Application of Graph Complements and Labeling in Secure Communication

Naik Sinchana Ganapathi, Medini H.R., and Sabitha D'Souza*

***Abstract*—The internet has become a permanent fixture in people's lives and has expanded significantly over the last few decades. As it has grown, data protection has become a critical concern to ensure that information is accessed only by intended recipients and remains secure from unauthorized modifications. Encryption plays a crucial role in data protection by transforming information into an unreadable format using a specific algorithm, requiring proper decryption to access and utilize the data. Graph theory, a field within discrete mathematics, is widely applied to strengthen encryption techniques for securing information. Harmonious labeling is utilized to assign values to edges in a graph, representing encrypted plaintext to ensure secure communication. Furthermore, incorporating the complement of a graph method along with the partial complement enhances encryption by adding an extra layer of complexity.**

***Index Terms*—Encryption and decryption, Harmonious Labeling, Secure communication, Complement of a graph, Partial complement.**

I. Introduction

Graph theory is a rapidly evolving field with diverse applications in cryptography. Various encryption methods have applied graph theory concepts to enhance security. One such approach, proposed by Etaiwi, integrates principles like minimum spanning tree, cycles, and complete graphs. [1]. Several techniques of graph theory, such as graph labeling, complement of a graph, and generalized complements have applications in the field of cryptography [2], [3], [4].

In this study, the proposed algorithm employs graph labeling and partial complement operations for plaintext encoding. Alexander Rosa first introduced the concept of graph labeling in 1967 [5], and extensive research on the subject was later conducted by Graham and Sloane in 1980. The comprehensive Gallian survey on graph labeling provides a vast collection of studies in this area [6]. Additionally, Deepa and Maheswari explored the relationship between various ciphers and graph labeling techniques for data encoding [7]. Harmonious labeling, a significant type of graph labeling introduced by Graham and Sloane in 1980, applies to a specific subset of graphs [8]. Their study established that a cycle (C_n) is harmonious if and only if it is of odd length and the wheel (W_n) is harmonious for any value of n. Vaidya et al. conducted additional research on odd harmonious labeling of certain graphs in 2012 [9]. The concept of partial complementation of a graph was introduced by Fomin et al. [10].

In encryption, harmonious graph labeling is performed to set edge values identical to different plaintext values from the encryption table. The process is executed on a general graph to find edge values for the input plaintext. Next, both the complement and partial complement of the graph are calculated to form a complex encryption-oriented graph (cipher graph). Decryption is a process of reversal, beginning with the partial complement of the input cipher graph, followed by the regular complementation. The original edge values (plaintext values) are retrieved by reapplying the original graph labeling procedure. The plaintext is then reconstructed from the assigned keys and the encryption table. The algorithm adopts a symmetric key cryptographic technique since the same keys are employed for encryption and decryption.

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II. Preliminaries

Definition 1. *Graph labeling is a function that maps a set of vertices or edges of a graph to a non-negative integer that fulfills specific properties/rules.*

Definition 2. *A graph G with q edges is said to be harmonious if there is a one-to-one (injective) function f that maps the set of vertices to {0,1,2,...,q-1} so that each edge gets the label of the sum of the two vertices labels incident with that edge in modulo q, which are all different.*

Definition 3. *The Partial complement of a graph G is the concept derived from the complement of a graph, where only a subset of edges (say S) is toggled (added or removed) instead of considering the entire edge set.*

Theorem 1. *Splitting graph of a bistar S'(B_n, n) is an odd harmonious graph [10].*

III. Reserach Approach

This study introduces a novel cryptographic technique that integrates graph complements with harmonious labeling methods. The approach involves constructing a general graph, defining an encryption table, and subsequently encrypting the plaintext using these techniques.

A. *Encryption table*

English alphabet, spaces between words, special characters up to '#', and numerals up to '2' are assigned odd values ranging from 1 to 67. The remaining special characters and numerals are assigned even values ranging from 0 to 68. Using Table 1, the plaintext is converted into its numeric representation. These values serve as the first level of encryption, where unique values are used as edge weights to construct a cipher graph for the given plaintext.

TABLE I
ENCRYPTION TABLE

A	B	C	D	E	F	G	H	I	J	K	L
1	3	5	7	9	11	13	15	17	19	21	23
M	**N**	**O**	**P**	**Q**	**R**	**S**	**T**	**U**	**V**	**W**	**X**
25	27	29	31	33	35	37	39	41	43	45	47
Y	**Z**	**space**	∼	\	**!**	**1**	**@**	**2**	#	**3**	**$**
49	51	53	55	57	59	61	63	65	67	0	2
4	**%**	**5**	^	**6**	**&**	**7**	∗	**8**	(	**9**	)
4	6	8	10	12	14	16	18	20	22	24	26
0	_	**-**	**+**	**=**	**{**	**}**	[	]	|	\	**:**
28	30	32	34	36	38	40	42	44	46	48	50
;	**’**	**”**	**,**	<	>	**.**	**?**	**/**			
52	54	56	58	60	62	64	66	68			

B. *Methodology for Constructing a Standard Graph*

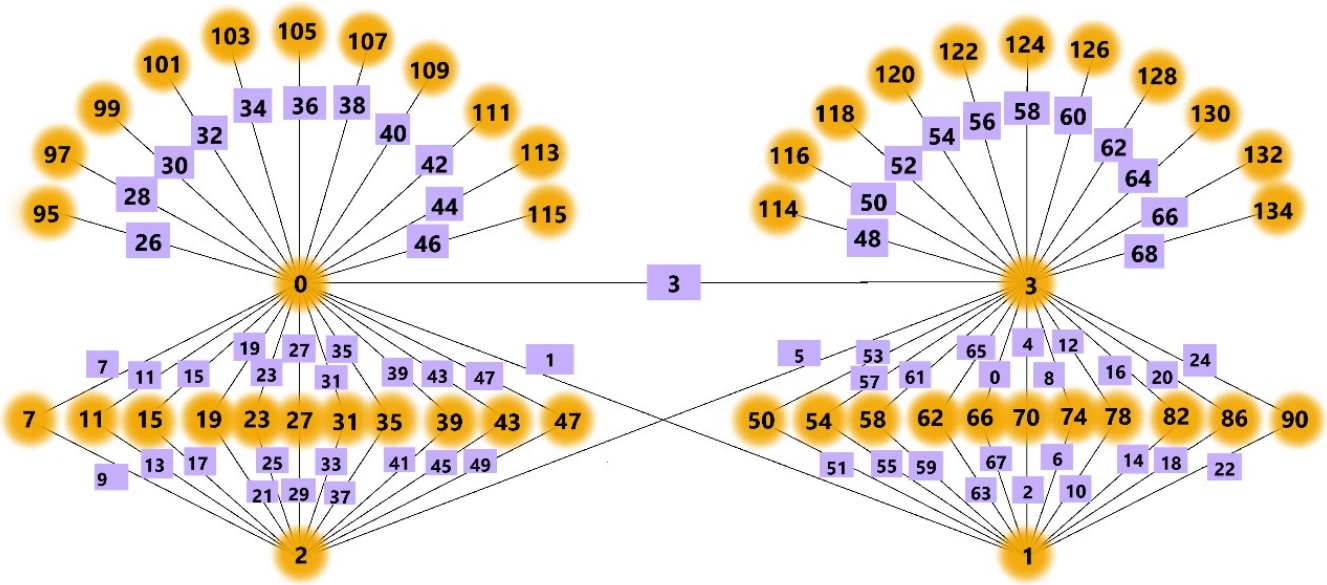


Fig. 1 General/Standard Graph

The vertices of the general graph are labeled according to Theorem 1 [7]. Edge values ranging from 0 to 68 are produced through harmonious labeling. The resulting graph forms a flexible structure suitable for encoding various types of plaintext. Table 1 presents the value assignments for 69 characters. If more characters are required, the graph can be extended by adding additional vertices and edges.

C. *Procedure for encryption and decryption*

1) Encryption procedure: Each character in the plaintext is mapped to a corresponding number in Table 1, which serves as the first level of encryption. From the encrypted values, distinct numbers are chosen and assigned as edge values within the general graph. Only the corresponding edges are then extracted from this graph. The cipher graph is then created by taking the complement of the extracted graph, followed by a partial complementation using a specific set of edges.

2) Procedure for key generation:
Key 1: Consider only adjacent vertices in both the extracted and complement graphs. Start with the smallest non-isolated vertex (if the smallest vertex is an isolated vertex, then switch to the complement graph and start) in the extracted graph, select the minimum among its neighbors, and form an edge. Then, switch to the complement graph and repeat the process. Continue selecting vertices from the extracted and complement graphs alternately until all vertices are covered. The total edges will be $\lfloor n/2 \rfloor$; if odd, the last vertex is ignored. This subset of edges **S** forms Key 1.

Key 2: To obtain the second encrypted values, the distinct numbers of order n from the first encrypted values are arranged in increasing order, and their positions are assigned prime number indices. Key 2 is formed by substituting the first encrypted values with their corresponding second encrypted values based on this mapping.

Key 3: In plaintext, it is essential to differentiate between capital and small letters. Uppercase and lowercase letters are given special characters, ? and #, respectively. The integration

of ? and produces Key 2. The keys, along with the vertex-labeled cipher graph and its associated edge set, are shared with the recipient.

3) Decryption Procedure:: The partial complement of the vertex-labeled cipher graph is taken using Key 1 sent by the sender. The resulting graph is then complemented once more, and its edge values are computed using harmonious labeling. These values are sorted in ascending order and assigned prime number positions. Using Key 2, the numbers from Table 1 are decrypted in the order of the original plaintext, converting them back into characters. Key 3 is used to distinguish between the uppercase and lowercase alphabets. As a result, the original plaintext is successfully reconstructed.

IV. ALGORITHMIC DESIGN

A. Encryption Mechanism:

Step 1: Use Table 1 to convert each character of the plaintext into its corresponding numerical value.
Step 2: Identify the distinct numerical values and extract the corresponding edges from the general graph.
Step 3: Compute the complement of the extracted subgraph.
Step 4: Follow the key generation steps to obtain keys 1,2, and 3.
Step 5: Derive the cipher graph by applying the partial complement operation to the complemented graph.
Step 6: Transmit the keys and the vertex-labeled cipher graph to the recipient, with harmonious labeling provided as a decoding hint.

B. Decryption Mechanism:

Step 1: Use Key 1 to obtain the partial complement of the graph sent by the sender.
Step 2: Take the complement of the obtained graph.
Step 3: Apply the harmonious labeling method to evaluate the edge values.
Step 4: Sort the edge values in ascending order and assign prime numbers as positional indices.
Step 5: Use Key 2 to retrieve the edge values in the order corresponding to the original plaintext.
Step 6: Refer to the encryption table to map the numerical values obtained in Step 5 back to their respective characters.
Step 7: Apply Key 3 to correctly represent characters in uppercase or lowercase, thereby reconstructing the plaintext.

V. ILLUSTRAIONS

A. Illustration 1

Consider the plaintext to be **Hello World**

Encryption:

By referring to Table 1, the first set of encrypted values for the plaintext is obtained.

Plaintext	H	e	l	l	o	
First encrypted values	15	9	23	23	29	53

W	o	r	l	d
45	29	35	23	7

From these first encrypted values, distinct values are taken and sorted in ascending order.

7	9	15	23	29	35	45	53

These specific edges are taken from the general graph to form the subgraph depicted below in Fig. 1.

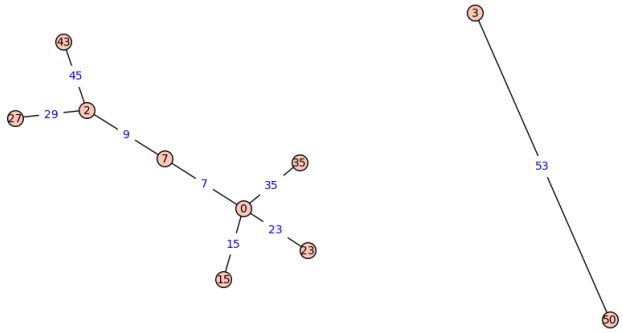


Fig. 2 Extracted Graph

Taking the complement of the extracted graph as shown in Fig. 3.

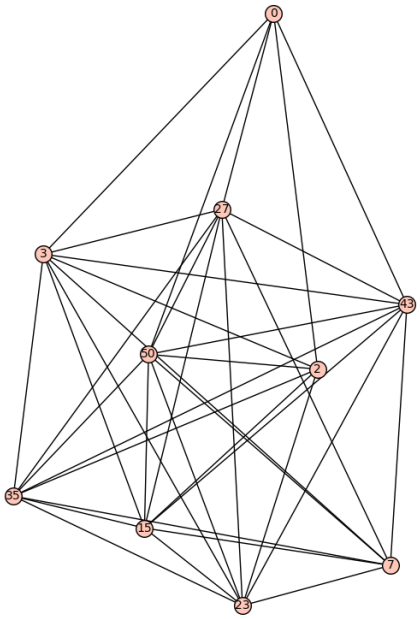


Fig. 3 Complement of the extracted graph

Key generation:

Consider the smallest non-isolated vertex in the extracted graph, which is '0', and the minimum among its neighbors is 7. Choose the edge **(0,7)**. Now, switch to the complement graph. Here, the next smallest vertex other than 0 and 7 is 2. And the minimum among its neighbors is 3. Choose the edge **(2,3)**. Again, switch to the extracted graph and repeat the process until all the vertices are covered.

Since the number of vertices is even (n = 10), the total number of edges in **S** is $\lfloor 10/2 \rfloor$ **= 5** and **S = {(0,7), (2,3), (15,23), (27,35), (43,50)}**. This set is considered as **Key 1** for the Decryption process.

The sorted distinct values are now numbered with prime numbers and are regarded as second-encrypted values.

Unique values	7	9	15	23	29
Second encrypted values	2	3	5	7	11

35	45	53
13	17	19

Key 2 is generated by mapping the second encrypted values to their corresponding first encrypted values. **Key 3** is formed by assigning '?' to indicate uppercase characters and '#' to represent lowercase characters.

Plaintext	H	e	l	l	o	
First encrypted values	15	9	23	23	29	53
Key 2	5	3	7	7	11	19
Key 3	?	#	#	#	#	#

W	o	r	l	d
45	29	35	23	7
17	11	13	7	2
?	#	#	#	#

Take a partial complement of the resulting graph using S. The graph obtained is considered as the cipher graph.

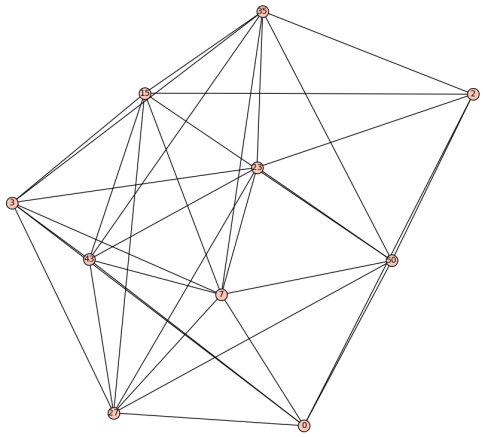


Fig. 4 Cipher Graph

Decryption:

The recipient uses Key 1 to compute the partial complement of the vertex-labeled cipher graph received from the sender. The resulting graph is then complemented, and harmonious labeling is applied to determine its edge values. These edge values are sorted in ascending order, and prime numbers are assigned as positional indices. Key 2 is then used to retrieve the values from Table 1 in the order of the original plaintext and convert them into characters. Key 3 is applied to correctly represent each character in uppercase or lowercase. As a result, the plaintext is successfully reconstructed as **Hello World**.

B. Illustration 2

Consider the plaintext to be **"Hell0, W0rld!"**

Encryption:

The first encrypted values for the given plaintext are obtained by referring to Table 1.

Plaintext	"	H	e	l	l	0
First encrypted values	56	15	9	23	23	28

,		W	0	r	l	d	!	"
58	53	45	28	35	23	7	59	56

From these first encrypted values, distinct values are taken and sorted in ascending order.

7	9	15	23	28	35	45	53	56	58	59

These edges are extracted from the general graph.

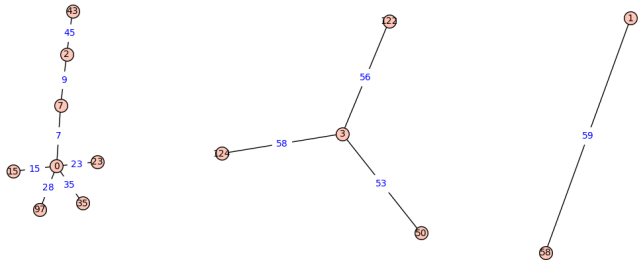


Fig. 5 Extracted graph

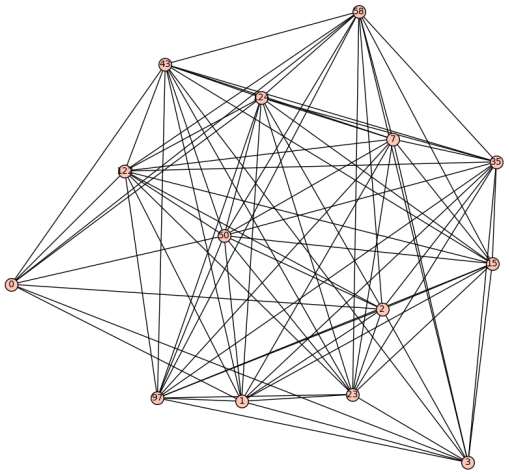


Fig. 6 Complement of the extracted graph

Key generation:

Consider the smallest non-isolated vertex in the extracted graph, which is '0' and the minimum among its neighbors is 7. Choose the edge **(0,7)**. Now, switch to the complement graph. Here, the next smallest vertex other than 0 and 7 is 1, and the minimum among its neighbors is 2. Choose the edge **(1,2)**. Again, switch to the extracted graph and repeat the process until all the vertices are covered.

Since the number of vertices is 14, the total number of edges in S is 7 and
S = {(0,7), (1,2), (3,50), (15,23), (35,43), (58,97),(122,124)}.
This set is considered as **Key 1** for the Decryption process. The sorted distinct values are now numbered with prime numbers and are referred to as the second encrypted values.

Unique values	7	9	15	23	28	35
Second encrypted values	2	3	5	7	11	13

45	53	56	58	59
17	19	23	29	31

Key 2 is created by mapping the second encrypted values to the first encrypted values. By allocating '?' for uppercase characters and '#' for lowercase characters, **Key 3** is created.

Plaintext	”	H	e	l	l	0
First encrypted values	56	15	9	23	23	28
Key 2	23	5	3	7	7	11
Key 3	#	?	#	#	#	#

,		W	0	r	l	d	!	”
58	53	45	28	35	23	7	59	56
29	19	17	11	13	7	2	37	23
#	#	?	#	#	#	#	#	#

The partial complement of the resulting graph is obtained using **S**. Hence, the graph obtained is the Cipher graph.

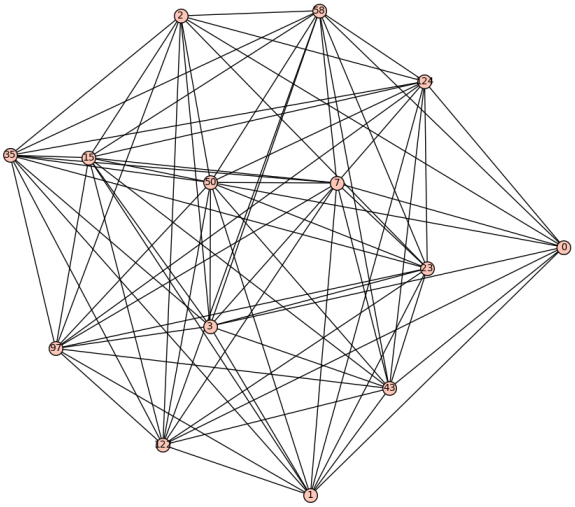


Fig. 7 **Cipher Graph**

Decryption:

By reversing the encryption process, the plaintext is retrieved as
”Hell0, W0rld!”.

C. Illustration 3

Consider the plaintext to be **Numbers never lie**

Encryption:

By referring to Table 1, the first set of encrypted values for the plaintext is obtained

Plaintext	N	u	m	b	e	r	s
First encrypted values	27	41	25	3	9	35	37

	n	e	v	e	r		l	i	e
53	27	9	43	9	35		23	17	9

The sorted distinct values are given below.

3	9	17	23	25	27	35	37	41	43	53

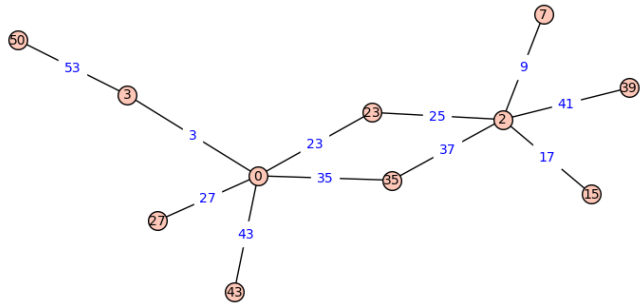


Fig. 8 Extracted graph

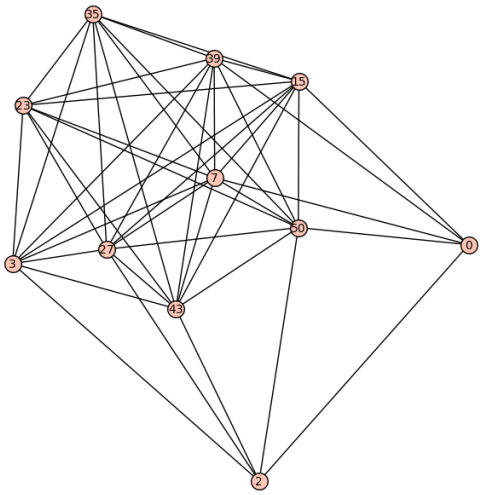


Fig. 9 Complement of the extracted graph

Key generation:

Key 1: {(0,3),(2,27),(7,15),(23,35),(39,43)}
Key 2: [13,29,11,2,3,17,23,37,13,3,31,3,17,37,7,5,3]
Key 3: [?,#,#,#,#,#,#,#,#,#,#,#,#,#,#,#]

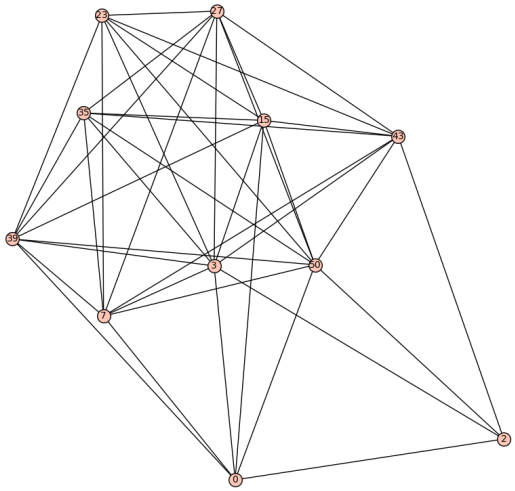


Fig 10. Cipher Graph

Decryption:

By reversing the encryption process, the plaintext is retrieved as
Numbers never lie.

D. Illustration 4

Consider the plaintext to be **N<mber$_ne^er_1!e**

Encryption:

By referring to Table 1, the first set of encrypted values for the plaintext is obtained.

Plaintext									
First encrypted values									
N	<	m	b	e	r	$			
27	60	25	3	9	35	2			
_	n	e	^	e	r	_	1	!	e
30	27	9	43	9	35	30	61	59	9

The sorted distinct values are given below.

2	3	9	10	25	27	30	35	59	60	61

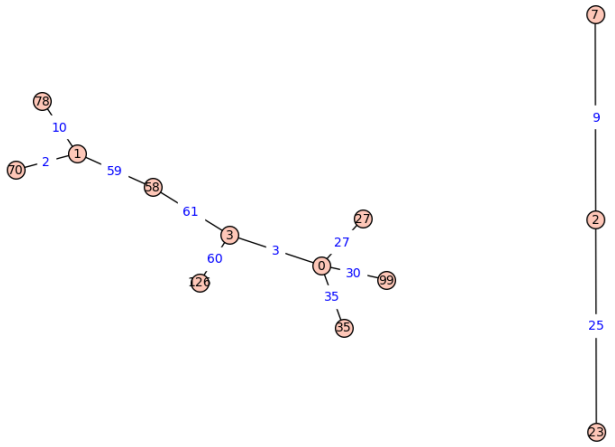


Fig. 11 Extracted graph

Key generation:

Key 1: {(0,3),(1,2),(7,23),(27,35),(58,70),(78,99)}
Key 2: [13,31,11,3,5,23,2,17,13,5,7,5,23,17,37,29,5]
Key 3: [?,#,#,#,#,#,#,#,#,#,#,#,#,#,#,#,#]

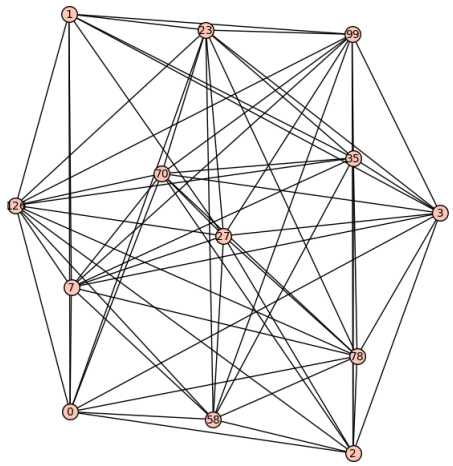


Fig. 12 Cipher Graph

Decryption:

By reversing the encryption process, the plaintext is retrieved as
N<mber$_ne^er_1!e.

VI. Empirical Analysis

Tables II and III display the cipher graphs generated for different plaintext sizes, emphasizing the effect of graph-based encryption. As the plaintext size increases, the resulting graph structures become more complex. The incorporation of special characters in plaintext also adds complexity to cipher graphs, influencing connectivity and encryption structure. In addition, the results emphasize how graph labeling and complement operations influence the encryption-decryption process.

TABLE II
SOME CIPHER GRAPHS WITHOUT SPECIAL CHARACTERS

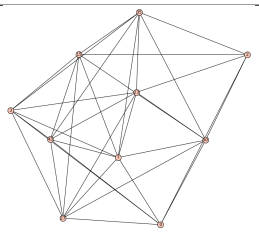
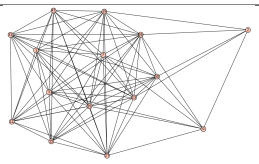
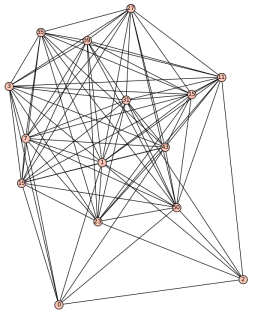
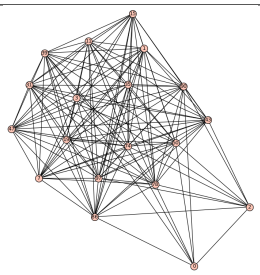
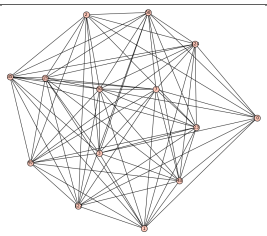
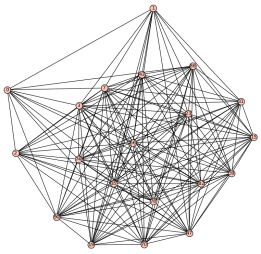
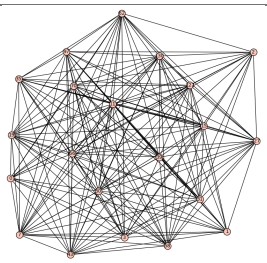
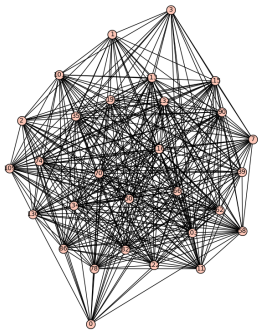
Sl. No.	Plaintext length excluding special characters	Cipher graphs
1	10	
2	20	
3	30	
4	40	

TABLE III
SOME CIPHER GRAPHS WITH SPECIAL CHARACTERS

Sl. No.	Plaintext length including special characters	Cipher graphs
1	10	
2	20	
3	30	
4	40	

VII. Conclusion

This study introduces a novel encryption and decryption approach utilizing the concepts of graph theory, focusing on the use of graph complementation and graph labeling techniques. By leveraging these graph-based methods, the proposed approach enhances data security and encryption complexity. By representing plaintext characters as numbers and including them in graph structures, this algorithm provides security. The suggested approach is confidential through the employment of various keys and graph transformation, thus resistant to usual cryptographic attacks.

Additionally, the research builds upon existing studies in graph labeling, such as harmonious labeling and partial complementation, further expanding its potential applications in cryptography. This method not only illustrates the real-world applicability of graph theory to cryptography but also suggests new directions for future research, including the optimization of key generation algorithms and further application of this method to big data. Through ongoing developments, this encryption model can be modified for use in secure communications across a variety of fields, including cybersecurity and secure messaging networks.

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