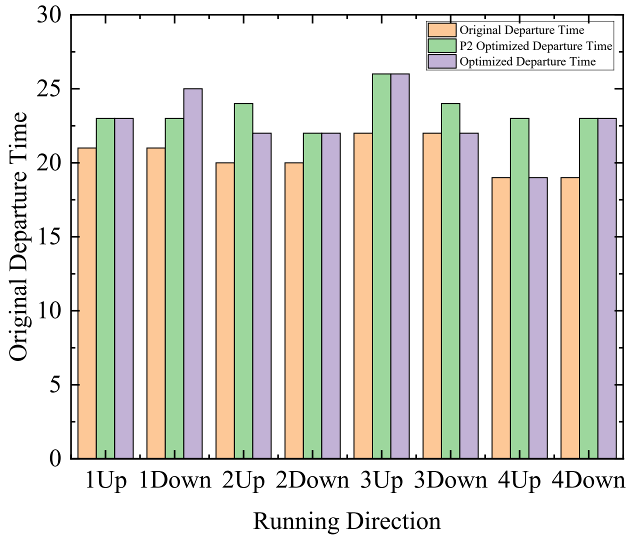


Fig.18 Optimization of departure times for upward and downward routes

As can be seen in Figures 16 and 17, based on the iterative curve, the algorithm satisfies convergence and the P2 model reaches the boundaries near 120 generations. The reachability can be adjusted to reduce supra-temporal arcs and improve passenger reachability. Compared with the original timetable, the evaluation metrics based on reachability can be greatly improved by the optimal solution provided by the two models' algorithms. As can be seen in Table 3, a 37% improvement in space-time accessibility is shown (from 30 to 41) and a decrease in the number of unreachable transfer passengers from 13 to 2 is shown. 3, it can be seen that the model improves accessibility based on flow by about 23% (from 6,580 to 8,120) and reduces the number of unreachable passenger flows from 1,810 to 270. An increase in the number of routes in the network leads to an increase in the complexity of the network and the number of passengers who make transfers. This results in an increase in the number of passengers who have an urgent need to travel. Some passengers are unable to complete their journey within the target time, which creates a situation where they cannot be contacted and are considered unreachable. This also results in passengers who make multiple transfers. Using a multi-composition operational organization mode can increase the accessibility of transfer trips in the network, which is conducive to meeting the demand for passenger transfers in networked operations. At the same time, it reduces some of the changes in non-reachable passenger flows becoming reachable, which alleviates the high passenger flow intensity during peak periods and reduces its impact on passengers travelling.

D. 5.4 Composition type parameters H analysis

The train composition type set R also has an important impact on the train running scheme. In order to better analyze the impact of the different train composition sets H, an additional 4-format train is added and then based on the feasible train composition types, which are H = {3}, H = {4}, H = {6}, H = {3,4}, H = {3,6}, H = {4,3}, and H = {3,4,6}, respectively. The number of available underframes for each composition type is set to a constant large enough to better analyze the effect of varying different

TABLE IV
CHANGES IN PASSENGER FLOW OF THE SMALL-SCALE CASE

Optimize objectives	Optimization comparison	Optimization result		
Changes in passenger accessibility	Before optimization (visits)	13	2	3
	Optimized (person-times)	30	41	3
Changes in passenger flow	Before optimization (visits)	6580	8030	8120
	Optimized (person-times)	1810	360	270

parameters in the model on the solution results. The influence of composition type on optimized plan is shown in Table V. Comparison of different composition type is shown in Fig.19

TABLE V
THE INFLUENCE OF COMPOSITION TYPE ON OPTIMIZED PLAN

Composition type	Optimization plan	Total cost/CNY
3- composition	no viable solution	
4- composition	no viable solution	
6- composition	1-12 5 pairs in 6 composition	369745.1
	10-12 2 pairs of 6 composition	
3, 4- composition	no viable solution	
3, 6- composition	1-12 6 pairs of 6 composition	351842.4
	10-12 3 pairs of 3 composition	
4., 6- composition	1-12 6 pairs of 6 composition	339253.7
	10-12 2 pairs of 4 composition	
3, 4, 6- composition	1-12 6 pairs of composition	339253.7
	10-12 2 pairs of 4 composition	

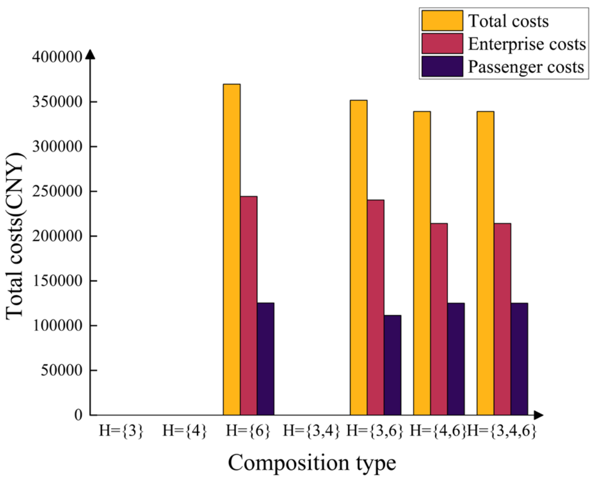


Fig.19 Comparison of different composition type

In the three cases H = {3}, H = {4}and H = {3,4}, there is no feasible solution. This is because in these three cases, even if the number of train pairs on the line is maximized, the capacity still cannot meet the passenger demand, so there is no feasible solution. Among the other cases, the total cost with passenger travel cost is highest at H = {6}. This is because for the same passenger demand, using only large train sets would mean that the number of train pairs is lower, which in turn leads to a higher waiting time cost for passengers and a higher total cost of the train program. At the same time the acquisition cost of the underframe will also increase the cost of the business will also increase. In contrast, the total cost of the train-running program is significantly lower with the addition of small-sized trains As can be seen from Fig. 19 the total cost of the H = {3,6} and H = {4,6} scenarios are reduced by 4.84% and 8.25%, respectively, compared to the total cost of the H = {6} scenario. This is because under the premise of meeting passenger demand and using as many small-sized trains as possible to increase the number of train pairs in each zone, the travel cost of passengers and the total cost of the train program are reduced. However, it should be noted that the introduction of small-sized trains in the set of optional train configurations does not necessarily reduce the total cost. H = {4,6} and H = {3,4,6} cases have the same total cost. It shows that the use of multiple train composition types can effectively reduce the total cost of the train-running scenario.

E. 5.5 Composition type parameters R analysis

Meanwhile, the number of route is also a great influencing factor for urban rail transit routes. The value of R is set to 1 to 6, respectively. As can be seen from Table VI, it can be seen that the total cost decreases with the increase of the number of routes R , among which the decrease of the transportation enterprise cost contributes the most to the decrease of the total cost. In addition, when the number of intersecting routes R is increased from 1 to 2, the total cost decreases most obviously, with a decrease of 18.94%, and then, the further increase of the number of intersecting routes R reduces the total cost, but with a smaller decrease or even no decrease at all. When R is 1, there is only one full major interchange on the line, passengers do not need to transfer, and the zones with lower passenger demand also have the same number of train pairs as the busy zones, so passenger travel costs are the lowest. In contrast, when multiple interchanges are operated on the line, passengers may have to change trains and the capacity and number of pairs of trains in the less-demanded zones are reduced, resulting in higher travel costs for passengers. Note that increasing the value of R does not necessarily reduce the total cost. As can be seen from Table VI, the total cost at R = 5 and 56 is the same. Consider that each route is subject to a minimum number of train pairs.

TABLE VI
THE INFLUENCE OF ROUTE TYPE ON OPTIMIZED PLAN

Route type	Optimization plan	Total cost/CNY
R =1	1-12 15 pairs of 6 composition	447621.4
	10-12 10 pairs of 3 composition	
R =2	1-12 12 pairs of 6 composition	362837.5
	10-12 9 pairs of 3 composition	
R =3	1-12 10 pairs in 6 composition	349621.2
	10-12 8 pairs of 3 composition	
R =4	1-12 9 pairs of 6 composition	336945.8
	10-12 8 pairs of 3 composition	
R =5	1-12 9 pairs of 6 composition	321753.2
	10-12 7 pairs of 3 composition	
R =6	1-12 9 pairs of 6 composition	3221753.2
	10-12 7 pairs of 3 composition	

Consequently, the introduction of an additional train crossing will result in a substantial increase in costs for the transportation company. Furthermore, the operation of an excessive number of train crossings on a given line constitutes a violation of the line's capacity constraints. These constraints are implemented with the objective of ensuring safe and efficient operations. It can thus be concluded that, even in the event of the maximum number of routes R being assigned a high value, the number of routes in the optimal train operation scheme will not be excessive. The findings indicate that the implementation of multiple crossings can serve as an effective strategy for achieving cost reductions.

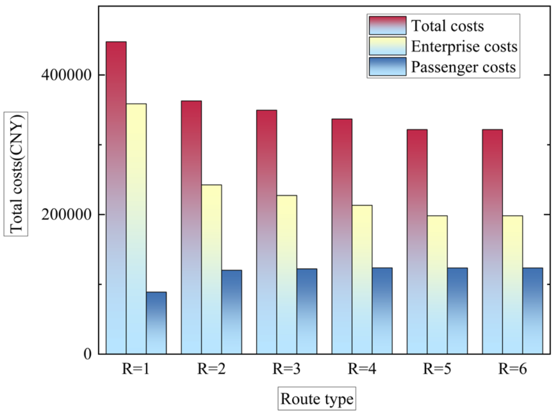


Fig.20 Comparison of different route type

VI. CONCLUSION

This paper applies space-time network theory as a research tool to analyze the optimization effect of the train running plan under the mode of adopting multiple compositions in comparison with the original design of the running plan, and at the same time analyses the network of the underground, and obtains the following conclusions.

A. Specifying transfer arc for passengers in space-time networks

In the space-time network, the passenger transfer arc is generally synonymous with the waiting arc. However, this paper, grounded in the objective realm of realism and the composition of the temporal cost of passenger travel, undertakes a multi-faceted consideration analysis. The passenger is regarded as a distinct composition layer, with the passenger's non-negligible waiting time being duly considered. In consideration of the aforementioned factors, the space-time network establishes a discrete transfer arc for passengers, with the objective of ensuring network realism. As illustrated by the representation of transfer nodes in different layers of the transfer, there is a phenomenon of node overlap. The construction of virtual transfer nodes is a proposed solution to enhance the comprehension of the transfer process across various composition layers. This approach is expected to facilitate a more accurate depiction of the transfer walk and waiting time.

B. Optimization analysis of multi-composition train operation plan

a) The present study proposes a multi-composition train departure optimization model based on a space-time network, and combines the La relaxation algorithm with the multi-component train departure plan optimization model to solve the case. It was determined that the underframe of 6-format trains underwent a reduction from 18 to 10, representing a 44% decrease, while the underframe of 3-format trains exhibited an increase from 26 to 36, indicative of a 38% rise. These modifications were implemented following a comparative analysis of the composition plan derived from the original running plan in the case and the enhanced composition plan, which yielded a series of satisfactory solutions. Nevertheless, the augmented deployment of small-format trains is anticipated to engender an escalation in the enterprise's initial operating expenditures. However, the operating time increases and the cost to the enterprise decreases from 261,407.2 to the objective function value of 243,141.9 for the current train running plan, which is 6.99% lower.

b) From the long-term analysis, multi-composition trains will bring higher corporate benefits because they can run more small-composition trains. During peak periods, additional small-composition trains can better meet passenger demand in large areas, while also reducing travel time for passengers. While the increase in small-composition trains can better meet the demand for passenger transfers, it also increases the operational pressure on turnback stations, which can cause cross-interference between operations during peak periods. This requires a more reasonable design of operational content and an optimal design of underframe scheduling for multi-composition trains.

C. Networkability of multi-composition under multiple routes

a) The space-time network is better at analyzing changes in line passengers' travel demand and changes in transfer flow under road network conditions because it is more difficult to characterize the network on a single line. In a multi-line network, there are different ways for passengers to travel on each line. The number of passenger transfer stations is also important. This creates a special travel arc. In a single line, the passengers' journey time is a fixed process and is relatively unique. The path choice is also relatively unique.

b) If the multi-line condition applies, the passenger OD path may require multiple transfers, which will increase transfer time. However, to meet the need to travel, the number of transfers can only increase. Passengers' travel psychology is affected by environmental factors such as congestion. This manifests as an irrational condition that affects arrival time. Some passenger flows will be infeasible, but designing a space-time network based on this can provide a more accurate portrayal of the travel process. This design can also more accurately portray the travel process for this part of the passenger flow. When combined with the small-scale case, applying space-time network theory improves space-time accessibility by around 37% (from 30 to 41) and reduces the number of unavailable transfer stations from 13 to two. The model also improves flow-based accessibility by around 23% (from 6,580 to 8,120) and reduces the number of unavailable flows from 1,810 to 270. Therefore, in multiline conditions, multiple trains can reduce unavailable flows and increase accessibility. This increases the number of non-viable and unreachable passenger flows while also increasing transfer diversity and travel convenience.

D. Composition type and route type analysis of multi-composition train operation plan

a) The utilizations of multi-composition trains, employing a variety of composition modes, has been demonstrated to engender disparities in the repercussions of the opening programs. It is anticipated that the financial expenditure pertaining to corporate costs will be reduced. Nevertheless, the utilizations of a diverse range of composition types have been demonstrated to engender a reduction in corporate expenditure. This is attributable to the line's capacity to return and the number of underframe limitations requirements. It is evident that utilizing three composition modes and two composition types will have a negligible impact. In order to achieve the desired composition, it is imperative to adhere to the specific conditions of the line and select the corresponding composition type.

b) Nevertheless, it is important to note that the number of crossings should not be set too high. Concomitantly, in practical implementation, the augmentation of the number of routes will engender an escalation in the operational intricacy and safety hazards of the line. It is therefore recommended that the operation department ascertain the number of routes in accordance with the operational level of the line.

In the future research, the passenger travel psychology will be considered. Passengers' finite rationality constraints, and then the multi-component train running plan will be investigated under different travel psychology.

REFERENCES

[1] X. Yang, C. Zhu, X. Wang, X. Ma, and L. Cheng, "Study on the train plan of the cross-line operation in the urban rail transit considering the multi- composition train," *IAENG International Journal of Applied Mathematics,* vol. 53, no. 4, pp. 1253-1265, 2023.

[2] Z. Li, J. Zhao, and Q. Peng, "Train service design in an urban rail transit line incorporating multiple service routes and multiple train compositions," *Transportation Research Part C: Emerging Technologies,* vol. 123, p. 102959, 2021.

[3] W. Li, S. Chen, W. Fan, and Q. Luo, "Optimization of Periodic Train Schedule with Flexible Train Composition during Off-Peak Periods for Urban Rail Transit," Journal of Transportation Engineering, Part A: Systems, vol. 150, no. 10, p. 04024057, 2024. Transit," *Journal of Transportation Engineering, Part A: Systems,* vol. 150, no. 10, p. 04024057, 2024.

[4] C. Zhu, X. Yang, Z. Wang, J. Fang, J. Wang, and L. Cheng, "Optimization for the Train Plan with Flexible Train Composition Considering Trainbon Emission," *Engineering Letters,* vol. 31, no. 2, pp. 562-573, 2023.

[5] T. Feng, R. M. Lusby, Y. Zhang, Q. Peng, P. Shang, and S. Tao, "An ADMM-based dual decomposition mechanism for integrating crew scheduling and rostering in an urban rail transit line," *Transportation Research Part C: Emerging Technologies,* vol. 149, p. 104081, 2023.

[6] Z. Feng, X. Li, J. Wu, and X. Chang, "Transport capacity optimization for high-speed rail network considering flexible train composition and additional capacity pool," *Incomposition Sciences,* vol. 680, p. 121175, 2024.

[7] L. Zhong, G. Xu, and W. Liu, "Energy-efficient and demand-driven train timetable optimization with a flexible train composition mode," *Energy,* vol. 305 , p. 132183, 2024.

[8] S. Zhuo, J. Miao, L. Meng, L. Yang, and P. Shang, "Demand-driven integrated train timetabling and rolling stock scheduling on urban rail transit line," *Transportmetrica A: Transport Science,* vol. 20, no. 3, p. 2181024, 2024.

[9] K. Zhao, Y. Wang, M. Ding, S. Li, E. Quaglietta, and L. Meng, "Train scheduling with flexible coupling and decoupling at stations for an urban rail transit line," in *2022 IEEE 25th International Conference on Intelligent Transportation Systems (ITSC)*, 2022: IEEE, pp. 1876-1881.

[10] P. Wang, X. Xiao, F. Wang, Y. Wang, and W. Zhu, "Rail transit express and local train schedule optimization considering hybrid rolling stock composition. " *International Journal of Rail Transportation,* vol. 12, no. 6, pp. 1104-1130, 2024.

[11] Y. Zhan, M. Ye, R. Zhang, S. He, and S. Ni, "Multi-objective optimization for through train service integrating train operation plan and type selection," *Transportation Letters,* vol. 16, no. 9, pp. 1039-1058, 2024.

[12] L. Lin, X. Meng, X. Cheng, Y. Ma, and S. Xu, "Train service plan design under the condition of multimodal rail transit systems integration and interconnection," *Journal of Advanced Transportation,* vol. 2023, no. 1, p. 8275191, 2023.

[13] A.-Q. Tian, X.-Y. Wang, H. Xu, J.-S. Pan, V. Snášel, and H.-X. Lv, "Multi-objective optimization model for railway heavy-haul traffic: Addressing trainbon emissions reduction and transport efficiency improvement," *Energy,* vol. 294, p. 130927, 2024.

[14] H. Zhou, J. Qi, L. Yang, J. Shi, H. Pan, and Y. Gao, "Joint optimization of train timetabling and rolling stock circulation planning: a novel flexible train composition mode," *Transportation Research Part B: Methodological,* vol. 162, pp. 352-385, 2022.

[15] L. Cadarso, Á. Marín, and G. Maróti, "Recovery of disruptions in rapid transit networks," *Transportation Research Part E: Logistics and Transportation Review,* vol. 53, pp. 15-33, 2013.

[16] Z. Tian, Z. Kong, H. Han, W. Wu, and J. Zeng, "Optimization of Train Operation Plan of Through Operation Between Urban Rail Transit and Suburban Railway," *IAENG International Journal of Applied Mathematics,* vol. 54, no. 12, pp. 2612-2619, 2024.

[17] Q. Dong, J. Tang, Y. Yuan, C. Li, and X. Shao, "Intelligent Train Scheduling Technology for Urban Rail Transit in Chinese Engineering: A Multi-Aspect Spatiotemporal Supply-Demand Equilibrium Theory," *Available at SSRN 5166161*.

[18] W. W. White and A. M. Bomberault, "A network algorithm for empty freight train allocation," *IBM Systems Journal,* vol. 8, no. 2, pp. 147-169, 1969.

[19] E. Wang, L. Yang, P. Li, C. Zhang, and Z. Gao, "Joint optimization of train scheduling and routing in a coupled multi-resolution space-time railway network," *Transportation Research Part C: Emerging Technologies,* vol. 147, p. 103994, 2023.

[20] S. Krygsman, M. Dijst, and T. Arentze, "Multimodal public transport: an analysis of travel time elements and the interconnectivity ratio," *Transport Policy,* vol. 11, no. 3, pp. 265-275, 2004.

[21] L. Cadarso and Á. Marín, "Improving robustness of rolling stock circulations in rapid transit networks," *Computers & Operations Research,* vol. 51, no. pp. 146-159, 2014.

[22] H. Pan, L. Yang, and Z. Liang, "Demand-oriented integration optimization of train timetabling and rolling stock circulation planning with flexible train compositions: A column-generation-based approach," *European Journal of Operational Research,* vol. 305, no. 1, pp. 184-206, 2023.

[23] H. Zheng, H. Sun, S. Zhu, L. Kang, and J. Wu, "Air traingo network planning and scheduling problem with minimum stay time: a matrix-based ALNS heuristic," *Transportation Research Part C: Emerging Technologies,* vol. 156, p. 104307, 2023.

[24] X. Bao, Q. Zhang, H. Yin, E. Wang, and Y. Xiao, "Integrated optimization of demand-oriented timetabling and rolling stock circulation planning with flexible train compositions and multiple service routes on urban rail lines," *Transportation Research Part C: Emerging Technologies,* vol. 174, p. 105071, 2025. 105071, 2025.

[25] Z. Liao, H. Li, J. Miao, and F. Corman, "Railway capacity estimation considering vehicle circulation: integrated timetable and trains scheduling on hybrid time-space networks," *Transportation Research Part C: Emerging Technologies,* vol. 124, p. 102961, 2021.

[26] Y. Wang, Z. Zhang, D. Huisman, A. D'Ariano, and J. Zhang, "A Lagrangian relaxation approach based on a time-space-state network for railway crew scheduling," *Computers & Industrial Engineering,* vol. 172, p. 108509, 2022.

[27] G. G. D. Santos, S. Birolini, and G. H. de Almeida Correia, "A space-time-energy flow-based integer programming model to design and operate a regional shared automated electric vehicle (SAEV) system and corresponding charging network," *Transportation Research Part C: Emerging Technologies,* vol. 147, p. 103997, 2023.

[28] Z. Yao *et al.* "Network periodic train timetabling with integrated stop planning and passenger routing: A periodic time-space network construct and ADMM algorithm," *Transportation Research Part C: Emerging Technologies,* vol. 153, p. 104201, 2023.

[29] Y. Qu and X. Zhou, "Large-scale dynamic transportation network simulation: a space-time-event parallel computing approach," *Transportation research part c: Emerging technologies,* vol. 75, pp. 1-16, 2017.

[30] L. Yang and X. Zhou, "Constraint reformulation and a Lagrangian relaxation-based solution algorithm for a least expected time path problem," *Transportation Research Part B: Methodological,* vol. 59, pp. 22-44, 2014.

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