Research on the Optimization of the Operation Plan for Suburban Railway and Urban Rail Trains under Cross-line Operation

Saibo Wang, Changfeng Zhu, Yunqi Fu, Jie Wang, Linna Cheng, Rongjie Kuang

Abstract—**The cross-line operation of suburban railway (SR) and urban rail transit (URT) is a key way to improve transportation efficiency and promote regional transportation integration. Based on the cross-line operation of SR and URT, this study introduces the event activity network theory, constructs a mixed logit model to quantify the passenger route choice behavior, and proposes the supply and demand capacity matching index (SDCIM), which is used to measure the matching degree of line capacity and passenger demand. On this basis, a bi-objective nonlinear integer programming model is further constructed with the objectives of minimizing SDCMI and the total costs of cross-line operation. The research results show that after implementing cross-line operation, SDCMI is reduced by 37.1%, and the total costs are reduced by 10.3%. This study presents a theoretical framework and provides practical guidance for decision-making in multi-level rail transit cross-line operations, taking into account both efficiency and cost.**

Index Terms—**Suburban railway, Urban rail transit, Cross-line operation, Event-activity network**

I. INTRODUCTION

WITH the development of metropolitan areas and urban agglomerations, the process of regional transportation integration has advanced. Building on this, some cities utilize existing surplus capacity in urban rail transit systems to organize cross-line operation of suburban railway trains. This approach combines the advantages of urban rail transit (URT) and suburban railway (SR), thereby enhancing transportation efficiency and optimizing resource allocation. However, while enabling rapid cross-line operation of trains, operators must also accommodate the intensive service demands of trains on the original line. Simultaneously, passengers face complex travel scenarios under cross-line operation and must make route selection decisions. Therefore, how to reasonably formulate the cross-line train operation scheme according to the complex operation mode and passenger demand has become an urgent problem to be solved.

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II. LITERATURE REVIEW

A. Cross-line operation

With the growth in demand for the integrated development of multi-mode rail transit networks, early research on cross-line operation schemes focused on optimizing the cross-line organization of rail transit lines at the same level. Wang [1] investigated strategies to maximize high-speed rail capacity utilization when introducing cross-line trains, providing theoretical support for the coordination of multi-level rail networks. Liang et al. [2] developed a multi-objective mixed-integer model of cross-line train operation to balance enterprise costs and passenger service level through the use of preference coefficients. Chen [3] developed an optimization model for cross-line train operation to balance vehicle usage costs, operational costs, and passenger transfer times. Yang et al. [4] developed a multi-objective programming model designed to minimize passenger travel costs and enterprise operational costs, and proposed an Improved Quantum Genetic Algorithm (IQGA) to solve the model. Zhao et al. [5] defined the passenger flow types under the "X" and "Y" line intersection modes. They constructed an optimization model for cross-line train operation, aiming to minimize passenger travel costs and enterprise operational costs while balancing the section's full load rate. Zhang [6] optimized timetables for cross-line routes under failure scenarios.

While studies as mentioned above have achieved notable progress in same-tier cross-line operations, research focus is shifting toward multi-level network coordination as demand grows for integrated rail systems. Li et al. [7] developed a mixed-integer nonlinear programming (MINLP) model for SR-URT cross-line operations targeting multi-stakeholder benefits. Peng et al. [8] developed an integrated service-planning and passenger-flow assignment model for SR-URT, optimizing cross-line routing through flexible stopping patterns.

Facing multiple alternative routes after cross-line operations, optimizing route choice and passenger flow assignment can improve operational efficiency and service quality. Yan [9] developed an optimization model for selecting cross-line train paths in high-speed rail systems. Yin et al. [10] proposed an optimization framework for the ART system that incorporates passenger path-choice behavior while minimizing the combined costs of operators and passengers. Tian [11] and Wang [122] emphasized improving direct connectivity by maximizing non-transfer passengers on cross-line services to optimize flow distribution. Chen [13] utilized spatio-temporal network diagrams to analyze passenger path choices, identifying transfer-induced sectional flow variations as critical factors in designing cross-line schemes. Sun [14] introduced the concepts of “effective” and “ineffective” travel time and applied α-fairness theory to optimize cross-line turnaround station layouts, thereby significantly improving travel equity. Huang [15] proposed a representative passenger path generation method for cross-line timetable optimization, demonstrating reduced transfer pressure and increased direct services.

B. Supply and demand capacity matching

In terms of transport capacity and volume matching, scholars have conducted systematic research on the dimensions of traffic organization, passenger flow distribution, and other related aspects.

Dynamic adjustment of train operation plans serves as the primary mechanism for matching capacity and demand. In this regard, Xu et al. [16][17] considered the time-varying demand of passenger flow and the phenomenon of passenger retention in peak hours. They developed a large and small routing schedule optimization model, as well as a double-layer planning model, for full and short routing within cross-line operations to coordinate the departure interval and the train marshaling scale. Guo et al. [18] further proposed a Y-shaped shared-track trunk-branch joint operation scheme, which reduces enterprise operation costs while shortening passenger waiting time, providing a novel approach for capacity-demand matching in complex rail network structures. In recent research, Huo et al. [19] introduced an integer programming model to optimize departure ratios and headways for cross-line routes in a dynamic manner. Yan [20] refined capacity-demand matching to the service-route level through a two-level planning model, enhancing spatial flexibility in cross-line scheduling. While these studies advance static scenario solutions, coordinated scheduling of multiple routes under fully networked operations requires further investigation.

Aiming to address the non-uniform characteristics of cross-line passenger flow in time and space, Wang et al. [21] developed a time-segmented nonlinear programming model to optimize the express/local service ratios on SR lines, thereby enhancing capacity-demand alignment. Zhang et al. [22] proposed a heterogeneous network timetable model that dynamically adjusts service frequencies based on boarding/alighting volumes, reducing station load factors. Huan et al. [23] further demonstrated how network-coordinated control models can alleviate bottleneck congestion through cross-line flow regulation. These studies confirm that incorporating dynamic demand into schedule optimization improves capacity adaptation efficiency under cross-line scenarios.

Accurate passenger-flow assignment and balance evaluation form the foundation of capacity-matching optimization. Wei et al. [24] developed a spatio-temporal path inference system using AFC data and timetables to reconstruct passenger trip chains. For balance assessment, Huang et al. [25] innovatively applied the Gini coefficient and Theil index, revealing the need for differentiated capacity allocation to address spatio-temporal heterogeneity. Jian et al. [26] integrated genetic algorithms to optimize cross-line service plans, reducing peak-period service costs while improving load-factor balance across routes. Wu et al. [27] proposed an SSAPSO-based freight-bus planning model with static/dynamic stages, though its application to complex passenger services remains unexplored.

C. Research gap and significant contributions

In summary, research on the cross-line operation of trains has accumulated to a certain extent. However, studies addressing cross-line operation between SR and URT remain limited. Research on capacity-volume matching following the cross-line operation predominantly examines scenarios on a single line or at a single service level， rarely integrating line capacity-volume matching with cross-line train operation schemes at different levels. Based on this, this paper takes the cross-line operation of URT and SR as the research object, characterizes the cross-line operation process of trains by combining an event-activity network, and quantifies passengers’ path choice behavior using a mixed logit model. In response to the problem of matching capacity and volume after the cross-line operation of trains, a supply-demand capacity matching index (SDCMI) is introduced. An optimization model for cross-line operation schemes is constructed to minimize the total costs of cross-line operation and the SDCMI. The model fully considers the unbalanced interval strategies and comprehensively optimizes the train service frequency and stop plan of each route. Finally, the validity and feasibility of the proposed model are verified through a numerical example, providing theoretical support for cross-line operation decision-making in the context of multi-network integration.

III. Problem Description

Define the set of alternative train operation routes on the line as $\Phi = \{\phi | \phi_1, \phi_2, \cdots, \phi_k\}$. Each route ϕ is defined by its set of potential stopping stations. The k-th operation route, denoted as $\phi_k = \{s_i^k, \cdots, s_j^k\}$, originates from station s_i^k and terminates at station s_j^k, with an operating frequency f_ϕ. Let S denote the set of all stations on the line, and X represent the set of available train units. Same-line trips (SLT) on the URT are denoted as $\phi_1 = \{s_1, \cdots s_n\}$ .SLT on the SR are denoted as $\phi_2 = \{s_o, \cdots, s_m\}$, and cross-line trips (CLT) as $\phi_3 = \{s_l, \cdots, s_o, \cdots, s_p\}$. Routes ϕ_1 and ϕ_2 connect at interchange stations s_o, with frequencies f_1, f_2, and f_3 $\chi(s)$ is a binary variable equal to 1 if the train stops at station s, and 0 otherwise. Fig. 1 illustrates the schematic diagram of cross-line operation between SR and URT.

To model the coordinated operation of multi-tier trains and complex passenger travel behavior in cross-line service, an event–activity network is constructed to represent the train operation process.[28] The network $G=(V, E, T)$ comprises a set of event nodes V, a set of activity arcs E, and time attributes T. Each event node $v \in V$ denotes a state of a train or passenger, covering key events such as departure, arrival, and turnback. Each activity arc $e \in E$ describes a transition between states, including train movement, dwell, passenger ticketing, waiting, or transferring actions. The time attributes T accurately quantifies the time parameters and activity duration of each event node. Detailed symbols and their definitions are listed in Table 1.

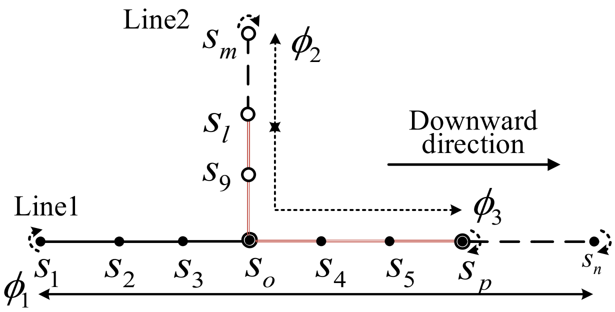

Fig. 1. Schematic diagram of cross-line operation between SR and URT.

TABLE I
SYMBOLS RELATED TO THE EVENT-ACTIVITY NETWORK

Symbol	Definition
$t_d(s)$	Arrival event time, i.e., the time when a train arrives at the station s, $s \in S$
$t_f(s)$	Departure event time, i.e., the time when a train departs from the station s, $s \in S$
$t_s(s)$	Dwell time activity, i.e., the duration a train stops at the station s, $s \in S$
$t_w(s)$	Passenger waiting activity time at the station s
t_r	Passenger onboard activity time
t_h	Passenger transfer activity time
Y_1	Cost of purchasing a URT ticket
Y_2	Cost of purchasing an SR ticket

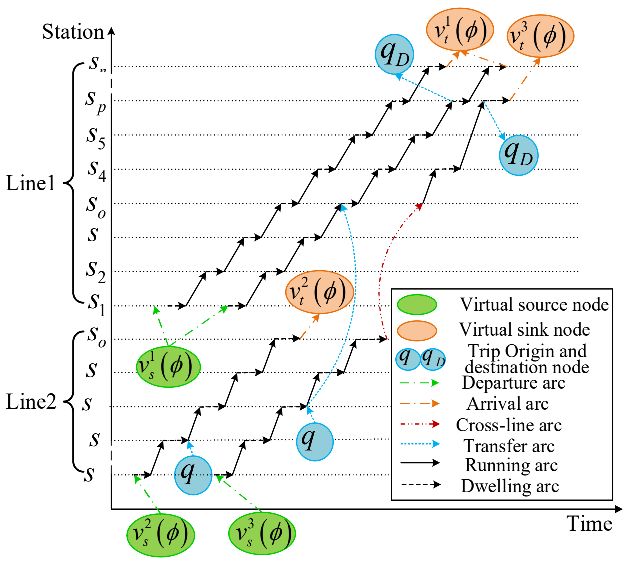

Fig. 2. Schematic diagram of cross-line operation between SR and URT.

To characterize each operation route, a virtual source event $v_s^k(\phi)$ is defined to represent the initial state of a route ϕ, connecting to its origin station. Similarly, a virtual sink event $v_t^k(\phi)$ is defined to represent the terminal state of a route ϕ, linking to its terminus station. A pair of origin-destination events (q_O, q_D) thus represents one feasible travel option for a passenger between those endpoints. The construction of this event–activity network is illustrated in Fig. 2.

IV. PASSENGER FLOW ASSIGNMENT

Existing studies frequently employ the "optimal path" concept to model passenger choice behavior, where passengers select the route with minimal generalized travel costs among feasible alternatives. Let $\Omega_{OD}=\{o_1,o_2,\cdots,o_r\}$ denote the set of feasible paths from origin O to destination D. For any OD pair, the utility $V_{\iota\tau}^r$ of a passenger ι choosing a path o_r under the scenario τ is expressed by the passenger's travel costs C_t. Therefore, the utility $V_{\iota\tau}^r$ is as follows:

$$V_{\iota\tau}^r=-C_t \tag{1}$$

The passenger's travel costs C_t comprise time costs from travel activities and fare differences across rail modes. Time components are calculated as follows.

Assuming uniform passenger arrival distributions at stations, the waiting time at the station s depends on the train departure frequency. Let f_ϕ be the operating frequency of the route ϕ. Therefore, the passenger's waiting activity time t_w at the station s is as follows:

$$t_w=\sum_{\phi\in\Phi}\frac{1}{2}\cdot\frac{60}{f_\phi} \tag{2}$$

The running time of a train within the activity arc e is related to the train's stoppage at the front and rear stations of the section. Therefore, for any train running within the activity arc e, the train's running time t_e is as follows:

$$t_e=\frac{l_e}{v_e}+\chi(s)\cdot\frac{v_e}{2a}+\chi(s+1)\cdot\frac{v_e}{2b},\forall s\in S \tag{3}$$

Where, l_e is the line length of the activity arc e, v_e is the average running speed of the train on the activity arc e, a, and b are the acceleration when the train starts and brakes, respectively. Then, the in-train time t_r of a passenger ι is as follows:

$$t_r=\sum_{e\in E}\sum_{\phi\in\Phi_e}\chi_e^q\cdot t_e(\phi) \tag{4}$$

Where, $t_e(\phi)$ is the running time of the train on the activity arc e within the route ϕ.

The passenger transfer activity time includes the walking time for the transfer and the waiting time. Let the average walking time for transfer be t_{walk}. Then the passenger transfer time t_{tr} is as follows:

$$t_{tr}=t_{walk}+t_w \tag{5}$$

Let γ_1 and γ_2 be the basic fares of URT and SR, respectively; α_1 and α_2 be the per capita cost per kilometer of URT and SR, respectively. Then, the cost for passengers to purchase tickets for different trains is as follows:

$$Y_1 = \gamma_1 + \alpha_1 \cdot \sum_{e \in E} l_e \tag{6}$$

$$Y_2 = \gamma_2 + \alpha_2 \cdot \sum_{e \in E} l_e \tag{7}$$

Where, E_q is the passenger activity set, θ is the value of passenger time. Then the generalized travel time costs C_t for a passenger ι is as follows:

$$C_t = \sum_{e \in E_q} \chi_e^q \cdot \left[\left(t_w + t_r + t_{tr} \cdot \varpi \right) \cdot \theta + Y_1 + Y_2 \right] \tag{8}$$

The utility $U_{\iota\tau}^r$ for an individual ι choosing the path o_r under the scenario τ is as follows:

$$U_{\iota\tau}^r = V_{\iota\tau}^r + \mu_\iota^r + \kappa_{\iota\tau}^r \tag{9}$$

Where, μ_ι^r is a random term, which follows a normal distribution among individuals with a mean of 0; $\kappa_{\iota\tau}^r$ is a random term with an IID extreme value distribution.

For the path o_r under the scenario τ, the choice probability of an individual ι can be allocated using the mixed logit model [29]

$$L_r = \prod_{\tau=1}^{T} \left[\frac{\exp\left(V_{\iota\tau}^r + \mu_\iota^r \right)}{\sum_{o_r \in \Omega_{OD}} \exp\left(V_{\iota\tau}^r + \mu_\iota^r \right)} \right] \tag{10}$$

Let $f\left(\mu \middle| \Omega\right)$ be the probability density function of μ. Let Ω be a fixed parameter. The choice probability P_τ^r of the mixed logit model is the integral of L_τ^r over the distribution of the random term $\mu_{\iota\tau}$

$$P_\tau^r = \int L_\tau^r(\mu) f\left(\mu \middle| \Omega\right) d\mu \tag{11}$$

Define the set of passenger flows $Q_e = \left\{ q_e^r \middle| r = 1,2,3,\cdots \right\}$ on each activity arc *e* of the route, and the passenger flow q_e^r choosing the path o_r under different scenarios can be obtained as

$$q_e^r = \left\lfloor Q_e \cdot P_\tau^r \right\rfloor \tag{12}$$

V. Model Construction

A. Assumptions

(1) The technical conditions for train crossing between lines have been met, and facilities such as signals, vehicles, communications, power supply, and line boundaries have been compatible or unified.

(2) Without considering the complex situation of multiple shared stations, it is assumed that there is a unique connection transfer station between the lines.

(3) The models and the number of train formations of SR trains and URT trains are the same, and both adopt a 6-car formation.

(4) Assuming that the operation of the up and down trains is consistent, only the optimization of the operation frequency of the unidirectional trains is studied.

B. Objective function

The primary objective of operational planning is to optimize the alignment between transportation capacity supply and passenger demand. Effective demand-capacity matching prevents both resource underutilization and service shortages. To ensure maximum matching of transportation capacity and volume while comprehensively considering the interests of passengers and operating enterprises, both the matching of line capacity and volume and the total costs of cross-line operation are selected as the dual objective functions for optimization.

(1) Minimize the supply-demand capacity matching index

The supply-demand level of the cross-line operation system can be represented by constructing the supply-demand capacity matching index (SDCMI). The SDCMI quantifies the equilibrium between route capacity and passenger demand in cross-line operations. To eliminate scale effects from absolute capacity differences, we construct this index using relative deviations through dimensionless normalization.

Let *B* denote the number of train formations. *Z* represents the train's capacity. $\beta_{\max}$ is the maximum load factor of the cross-section. Φ_e is the set of all train running routes in the active arc *e*. χ_e^q is a 0-1 variable of passenger flow q in the active arc *e*; if the route ϕ includes arc e, take 1. The road transportation capacity is as follows:

$$H_e = \sum_{\phi \in \Phi_e} \left(BZ\beta_{\max} \cdot f_\phi \cdot \chi_e^q \right) \tag{13}$$

Define the cross-sectional passenger flow of the active arc *e* in the network as Q_e. Let the SDCMI be recorded as Z_1, and minimize the SDCMI.

$$\min Z_1 = \sum_{e \in E} \frac{\left| H_e - Q_e \right|}{\max\left(H_e, Q_e \right)} \tag{14}$$

Where, $Z_1 \in [0,1]$, the value of 0 indicates perfect capacity-demand alignment, while 1 signifies complete imbalance. Lower index values reflect better supply-demand matching.

(2) Minimize the Total Costs of Cross-Line Operation

The total costs comprise passenger travel costs and operator costs. Operator costs include train operation costs and additional stopping costs. Train operation costs depend on service frequency, train-kilometers, and per-kilometer operational expenses. The train operation cost C_{op} can be calculated as

$$C_{op} = 2 \cdot \sum_{\phi \in \Phi} \sum_{e \in E} l_e \cdot f_\phi \cdot c_l \tag{15}$$

Where, c_l denotes the per-kilometer operational cost.

The extra stop cost represents the additional expense incurred by the cross-line train stopping at the station, which includes the increased energy consumption resulting from the frequent acceleration and deceleration required for these stops. Let C_{st} represent the additional stop cost, then C_{st} is calculated as

$$C_{st} = 2 \cdot f_3 \cdot \sum_{s \in \phi_3} \chi(s) \cdot c_s, \forall s \in S \tag{16}$$

Where, c_s represents the average cost per stop.

Let Z_2 denote the comprehensive cross-line operation total costs, with q_e representing the passenger flow on activity arc *e*. The minimized total costs are expressed as

$$\min Z_2 = \sum_{q_e \in Q_e} q_e \cdot C_t + C_{op} + C_{st} \tag{17}$$

C. Constraint Conditions

To ensure the service level of cross-line operation and the feasibility of cross-line operation plans, the basic constraints of cross-line operation should include service level constraints and cross-line capacity constraints.

(1) Basic constraints of cross-line operation

To ensure the cross-line service level, the maximum load factor within each area should be set within a reasonable range to avoid excessive congestion and waste of resources. When the waiting time exceeds the passengers' maximum acceptable waiting time, it will affect the travel level of passengers. Let $q_{e,\phi}$ be the passenger flow of the active arc *e* on the route ϕ, then the full load factor constraint of each route is as follows:

$$\beta_{\min} \le \sum_{e \in E} \frac{q_{e,\phi}}{f_\phi BZ} \le \beta_{\max},\ q_{e,\phi} \in Q_e \tag{18}$$

The waiting time limit for passengers is as follows:

$$\frac{30}{f_\phi} \le w_{\max}, \forall \phi \in \Phi \tag{19}$$

Where, $w_{\max}$ is the longest waiting time for passengers.

To ensure the feasibility of the cross-line operation plan, the operation frequencies of SR trains and URT trains in each section and at the turnaround stations in the operation plan must comply with the infrastructure capacity limitations. The constraint is as follows:

$$f_\phi \le C_\phi, \forall \phi \in \Phi \tag{20}$$

$$f_\phi \in N^+, \forall \phi \in \Phi \tag{21}$$

$$\chi(s_i) = \chi(s_j) = \chi(s_o) = 1 \tag{22}$$

Where, Equation(20) is the constraint of the line's passing capacity. Equation(21) is the integer constraint of the opening frequency. Equation(22) is the constraint on stopping at the first and last stations and transfer stations.

(2) Unbalanced departure interval constraints

The non-stop passage of cross-line trains will lead to an increase in the departure interval. However, adopting an unbalanced departure strategy allows for interval adjustments while satisfying minimum separation requirements, enhancing operational flexibility, and mitigating the impact on main-line capacity.

The departure intervals of trains at the starting station are classified into three types: the departure interval TF_{B-A} between a train on the same line ahead and a cross-line train behind; the departure interval TF_{A-B} between a cross-line train ahead and a train on the same line behind; and the departure interval TF_{x-x} between trains of the same type. The schematic diagram of the unbalanced departure mode is shown in Fig. 3.

The intervals TF_{B-A}, TF_{A-B}, and TF_{x-x} are constrained as follows:

$$TF_{\min} \le TF_{B-A} = \frac{60}{f_3} + \Delta TF \le TF_{\max} \tag{23}$$

$$\begin{aligned} TF_{\min} &\le TF_{A-B} = 60 - TF_{B-A} \\ &-\left(f_k - f_3 + 1\right) \cdot \frac{60}{f_3} \le TF_{\max}, k = 1,2 \end{aligned} \tag{24}$$

$$TF_{\min} \le TF_{x-x} = \frac{60}{f_3} \le TF_{\max} \tag{25}$$

Where, $TF_{\min}$ and $TF_{\max}$ denote the minimum and maximum allowable departure intervals, respectively, and ΔTF represents the additional interval time required for non-stopping cross-line trains.

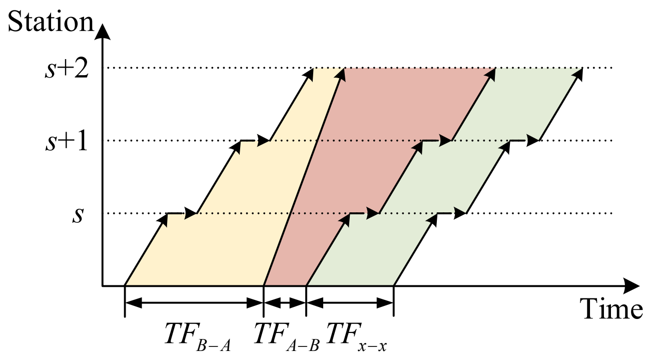

Fig. 3. Schematic diagram of the unbalanced departure mode.

(3) Events-Activity Constraints

Within the event-activity network, each train activity or event permits at most one train flow route. Any train event node on each route must satisfy the train flow balance constraint. Let E_v^+ and E_v^- denote the train activity sets of the inflow event *v* and the outflow event *v* respectively. The event-activity constraints are expressed as

$$\sum_{\phi=1}^{3} y_e^\phi \le 1, \forall e \in E \tag{26}$$

$$\sum_{e \in E_v^-} \sum_{\phi=1}^{3} y_e^\phi \le 1, \forall e \in E, \forall v \in V \tag{27}$$

$$\sum_{e \in E_v^- \cap E^\phi} y_e^\phi = \sum_{e \in E_v^+ \cap E^\phi} y_e^\phi, \forall e \in E, \forall v \in V, \phi \in \Phi \tag{28}$$

Where, y_e^ϕ is a 0-1 variable. If the train starts at the route ϕ, it is 1; otherwise, it is 0. E^ϕ is the collection of train activities at the route ϕ.

VI. Algorithm Design

The model above is essentially a large-scale, nonlinear, multi-objective integer programming problem. Considering that the Particle Swarm Optimization (PSO) algorithm is prone to falling into local optima when dealing with high-dimensional, complex issues and has insufficient global search ability, the chaotic concept is introduced into PSO to construct the Chaotic Particle Swarm Optimization (CPSO) algorithm, thereby improving the global optimization ability of PSO. The algorithm steps for solving the chaotic particle swarm model are as follows.

Step 1: Initialize particle swarm size *N*. Define candidate solutions $X_{solution} = f_1, f_2, f_3, n_{stop}$, which respectively represent the departure frequency of URT trains, the operation frequency of SLT trains, the operation frequency of CLT trains, and the stop schemes.

Step 2: Chaotic sequences are generated by using logistic chaotic mapping. Map the chaotic variables to the velocity and position value ranges, and initialize the velocity and position of the particles.

Step 3: Update the velocity and position of the particle h based on its current position and historical optimal position.

Step 4: For each particle, construct a passenger flow distribution model, compute objective function values, calculate the new fitness value of each particle, and conduct particle fitness evaluation.

Step 5: To prevent premature convergence of the algorithm, chaotic perturbations are introduced to a subset of particles at regular intervals, forcing them out of local optimal regions. The dynamic disturbance $V_h(d+1)$ can be expressed as follows:

$$V_h(d+1) = \alpha \cdot V_h(d) + c_1 \cdot r_1 \cdot \left(P_{Best_h} - X_h(d)\right) + c_2 \cdot r_2 \cdot \left(G_{Best} - X_h(d)\right) + \partial \cdot Chaos(d) \tag{29}$$

$$X_h(d+1) = X_h(d) + V_h(d+1) \tag{30}$$

Where, d is the number of iterations, α is the inertia weight, c_1 and c_2 are the learning factors, r_1 and r_2 are random numbers, $Chaos(d)$ is the perturbation term selected from the chaotic sequence; ∂ is the disturbance intensity coefficient.

Step 6: Check whether the maximum number of iterations has been reached or whether other convergence conditions have been met. If satisfied, the algorithm ends. Otherwise, return to Step 3 to continue the iteration.

Step 7: Output the global optimal position G_{Best} and the corresponding fitness value as the final solution of the algorithm.

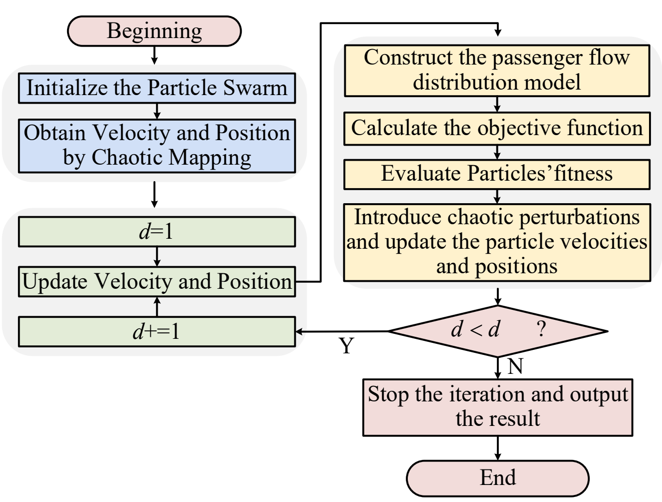

Fig. 4. Algorithm process.

To summarize, the process of the CPSO algorithm considering passenger flow distribution is shown in Fig. 4.

VII. Case Analysis

To verify the validity and rationality of the proposed model and algorithm, a case study was conducted on two Y-shaped interconnected rail lines implementing cross-line operations during the morning rush hour (8:00–9:00). Line 1 is a URT line, while Line 2 is an SR line. The line consists of 16 operating stations and runs 6B formation trains with a rated passenger capacity of 1,460 people per train. The minimum load factor is 0.5, the maximum load factor is 0.9, and the minimum departure interval is 2 min. The cross-line train runs from Line 2 to Line 1, and CLT includes 12 stations, which are set $\phi_3 = \{P, \cdots, M, C, \cdots I, J\}$. Among them, stations A, C, J, L and P are turnback stations. The operating cost of the train per kilometer is 60 CNY/km, and the cost of a single stop is 200 CNY/time. Trains require 2 minutes for turnaround operations at terminal stations. The walking time for transfer passengers is 0.5 min. The minimum departure frequency is 5 pairs per hour. The designed line capacity is 30 pairs per hour. The value of passenger time is 0.6 CNY/min. The base fare for URT is 2 CNY; for SR, it is 4 CNY. Detailed train dwell times and inter-station distances are provided in Tables 2 and 3.

TABLE 2
PARAMETERS OF ROUTE 1

Station	Stop time/s	Distance from the next station/km
A	30	2.50
B	25	1.06
C	25	1.88
D	25	2.43
E	30	3.99
F	30	4.37
G	25	5.34
H	25	4.49
I	40	3.64
J	30	0.96
K	30	2.33
L	30	--

TABLE 3
PARAMETERS OF ROUTE 2

Station	Stop time/s	Distance from the next station/km
C	35	9.89
M	30	8.52
N	40	10.05
O	30	7.18
P	30	--

Given the initial OD passenger flow data, the spatial distribution characteristics of passenger flow were uncovered by developing a heat map of the OD passenger flow data, as depicted in Fig. 5. $d < d_{\max}$?

A. Result Analysis

The proposed algorithm was implemented using the Python programming environment. The model parameters were derived from the given operation data and existing research, after which the model was solved. According to the relevant research in reference [30], the parameter values are shown in Table 4. Ten high-quality solutions were uniformly selected from the Pareto solution set obtained. The algorithm iteration curve and the Pareto optimal front comprising these ten Pareto solutions are presented in Fig. 6 and Fig. 7.

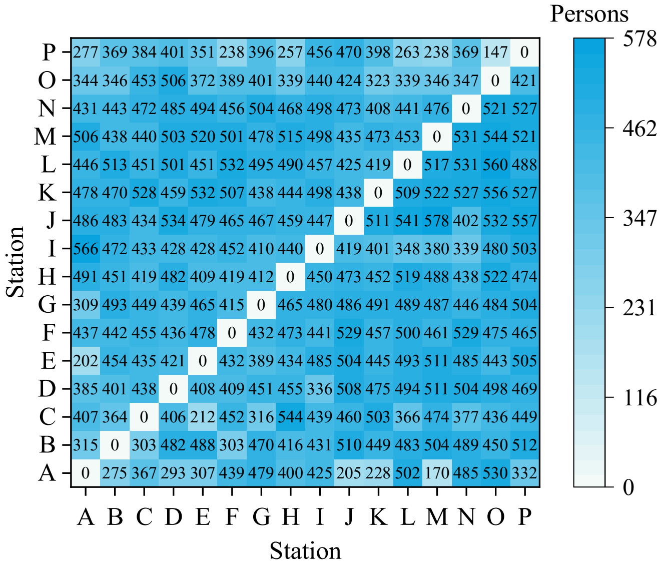

Fig. 5. OD passenger flow data.

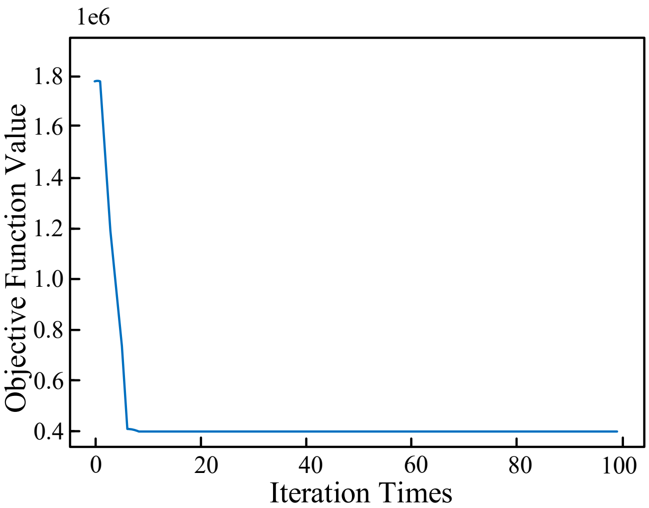

Fig. 6. Algorithm convergence curve.

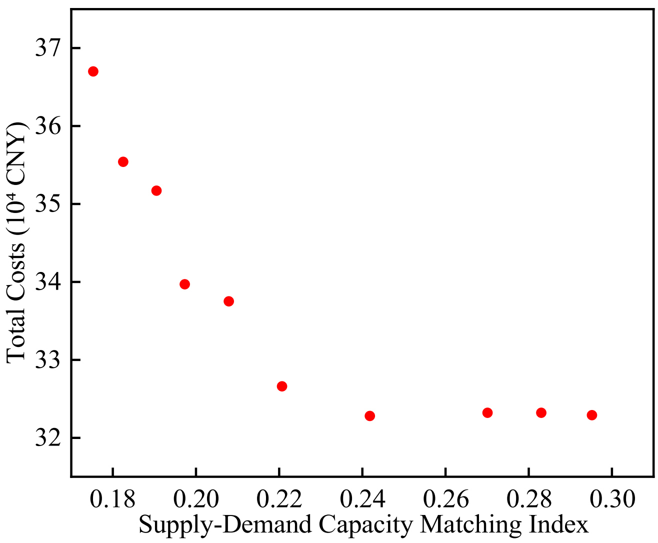

Fig. 7. Pareto frontier solution set.

Fig. 6 indicates that the algorithm converges stably after 20 iterations, verifying its convergence characteristics, which suggests that the algorithm can effectively approach the optimal solution and achieve convergence to the target value during iterations. The Pareto solution set in Fig. 7 exhibits a typical objective trade-off relationship inherent in multi-objective optimization. SDCMI optimization is often accompanied by an increase in total costs, and vice versa, highlighting the inherent conflicts among the objective functions.

TABLE 4
PARAMETER VALUES

Parameters	value
Particle quantity	100
Maximum number of iterations	100
Decision variable dimension	3
Chaotic sequence length	100
Inertia weight	0.9
Individual learning factor	2
Social learning factor	2

B. Selection of the Pareto front optimal solution

Considering scheme diversity and the trade-off relationships among objectives, multiple solutions at different positions were separately chosen for comparison. Schemes with minimum objective values and balanced schemes with the objective value were selected to represent the Pareto solution set. The points on the left, middle, and right in Fig. 8 serve as the primary chosen solutions.

Fig. 8 shows that the total costs decrease from left to right while the SDCMI increases. This trend persists to the far right. When optimizing solely for SDCMI, the CLT train stop scheme P-O-N-M-D-E-F-G-H-J yields the lowest, which is 0.1668. At this point, the total cost is 367,400 CNY. The optimal operation scheme is shown in Fig. 8.

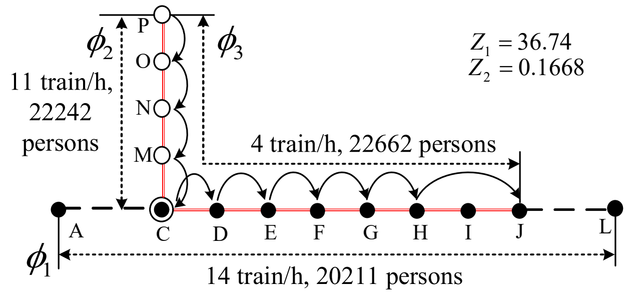

Fig. 8. Train operation plan with the smallest SDCMI.

When optimizing solely for total costs, the cross-line train stop scheme P-O-N-M-D-E-J achieves the lowest total cost for the entire line at 321,200 CNY. At this point, the SDCMI is 0.3306. The optimal operation scheme is shown in Fig. 9.

When optimizing for both SDCMI and total costs balance, the CLT train stop scheme P-O-N-M-C-D-F-J achieves a total cost of 323,200 CNY with an SDCMI of 0.2418. The optimal operation scheme is shown in Fig. 10.

The three optimization schemes reveal that extreme optimization of any one target would lead to a significant deterioration of the other target. The balanced value is critical to resolving this conflict.

TABLE 5
OPTIMIZATION RESULTS

Route	Operation Frequency (Trains per Hour)	Stopping Scheme	Passenger Flow (Persons)	Z_1	Z_2 (10^4 CNY)
Urban Rail Route	16	All stations Stopping	21354		
Suburban Route	12	All stations Stopping	22242	0.2418	32.32
Cross-line Route	9	P-O-N-M-C-D-F-J	18369		

Therefore, taking the balance between SDCMI and total costs as the optimization objectives, the Pareto solution set is obtained to obtain the optimal scheme. The values of each index are shown in Table 5.

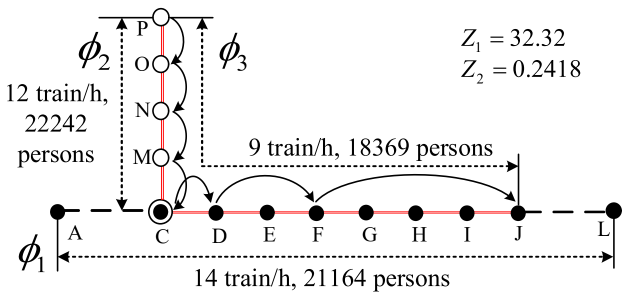

Fig. 9. Train operation plan with the lowest total cost.

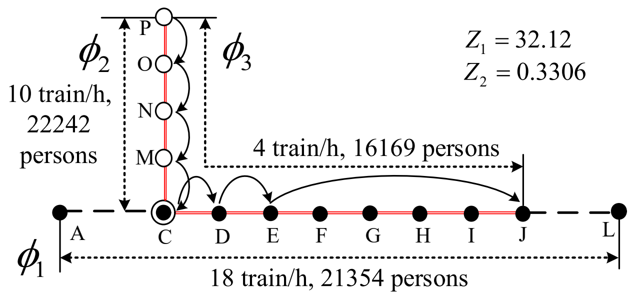

Fig. 10. Train operation plan when the SDCMI and the total cost are balanced.

C. Comparative analysis before and after the operation of cross-line trains

The introduction of cross-line trains may significantly impact the frequency of train service, total costs, and SDCMI of the original line. To explore its impact on the operational efficiency of the original line, a comparative analysis of the train operation frequencies before and after cross-line operation was conducted, as shown in Fig. 11.

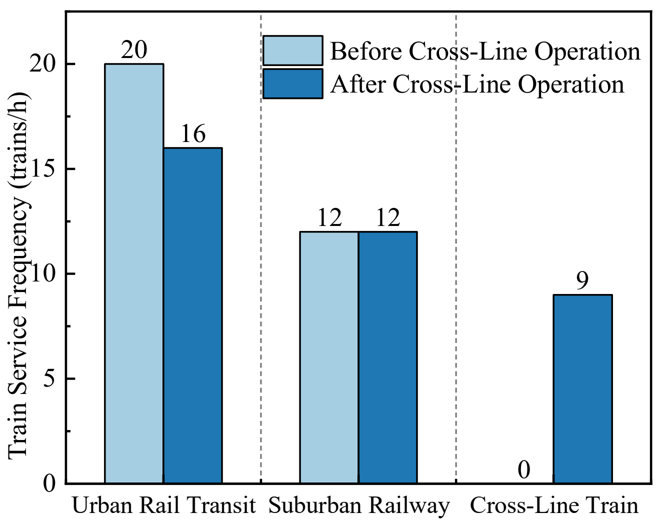

Fig. 11. Train service frequency before and after cross-line operation.

As shown in Fig. 11, cross-line operations have introduced 9 trains per hour where none previously existed. These CLT trains now partially absorb passenger demand originally served by SLT trains, alleviating demand pressure on the latter. However, CLT trains also occupy a portion of track resources in shared sections of the URT line. Consequently, the operation frequency of URT's SLT trains has been adjusted downward from 20 to 16 trains per hour, a 20% reduction. In contrast, SR's SLT trains maintain a stable frequency of 12 trains per hour. This stability stems from CLT trains attracting long-distance passengers through regional connectivity, leaving short-distance demand relatively unaffected. Passenger demand on routes served by SLT trains remains unchanged mainly, preserving their original frequency. The changes in the objective function values before and after cross-line operation were further analyzed, with comparison results shown in Fig. 12.

Fig. 12 shows that cross-line operations reduced the SDCMI from 0.3847 to 0.2418, indicating a 37.1% reduction in SDCMI, which demonstrates how CLT trains alleviate localized supply-demand imbalances through resource sharing and achieve capacity complementarity among different types of lines. The Total costs decreased by 10.3%, confirming the effectiveness of cross-line operations in reducing passenger travel time and improving system efficiency. However, the operator costs increased from 45,600 CNY to 48,900 CNY, suggesting a trade-off between short-term economic efficiency and long-term network performance. Operators must balance peak-period service investments with off-peak cost controls by adjusting cross-line frequency.

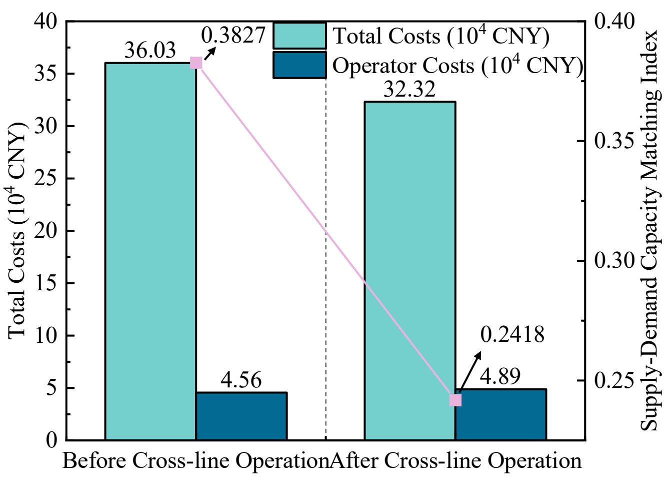

Fig. 12. Objective function values before and after cross-line operation.

D. Comparative analysis of unbalanced and balanced departure constraints

Departure interval constraints have a significant impact on system costs and operational efficiency. To thoroughly explore the performance differences of algorithms under varying constraint conditions, a comparative analysis of unbalanced and balanced departure interval strategies is conducted. Fig. 13 presents the algorithm convergence curves under different constraints.

Fig. 13 reveals distinct convergence patterns between unbalanced and balanced departure interval constraints. The unbalanced constraint curve shows a steeper descent, achieving faster convergence to lower objective values. The cost impact comparison under different constraints is further analyzed in Table 6.

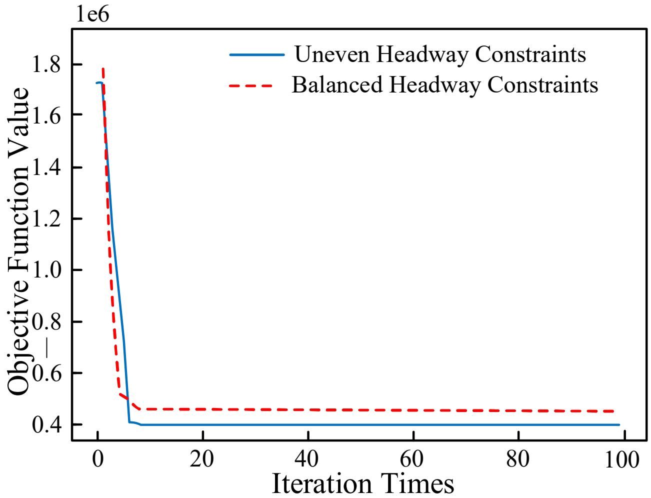

Fig. 13. Algorithm convergence curves under different constraints.

The SDCMI under unbalanced departure intervals shows a 32.5% improvement compared to balanced intervals, with total costs reduced by 7.1%. These results demonstrate that the non-equilibrium headway constraint can accelerate the convergence of the algorithm, reduce costs, and enhance the matching degree between transport capacity supply and demand. Therefore, it is recommended to adopt the unbalanced departure interval mode in cross-line operation.

TABLE 6
COST IMPACT COMPARISON UNDER DIFFERENT CONSTRAINTS

Constraint type	Z_1	Z_2
Balanced departure interval constraints	0.3580	34.79
Unbalanced departure interval constraints	0.2418	32.32

E. Sensitivity analysis of the maximum load factor β_{max}

The maximum load factor plays a crucial role in balancing capacity and enhancing the passenger experience. The sensitivity of the objective function to the variation of β_{max} was analyzed, and the influence of β_{max} on the objective function is shown in Fig. 14.

Fig. 14 shows that $\beta_{max} = 0.8$ is a critical point. At this value, the total cost is 322,300 CNY and the SDCMI is 0.2314. Below this crucial point, increasing yields provides dual benefits of cost reduction and degree improvement matching. When β_{max} exceeds the critical point, adverse effects on optimization occur. The research results guide decision-makers in setting the optimal β_{max} to balance achieve cost efficiency and supply-demand consistency in the operational plan.

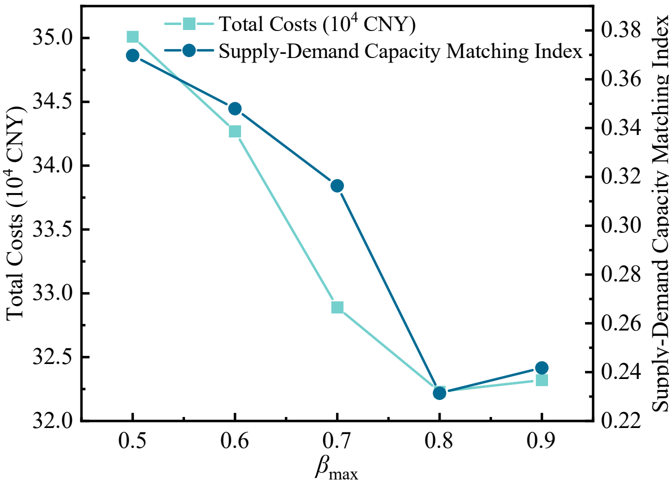

Fig. 14. The influence of β_{max} on the objective function.

F. Sensitivity analysis of the value of passenger time θ

The value of passenger time is primarily determined by local GDP. Analysis of the influence of different values of passenger time on the model helps verify the model's adaptability under varying economic levels. The influence of θ on the objective function is presented in Fig. 15.

Fig. 15 shows that when the Value of Passenger Time increased from 0.4 CNY/min to 1.2 CNY/min, total cost rose from 262,700 to 629,000 CNY, and the SDCMI increased from 0.2329 to 0.3132. This demonstrates that higher passenger time values elevate passenger travel costs and transportation efficiency. θ impact on the number of stops of cross-line trains is shown in Table 7.

When the Value of Passenger Time increases from 0.4 CNY/min to 0.8 CNY/min, the optimal scheme reduces cross-line train stops, indicating that passengers within this economic level range increasingly prefer direct services as the time value rises. When the Value of Passenger Time increases from 0.8 CNY/min to 1.2 CNY/min, cross-line train stops increase, showing that among passengers in this economic level range, the long-distance passenger flow proportion rises. The model must cover more OD pairs by increasing the number of stops to reduce the time from station departure to destination.

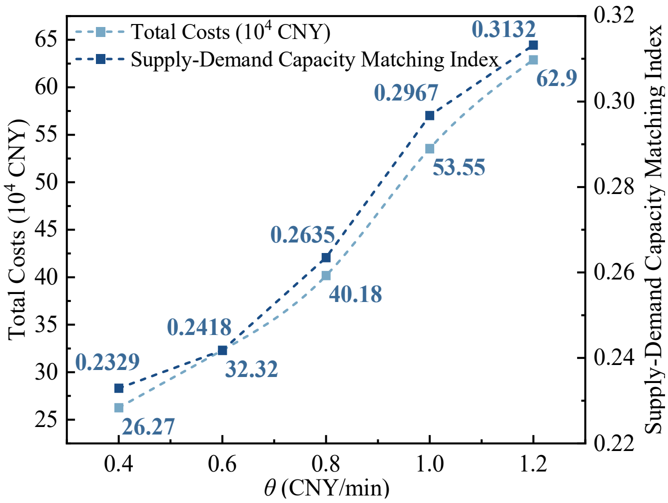

Fig. 15. The influence of θ on the objective function.

TABLE 7
θ IMPACT ON THE NUMBER OF STOPS ON CLT

θ	0.4	0.6	0.8	1.0	1.2
Number of Stops on CLT	9	8	7	8	9

V. CONCLUSION AND DISCUSSION

(1) Focusing on cross-line operation between SR and URT, this paper establishes a bi-objective programming model. This model prioritizes optimizing the SDCMI and minimizing the total costs. Optimization results demonstrate a 37.1% reduction in the SDCMI and a 10.3% decrease in total costs, enhancing both efficiency and operational economy. By integrating a mixed logit model with event-activity network theory, the model accurately describes multi-tier train coordination and the complex behavior of passenger route choice.

(2) Comparative analysis shows cross-line operation trains reduce overall line operation frequency by consolidating long-distance passenger flow. However, the enterprise experiences minor short-term cost increases, which provides a basis for operators to implement differentiated peak/off-peak strategies. Implementing non-equilibrium departure intervals accelerates algorithm convergence, reduces costs, and improves the SDCMI. Therefore, asymmetric headways are recommended for cross-line operation.

(3) Sensitivity analysis of key parameters indicates that moderately increasing the maximum load factor improves supply and demand capacity matching precision while lowering operational costs. The objective function achieves an optimal value at a maximum load factor of 0.8. Excessively high load factors cause negative impacts. Passengers from differing economic regions exhibit distinct travel preferences. To accommodate these variations, dynamic adjustment of station-specific stopping patterns is necessary.

(4) There are still some limitations in this study. The current framework does not incorporate dynamic passenger flow real-time adjustment mechanisms, leaving the model's adaptability to sudden passenger flow fluctuations requiring verification. Future research should account for passengers' bounded rationality while integrating dynamic flow prediction to extend the model's adaptability to more complex contexts.

REFERENCES

[1] Y. Q. Wang, Y. G. Wei, P. Shang, and J. C. Zhang, "Research on maximum reasonable utilization of high-speed railway capacity considering off-line trains," *Journal of the China Railway Society*, vol. 42, no. 10, pp. 23-29, 2020.

[2] Q. H. Liang, S. S. Chai, and J. W. Deng, "Optimization model of train operation density for cross-line in regional rail transit," *Journal of Beijing Jiaotong University*, vol. 44, no. 1, pp. 20-26, 2020.

[3] L. Chen, X. Y. Duan, and Y. Bai, "Optimizing cross-line train service planning in urban rail transit network," *Urban Rapid Rail Transit*, vol. 36, no. 3, pp. 59-64, 2023.

[4] X. E. Yang, C. F. Zhu, X. G. Wang, X. J. Ma, and L. N. Cheng, "Study on the train plan of the cross-line operation in the urban rail transit considering the multi-composition train," *IAENG International Journal of Applied Mathematics*, vol. 53, no. 4, pp. 1253-1265, 2023.

[5] K. W. Zhao, C. F. Zhu, X. L. Li et al. "Train service design for cross-line operation during peak passenger flow period," *IAENG International Journal of Applied Mathematics*, vol. 54, no. 9, pp. 1797-1806, 2024.

[6] X. R. Zhang, S. K. Chen, X. D. Zhao, and Z. Wang, "A rescheduling optimization method for metro trains under cross-line operation," *Journal of Transportation Systems Engineering and Information Technology*, vol. 23, no. 4, pp. 164-174, 2023.

[7] X. P. Li, Y. Bai, F. Li, Y. Chen, and J. F. Liu, "Compilation of train cross-line service scheme of rail transit with multi-operators," *Journal of Railway Science and Engineering*, vol. 20, no. 9, pp. 3287-3297, 2023.

[8] Q. Y. Peng, S. Y. Liu, S. Jiang et al. "Collaborative optimization of suburban railway and metro train operation plans under the interconnection operational mode," *Journal of Transportation Systems Engineering and Information Technology*, vol. 25, no. 2, pp. 36-47, 2025.

[9] S. H. Yan, T. W. Zhang, and L. J. Dong, "Optimization model of operation route selection of high speed railway cross-line trains," *Railway Transport and Economy*, vol. 43, no. 5, pp. 109-116, 2021.

[10] Y. Yin, C. Z. Jiang, C. Liang, J. Q. Chen, and B. Z. Li, "Operation plan optimization of cross-line train in autonomous-rail rapid transit," *Railway Transport and Economy*, vol. 47, no. 1, pp. 181-190, 2025.

[11] P. N. Tian, R. Y. Tong, H. P. Wang et al. "Optimization of cross-line train operation across the urban rail transit loop line," *Journal of Ji-lin University (Engineering and Technology Edition),* vol. 55, no. 1, pp. 132-140, 2025.

[12] R. X. Wang, L. Nie, and Y. Y. Tan, "Integrated optimization of train line planning and timetabling: a new method of changing train operation zone and direct-service of cross-line ODs," *Transportation Letters*, vol. 17, no. 4, pp. 1-21, 2024.

[13] Y. Chen, Y. Bai, B. H. Mao, and F. Wen, "Optimization of train routing plan for interconnected lines in urban rail transit," *Journal of the China Railway Society*, vol. 46, no. 8, pp. 21-29, 2024.

[14] L. S. Sun, H. B. Lu, Y. Xu, D. W. Kong, and J. Shao, "Fairness-oriented train service design for urban rail transit cross-line operation," *Physica A: Statistical Mechanics and its Applications*, vol. 606, no. Suppl C, p. 128124, 2022.

[15] J. S. Huang, Q. Luo, and K. Huang, "Optimizing the train timetable considering cross-line operation in the urban rail transit network," *Transportation Letters*, pp. 1-22, 2025.

[16] D. J. Xu, L. Gong, and J. W. Zeng, "Modeling of train timetable with full-length and short-turn routing considering the time-varying demand," *Journal of Transportation Systems Engineering and Information Technology*, vol. 19, no. 2, pp. 122-129, 2019.

[17] D. J. Xu, J. W. Zeng, C. R. Ma, and S. K. Chen, "Optimization for train plan of full-length and short-turn routing considering the equilibrium of load factor," *Journal of Transportation Systems Engineering and Information Technology*, vol. 17, no. 6, pp. 185-192, 2017.

[18] J. M. Guo, L. F. Han, J. G. Zhou, J. Lv, and J. Liu, "Research on the operation scheme for rail transit with the Y-shaped collinear mode," *Journal of Railway Engineering Society*, vol. 34, no. 2, pp. 87-92, 2017.

[19] J. G. Huo, Z. J. Wang, Y. Chen, Z. W. Chen, and Y. Zhou, "The matching method between line transport capacity and passenger flow in urban rail transit based on integer programming," *Science and Technology Innovation*, no. 18, pp. 50-53, 2024.

[20] F. Yan, X. M. Yao, P. Zhao, M. Han, and K. Qiao, "Modelling train operation plan for urban rail transit under interconnection operational mode," *Journal of the China Railway Society*, vol. 45, no. 2, pp. 13-21, 2023.

[21] R. Wang and S. Ma, "Regional rail transit network capacity matching method," *Journal of Transportation Engineering and Information*, vol. 18, no. 2, pp. 103-110, 2020.

[22] H. Zhang, M. M. Lv, and S. Q. Ni, "Train timetable optimization model for full-length and short-turn routings with irregular departure intervals," *Journal of Transportation Systems Engineering and Information Technology*, vol. 22, no. 6, pp. 224-233, 2022.

[23] N. Huan, J. M. Zhang, and E. J. Yao, "Coordinated optimization model for passenger flow control in metro network considering both efficiency and equity," *Journal of Southeast University (Natural Science Edition)*, vol. 50, no. 6, pp. 1148-1155, 2020.

[24] R. B. Wei, S. P. Jia, B. H. Mao, Z. S. Xiao, and Y. Q. Wang, "Train planning optimization for through operation of urban rail transit considering load balance," *Journal of Harbin Institute of Technology*, vol. 55, no. 3, pp. 68-77, 2023.

[25] Z. Y. Huang, R. H. Xu, R. D. Yang, and W. Liu, "Evaluation on equilibrium of passenger flow distribution on urban rail transit network," *Journal of Transportation Systems Engineering and Information Technology*, vol. 18, no. 3, pp. 139-145, 2018.

[26] M. Jian, Z. Wang, Z. X. Chen et al. "Passenger flow assignment method for urban rail transit networks based on inference of spatiotemporal path," *Journal of Southwest Jiaotong University*, vol. 58, no. 5, pp. 1117-1125, 2023.

[27] H. R. Wu, Y. Y. Wang, and C. T. Sheng, "Optimization model of train timetable with different routing considering time-varying passenger demand," *Journal of Transportation Systems Engineering and Information Technology*, vol. 25, no. 2, pp. 214-226, 2025.
[28] F. Wen, Y. Bai, and Y. Chen, "Optimization of operation scheme and timetable for interconnected lines in urban rail transit considering express-local train," *Journal of Transportation Systems Engineering and Information Technology*, vol. 24, no. 3, pp. 172-183, 2024.
[29] K. E. Train, *Discrete Choice Methods with Simulation*. Cambridge: Cambridge University Press, 2003.
[30] D. P. Tian, X. F. Zhao, and Z. Z. Shi, "Chaotic particle swarm optimization with sigmoid-based acceleration coefficients for numerical function optimization," *Swarm and Evolutionary Computation*, vol. 51, p. 100573, 2019.

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