Fuzzy Adaptive Control for Strict-Feedback Uncertain Nonlinear Systems with Input Delays and Saturations

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Abstract—**This paper devotes to study the strict-feedback uncertain nonlinear systems with input delays and saturations, a fuzzy adaptive tracking control strategy is offered in an adaptive backstepping recursive framework. Firstly, the Pade approximation method is introduced to overcome the difficulty of input delays in the network systems. Then, the fuzzy-logic systems (FLSs) are invoked to deal with the unknown nonlinear function, and the output of the system does not violate the qualification by using the barrier Lyapunov function (BLF). In addition, the boundary estimation method is flexibly used for the issue of unknown control coefficients, also the hyperbolic tangent function is combined to realize fast compensation for input saturations. Finally, the simulation results indicate that all signals within the closed-loop are uniformly bounded, and the tracking error converges to a small vicinity of the origin, which demonstrates the rationality of the method.**

Index Terms—**Fuzzy-logic systems, Input delays, Barrier Lyapunov function, Output constraint, Input saturations**

I. Introduction

ALL the time, the research on nonlinear systems has consistently been a central concern for a great many scholars. The extensive application of nonlinear systems in diverse fields such as industrial control [1], power systems [2], and aerospace control [3] makes it extremely crucial to ensure the safety and reliability of these systems. For this reason, numerous advanced control strategies have been developed to tackle the nonlinear problems present in engineering practice. Among them, FLSs have become the primary means for modeling uncertain functions and achieved accurate estimation of unmeasurable states by designing state observers [4]. In view of reducing the complexity of online computation, several adaptive-based optimal control schemes were discussed in [5]-[6]. Furthermore, the method described in [7] was applicable to the control of high-dimensional systems. The proposed strategies in this works did not require the system to possess a specific parametric form and can effectively overcome the challenges of stability loss and poor transient performance. It is essential to note that the unknown control direction is also one of the significant factors contributing to system performance degradation. By introducing micro-adjustable auxiliary functions [8] and Nussbaum functions [9], the obstacles posed by the unknown control direction were effectively circumvented, thereby improving the robustness of the system in complex environments.

Meanwhile, input delays cannot be avoided in real industrial systems due to communication bandwidth and signal transmission. It may significantly weaken the dynamic performance of the system, thereby affecting the production of real industrial systems, so exploring the control system with input delays is of profound practical importance. Over the past few decades, many scholars have been working on solving the input delay issue. Specifically, with the aid of auxiliary systems, the difficulties created by input delays can be quickly tackled in [10]-[11]. In addition, the introduction of the Pade approximation [12]-[13] gave an alternative idea for solving the input delay issues in nonlinear networks. Based on this, a condition for the finite-time stability of the system was brought up in [14], and the state feedback control rate was designed in combination with a power integrator. However, since the convergence within a finite time is influenced by the initial state of the system and is not feasible to predict in advance, a class of fixed-time fault tolerant control methods relying on fuzzy adaptation was proposed for nonlinear systems with input delays [15]-[17], which provides some new ideas for the study of this problem. It should be considered that although some meaningful findings have been achieved in dealing with the input delay problem, there are still certain challenges in handling the effects of input saturations in practical engineering applications.

As an inherent and prevalent characteristic of nonlinear systems, input saturation not only poses certain obstacles to the design of controllers but also potentially precipitate system instability. To confront this formidable challenge, there have been some results that approximate input saturation with the help of smoothing functions [18]-[19], or compensate it by constructing auxiliary systems [20]. Furthermore, by invoking the predefined time performance functions and smooth functions, the stability of the system under input saturations [21] was guaranteed, and the precise tracking of the state trajectory was realized within a predefined time. However, when the saturation function exhibits asymmetric features, the complexity of control escalates significantly. Therefore, a Gaussian function was introduced to construct the saturation model, and a robust adaptive control method was devised for the pure feedback system [22] by combining the implicit function and the BLF. It can be found that limitations on the state of the system are frequently desirable in practical systems. The BLF has emerged as an effective technique for addressing output constraint issues, and there have been some notable results in [23]-[25]. In subsequent

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work, the asymmetric BLF has turned into an effective instrument to tackle systems with asymmetric output constraints. Building on these advancements, Yang et al. [26] relaxed the Lipschitz continuity condition by designing a high-gain observer, thus reducing the complexity of the algorithm. It should be emphasised that although numerous control methods have been developed for input saturations and state limitations, solving the general nonlinear problem of input delays on this basis is still an unexplored area, especially when considering systems with unknown control coefficients, which creates considerable challenges for control design, thus further motivating our investigation.

Inspired by the above findings, to rapidly address the influence of the input delays and saturations on systems with unknown control direction, while maintaining the output tracking within the specified bounds, an effective fuzzy adaptive control scheme is presented in this article. In addition, the specific contributions are listed below:

1) Compared with the work in [11], [12] and [13], a fuzzy adaptive controller is developed by applying the hyperbolic tangent function and the Pade approximation method, which are capable of being applied in complex systems with input delays and saturations, thus having higher practical engineering value.

2) Different from the traditional adaptive control strategies in [11], [12] and [14], the BLF is introduced as an effective constrained control strategy that guarantees the transient and steady state responses of the system, and meanwhile realizes the accurate tracking of outputs.

3) Unlike the existing error transformation methods proposed in [11], [13], [16] and [23], the upper and lower bounds of the unknown coefficients are constructed by combining the boundary estimation methods, which can be directly used to tackle nonlinear systems having unknown control directions.

The paper is structured as follows. Section 2 provides the system description and preliminaries. Section 3 focuses on building the adaptive controller and analyzing its stability. In the end, two examples are drawn out to enhance the persuasive power of the theoretical results.

II. System description and preliminaries

Introduce the following SFNSs with unknown control coefficients

$$\begin{aligned} \dot{x}_i &= \Psi_i(\bar{x}_i)x_{i+1} + f_i(\bar{x}_i) + \beta_i(x,t), \\ & i = 1,2,\ldots,n-1 \\ \dot{x}_n &= \Psi_n(\bar{x}_n)u(t-\tau) + f_n(\bar{x}_n) + \beta_n(x,t) \\ y &= x_1 \end{aligned} \tag{1}$$

where $\bar{x}_i = [x_1, x_2, \ldots, x_i]^T \in R^i$, $x \in R_n$ and $y \in R$ denote the input and output variables. $f_i(\cdot)$ and $\beta_i(x,t)$ represent the unknown nonlinear function and the disturbance, respectively. $\tau_{\max}$ is the upper limit of the unknown delay constant τ, and the output vector y satisfies $|y| \leq k_{m1}$ with $k_{m1} > 0$.

In view of dealing with input delays, the pade approximate method in [13] is introduced here as follows

$$\begin{aligned} \omega\{u(t-\tau)\} &= e^{(-\tau\aleph)}\omega\{u(t)\} = \frac{e^{(\frac{-\tau\aleph}{2})}}{e^{(\frac{\tau\aleph}{2})}}\omega\{u(t)\} \\ &\approx \frac{1-\frac{\tau\aleph}{2}}{1+\frac{\tau\aleph}{2}}\omega\{u(t)\} \end{aligned} \tag{2}$$

where $\aleph$ denotes the variable, $\omega\{u(t)\}$ represents the Laplace transform of $u(t)$, and the additional variable x_{n+1} is specifically described by

$$\frac{1-\frac{\tau\aleph}{2}}{1+\frac{\tau\aleph}{2}}\omega\{u(t)\} = \omega\{x_{n+1}(t)\} - \omega\{u(t)\} \tag{3}$$

Next, a simple calculation gives

$$2\omega\left\{u_i^F(t)\right\} = \omega\{x_{i,n+1}(t)\} + \frac{\tau\aleph}{2}\omega\{x_{i,n+1}(t)\} \tag{4}$$

Combining the Laplace inverse transform, we get

$$\dot{x}_{i,n+1} = -\bar{\tau}x_{i,n+1} + 2\bar{\tau}u_i^F \tag{5}$$

where $\bar{\tau} = \frac{2}{\tau}$.

Next, the saturation model with respect to v can be formulated by

$$u(v) = \mathrm{sat}(v) = \begin{cases} F_1, & v \geq F_1 \\ v, & f_1 < v < F_1 \\ f_1, & v \leq f_1 \end{cases} \tag{6}$$

where $u(v)$ means the saturated input, f_1 and F_1 stand for the unknown lower and upper bounds of v.

With the aid of the hyperbolic tangent function, it holds that

$$c(v) = \begin{cases} F_1\tanh(\frac{v}{F_1}), & v \geq 0 \\ f_1\tanh(\frac{v}{f_1}), & v < 0 \end{cases} \tag{7}$$

where $\mathrm{sat}(v)$ satisfies $\mathrm{sat}(v) = c(v) + \zeta(v)$, and $|\zeta(v)| \leq Q$ is the boundary of $\zeta(v)$.

For this, invoking the mean value theorem yields

$$c(v) = c(v_0) + c_\mu(v - v_0) \tag{8}$$

where μ satisfies $0 < \mu < 1$, and it gives

$$c_\mu = \frac{\partial c_v}{\partial v}|_{v=v_\mu} = \frac{4}{(e^{v/M_\mu}+e^{-v/M_\mu})^2}|_{v=v_\mu} \tag{9}$$

Then, by setting $v_0 = 0$, we arrive at

$$u(v) = c_\mu(v) + \varsigma(v) \tag{10}$$

By transformation analysis, the system (1) becomes

$$\begin{aligned} \dot{x}_i &= \Psi_i(\bar{x}_i)x_{i+1} + f_i(\bar{x}_i) + \beta_i(x,t), \\ & i = 1,2,\ldots,n-1 \\ \dot{x}_n &= \Psi_n(\bar{x}_n)x_{n+1} - \Psi_n(\bar{x}_n)u(v) + f_n(\bar{x}_n) + \beta_n(x,t) \\ \dot{x}_{n+1} &= -\bar{\tau}x_{n+1} + 2\bar{\tau}u \\ y &= x_1 \end{aligned} \tag{11}$$

Remark 1. To cope with the input delays in networked systems, we define a variable x_{n+1} that does not represent the actual state variable. In particular, due to the existence of error variable x_{n+1}, how to construct an effective coordinate transformation to make the selection of the controller meet the actual requirements is also a difficulty to be overcome in this article.

Firstly, in view of achieving the desired control objectives, we provide a few assumptions:

Assumption 1. [17] There exists a bound for the disturbance $\beta_i(x,t)$, such that $|\beta_i(x,t)| \leq \bar{\beta}_i$.

Assumption 2. [13] If the reference signal $y_d(t)$ and its time derivatives $y_d^{(k)}(t)$ for $1 \leq k \leq n$ are bounded and continuous, then there exist positive constants $\chi_0, \chi_1, \chi_2, \ldots, \chi_n$ such that $|y_d(t)| \leq \chi_0$ and $\left|y_d^{(k)}(t)\right| \leq \chi_k$, $\forall t \geq 0$.

Assumption 3. [15] The uncertain nonlinear vectors $\Psi_i(\bar{x}_i)$ in system (1) are well-defined, and its lower and upper bounds are chosen as $0 < h_i = \underline{\Psi}_i \leq \Psi_i(t) \leq \overline{\Psi}_i = H_i$.

Assumption 4. For a given small positive number c_m, one can find a positive number c_μ that satisfies $0 < c_m < c_\mu < 1$.

Remark 2. Assumptions 1 and 2 are typical and evidently quite reasonable preconditions. In particular, it is noted that $\overline{\Psi}_i$ is unknown in Assumption 3, so it only applies to the stability analysis and does not participate in the controller design. Assumption 4 establishes the range of the gain c_μ and its lower limit, which quickly compensates for the negative influences of input saturation, and prevents the degradation of system performance resulting from excessive gains.

Next, we give the specific form of FLSs

$$y(x) = \frac{\sum_{i=1}^{N} \tilde{y}_l \prod_{i=1}^{n} \mu_{F_i^l}(x_i)}{\sum_{l=1}^{N} \prod_{i=1}^{n} \mu_{F_i^l}(x_i)} \tag{12}$$

where $x = [x_1, x_2, \ldots, x_n]^T$ denotes the input of the system, y is a result of the system output, $\tilde{y}_l = \max_{y \in R} \mu_{P^l}(y)$, $\mu_{F_i{}^l}(x_i)$ and $\mu_{P^l}(y)$ represent the functions defined in the fuzzy sets F_i^l and P^l.

Accordingly, the fuzzy basis function complies with

$$\varphi_l = \frac{\prod_{i=1}^{n} \mu F_i^l(x_i)}{\sum_{l=1}^{N} \prod_{i=1}^{n} \mu F_i^l(x_i)} \tag{13}$$

Then, the final output of (12) is organized as

$$y(x) = \eta^T \phi(x) \tag{14}$$

To simplify the design of the system (1), a few lemmas are given here:

Lemma 1. [15] Take $f(x)$ as a continuous function on the compact set Ξ, and it follows that

$$\sup_{x \in \Xi} \left| f(x) - \eta^T \phi(x) \right| < \xi \tag{15}$$

where $\xi > 0$ is the approximation error.

Lemma 2. [22] Postulate that there is an upper bound $k_{b1} > 0$, and for any $z_1 \in R$, the inequality $|z_1| < k_{b1}$ is fulfilled. Then, we have

$$\log \frac{k_{b1}^2}{k_{b1}^2 - z_1^2} < \frac{z_1^2}{k_{b1}^2 - z_1^2} \tag{16}$$

III. Design of adaptive fuzzy controller

This section presents a fuzzy adaptive control strategy that relies on the Lyapunov stability theory, and the stability proof of model (1) is completed by introducing BLF. Subsequently, the variable changes are set as follows

$$\begin{aligned} z_1 &= x_1 - y_d \\ z_i &= x_i - \alpha_{i-1}, \quad i = 2, 3, \cdots, n-1 \\ z_n &= x_n - \alpha_{n-1} + \frac{1}{\overline{\tau}} H_n x_{n+1} \end{aligned} \tag{17}$$

Remark 3. The virtual control α_{i-1} appears at each step and the actual control input v is given at step n. In particular, $\frac{1}{\overline{\tau}} H_n x_{n+1}$ is regarded as a way to get rid of the previously introduced x_{n+1} and prepare for the subsequent processing of the unknown control direction.

Step 1: Exploiting the coordinate transformation (17), one has

$$\dot{z}_1 = \Psi_1 x_2 + f_1(\bar{x}_1) + \beta_1(x, t) - \dot{y}_d \tag{18}$$

To guarantee that z_1 does not violate the expected constraint bounds, the BLF is invoked as

$$V_1 = \frac{1}{2h_1} \log \frac{k_{b1}^2}{k_{b1}^2 - z_1^2} + \frac{1}{2r_1} \tilde{\theta}_1^2 \tag{19}$$

where $r_1 > 0$, $h_1 > 0$, $\hat{\theta}_1$ is the estimated value of θ_1, and the estimation error $\tilde{\theta}_1 = \theta_1 - \hat{\theta}_1$.

From (18) and (19), we have

$$\begin{aligned} \dot{V}_1 =& \frac{z_1 \dot{z}_1}{h_1(k_{b1}^2 - z_1^2)} - \frac{\tilde{\theta}_1 \dot{\theta}_1}{r_1} \\ =& \frac{z_1}{h_1(k_{b1}^2 - z_1^2)} (\Psi_1 x_2 + f_1(\bar{x}_1) + \beta_1(x, t) \\ &- \dot{y}_d) - \frac{\tilde{\theta}_1 \dot{\theta}_1}{r_1} \end{aligned} \tag{20}$$

Invoking Young's inequality, the following holds

$$\frac{z_1}{h_1(k_{b1}^2 - z_1^2)} \beta_1(x, t) \leq \frac{z_1^2}{2h_1^2(k_{b1}^2 - z_1^2)^2} + \frac{\bar{\beta}_1^2}{2} \tag{21}$$

Obviously, a straightforward calculation shows that

$$\begin{aligned} \dot{V}_1 \leq& \frac{z_1}{h_1(k_{b1}^2 - z_1^2)} \Bigg(\Psi_1 z_2 + \Psi_1 \alpha_1 + \frac{z_1}{2h_1(k_{b1}^2 - z_1^2)} \\ &- \dot{y}_d + f_1(\bar{x}_1) \Bigg) + \frac{\bar{\beta}_1^2}{2} - \frac{\tilde{\theta}_1 \dot{\hat{\theta}}_1}{r_1} \end{aligned} \tag{22}$$

Similarly, we can get

$$\begin{aligned} \frac{z_1 z_2}{h_1(k_{b_1}^2 - z_1^2)} \Psi_1 \leq& \frac{z_1^2 H_1^2}{2h_1^2(k_{b_1}^2 - z_1^2)^2} + \frac{z_2^2}{2} \\ \leq& \frac{z_1^2}{2(k_{b_1}^2 - z_1^2)^2} + \frac{z_2^2}{2} \end{aligned} \tag{23}$$

Then, (22) is recalculated as

$$\begin{aligned} \dot{V}_1 \leq& \frac{z_1}{h_1(k_{b1}^2 - z_1^2)} \Bigg(\Psi_1 \alpha_1 + f_1(\bar{x}_1) + \frac{h_1 z_1}{2(k_{b1}^2 - z_1^2)} \\ &+ \frac{z_1}{2h_1(k_{b1}^2 - z_1^2)} - \dot{y}_d \Bigg) + \frac{\bar{\beta}_1^2}{2} - \frac{\tilde{\theta}_1 \dot{\hat{\theta}}_1}{r_1} + \frac{z_2^2}{2} \end{aligned} \tag{24}$$

Let $\bar{f}_1(x_1) = f_1(\bar{x}_1) + \frac{h_1 z_1}{2(k_{b1}^2 - z_1^2)} + \frac{z_1}{2h_1(k_{b1}^2 - z_1^2)} - \dot{y}_d$. Next, the Lemma 1 is invoked to fuzzy approximate

$$\bar{f}_1(x_1) = \eta_1^T \phi_1(x_1) + \xi_1(x_1) \tag{25}$$

where $\bar{\xi}_1 > 0$ represents the upper bound of the error, with $|\xi_1(x_1)| \leq \bar{\xi}_1$ holding.

From Young's inequality, one has

$$\begin{aligned} \frac{z_1}{h_1(k_{b1}^2 - z_1^2)} \eta_1^T \phi_1(x_1) \leq& \frac{z_1^2 \left\| \eta_1 \right\|^2 \phi_1^T(x_1) \phi_1(x_1)}{2a_1^2(k_{b1}^2 - z_1^2)^2} + \frac{a_1^2}{2h_1^2} \\ \frac{z_1}{h_1(k_{b1}^2 - z_1^2)} \xi_1(x_1) \leq& \frac{z_1^2}{2\rho_1^2(k_{b1}^2 - z_1^2)^2} + \frac{\rho_1^2 \bar{\xi}_1^2}{2h_1^2} \end{aligned} \tag{26}$$

Then, there exists

$$\begin{aligned} \dot{V}_1 \leq& \frac{z_1}{h_1(k_{b1}^2 - z_1^2)} \Psi_1 \alpha_1 + \frac{z_1^2 \theta_1 \phi_1^T(x_1) \phi_1(x_1)}{2a_1^2(k_{b1}^2 - z_1^2)^2} \\ &+ \frac{z_1^2}{2\rho_1^2(k_{b1}^2 - z_1^2)^2} + \frac{a_1^2}{2h_1^2} + \frac{z_2^2}{2} + \frac{\bar{\beta}_1^2}{2} \\ &+ \frac{\rho_1^2 \bar{\xi}_1^2}{2h_1^2} - \frac{\tilde{\theta}_1 \dot{\hat{\theta}}_1}{r_1} \end{aligned} \tag{27}$$

where a_1 and ρ_1 are positive design constants. The parameter θ_1 satisfies $\theta_1 = \|\eta_1\|^2$.

Naturally, the control signal α_1 and adaptive law $\dot{\hat{\theta}}_1$ are designed as

$$\begin{aligned}\alpha_1 &= \frac{-z_1\hat{\theta}_1\phi_1^T(x_1)\phi_1(x_1)}{2a_1^2(k_{b1}^2-z_1^2)} - \frac{z_1}{2\rho_1^2(k_{b1}^2-z_1^2)} - c_1z_1 \\ \dot{\hat{\theta}}_1 &= \frac{r_1z_1^2\phi_1^T(x_1)\phi_1(x_1)}{2a_1^2(k_{b1}^2-z_1^2)^2} - \sigma_1\hat{\theta}_1\end{aligned} \tag{28}$$

Substituting (28) into (27) yields

$$\dot{V}_1 \le \frac{-c_1z_1^2}{k_{b1}^2-z_1^2} + \frac{\sigma_1\tilde{\theta}_1\hat{\theta}_1}{r_1} + \frac{a_1^2}{2h_1^2} + \frac{z_2^2}{2} + \frac{\bar{\beta}_1^2}{2} + \frac{\rho_1^2\bar{\xi}_1^2}{2h_1^2} \tag{29}$$

It is well known that

$$\tilde{\theta}_1\hat{\theta}_1 = \tilde{\theta}_1(\theta_1 - \tilde{\theta}_1) \le \frac{1}{2}\theta_1^2 - \frac{1}{2}\tilde{\theta}_1^2 \tag{30}$$

Therefore, we can easily get that

$$\begin{aligned}\dot{V}_1 \le& \frac{-c_1z_1^2}{k_{b1}^2-z_1^2} + \frac{\sigma_1\theta_1^2}{2r_1} - \frac{\sigma_1\tilde{\theta}_1^2}{2r_1} + \frac{a_1^2}{2h_1^2} \\ &+ \frac{z_2^2}{2} + \frac{\bar{\beta}_1^2}{2} + \frac{\rho_1^2\bar{\xi}_1^2}{2h_1^2} \\ =& \frac{-c_1z_1^2}{k_{b1}^2-z_1^2} - \frac{\sigma_1\tilde{\theta}_1^2}{2r_1} + \frac{z_2^2}{2} + d_1\end{aligned} \tag{31}$$

where $d_1 = \frac{a_1^2}{2h_1^2} + \frac{\sigma_1\theta_1^2}{2r_1} + \frac{\bar{\beta}_1^2}{2} + \frac{\rho_1^2\bar{\xi}_1^2}{2h_1^2}$.

Step $\mathbf{i}(\mathbf{2} \le \mathbf{i} \le \mathbf{n-1})$: Design the ensuing dynamics for V_i

$$V_i = V_{i-1} + \frac{1}{2h_i}z_i^2 + \frac{\tilde{\theta}_i^2}{2r_i} \tag{32}$$

Its derivative is described as

$$\dot{V}_i = \dot{V}_{i-1} + \frac{z_i\dot{z}_i}{h_i} - \frac{\tilde{\theta}_i\dot{\hat{\theta}}_i}{r_i} \tag{33}$$

Noting the transformation (17), we have

$$\dot{z}_i = \Psi_ix_{i+1} + f_i(\bar{x}_i) + \beta_i(x,t) - \dot{\alpha}_{i-1} \tag{34}$$

where

$$\begin{aligned}\dot{\alpha}_{i-1} =& \sum_{k=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_k}(f_k(\bar{x}_k) + \Psi_kx_{k+1} + \beta_k(x,t)) \\ &+ \sum_{k=0}^{i-1}\frac{\partial\alpha_{i-1}}{\partial y_d^{(k)}}y_d^{(k+1)} + \sum_{k=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\hat{\theta}_k}\dot{\hat{\theta}}_k\end{aligned} \tag{35}$$

Obviously, there exists

$$\begin{aligned}\frac{z_i}{h_i}\left(\frac{\partial\alpha_{i-1}}{\partial x_k}\right)\beta_k(x,t) &\le \frac{z_i^2}{2h_i^2}\left(\frac{\partial\alpha_{i-1}}{\partial x_k^2}\right)^2 + \frac{\bar{\beta}_k^2}{2} \\ \frac{z_i}{h_i}\beta_i(x,t) &\le \frac{z_i^2}{2h_i^2} + \frac{\bar{\beta}_i^2}{2}\end{aligned} \tag{36}$$

Consequently, one can obtain

$$\begin{aligned}\dot{V}_{i-1} \le& \frac{z_{i-1}z_i}{h_i} + \frac{-c_1z_1^2}{k_{b1}^2-z_1^2} - \sum_{k=1}^{n-1}\frac{\sigma_k\tilde{\theta}_k^2}{2r_k} \\ &- \sum_{k=2}^{n-1}c_kz_k^2 + \sum_{k=1}^{i-1}d_k\end{aligned} \tag{37}$$

where $d_k = \frac{a_k^2}{2h_k^2} + \frac{\sigma_k\theta_k^2}{2r_k} + \sum_{l=1}^{k}\frac{\bar{\beta}_k^2}{2} + \frac{\rho_k^2\bar{\xi}_k^2}{2h_k^2}$.

By leveraging (33)-(37), we arrive at

$$\begin{aligned}\dot{V}_i \le& \frac{-c_1z_1^2}{k_{b1}^2-z_1^2} - \sum_{k=2}^{i-1}c_kz_k^2 + \sum_{k=1}^{i-1}d_k - \sum_{k=1}^{i-1}\frac{\sigma_k\tilde{\theta}_k^2}{2r_k} \\ &+ \sum_{l=1}^{i}\frac{\bar{\beta}_i^2}{2} - \frac{\tilde{\theta}_i\dot{\hat{\theta}}_i}{r_i} + \frac{z_i}{h_i}\Bigg(z_{i-1} + f_i(\bar{x}_i) + \frac{z_i}{2} \\ &+ \frac{z_i}{2}\sum_{k=1}^{i-1}\left(\frac{\partial\alpha_{i-1}}{\partial x_k}\right)^2 - \sum_{k=0}^{i-1}\frac{\partial\alpha_{i-1}}{\partial y_d^{(k)}}y_d^{(k+1)}\Bigg) \\ &- \frac{z_i}{h_i}\Bigg(\sum_{k=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_k}(f_k(\bar{x}_k) + \Psi_kx_{k+1}) \\ &- \sum_{k=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\hat{\theta}_k}\dot{\hat{\theta}}_k + \Psi_i(z_{i+1}+\alpha_i)\Bigg) \\ =& \frac{-c_1z_1^2}{k_{b1}^2-z_1^2} - \sum_{i=2}^{i-1}c_kz_k^2 + \sum_{k=1}^{i-1}d_k - \sum_{k=1}^{i-1}\frac{\sigma_k\tilde{\theta}_k^2}{2r_k} \\ &- \frac{\tilde{\theta}_i\dot{\hat{\theta}}_i}{r_i} + \sum_{l=1}^{i}\frac{\bar{\beta}_i^2}{2} + \frac{z_i}{h_i}(\Psi_iz_{i+1} + \Psi_i\alpha_i + \bar{f}_i(x_i))\end{aligned} \tag{38}$$

where

$$\begin{aligned}\bar{f}_i(x_i) =& z_{i-1} + f_i(\bar{x}_i) + \frac{z_i}{2} + \frac{z_i}{2}\sum_{k=1}^{i-1}\left(\frac{\partial\alpha_{i-1}}{\partial x_k}\right)^2 \\ &- \sum_{k=0}^{i-1}\frac{\partial\alpha_{i-1}}{\partial y_d^{(k)}}y_d^{(k+1)} - \sum_{k=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\hat{\theta}_k}\dot{\hat{\theta}}_k \\ &- \sum_{k=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_k}(f_k(\bar{x}_k) + H_kx_{k+1})\end{aligned}$$

By referring to Young's inequality, it leads to

$$\begin{aligned}\frac{z_i}{h_i}\bar{f}_i(x_i) \le& \frac{z_i^2\|\eta_i\|^2\phi_i^T(x_i)\phi_i(x_i)}{2a_i^2} + \frac{a_i^2}{2h_i^2} \\ &+ \frac{z_i^2}{2\rho_i^2} + \frac{\rho_i^2\bar{\xi}_i^2}{2h_i^2}\end{aligned} \tag{39}$$

Further more

$$\begin{aligned}\dot{V}_i \le& \frac{-c_1z_1^2}{k_{b1}^2-z_1^2} - \sum_{k=2}^{i-1}c_kz_k^2 + \sum_{k=1}^{i-1}d_k - \sum_{k=1}^{i-1}\frac{\sigma_k\tilde{\theta}_k^2}{2r_k} \\ &- \frac{\tilde{\theta}_i\dot{\hat{\theta}}_i}{r_i} + \sum_{l=1}^{i}\frac{\bar{\beta}_i^2}{2} + \frac{z_i^2\theta_i\phi_1^T(x_1)\phi_1(x_1)}{2a_i^2} + \frac{a_i^2}{2h_i^2} \\ &+ \frac{z_i^2}{2\rho_i^2} + \frac{\rho_i^2\bar{\xi}_i^2}{2h_i^2} + \frac{z_i}{h_i}(\Psi_iz_{i+1} + \Psi_i\alpha_i)\end{aligned} \tag{40}$$

where $\theta_i = \|\eta_i\|^2$.

Take the control signal α_i and the adaptive law $\dot{\hat{\theta}}_i$ as

$$\begin{aligned}\alpha_i &= \frac{-z_i\hat{\theta}_i\phi_i^T(x_i)\phi_i(x_i)}{2a_i^2} - \frac{z_i}{2\rho_i^2} - c_iz_i \\ \dot{\hat{\theta}}_i &= \frac{r_iz_i^2\phi_i^T(x_i)\phi_i(x_i)}{2a_i^2} - \sigma_i\hat{\theta}_i\end{aligned} \tag{41}$$

Next, combining (40) and (41), a simple calculation gives

$$\begin{aligned}\dot{V}_i \le& \frac{-c_1z_1^2}{k_{b1}^2-z_1^2} - \sum_{k=2}^{i-1}c_kz_k^2 + \sum_{k=1}^{i-1}d_k - \sum_{k=1}^{i-1}\frac{\sigma_k\tilde{\theta}_k^2}{2r_k} + \frac{a_i^2}{2h_i^2} \\ &+ \frac{\rho_i^2\bar{\xi}_i^2}{2h_i^2} + \sum_{l=1}^{i}\frac{\bar{\beta}_i^2}{2} + \frac{\sigma_i\tilde{\theta}_i\hat{\theta}_i}{r_i} - c_iz_i^2 + z_iz_{i+1}\end{aligned} \tag{42}$$

and

$$\tilde{\theta}_i\hat{\theta}_i = \tilde{\theta}_i(\theta_i - \tilde{\theta}_i) \leq \frac{\theta_i^2}{2} - \frac{\tilde{\theta}_i^2}{2} \tag{43}$$

Then, we deduce that

$$\begin{aligned}\dot{V}_i \leq & \frac{-c_1 z_1^2}{k_{b1}^2 - z_1^2} - \sum_{k=2}^{i} c_k z_k^2 + \sum_{k=1}^{i} d_k \\ & - \sum_{k=1}^{i} \frac{\sigma_k \tilde{\theta}_k^2}{2r_k} + z_i z_{i+1}\end{aligned} \tag{44}$$

where $d_i = \frac{a_i^2}{2h_i^2} + \frac{\sigma_i\theta_i^2}{2r_i} + \sum_{l=1}^{i}\frac{\bar{\beta}_l^2}{2} + \frac{\rho_i^2\bar{\xi}_i^2}{2h_i^2}$.

$\mathbf{Step\ n}$: It is worth noting that the actual controller v only occurs in the last step, and the following Lyapunov function holds

$$V_n = V_{n-1} + \frac{1}{2h_n}z_n^2 + \frac{\tilde{\theta}_n^2}{2r_n} \tag{45}$$

Based on (11) and (17), one arrives at

$$\begin{aligned}\dot{V}_n =& \dot{V}_{n-1} + \frac{z_n}{h_n}(\dot{x}_n - \dot{\alpha}_{n-1} + \frac{1}{\tau}H_n\dot{x}_{n+1}) - \frac{\tilde{\theta}_n\dot{\hat{\theta}}_n}{r_n} \\ =& \dot{V}_{n-1} + \frac{z_n}{h_n}(\Psi_n x_{n+1} - \Psi_n u(v) + f_n(\bar{x}_n) + \beta_n(x,t) \\ & - \dot{\alpha}_{n-1} - H_n x_{n+1} + 2H_n u(v)) - \frac{\tilde{\theta}_n\dot{\hat{\theta}}_n}{r_n} \\ \leq & \dot{V}_{n-1} + \frac{z_n}{h_n}((2H_n - h_n)c(v) + f_n(\bar{x}_n) \\ & + \beta_n(x,t) - \dot{\alpha}_{n-1}) - \frac{\tilde{\theta}_n\dot{\hat{\theta}}_n}{r_n}\end{aligned} \tag{46}$$

where

$$\begin{aligned}\dot{\alpha}_{n-1} =& \sum_{k=0}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_k}(f_k(\bar{x}_k) + \Psi_k x_{k+1} + \beta_k(x,t)) \\ & + \sum_{k=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial\hat{\theta}_k}\dot{\hat{\theta}}_k + \sum_{k=0}^{n-1}\frac{\partial\alpha_{n-1}}{\partial y_d^{(k)}}y_d^{(k+1)}\end{aligned} \tag{47}$$

Furthermore, there exists

$$\begin{aligned}\frac{z_n}{h_n}\left(\frac{\partial\alpha_{n-1}}{\partial x_k}\right)\beta_k(x,t) \leq & \frac{z_n^2}{2h_n^2}\left(\frac{\partial\alpha_{n-1}}{\partial x_k}\right)^2 + \frac{\beta_k^2}{2} \\ \frac{z_n}{h_n}\beta_n(x,t) \leq & \frac{z_n^2}{2h_n^2} + \frac{\bar{\beta}_n^2}{2}\end{aligned} \tag{48}$$

Substituting (47) and (48) into (46) yields

$$\begin{aligned}\dot{V}_n \leq & \frac{-c_1 z_1^2}{k_{b1}^2 - z_1^2} - \sum_{k=2}^{i} c_k z_k^2 + \sum_{k=1}^{i} d_k - \sum_{k=1}^{i}\frac{\sigma_k\tilde{\theta}_k^2}{2r_k} \\ & - \frac{\tilde{\theta}_n\dot{\hat{\theta}}_n}{r_n} + \sum_{l=1}^{n}\frac{\bar{\beta}_l^2}{2} + \frac{z_n}{h_n}\Big(z_{n-1} + \frac{z_n}{2} \\ & + f_n(\bar{x}_n) - \sum_{k=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_k}(f_k(\bar{x}_k) + \Psi_k x_{k+1}) \\ & - \sum_{k=0}^{i-1}\frac{\partial\alpha_{n-1}}{\partial y_d^{(k)}}y_d^{(k+1)}\Big) + \frac{z_n}{h_n}\Big(\frac{z_n}{2}\left(\frac{\partial\alpha_{n-1}}{\partial x_k}\right)^2 \\ & - \sum_{k=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial\hat{\theta}_k}\dot{\hat{\theta}}_k + (2H_n - h_n)(c_\mu v + \varsigma)\Big) \\ = & \frac{-c_1 z_1^2}{k_{b1}^2 - z_1^2} - \sum_{k=2}^{n-1} c_k z_k^2 + \sum_{k=1}^{n-1} d_k - \sum_{k=1}^{n-1}\frac{\sigma_k\tilde{\theta}_k^2}{2r_k} \\ & + \sum_{l=1}^{n}\frac{\bar{\beta}_l^2}{2} - \frac{\tilde{\theta}_n\dot{\hat{\theta}}_n}{r_n} + \frac{z_n}{h_n}((2H_n - h_n)c_\mu v + \bar{f}_n(x_n))\end{aligned} \tag{49}$$

By virtue of Assumption 3, it gives

$$\begin{aligned}\bar{f}_n(x_n) =& z_{n-1} + \frac{z_n}{2} + f_n(\bar{x}_n) + \frac{z_n}{2}\left(\frac{\partial\alpha_{n-1}}{\partial x_k}\right)^2 \\ & + H_n\varsigma - \sum_{k=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial\hat{\theta}_k}\dot{\hat{\theta}}_k - \sum_{k=0}^{n-1}\frac{\partial\alpha_{n-1}}{\partial y_d^{(k)}}y_d^{(k+1)} \\ & - \sum_{k=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_k}(f_k(\bar{x}_k) + H_k x_{k+1})\end{aligned}$$

Similarly, one can get

$$\begin{aligned}\frac{z_n}{h_n}\eta^T\phi_n(x_n) \leq & \frac{z_n^2\|\eta_n\|^2\phi_n^T(x_n)\phi_n(x_n)}{2a_n^2} + \frac{a_n^2}{2h_n^2} \\ \frac{z_n}{h_n}\xi_n \leq & \frac{z_n^2}{2\rho_n^2} + \frac{\rho_n^2\bar{\xi}_n^2}{2h_n^2}\end{aligned} \tag{50}$$

Additionally, let $\theta_n = \frac{\|\eta_n\|^2}{c_m}$, under Assumption 4, the virtual controller v and the adaptive law $\dot{\hat{\theta}}_n$ are produced here

$$\begin{aligned}v =& \frac{-c_n z_n}{c_m} - \frac{z_n\hat{\theta}_n\phi_n^T(x_n)\phi_n(x_n)}{2a_n^2} - \frac{z_n}{2c_m\rho_n^2} \\ \dot{\hat{\theta}}_n =& \frac{r_n z_n^2 c_m\phi_n^T(x_n)\phi_n(x_n)}{2a_n^2} - \sigma_n\hat{\theta}_n\end{aligned} \tag{51}$$

and

$$\tilde{\theta}_n\hat{\theta}_n = \tilde{\theta}_n(\theta_n - \tilde{\theta}_n) \leq \frac{\theta_n^2}{2} - \frac{\tilde{\theta}_n^2}{2} \tag{52}$$

Based on (50)-(52), (49) becomes

$$\dot{V}_n \leq \frac{-c_1 z_1^2}{k_{b1}^2 - z_1^2} - \sum_{k=2}^{n} c_k z_k^2 + \sum_{k=1}^{n} d_k - \sum_{k=1}^{n}\frac{\sigma_k\tilde{\theta}_k^2}{2r_k} \tag{53}$$

where $d_n = \frac{a_n^2}{2h_n^2} + \frac{\sigma_n\theta_n^2}{2r_n} + \sum_{l=1}^{n}\frac{\bar{\beta}_n^2}{2} + \frac{\rho_n^2\bar{\xi}_n^2}{2h_n^2}$.

Noting Lemma 2, one can obtain

$$\frac{-c_1 z_1^2}{k_{b1}^2 - z_1^2} \leq -c_1\log\frac{k_{b1}^2}{k_{b1}^2 - z_1^2} \tag{54}$$

Finally, the above discussion gives us

$$\dot{V}_n \leq -cV_n + \varpi \tag{55}$$

where $c = \min\{2c_k, \sigma_k, k = 1, 2, \ldots, n\}$, and $\varpi = \sum_{k=1}^{n} d_k$.

Relying on the aforementioned calculations, owing to the boundedness of $x_i(t)$, $z_i(t)$ and $\theta_i(t)$ for $i = 1, 2, \ldots, n$, the actual control signal $u(t)$ is bounded. By applying the conditions stated in Assumption 2 and Lemma 2, considering that $|z_1| \leq k_{b1}$, and $y(t) = y_d(t) + z_1(t)$ with $|y_d(t)| \leq \chi_0$. Therefore, we can deduce that $|y(t)| \leq |y_d(t)| + |z_1(t)| < \chi_0 + k_{b1} = k_{m1}$ for all $t \geq 0$. In this way, the state constraints are not transgressed.

Remark 4. Although the control of nonlinear systems with input delays has been researched in [10], the obstacles posed by unknown control directions were not sufficiently considered. In addition, input delays are separated from the input signal by the Pade approximation, which eases the analytical process and can be applied to complex system

(1) with input saturations and state constraints at the same time, thus effectively meeting the requirements of control engineering.

IV. Simulation Studies

This section verifies the credibility and practicality of the theoretical results through two simulation examples.

Example 1: Consider a third-order nonlinear system

$$\begin{aligned}\dot{x}_1 &= x_2 + f_1(\bar{x}_1) + \beta_1(x,t)\\ \dot{x}_2 &= x_3 + f_2(\bar{x}_2) + \beta_2(x,t)\\ \dot{x}_3 &= u + f_3(\bar{x}_3) + \beta_3(x,t)\\ \dot{x}_4 &= -\tau x_4 + 2\tau u\end{aligned} \tag{56}$$

where

$$\begin{aligned}f_1(\bar{x}_1) =& -0.1\sin(x_1)\\ f_2(\bar{x}_2) =& 0.01\sin(x_1 x_2)\\ f_3(\bar{x}_3) =& -e^{-x_3^2}\sin x_1 \cos x_2\\ \beta_i(x,t) =& 0.1\sin(x_1 x_2 x_3)\cos(t), \quad i = 1,2,3\end{aligned}$$

On this basis, a subordinate function is chosen as

$$u_{F_j^i}(x_i) = e^{-\frac{(x_i - 2 + 0.5j)^2}{8}}, j = 1,2,\ldots,7, i = 1,2,3 \tag{57}$$

Now, we present the remaining parameters of the system. Specifically, c_1=5, c_2=4, c_3=3, $a_i = 0.5$, $r_i = 1$ and $\sigma_i = 0.5$ for $i = 1,2,3$, $\tau = 0.8$, $c_m = 0.5$, $k_{b1} = 1$, $v_{\min} = -2$, $v_{\max} = 0.5$. The initial conditions are selected as $x_1 = x_4 = 0$, $x_2 = 0.2$, $x_3 = 0.3$, $\hat{\theta}_1 = \hat{\theta}_2 = 0.1$, $\hat{\theta}_3 = 0.2$. Design the reference signal $y_d = 0.2\sin(t)$, and the state variable x_1 is constrained such that $|x_1| \le 0.3$.

In accordance with the designed tracking control strategy, Figs. 1-6 indicate the corresponding simulation findings. Fig. 1 depicts the output signal y and the predetermined trajectory y_d. The system states x_2 and x_3 are illustrated in Fig. 2. Fig. 3 shows the trajectory of variable x_4. The response curves of the adaptive laws $\hat{\theta}_1$, $\hat{\theta}_2$ and $\hat{\theta}_3$ are presented in Fig. 4. Moreover, the actual control signal v and the saturation input $\mathrm{sat}(v)$ in Figs. 5 and 6 are bounded respectively.

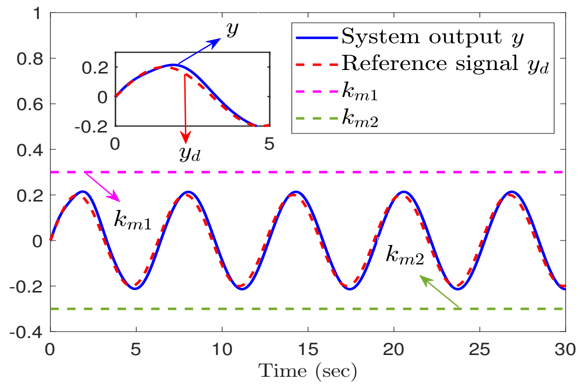

Fig. 1: State trajectories y and y_d in Example 1.

Example 2: To further demonstrate the utility of the strategy, we studied a one-link manipulator having the following dynamics in an industrial system

$$\begin{aligned}\mathcal{A}\ddot{p} + \mathcal{C}\dot{p} + \mathcal{N}\sin(p) &= v\\ \mathcal{F}\dot{v} + \mathcal{D}v &= u - \mathcal{L}\dot{q}\end{aligned} \tag{58}$$

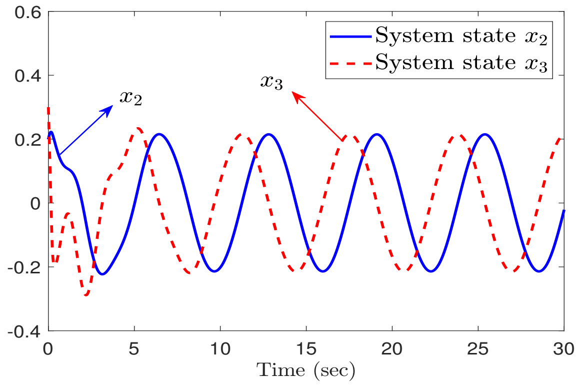

Fig. 2: System states x_2 and x_3 in Example 1.

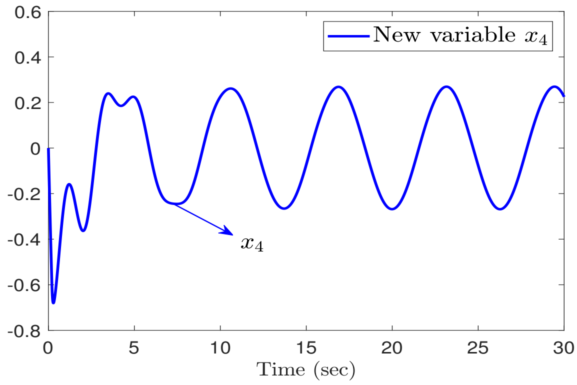

Fig. 3: New variable x_4 in Example 1.

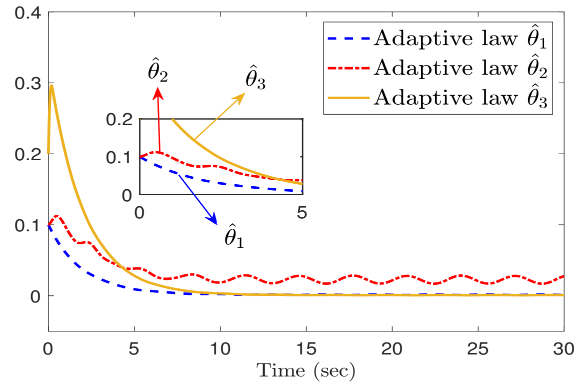

Fig. 4: Adaptive laws $\hat{\theta}_1$, $\hat{\theta}_2$ and $\hat{\theta}_3$ in Example 1.

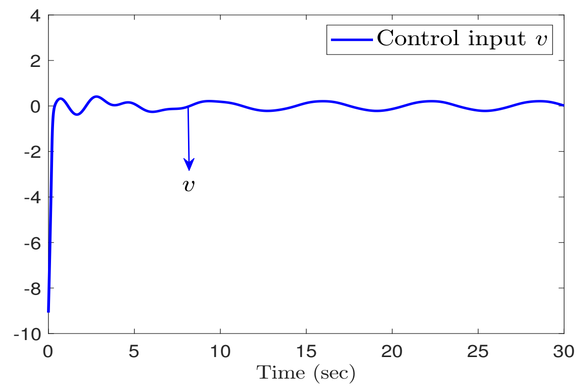

Fig. 5: Control input v in Example 1.

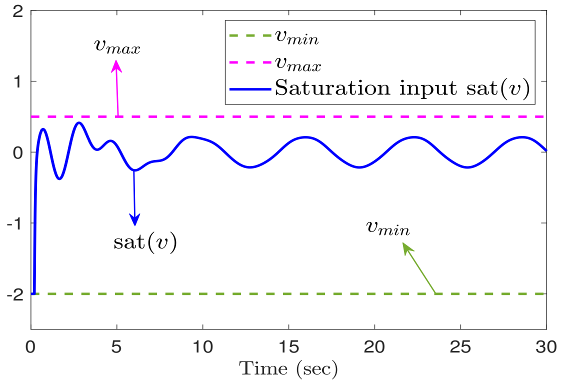

Fig. 6: Saturation input $\mathrm{sat}(v)$ in Example 1.

where $\ddot{p}$, $\dot{p}$, p, u are joint acceleration, velocity, position and system input, $\mathcal{A} = 1\mathrm{kg}\cdot\mathrm{m}^2$ is the rotor moment of inertia, $\mathcal{C} = 1\mathrm{N}\cdot\mathrm{s/m}^2$ is coefficient of viscous friction, $\mathcal{L}$ represents the coefficient of reverse electromotive force, $\mathcal{D} = 1\Omega$ and $\mathcal{F} = 0.1\mathcal{D}$ are the armature resistance and armature inductance respectively.

Then, let $x_1 = p$, $x_2 = \dot{p}$, $x_3 = v$, we then have

$$\begin{aligned} \dot{x}_1 &= x_2 + \beta_1(x,t) \\ \dot{x}_2 &= x_3 + f_2(\bar{x}_2) + \beta_2(x,t) \\ \dot{x}_3 &= 10u + f_3(\bar{x}_3) + \beta_3(x,t) \\ \dot{x}_4 &= -\tau x_4 + 2\tau u \end{aligned} \tag{59}$$

where

$$\begin{aligned} f_2(\bar{x}_2) =& -x_2 - 0.8\sin(x_1) \\ f_3(\bar{x}_3) =& -2x_2 - 10x_3 \\ \beta_i(x,t) =& 0.01\sin(x_1x_2x_3)\cos(t), \quad i = 1,2,3 \end{aligned}$$

Accordingly, the values of other parameters are taken as $c_1 = 8$, $c_2 = c_3 = 5$, $a_i = 0.4$, $r_i = 1$ and $\sigma_i = 0.5$ for $i = 1,2,3$, $c_m = 0.5$, $k_{b1} = 1$, and $\tau = 2 + 0.1\sin(t)$. The initial values are specified as $x_1 = 0$, $x_2 = x_4 = 0.1$, $x_3 = 0.2$, $\hat{\theta}_1 = 0.1$, $\hat{\theta}_2 = \hat{\theta}_3 = 0.2$. Given the reference signal $y_d = 0.2\sin(t)$ and the constraint that the system state x_1 is constrained to be $|x_1| \leq 0.3$. Furthermore, we define $v_{\min} = -2$ and $v_{\max} = 0.7$.

Similarly, the analysis findings are illustrated in Figs. 7-12. Of these, Fig. 7 displays the tracking curves for y and y_d. The system states x_2, x_3 and the variable x_4 are shown in Figs. 8 and 9. Additionally, the response curves of the adaptive laws $\hat{\theta}_1$, $\hat{\theta}_2$, $\hat{\theta}_3$, the actual control input v and the saturation input $\mathrm{sat}(v)$ depicted in Figs. 10-12 are bounded, respectively.

V. Conclusion

This paper is concerned with the fuzzy adaptive tracking control issue of strict-feedback nonlinear systems with input delays and saturations. By introducing a hyperbolic tangent function into the Pade approximation method, it is guaranteed that the control signal does not to violate the saturation bound, even if input delays occur. Then, a fuzzy adaptive controller is constructed in accordance with the boundary estimation method and the BLF. Furthermore, the stability analysis shows that all closed-loop signals are uniformly bounded, and the output error converges to a small area near

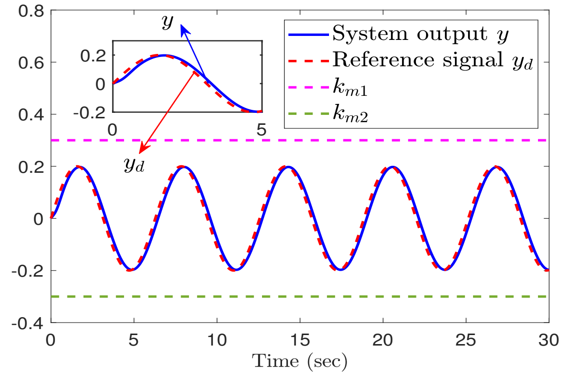

Fig. 7: State trajectories y and y_d in Example 2.

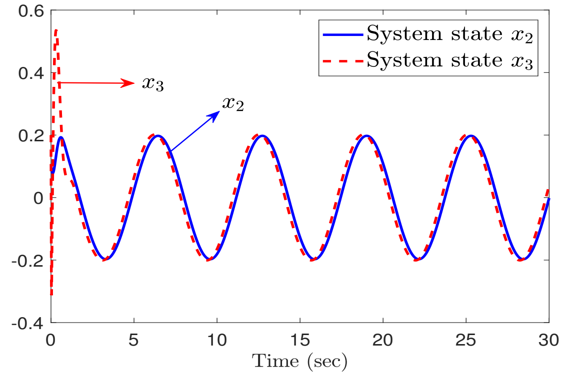

Fig. 8: System states x_2 and x_3 in Example 2.

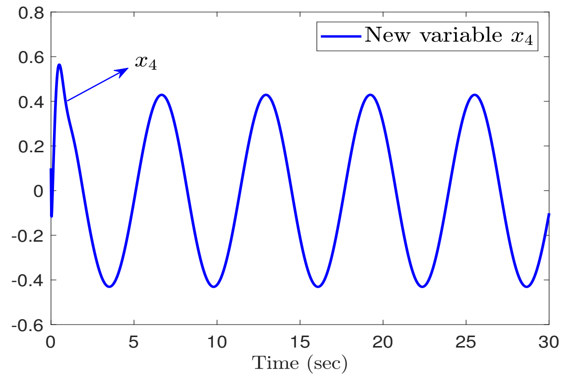

Fig. 9: New variable x_4 in Example 2.

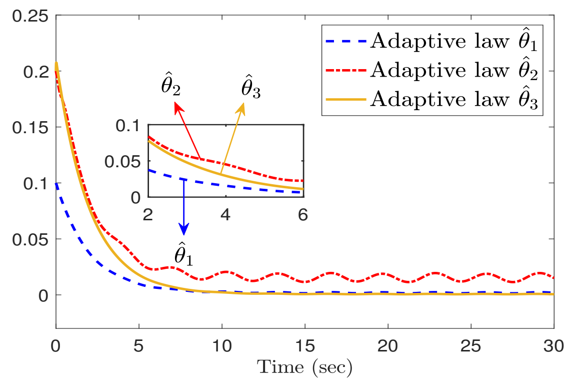

Fig. 10: Adaptive laws $\hat{\theta}_1$, $\hat{\theta}_2$ and $\hat{\theta}_3$ in Example 2.

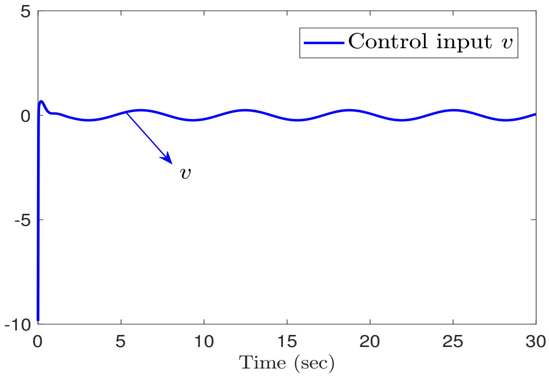

Fig. 11: Control input v in Example 2.

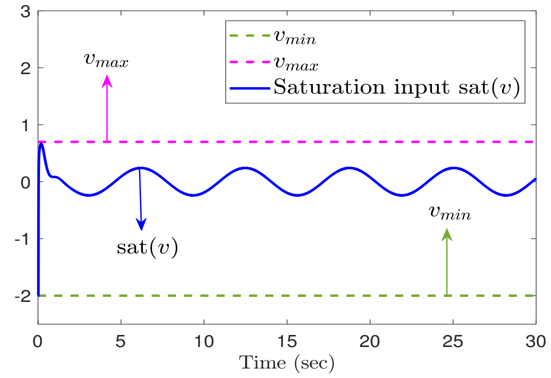

Fig. 12: Saturation input $\mathrm{sat}(v)$ in Example 2.

the origin. However, continuous updates of the control signal result in significant resource wastage, and ensuring the convergence of system states to stability within a predefined time remains a challenge. These issues will be further investigated in future work.

References

[1] S. Pulgarin-Correa, S. Lopez-Blandon, and E. Giraldo, "A comparison analysis of adaptive control techniques applied over coupled DC motors under disturbances," *Engineering Letters*, vol. 32, no. 11, pp. 2052-2062, 2024.

[2] L. Ma, C. Cheng, J. F. Guo, et al, "Direct yaw-moment control of electric vehicles based on adaptive sliding mode," *Mathematical Biosciences and Engineering*, vol. 20, no. 7, pp. 13334-3355, 2023.

[3] X. N. Song, Q. Y. Zhang, M. Wang, and S. Song, "Distributed estimation for nonlinear PDE systems using space-sampling approach: applications to high-speed aerospace vehicle," *Nonlinear Dynamics*, vol. 106, pp. 3183-3198, 2021.

[4] N. Wang and S. C. Tong, "Observer-based adaptive fuzzy control of a class of MIMO non-strict feedback nonlinear systems," *Journal of the Franklin Institute*, vol. 355, no. 12, pp. 4873-4896, 2018.

[5] C. Y. Liu, R. Loxton, K. L. Teo, S. Wang, "Optimal state-delay control in nonlinear dynamic systems," *Automatica*, vol. 135, 2022.

[6] Y. Zhang, M. Chadli, and Z. R. Xiang, "Prescribed-time adaptive fuzzy optimal control for nonlinear systems," *IEEE Transactions on Fuzzy Systems*, vol. 32, no. 4, pp. 2403-2412, 2024.

[7] I. Michailidis, S. Baldi, E. B. Kosmatopoulos, and P. A. Ioannou, "Adaptive optimal control for large-scale nonlinear systems," *IEEE Transactions on Automatic Control*, vol. 62, no. 11, pp. 5567-5577, 2017.

[8] C. L. Wang, L. Guo, C. Y. Wen, et al, "Adaptive neural network control for a class of nonlinear systems with unknown control direction," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 50, no. 11, pp. 4708-4718, 2020.

[9] Z. S. Huang, T. S. Li, Y. Long, et al, "Improved prescribed performance control for nonlinear systems with unknown control direction and input dead-zone," *International Journal of Robust and Nonlinear Control*, vol. 34, no. 7, pp. 4489-4508, 2024.

[10] H. Chen, Y. Tang, R. Xu, et al, "Observer-based finite-time adaptive fault-tolerant control for nonlinear system with unknown time-varying delay," *Circuits, Systems, and Signal Processing*, vol. 43, pp. 6254-6278, 2024.

[11] H. Q. Wang, S. W. Liu, and X. B. Yang, "Adaptive neural control for non-strict-feedback nonlinear systems with input delay," *Information Sciences*, vol. 514, pp. 605-616, 2020.

[12] C. W. Wu, J. X. Liu, X. J. Jing, et al, "Adaptive fuzzy control for nonlinear networked control systems," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 47, no. 8, pp. 2420-2430, 2017.

[13] H. Y. Li, L. J. Wang, H. P. Du, and A. Boulkroune, "Adaptive fuzzy backstepping tracking control for strict-feedback systems with input delay," *IEEE Transactions on Fuzzy Systems*, vol. 25, no. 3, pp. 642-652, 2017.

[14] X. Wang, S. Huang, W. Zou and Z. Xiang, "Finite-time stabilization for a class of nonlinear systems with time-varying delay," *International Journal of Robust and Nonlinear Control*, vol. 30, no. 8, pp. 3164-3178, 2020.

[15] J. H. Song, Y. Chen, Y. Z. Liu, and L. J. Zhang, "Fixed-time fuzzy adaptive fault-tolerant control for strict-feedback nonlinear systems with input delay," *IEEE Transactions On Systems*, vol. 53, no. 11, pp. 6999-7010, 2023.

[16] X. Zhang, J. Q. Tan, Y. G. Yao, and J. Wu, "Adaptive practical fixed-time tracking control for uncertain nonstrict-feedback systems with input delay and prescribed boundary constraints," *International Journal of Adaptive Control and Signal Processing*, vol. 36, no. 3, pp. 653-669, 2023.

[17] Z. W. Hua, "Fixed-time prescribed performance tracking for nonlinear systems with unknown time-varying input delay," *Nonlinear Dynamics*, vol. 113, pp. 661-680, 2025.

[18] C. G. Liu, H. Q. Wang, X. P. Liu, and Y. C. Zhou, "Adaptive fuzzy funnel control for nonlinear systems with input deadzone and saturation," *International Journal of Systems Science*, vol. 51, no. 9, pp. 1542-1555, 2020.

[19] Y. Jiang and S. Q. Yi, "Finite-time adaptive practical tracking control for high-order nonlinear systems with full-state constraints and input saturation," *International Journal of Control*, vol. 97, no. 8, pp. 1704-1716, 2023.

[20] H. X. Xie, Y. W. Jing, G. Dimirovski and J. Q. Chen, "Adaptive fuzzy prescribed time tracking control for nonlinear systems with input saturation," *ISA Transactions*, vol. 143, pp. 370-384, 2023.

[21] R. Chang, T. T. Hou, Z. Z. Bai, and C. Y. Sun, "Event-triggered adaptive tracking control for nonlinear systems with input saturation and unknown control directions," *International Journal of Robust and Nonlinear Control*, vol. 34, no. 6, pp. 3891-3911, 2024.

[22] W. J. Si and X. D. Dong, "Adaptive neural control for MIMO stochastic nonlinear pure-feedback systems with input saturation and full-state constraints," *Neurocomputing*, vol. 275, pp. 298-307, 2018.

[23] H. F. Min, S. Y. Xu, Y. M. Li, and Z. Q. Zhang, "Adaptive stabilization of uncertain nonlinear systems under output constraint," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 52, no. 6, pp. 3957-3966, 2022.

[24] Q. K. Yu, J. X. Ding, L. B. Wu, and X. Q. He, "Event-triggered prescribed time adaptive fuzzy fault-tolerant control for nonlinear systems with full state constraints," *Engineering Letters*, vol. 32, no. 8, pp. 1577-1584, 2024.

[25] C. C. Chen and Z. Y. Sun, "A unified approach to finite-time stabilization of high-order nonlinear systems with an asymmetric output constraint," *Automatica*, vol. 111, 2020.

[26] Y. L. Yang, Z. J. Liu, Q. Li, and D. C. Wunsch, "Output constrained adaptive controller design for nonlinear saturation systems," *IEEE/CAA Journal of Automatica Sinica*, vol. 8, no. 2, pp. 441-454, 2021.