Observer-based Fuzzy Adaptive Finite-time Fault-tolerant Control for Switched Stochastic Nonlinear Systems

Haoran Qi, Lidong Wang*, Zhongfeng Li* and Aosen Zhang

Abstract—**This research investigates finite-time fault-tolerant control (FTC) for switched stochastic nonlinear systems under arbitrary switching laws. To address the challenges posed by unmeasured state variables, a fuzzy observer is developed for state estimation. Additionally, fuzzy logic systems (FLS) are employed to approximate unknown nonlinear functions within the system. The issue of differential explosion is mitigated through the use of command filters, and compensation signals are introduced to correct the errors introduced by these filters, thereby enhancing the overall control performance. Considering actuator faults such as bias faults and loss of effectiveness, an adaptive fault-tolerant controller is proposed. It is shown that, under arbitrary switching conditions, the proposed method ensures finite-time closed-loop stability and rapid convergence of the tracking error to a small neighborhood around the origin. A simulation example is provided to validate the effectiveness of the theoretical results.**

Index Terms—**Switched stochastic nonlinear system, Fault-tolerant control, Arbitrary switching law, Fuzzy logic system, Command filter.**

I. Introduction

STOCHASTIC disturbances are commonly found in actual systems, including electricity generation systems, thermal processes, and network control systems. These random factors can have unpredictable effects on the controlled objects or controlled processes within the system. Numerous investigations into the adaptive control of stochastic systems have been conducted, with notable contributions including those in [1] and [2]. Researchers typically address problems involving gradient and higher-order terms in $It\hat{o}$ stochastic differential equations by constructing quadratic or quartic Lyapunov functions [3]. In reference [4], the author introduced a stochastic system control method that leverages backstepping technology, incorporating a quadratic Lyapunov function and a cost-sensitive function. The work of [5] marked a significant advancement by integrating the quadratic Lyapunov function into the backstepping framework, thereby developing an adaptive control strategy for a class of stochastic nonlinear systems with strict feedback. Despite this progress, traditional control methods frequently encounter difficulties in effectively handling unknown terms or functions within the system. Consequently, the integration of fuzzy logic systems or neural network technology with adaptive control methodologies has been proposed to address the challenges and enhance the performance of complex control systems. Significant progress has been made in researching these approaches [6, 7]. In these studies, the parameters of fuzzy logic systems or neural networks are dynamically tuned through adaptive control mechanisms to approximate unknown functions. Meanwhile, the stability of the resulting systems is rigorously analyzed and established using Lyapunov theory.

In practical engineering applications, many systems require finite time to achieve control objectives, such as spacecraft systems, robotic systems, and advanced missile systems [8–10]. This goal can be achieved by employing finite time control methods while improving the speed of convergence, robustness, and precision of the system. In [11], it was demonstrated that the tracking error can be confined to a small neighborhood around the origin within a finite time, while ensuring the boundedness in probability of the closed-loop system. Sui et al. developed a stochastic finite-time stability theorem by integrating finite-time theory and $It\hat{o}$ differential equations. This theorem addresses the finite-time output feedback control problem for a non-standard lower triangular stochastic nonlinear system with uncertain parameters [12]. In [13], a disturbance observer which can control disturbance attenuation was designed for a class of multi-disturbance stochastic systems.

Nonetheless, the aforementioned control strategies are fundamentally contingent upon the underlying assumptions that there are no faults in the system actuators. In fact, the control system is prone to various problems such as equipment wear, aging, or component failure under long-term working conditions, which may lead to the failure of the actuator [14–16]. Therefore, it is important to study the problem of maintaining system stability in the case of actuator failure. In [17], the FTC problem for stochastic multi-fault nonlinear systems was examined by Wang. In [18], Bai et al. proposed an innovative approach known as the adaptive fixed-time channel control technique, specifically tailored for a particular class of nonlinear systems. This method solves the problem of actuator failure and ensures that the tracking error remains within the bounds of the specified performance funnel and achieves convergence within

Manuscript received October 26, 2024; revised March 11, 2025. This work was supported in part by the Fundamental Research Funds for the Liaoning Universities(LJ212410146009), in part by the Liaoning Provincial Joint Funds Project-General Funding Program (Grant No. 2023-MSLH-323).

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the designated time interval. In [19], a fuzzy controller was designed for stochastic systems experiencing actuator failures using backstepping and fuzzy logic control.

In recent times, adaptive FTC has also been extended from general systems to switching systems[20]. Under the concept of the Generalized Separation Principle (GSP), Yang et al. designed an active FTC that can be applied to high-mobility vehicles utilizing the average dwell time criterion. In addressing actuator faults in switching nonlinear systems, Zhang et al. proposed a static output feedback (SOF) control strategy to effectively mitigate the impact of such failures, as detailed in [21], and the controller gain is determined through iterative algorithms to avoid the conservatism of traditional singular value decomposition methods. Through the application of the average dwell time concept, it can ensure good tracking performance under specified constraints. In [22], an adaptive fault-tolerant control technique was developed for a class of switching nonlinear systems with uncertain functions and unobservable states. This approach integrates an enhanced mean residence time method with the backstepping technique to address the challenges posed by these systems.

This paper proposes a novel command filtering-based output feedback fault-tolerant controller for a class of switched stochastic nonlinear systems, leveraging an adaptive backstepping control framework. The designed controller achieves fast convergence of the control system under arbitrary switching laws and in finite time. A major highlight of this scheme is the introduction of command filtering methods in each step, which reduces the differential calculation in the traditional backstepping method. The primary advantages can be summarized as:

(1)Unlike the fault model in [23], our proposed model accounts for the concurrent occurrence of both loss of effectiveness and bias faults. This enhancement significantly broadens the applicability of the model.

(2)In the process of developing the virtual control law, the differential explosion problem is resolved by using the command filtering technique, and the filtering error problem is resolved by incorporating an error compensation mechanism. This method not only compensates for the DSC method's deficiency but also enhances the system's tracking precision.

(3)Theoretical research demonstrates that when the system actuator fails, the suggested control strategy may guarantee that the output signal stays stable and that the tracking error converges to a neighborhood around the origin within a finite time. The semiglobal finite-time stable in probability (SGFSP) of switched stochastic systems is guaranteed.

II. System Description and Preliminaries

Examine the following strictly-feedback switched stochastic nonlinear system with actuator faults:

$$\begin{cases} \dot{\mathcal{X}}_j = \left(f_{\sigma j}\left(\bar{\mathcal{X}}_j\right) + \mathcal{X}_{j+1}\right)dt + \phi_{\sigma j}\left(\bar{\mathcal{X}}_j\right)d\omega \\ \dot{\mathcal{X}}_m = \left(f_{\sigma m}(\bar{\mathcal{X}}_m) + u_\sigma(t)\right)dt + \phi_{\sigma m}(\bar{\mathcal{X}}_m)d\omega \\ y = \mathcal{X}_1 \end{cases} \tag{1}$$

where $\bar{\mathcal{X}}_j = [\mathcal{X}_1, \cdots \mathcal{X}_j]^T \in R^j$ is the system state vector. $y \in R$ is the system output. $\sigma : [0, \infty) \to N = \{1, \ldots, I_N\}$ is the switching signal. $f_{kj}(\cdot)$ and $\phi_{kj}(\cdot)$, $j = 1, \ldots, m, k \in N$ stand for the unknown smooth function of the kth subsystem. Here, it is assumed that only $\mathcal{X}_1$ is available for measurement.

This paper considers time-varying actuator faults, including losing effectiveness and bias faults. Assuming an actuator failure occurs at time t, the control input then be characterized by the following fault model:

$$u_k(t) = \rho_k v_k(t) + \bar{u}_k(t) \tag{2}$$

with $v_k(t)$ reflecting the actual input signal of the subsystem. $\bar{u}_k(t)$ is an unknown function representing bias faults. $\rho_k \in (0, 1]$ being the known factor, when $\rho_k = 1$, indicates that the actuator does not have any losing effectiveness failure, only considering bias faults. If $0 < \rho_k < 1$, it means that need to consider both losing effectiveness and bias faults simultaneously.

Assumption 1: There exist normal numbers u^* that satisfy $|\bar{u}_k(t)| \leq u^*$.

Now consider a class of stochastic nonlinear systems:

$$dx = f(\mathcal{X})dt + \phi(\mathcal{X})d\omega \tag{3}$$

where $\mathcal{X}$, $f(\mathcal{X})$ and $\phi(\mathcal{X})$ are defined in (1). The variable ω represents an r-dimensional standard Brownian motion, which is defined as $E\{d\omega \cdot d\omega^{\mathrm{T}}\} = \varsigma(t)\varsigma(t)^{\mathrm{T}}dt$.

Definition 1: For $\forall V(\mathcal{X}) \in C^2$, give the following definition for the differential operator $\mathcal{L}$:

$$\mathcal{L}V = \frac{\partial V}{\partial \mathcal{X}}f + \frac{1}{2}Tr\{\phi^{\mathrm{T}}\frac{\partial^2 V}{\partial \mathcal{X}^2}\phi\} \tag{4}$$

where $Tr(\cdot)$ represents the trace operation applied to a matrix.

Lemma 1: [24] For $s_j \in R, 0 < r \leq 1$, there is

$$\left(\sum_{j=1}^{m}|s_j|\right)^r \leq \sum_{j=1}^{m}|s_j|^r \leq m^{1-r}\left(\sum_{j=1}^{m}|s_j|\right)^r \tag{5}$$

Lemma 2: [25] For the system (3), there exist $\forall V(\mathcal{X}) \in C^2$ and $\psi_1, \psi_2 \in K_\infty$, positive constants $\mu_1, \mu_2, \gamma, \kappa$ and $0 < \gamma < 1, \quad 0 < \kappa < \infty$, such that

$$\begin{gathered} \psi_1(\|\mathcal{X}\|) \leq V(\mathcal{X}) \leq \psi_2(\|\mathcal{X}\|) \\ \mathcal{L}V(\mathcal{X}) \leq -\mu_1 V(\mathcal{X}) - \mu_2 V(\mathcal{X})^\gamma + \kappa \end{gathered} \tag{6}$$

Then, the system (3) is SGFSP, and a setting time $T(\mathcal{X}_0)$ meets the following relation

$$E[T(\mathcal{X}_0)] \leq \frac{1}{(1-\gamma)\mu_1}\ln\left[\frac{\mu_1 V(\mathcal{X}_0)^{1-\gamma} + \mu_2\nu}{\mu_1\left(\frac{\kappa}{(1-\nu)\mu_2}\right)^{\frac{1-\gamma}{\gamma}} + \mu_2\nu}\right] \tag{7}$$

where $0 < \nu < 1$ is a constant.

Lemma 3: [26] For the parameters F_1 and F_2, the following equation holds without input noise process: $\Phi_1 = \bar{\alpha}_r, \gamma_1 = \dot{\bar{\alpha}}_r$. If input noise satisfies $|\alpha_r - \bar{\alpha}_r| \leq \rho$, the following inequalities are satisfied within a finite time interval with scalars $p_1 > 0$ and $p_2 > 0$

$$\begin{aligned} |\Phi_1 - \overline{\alpha}_r| &\leq p_1\rho = h_1 \\ \left|\gamma_1 - \dot{\bar{\alpha}}_r\right| &\leq p_2\rho^{\frac{1}{2}} = h_2 \end{aligned} \tag{8}$$

Lemma 4: [27] For $\forall(l_1, l_2) \in R^2$, and positive constants x, y, p, one has

$$|l_1|^x|l_2|^y \leq \frac{x}{x+y}p|l_1|^{x+y} + \frac{y}{x+y}p^{-x/y}|l_2|^{x+y} \tag{9}$$

Lemma 5: [28] For a continuous function $f(Z)$, it possesses the following property:

$$\sup_{Z\in\Omega}\left|f(Z)-\theta^{\mathrm{T}}S(Z)\right|\leq\varepsilon,\varepsilon>0 \tag{10}$$

where $S(Z) = [p_1(Z),\cdots,p_N(Z)]^T$, $p_j(z)(=\prod_{i=1}^{n}\mu_{F_i^j}(z_i)/\sum_{j=1}^{N}[\prod_{i=1}^{n}\mu_{F_i^j}(z_i)])$ is the fuzzy basis functions. $\theta=[\theta_1,\cdots,\theta_N]^T$, the point θ_i is where the fuzzy membership function $\mu_{G^i}(\theta_i)$ attains its maximum value.

Assumption 2: The covariance is bounded: $\phi^T\varsigma(t)\varsigma(t)^T\phi=\overline{\varsigma\varsigma}^T$, $\phi=[\phi_{\sigma1}(\mathcal{X}_1),\phi_{\sigma2}(\overline{\mathcal{X}}_2),...,\phi_{\sigma m}(\overline{\mathcal{X}})]^T$.

Assumption 3: In this paper, the desired signal y_d of the system and its first time derivative are continuous and bounded.

III. Fuzzy State Observer Design

To address the issue of unmeasured states, a fuzzy state observer is introduced to provide accurate state estimates. Subsequently, an observer-based controller is constructed utilizing the backstepping methodology.

According to Lemma 5, we have $\hat{f}_{kj}(\hat{\overline{\mathcal{X}}}_j\mid\hat{\theta}_{kj})=\hat{\theta}_{kj}^{\mathrm{T}}S_j(\hat{\overline{\mathcal{X}}}_j)$, where $\hat{\theta}_{kj}=\theta_{kj}-\tilde{\theta}_{kj}$ is the estimate of the unknown ideal vector θ_{kj}.

$$\theta_{kj}=\arg\min_{\hat{\theta}_{kj}\in\Omega_{kj}}|\hat{f}_{kj}(\hat{\overline{\mathcal{X}}}_j\mid\hat{\theta}_{kj})-f_{kj}(\hat{\overline{\mathcal{X}}}_j)\,|] \tag{11}$$

There exist the fuzzy estimation error such that

$$\begin{aligned}\delta_{kj}&=f_{kj}(\overline{\mathcal{X}}_j)-\hat{f}_{kj}(\hat{\overline{\mathcal{X}}}_j\mid\hat{\theta}_{kj})\\ \varepsilon_{kj}&=f_{kj}(\overline{\mathcal{X}}_j)-\hat{f}_{kj}(\hat{\overline{\mathcal{X}}}_j\mid\theta_{kj})\end{aligned} \tag{12}$$

where $|\delta_{kj}|<\overline{\delta}_{kj},|\varepsilon_{kj}|<\overline{\varepsilon}_{kj},\overline{\delta}_{kj}>0$, and $\bar{\varepsilon}_{kj}>0$ are the constant.

Subsequently, system (1) can be reformulated as follows:

$$\begin{aligned}\dot{\mathcal{X}}&=[A_k\mathcal{X}+k_ky+\sum_{j=1}^{m}B_jf_{kj}\left(\overline{\mathcal{X}}_j\right)+Du_k(t)]dt+\phi d\omega\\ y&=C^T\mathcal{X}\end{aligned} \tag{13}$$

where $A_k=\begin{bmatrix}-k_{k1} & & \\ \vdots & I_{m-1} & \\ -k_{km} & 0 & \cdots & 0\end{bmatrix}, k_k=\begin{bmatrix}k_{k1}\\ \vdots\\ k_{km}\end{bmatrix}$, $B_j=[0\cdots1\cdots0]^T$, $C=[1\cdots0\cdots0]$, $D=[0\cdots0\cdots1]$, $\phi=[\phi_{k1}(\mathcal{X}_1),\phi_{k2}(\overline{\mathcal{X}}_2),\ldots,\phi_{km}(\overline{\mathcal{X}})]^T$.

A_k can be designed as a Hurwitz matrix by choosing the appropriate vector k_k

$$A_k^TP_k+P_kA_k=-2Q_k \tag{14}$$

where P_k and Q_k are required to be symmetric and positive definite.

The following adaptive fuzzy observer is constructed:

$$\begin{aligned}\dot{\hat{\mathcal{X}}}&=A_k\hat{\mathcal{X}}+k_ky+\sum_{j=1}^{m}B_j\hat{f}_{kj}(\hat{\overline{\mathcal{X}}}_j|\hat{\theta}_{kj})+Du_k(t)\\ \hat{y}&=C^T\hat{\mathcal{X}}\end{aligned} \tag{15}$$

Let $e=\mathcal{X}-\hat{\mathcal{X}},e=[e_1,...,e_m]^T$. According to (13) and (15), one can be obtained

$$de=[A_ke+\sum_{j=1}^{m}B_j(f_{kj}\left(\overline{\mathcal{X}}_j\right)-\hat{f}_{kj}(\hat{\overline{\mathcal{X}}}_j|\hat{\theta}_{kj}))]dt+\phi d\omega \tag{16}$$

Choose the Lyapunov function as $V_0=\frac{1}{2}e^TPe$, so

$$\begin{aligned}\mathcal{L}V_0&=\frac{1}{2}e^TP\dot{e}+\frac{1}{2}\dot{e}^TPe\\ &=-e^TQ_ke+e^TP\sum_{j=1}^{m}B_j\delta_{kj}+Tr\{\varsigma\phi^TP\phi\varsigma^T\}\end{aligned} \tag{17}$$

where $P=\{P_k\mid\max_k\|P_k\|\}$.

Applying Young's inequality and leveraging Assumption 2, it follows that

$$e^TP\sum_{j=1}^{m}B_j\delta_{kj}\leq\frac{m}{2}e^Te+\frac{1}{2}\|P\|^2\sum_{j=1}^{m}\delta_j^{*2} \tag{18}$$

$$Tr\{\varsigma\phi^TP\phi\varsigma^T\}\leq\frac{1}{2}\|P\|^2+\frac{1}{2}\left|\overline{\varsigma\varsigma}^T\right|^2 \tag{19}$$

where $\delta_j^*=\max_{k\in K}\{\overline{\delta}_{kj}\}$.

By using (13), (14) and let $\lambda_{\min}(Q)$ being the smallest eigenvalue of matrix Q with $Q=\{Q_k\mid\min_k\|Q_k\|\}$, the following result can be obtained:

$$\begin{aligned}\mathcal{L}V_0\leq&-(\lambda_{\min}(Q)-\frac{1}{2}m)e^Te+\frac{1}{2}\|P\|^2\sum_{j=1}^{m}\delta_j^{*2}+\frac{1}{2}\|P\|^2\\ &+\frac{1}{2}\left|\overline{\varsigma\varsigma}^T\right|^2\leq-\eta_0e^Te+\mu_0\end{aligned} \tag{20}$$

where $\eta_0=\lambda_{\min}(Q)-\frac{1}{2}m,\mu_0=\frac{1}{2}\|P\|^2\sum_{j=1}^{m}\delta_j^{*2}+\frac{1}{2}\|P\|^2+\frac{1}{2}\left|\overline{\varsigma\varsigma}^T\right|^2$.

IV. Fault Tolerant Controller Design

In this section, the actual input for each subsystem is designed based on backstepping method to guarantee that the system output $y(t)$ converges to the desired trajectory y_d within a finite time. Subsequently, the stability properties of the system are examined.

Introduce the tracking error for the command filtered backstepping approach as

$$z_1=\mathcal{X}_1-y_d,z_j=\hat{\mathcal{X}}_j-\mathcal{X}_{j,c} \tag{21}$$

for $j=2,...,m$, where y_d is the desired target signal and $\mathcal{X}_{j,c}$ denotes the output of the command filter with α_{j-1} as its input.

To avoid the problem of differential explosion, the following command filtering method is introduced as follows

$$\begin{cases}\dot{\Phi}_{j,1}=\gamma_{j,1}\\ \gamma_{j,1}=-F_1\left|\Phi_{j,1}-\alpha_j\right|^{\frac{1}{2}}\operatorname{sign}\left(\Phi_{j,1}-\alpha_j\right)+\Phi_{j,2}\\ \dot{\Phi}_{j,2}=-F_2\operatorname{sign}\left(\Phi_{j,2}-\gamma_{j,1}\right),1\leq j\leq m-1\end{cases} \tag{22}$$

where $\mathcal{X}_{j+1,c}(t)=\Phi_{j,1}(t)$ and $\dot{\mathcal{X}}_{j+1,c}(t)=\gamma_{j,1}(t)$, F_1 and F_2 are positive constants.

To mitigate the adverse effects of filtering errors caused by the introduction of command filtering on control accuracy, we have designed the following error compensation mechanism:

$$\begin{cases}\dot{\chi}_1=-c_1\chi_1+\chi_2+(\mathcal{X}_{2,c}-\alpha_1)-\iota_1\operatorname{sign}(\chi_1)\\ \dot{\chi}_j=-c_j\chi_j+\chi_{j+1}+(\mathcal{X}_{j+1,c}-\alpha_j)\\ \qquad-\iota_j\operatorname{sign}(\chi_j)-\frac{1}{4}\chi_j\\ \dot{\chi}_m=-c_m\chi_m-\iota_m\operatorname{sign}(\chi_m)-\frac{1}{4}\chi_m\end{cases} \tag{23}$$

where c_j and ι_j are positive constants. Define the signals for compensation tracking error as $\nu_j=z_j-\chi_j,j=1,2\ldots,m$.

Step 1: Based on $z_1 = \mathcal{X}_1 - y_d, \nu_1 = z_1 - \chi_1$, we can get the following equation

$$d\nu_1 =(z_2 + f_{k1}(\hat{\mathcal{X}}_1|\theta_{k1}) + \varepsilon_{k1} + \mathcal{X}_{2,c} + e_2 - \dot{y}_d - \dot{\chi}_1)dt + \phi_{k1} d\omega \tag{24}$$

Select the Lyapunov function as $V_1 = V_0 + \frac{1}{4}\nu_1^4 + \frac{1}{2r_1}\tilde{\vartheta}_1^2$, where $\tilde{\vartheta}_j = \vartheta_j - \hat{\vartheta}_j$ is the estimation error with $\vartheta_j = \max_{k\in K}\{\|\theta_{kj}\|^2\}$ and $\hat{\vartheta}_j$ being the estimation of ϑ_j and r_1 is a positive constant. According to (4) and (24), we get

$$\mathcal{L}V_1 =\mathcal{L}V_0 + \nu_1^3(z_2 + f_{k1}(\hat{\mathcal{X}}_1|\theta_{k1}) + \varepsilon_{k1} + \mathcal{X}_{2,c} + e_2 - \dot{y}_d - \dot{\chi}_1) + \frac{3}{2}\nu_1^2\phi_{k1}^T\varsigma_1\varsigma_1^T\phi_{k1} - \frac{1}{r_1}\tilde{\vartheta}_1\dot{\hat{\vartheta}}_1 \tag{25}$$

Substituting $\dot{\chi}_1$ into (25), based on Assumption 2 and Young's inequality, we get the following inequalities

$$\nu_1^3(\varepsilon_{k1} + e_2) \le \nu_1^6 + \frac{1}{2}\varepsilon_1^{*2} + \frac{1}{2}e^Te \tag{26}$$

$$\nu_1^3\iota_1 sign(\chi_1) \le \frac{1}{2}\nu_1^6 + \frac{1}{2}\iota_1^2 \tag{27}$$

$$\frac{3}{2}\nu_1^2\phi_{k1}^T\varsigma_1\varsigma_1^T\phi_{k1} \le \frac{3}{4}\nu_1^4 + \frac{3}{4}\left|\overline{\varsigma\varsigma}^T\right|^2 \tag{28}$$

where $\varepsilon_1^* = \max_{k\in K}\{\bar{\varepsilon}_{k1}\}$.

Applying FLS, we have

$$\nu_1^3 f_{k1}(\hat{\mathcal{X}}_1|\theta_{k1}) = \nu_1^3\theta_{k1}^T S_1(\hat{\mathcal{X}}_1) \le \frac{1}{2a_1^2}\nu_1^6\vartheta_1 S_1^T(\hat{\mathcal{X}}_1)S_1(\hat{\mathcal{X}}_1) + \frac{a_1^2}{2} \tag{29}$$

where $a_j > 0, j = 1, 2, \ldots, m$, are designed constants.

Substituting (26)-(29) into (25), and there is an inequality $\nu_1^3\nu_2 \le \frac{3}{4}\nu_1^4 + \frac{1}{4}\nu_2^4$, we have

$$\begin{aligned}\mathcal{L}V_1 \le&\mathcal{L}V_0 + \nu_1^3(\frac{1}{2a_1^2}\nu_1^3\vartheta_1 S_1^T(\hat{\mathcal{X}}_1)S_1(\hat{\mathcal{X}}_1) + \alpha_1 + \frac{3}{2}\nu_1 \\ &+ \frac{3}{2}\nu_1^3 + c_1\chi_1 - \dot{y}_d) + \frac{1}{2}e^Te + \frac{3}{4}\left|\overline{\varsigma\varsigma}^T\right|^2 + \frac{a_1^2}{2} \\ &+ \frac{1}{2}\varepsilon_1^{*2} + \frac{1}{2}\iota_1^2 + \frac{1}{4}\nu_2^4 - \frac{1}{r_1}\tilde{\vartheta}_1\dot{\hat{\vartheta}}_1\end{aligned} \tag{30}$$

Next, we define the virtual control function and the adaptive parameter update law as follows:

$$\begin{aligned}\alpha_1 =& - c_1 z_1 - \frac{1}{2a_1^2}\nu_1^3\hat{\vartheta}_1 S_1^T(\hat{\mathcal{X}}_1)S_1(\hat{\mathcal{X}}_1) - \frac{3}{2}\nu_1 \\ &- \frac{3}{2}\nu_1^3 + \dot{y}_d - s_1\nu_1^{4\beta-3}\end{aligned} \tag{31}$$

$$\dot{\hat{\vartheta}}_1 =\frac{r_1}{2a_1^2}\nu_1^6 S_1^T(\hat{\mathcal{X}}_1)S_1(\hat{\mathcal{X}}_1) - \lambda_1\hat{\vartheta}_1 \tag{32}$$

where s_1 and λ_1 are positive constants.

By using (31) and (32), the formula (30) can be expressed as

$$\begin{aligned}\mathcal{L}V_1 \le& \mathcal{L}V_0 - c_1\nu_1 - s_1\nu_1^{4\beta} + \frac{1}{2}e^Te + \frac{3}{4}\left|\overline{\varsigma\varsigma}^T\right|^2 + \frac{a_1^2}{2} \\ &+ \frac{1}{4}\varepsilon_1^{*2} + \frac{1}{4}\nu_2^4 + \frac{\lambda_1}{r_1}\tilde{\vartheta}_1\hat{\vartheta}_1 \\ \le& -\eta_1 e^Te - c_1\nu_1 - s_1\nu_1^{4\beta} + \frac{1}{4}\nu_2^4 + \frac{\lambda_1}{r_1}\tilde{\vartheta}_1\hat{\vartheta}_1 + \mu_1\end{aligned} \tag{33}$$

where $\eta_1 = \eta_0 + \frac{1}{2}, \mu_1 = \mu_0 + \frac{3}{4}\left|\overline{\varsigma\varsigma}^T\right|^2 + \frac{a_1^2}{2} + \frac{1}{4}\varepsilon_1^{*2} + \frac{1}{2}\iota_1^2$.

Step j ($2 \le j \le m-1$): From (21) and $\nu_j = z_j - \chi_j$, we can get

$$d\nu_j =(z_{j+1} + f_{kj}(\hat{\bar{\mathcal{X}}}_j|\theta_{kj}) + \delta_{kj} - \varepsilon_{kj} + \mathcal{X}_{j+1,c} + k_j e_1 - \dot{\mathcal{X}}_{j,c} - \dot{\chi}_j)dt + \phi_{kj} d\omega \tag{34}$$

Construct the Lyapunov function as $V_j = V_{j-1} + \frac{1}{4}\nu_j^4 + \frac{1}{2r_j}\tilde{\vartheta}_j^2$, and let $r_j > 0$ be the design parameter, then we derive

$$\begin{aligned}\mathcal{L}V_j =&\mathcal{L}V_{j-1} + \nu_j^3(z_{j+1} + f_{kj}(\hat{\bar{\mathcal{X}}}_j|\theta_{kj}) + \delta_{kj} - \varepsilon_{kj} \\ &+ \mathcal{X}_{j+1,c} + k_j e_1 - \dot{\mathcal{X}}_{j,c} - \dot{\chi}_j) + \frac{3}{2}\nu_j^2\phi_{kj}^T\varsigma_j\varsigma_j^T\phi_{kj} \\ &- \frac{1}{r_j}\tilde{\vartheta}_j\dot{\hat{\vartheta}}_j\end{aligned} \tag{35}$$

Applying Young's inequality, the same as in step 1, one has

$$\nu_j^3(\delta_{kj} - \varepsilon_{kj}) \le \nu_j^6 + \frac{1}{2}\delta_j^{*2} + \frac{1}{2}\varepsilon_j^{*2} \tag{36}$$

$$\nu_j^3\iota_j sign(\chi_j) \le \frac{1}{2}\nu_j^6 + \frac{1}{2}\iota_j^2 \tag{37}$$

$$\frac{3}{2}\nu_j^2\phi_{kj}^T\varsigma_j\varsigma_j^T\phi_{kj} \le \frac{3}{4}\nu_j^4 + \frac{3}{4}\left|\overline{\varsigma\varsigma}^T\right|^2 \tag{38}$$

Applying FLS, we have

$$\begin{aligned}\nu_j^3 f_{kj}(\hat{\bar{\mathcal{X}}}_j|\theta_{kj}) =&\nu_j^3\theta_{kj}^T S_j(\hat{\bar{\mathcal{X}}}_j) \\ \le&\frac{1}{2a_j^2}\nu_j^6\vartheta_j S_j^T(\hat{\bar{\mathcal{X}}}_j)S_j(\hat{\bar{\mathcal{X}}}_j) + \frac{a_j^2}{2}\end{aligned} \tag{39}$$

Substituting (36)-(39) into (35), and there is an inequality $\nu_j^3\nu_{j+1} \le \frac{3}{4}\nu_j^4 + \frac{1}{4}\nu_{j+1}^4$, we have

$$\begin{aligned}\mathcal{L}V_j \le&\mathcal{L}V_{j-1} + \nu_j^3(\frac{1}{2a_j^2}\nu_j^3\vartheta_j S_j^T(\hat{\bar{\mathcal{X}}}_j)S_j(\hat{\bar{\mathcal{X}}}_j) + \alpha_j - \frac{1}{4}\chi_{j-1} \\ &+ \frac{3}{2}\nu_j + \frac{3}{2}\nu_j^3 + c_j\chi_j - \dot{\mathcal{X}}_{j,c} + k_j e_1) + \frac{3}{4}\left|\overline{\varsigma\varsigma}^T\right|^2 \\ &+ \frac{\alpha_j^2}{2} + \frac{1}{2}\delta_j^{*2} + \frac{1}{2}\varepsilon_j^{*2} + \frac{1}{2}\iota_j^2 + \frac{1}{4}\nu_{j+1}^4 - \frac{1}{r_j}\tilde{\vartheta}_j\dot{\hat{\vartheta}}_j\end{aligned} \tag{40}$$

Establish the virtual control function and the adaptive parameter update law in the following manner:

$$\begin{aligned}\alpha_j =& - c_j z_j - \frac{1}{4}z_{j-1} - \frac{1}{4}\nu_{j-1} - \frac{1}{2a_j^2}\nu_j^3\hat{\vartheta}_j S_j^T(\hat{\bar{\mathcal{X}}}_j)S_j(\hat{\bar{\mathcal{X}}}_j) \\ &- \frac{7}{4}\nu_j - \frac{3}{2}\nu_j^3 + \dot{\mathcal{X}}_{j,c} - k_j e_1 - s_j\nu_j^{4\beta-3}\end{aligned} \tag{41}$$

$$\dot{\hat{\vartheta}}_j =\frac{r_j}{2a_j^2}\nu_j^6 S_j^T(\hat{\bar{\mathcal{X}}}_j)S_j(\hat{\bar{\mathcal{X}}}_j) - \lambda_j\hat{\vartheta}_j \tag{42}$$

where s_j and λ_j are positive constants.

Then, the formula (34) can be expressed as

$$\begin{aligned}\mathcal{L}V_j \le& \mathcal{L}V_{j-1} - c_j\nu_j - s_j\nu_j^{4\beta} + \frac{3}{4}\left|\overline{\varsigma\varsigma}^T\right|^2 + \frac{a_j^2}{2} \\ &+ \frac{1}{2}\delta_j^{*2} + \frac{1}{2}\varepsilon_j^{*2} + \frac{1}{2}\iota_j^2 + \frac{1}{4}\nu_{j+1}^4 + \frac{\lambda_j}{r_j}\tilde{\vartheta}_j\hat{\vartheta}_j \\ \le& -\eta_1 e^Te - \sum_{i=1}^{j} c_i\nu_i - \sum_{i=1}^{j} s_i\nu_i^{4\beta} + \frac{1}{4}\nu_{j+1}^4 \\ &+ \sum_{i=1}^{j}\frac{\lambda_i}{r_i}\tilde{\vartheta}_i\hat{\vartheta}_i + \mu_j\end{aligned} \tag{43}$$

where $\mu_j = \mu_{j-1} + \frac{3}{4}\left|\overline{\varsigma\varsigma}^T\right|^2 + \frac{\alpha_j^2}{2} + \frac{1}{2}\delta_j^{*2} + \frac{1}{2}\varepsilon_j^{*2} + \frac{1}{2}\iota_j^2$.

Step m: According to $z_m = \hat{x}_m - x_{m,c}, \nu_m = z_m - \chi_m$, one has

$$d\nu_m = (u_k(t) + f_{km}(\hat{\bar{\mathcal{X}}}_m|\theta_{km}) + \delta_{km} - \varepsilon_{km} + k_m e_1 - \dot{\mathcal{X}}_{m,c} - \dot{\chi}_m)dt + \phi_{km}d\omega \tag{44}$$

Consider the following Lyapunov function as

$$V_m = V_{m-1} + \frac{1}{4}\nu_m^4 + \frac{1}{2r_m}\tilde{\vartheta}_m^2 + \sum_{k=1}^{I_N}\tilde{u}_k^2(t) \tag{45}$$

where r_m is a positive constant, $\tilde{u}_k(t) = \overline{u}_k(t) - \hat{u}_k(t)$, $\hat{u}_k(t)$ is the estimation of $\overline{u}_k(t)$, then we can get

$$\begin{aligned}\mathcal{L}V_m =& \mathcal{L}V_{m-1} + \nu_m^3(\rho_k\nu_k + \overline{u}_k + f_{km}(\hat{\bar{\mathcal{X}}}_m|\theta_{km}) \\ &+ \delta_{km} - \varepsilon_{km} + k_m e_1 - \dot{\mathcal{X}}_{m,c} - \dot{\chi}_m) \\ &+ \frac{3}{2}\nu_m^2\phi_{km}^T\varsigma_m\varsigma_m^T\phi_{km} - \frac{1}{r_m}\tilde{\vartheta}_m\dot{\hat{\vartheta}}_m \\ &- \sum_{k=1}^{I_N}\tilde{u}_k(t)\dot{\hat{u}}_k(t)\end{aligned} \tag{46}$$

Similar to inequality (36)-(38), we have

$$\nu_m^3(\delta_{km} - \varepsilon_{km}) \leq \nu_m^6 + \frac{1}{2}\delta_m^{*2} + \frac{1}{2}\varepsilon_m^{*2} \tag{47}$$

$$\nu_m^3\iota_m sign(\chi_m) \leq \frac{1}{2}\nu_m^6 + \frac{1}{2}\iota_m^2 \tag{48}$$

$$\frac{3}{2}\nu_m^2\phi_{km}^T\varsigma_m\varsigma_m^T\phi_{km} \leq \frac{3}{4}\nu_m^4 + \frac{3}{4}\left|\overline{\varsigma\varsigma}^T\right|^2 \tag{49}$$

Applying FLS, we have

$$\begin{aligned}\nu_m^3 f_{km}(\hat{\bar{\mathcal{X}}}_m|\theta_{km}) &= \nu_m^3\theta_{km}^T S_m(\hat{\bar{\mathcal{X}}}_m) \\ &\leq \frac{1}{2a_m^2}\nu_m^6\vartheta_m S_m^T(\hat{\bar{\mathcal{X}}}_m)S_m(\hat{\bar{\mathcal{X}}}_m) + \frac{a_m^2}{2}\end{aligned} \tag{50}$$

Substituting (47)-(50) into (46), we have

$$\begin{aligned}\mathcal{L}V_m \leq& \mathcal{L}V_{m-1} + \nu_m^3(\rho_k\nu_k + \overline{u}_k - \frac{1}{4}\chi_{m-1} \\ &+ \frac{1}{2a_m^2}\nu_m^3\vartheta_m S_m^T(\hat{\bar{\mathcal{X}}}_m)S_m(\hat{\bar{\mathcal{X}}}_m) + \frac{3}{4}\nu_m + \frac{3}{2}\nu_m^3 \\ &+ c_m\chi_m - \dot{\mathcal{X}}_{m,c} + k_m e_1) + \frac{3}{4}\left|\overline{\varsigma\varsigma}^T\right|^2 + \frac{a_m^2}{2} + \frac{1}{2}\delta_m^{*2} \\ &+ \frac{1}{2}\varepsilon_m^{*2} + \frac{1}{2}\iota_m^2 - \frac{1}{r_m}\tilde{\vartheta}_m\dot{\hat{\vartheta}}_m - \sum_{k=1}^{I_N}\tilde{u}_k(t)\dot{\hat{u}}_k(t)\end{aligned} \tag{51}$$

Design the actual fault-tolerant controller and the adaptive laws as

$$\begin{aligned}v_k =& \frac{1}{\rho_k}\big(- c_m z_m - \frac{1}{4}z_{m-1} - \frac{1}{4}\nu_{m-1} - \frac{1}{2a_m^2}\nu_m^3\hat{\vartheta}_m \\ & S_m^T(\hat{\bar{\mathcal{X}}}_m)S_m(\hat{\bar{\mathcal{X}}}_m) - \nu_m - \frac{3}{2}\nu_m^3 + \dot{\mathcal{X}}_{m,c} - k_m e_1 \\ &- \hat{u}_k - s_m\nu_m^{4\beta-3}\big)\end{aligned} \tag{52}$$

$$\dot{\hat{\vartheta}}_m = \frac{r_m}{2a_m^2}\nu_m^6 S_m^T(\hat{\bar{\mathcal{X}}}_m)S_m(\hat{\bar{\mathcal{X}}}_m) - \lambda_m\hat{\vartheta}_m \tag{53}$$

$$\dot{\hat{u}}_k = \begin{cases}-\varpi_k\hat{u}_k + \nu_n^3, & \sigma = k \\ -\varpi_k\hat{u}_k, & \sigma \neq k\end{cases} \tag{54}$$

where ϖ_k is designed constants.

Combing (52)-(54), we can conclude that

$$\begin{aligned}\mathcal{L}V_m \leq& -\eta_1 e^T e - \sum_{j=1}^m c_j\nu_j - \sum_{j=1}^m s_j\nu_j^{4\beta} + \sum_{j=1}^m\frac{\lambda_j}{r_j}\tilde{\vartheta}_j\hat{\vartheta}_j \\ &+ \sum_{k=1}^{I_N}\varpi_k\tilde{u}_k(t)\hat{u}_k(t) + \mu_m\end{aligned} \tag{55}$$

where $\mu_m = \mu_{m-1} + \frac{3}{4}\left|\overline{\varsigma\varsigma}^T\right|^2 + \frac{\alpha_m^2}{2} + \frac{1}{2}\delta_m^{*2} + \frac{1}{2}\varepsilon_m^{*2} + \frac{1}{2}\iota_m^2$.

Utilizing the Young's inequality again, one obtains

$$\frac{\lambda_j}{r_j}\tilde{\vartheta}_j\hat{\vartheta}_j \leq -\frac{\lambda_j}{2r_j}\tilde{\vartheta}_j^2 + \frac{\lambda_j}{2r_j}\vartheta_j^2 \tag{56}$$

$$\varpi_k\tilde{u}_k(t)\hat{u}_k(t) \leq -\frac{\varpi_k}{2}\tilde{u}_k^2(t) + \frac{\varpi_k}{2}u_k^{*2} \tag{57}$$

Then, we have

$$\begin{aligned}\mathcal{L}V_m \leq& -\eta_1 e^T e - \sum_{j=1}^m c_j\nu_j - \sum_{j=1}^m s_j\nu_j^{4\beta} - \sum_{j=1}^m\frac{\lambda_j}{2r_j}\tilde{\vartheta}_j^2 \\ &- \sum_{k=1}^{I_N}\frac{\varpi_k}{2}\tilde{u}_k^2(t) + \mu\end{aligned} \tag{58}$$

where $\mu = \mu_m + \sum_{j=1}^m\frac{\lambda_j}{2r_j}\vartheta_j^2 + \sum_{k=1}^{I_k}\frac{\varpi_k}{2}u_k^{*2}$.

Theorem 1: Consider a switched stochastic nonlinear system (1) that is affected by actuator faults (2), including actuator effectiveness loss and bias failure. Under Assumptions 1-3, the designed virtual controllers (31), (41), subsystem controllers (52), and adaptive updating laws (32), (42), (53) ensure that all variables in the closed-loop system are bounded in the frame of arbitrary switching signals. The tracking error converges to a small neighborhood near the origin in finite time. The system is SGFSP.

Proof: Equation (58) can be rewritten as

$$\begin{aligned}\mathcal{L}V_m \leq& -\eta_1 e^T e + (\frac{\lambda_{\min}(Q)}{2}e^T e)^\beta - (\frac{\lambda_{\min}(Q)}{2}e^T e)^\beta \\ &- \sum_{j=1}^m c_j\nu_j - \sum_{j=1}^m s_j\nu_j^{4\beta} + \sum_{j=1}^m[-\frac{\lambda_j}{2r_j}\tilde{\vartheta}_j^2 - (\frac{1}{2r_j}\tilde{\vartheta}_j^2)^\beta \\ &+ (\frac{1}{2r_j}\tilde{\vartheta}_j^2)^\beta] + \sum_{k=1}^{I_N}[-\frac{\varpi_k}{2}\tilde{u}_k^2(t) - (\frac{1}{2}\tilde{u}_k^2(t))^\beta \\ &+ (\frac{1}{2}\tilde{u}_k^2(t))^\beta] + \mu\end{aligned} \tag{59}$$

Based on Lemma 4, we choose $l_1 = 1, l_2 = \frac{\lambda_{\min}(Q)}{2}e^T e, \frac{1}{2r_i}\tilde{\vartheta}_i^2, \frac{1}{2}\tilde{u}_k^2(t), x = 1-\beta, y = \beta, p = \beta^{\beta/(1-\beta)}$. Then, the following relationship is obtained

$$(\frac{\lambda_{\min}(Q)}{2}e^T e)^\beta \leq (1-\beta)p + \frac{\lambda_{\min}(Q)}{2}e^T e \tag{60}$$

$$(\frac{1}{2r_j}\tilde{\vartheta}_j^2)^\beta \leq (1-\beta)p + \frac{1}{2r_j}\tilde{\vartheta}_j^2 \tag{61}$$

$$(\frac{1}{2}\tilde{u}_k^2(t))^\beta \leq (1-\beta)p + \frac{1}{2}\tilde{u}_k^2(t) \tag{62}$$

According to (60)-(62), further sorting can get

$$\begin{aligned}\mathcal{L}V_m \leq &-(\eta_1-\frac{\lambda_{\min}(Q)}{2})e^Te-\sum_{j=1}^m c_j\nu_j-\sum_{j=1}^m\frac{\lambda_j-1}{2r_j}\tilde{\vartheta}_j^2\\&-\sum_{k=1}^{I_N}\frac{\varpi_k-1}{2}\tilde{u}_k^2(t)-(\frac{\lambda_{\min}(Q)}{2}e^Te)^\beta-\sum_{j=1}^m s_j\nu_j^{4\beta}\\&-\sum_{j=1}^m(\frac{1}{2r_j}\tilde{\vartheta}_j^2)^\beta-\sum_{k=1}^{I_N}(\frac{1}{2}\tilde{u}_k^2(t))^\beta+\mu \end{aligned}\tag{63}$$

Then define $C=\min\left\{\frac{\eta_1-\frac{\lambda_{\min}(Q)}{2}}{\lambda_{\max}(P)},4c_j,\lambda_j-1,\frac{\varpi_k-1}{2}\right\}$, $D=\min\left\{\left(\frac{\lambda_{\min}(Q)}{\lambda_{\max}(P)}\right)^\beta,4^\beta s_j,1,\left(\frac{1}{2}\right)^\beta\right\}$, $\overline{\mu}=\mu+3(1-\beta)p$, the following inequality can be obtained

$$\mathcal{L}V_m\leq -CV_m-DV_m^\beta+\overline{\mu}\tag{64}$$

Step $n+1$: Choose the Lyapunov function for χ_j:

$$V_{m+1}=\frac{1}{4}\sum_{j=1}^m\chi_j^4\tag{65}$$

Differentiating V_{m+1} yields that

$$\begin{aligned}\dot{V}_{m+1}=&\chi_1^3\left(-c_1\chi_1+(\mathcal{X}_{2,c}-\alpha_1)+\chi_2-\iota_1 sign(\chi_1)\right)\\&+\sum_{j=2}^{m-1}\chi_j^3(-c_j\chi_j+(\mathcal{X}_{j+1,c}-\alpha_j)+\chi_{j+1}\\&-\iota_j sign(\chi_j)-\frac{1}{4}\chi_j)+\chi_m^3(-c_m\chi_m\\&-\iota_m sign(\chi_m)-\frac{1}{4}\chi_m)\end{aligned}\tag{66}$$

where

$$\chi_j^3\chi_{j+1}\leq\frac{3}{4}\chi_j^4+\frac{1}{4}\chi_{j+1}^4\tag{67}$$

Then, it follows

$$\begin{aligned}\dot{V}_{m+1}\leq&\sum_{j=1}^{m-1}((-c_j+\frac{3}{4})\chi_j^4+\left|\left|\chi_j^3\right|(\mathcal{X}_{j+1,c}-\alpha_j)\right|)\\&-c_m\chi_m^4-\sum_{j=1}^m\iota_j\left|\chi_j^3\right|\\\leq&\sum_{j=1}^{m-1}(-c_j+\frac{3}{4})\chi_j^4+\sum_{j=1}^{m-1}\left|\left|\chi_j^3\right|(\mathcal{X}_{j+1,c}-\alpha_j)\right|\\&-c_m\chi_m^4-\sum_{j=1}^m\iota_j\left|\chi_j^3\right|+|h_{m1}|\left|\chi_m^3\right|\end{aligned}\tag{68}$$

According to Lemmas 3, one has $|\mathcal{X}_{j+1,c}-\alpha_j|\leq h_{j1}$

$$\begin{aligned}\dot{V}_{m+1}\leq&\sum_{j=1}^{m-1}(-c_j+\frac{3}{4})\chi_j^4+\sum_{j=1}^m|h_{j1}|\left|\chi_j^3\right|-c_m\chi_m^4\\&-\sum_{j=1}^m\iota_j\left|\chi_j^3\right|\\\leq&\sum_{j=1}^{m-1}(-c_j+\frac{3}{4})\chi_j^4+h\sum_{j=1}^m\left|\chi_j^3\right|-c_m\chi_m^4\\&-\frac{\iota}{2\sqrt{2}}\sum_{j=1}^m\left|\chi_j^3\right|\\\leq&\sum_{j=1}^{m-1}(-c_j+\frac{3}{4})\chi_j^4-(\frac{\iota}{2\sqrt{2}}-h)\sum_{j=1}^m\left|\chi_j^3\right|\\&-c_m\chi_m^4\end{aligned}\tag{69}$$

where $h=\max\{h_{j1}\}$ and $\iota=2\sqrt{2}\min\{\iota_j\},j=1,\ldots,m$.

Based on Lemma 1, we have

$$\begin{aligned}\dot{V}_{m+1}\leq&\sum_{j=1}^{m-1}(-c_j+\frac{3}{4})\chi_j^4-c_m\chi_m^4-(\iota-2\sqrt{2}m^{\frac{1}{4}}h)\\&(\frac{1}{4}\sum_{j=1}^m\chi_j^4)^{\frac{3}{4}}\\\leq&-q_1V_{m+1}-q_2V_{m+1}^{\frac{3}{4}}\end{aligned}\tag{70}$$

where $q_1=\min\left\{4(c_1-\frac{3}{4}),\ldots,4(c_{m-1}-\frac{3}{4}),4c_m\right\},q_2=1-2\sqrt{2}m^{\frac{1}{4}}h$. We can be certain that χ_j will attain finite-time stability based on Lemma 2. ■

V. Simulation Example

Example 1: Consider the following switched second-order system, which can be described as

$$\begin{cases}d\mathcal{X}_1=\left(f_{\sigma 1}\left(\mathcal{X}_1\right)+\mathcal{X}_2\right)dt+\phi_{\sigma 1}\left(\mathcal{X}_1\right)d\omega\\d\mathcal{X}_2=\left(f_{\sigma 2}\left(\bar{\mathcal{X}}_2\right)+u_\sigma(t)\right)dt+\phi_{\sigma 2}\left(\bar{\mathcal{X}}_2\right)d\omega\\y=\mathcal{X}_1\end{cases}\tag{71}$$

$\sigma=\{1,2\}$, when $\sigma=1,f_{11}=0.5\mathcal{X}_1^2,\phi_{11}=0.2\mathcal{X}_1^3,f_{12}=2\mathcal{X}_1^2\mathcal{X}_2,\phi_{12}=0.5\mathcal{X}_1^2+\mathcal{X}_2^2,u_1(t)=0.6\nu_1+0.5\sin(t)$, when $\sigma=2,f_{21}=0.1\sin(\mathcal{X}_1),\phi_{21}=0.2\mathcal{X}_1\sin(2\mathcal{X}_1),f_{22}=1.5\cos(\mathcal{X}_1)\mathcal{X}_2$, $\phi_{22}=0.25\mathcal{X}_1\sin(\mathcal{X}_2)$, $u_2(t)=0.4\nu_2+1-0.25\sin^2(t)$. y_d is given: $y_d=\sin(t)$.Based on the (31),(32),(52),(53)and (54), we can obtain

$$\dot{\hat{\vartheta}}_1=\frac{r_1}{2a_1^2}\nu_1^6S_1^T(\hat{\mathcal{X}}_1)S_1(\hat{\mathcal{X}}_1)-\lambda_1\hat{\vartheta}_1\tag{72}$$

$$\dot{\hat{\vartheta}}_2=\frac{r_2}{2a_2^2}\nu_2^6S_2^T(\hat{\bar{\mathcal{X}}}_2)S_2(\hat{\bar{\mathcal{X}}}_2)-\lambda_2\hat{\vartheta}_2\tag{73}$$

$$\begin{aligned}\alpha_1=&-c_1z_1-\frac{1}{2a_1^2}\nu_1^3\hat{\vartheta}_1S_1^T(\hat{\mathcal{X}}_1)S_1(\hat{\mathcal{X}}_1)-\frac{3}{2}\nu_1-\frac{3}{2}\nu_1^3\\&+\dot{y}_d-s_1\nu_1^{4\beta-3}\end{aligned}\tag{74}$$

$$\begin{aligned}v_k=&\frac{1}{\rho_k}\big(-c_2z_2-\frac{1}{4}z_1-\frac{1}{4}\nu_1-\frac{1}{2a_2^2}\nu_2^3\hat{\vartheta}_2S_2^TS_2-\nu_2\\&-\frac{3}{2}\nu_2^3+\dot{\mathcal{X}}_{2,c}-k_2e_1-\hat{u}_k-s_2\nu_2^{4\beta-3}\big)\end{aligned}\tag{75}$$

$$\dot{\hat{u}}_k=\begin{cases}-\varpi_k\hat{u}_k+\nu_2^3, & \sigma=k\\-\varpi_k\hat{u}_k, & \sigma\neq k\end{cases}\tag{76}$$

with the parameters $c_1 = 20, c_2 = 30, a_1 = a_2 = 100, r_1 = r_2 = 20, \lambda_1 = \lambda_2 = 1, \beta = 99/100$, $\varpi_1 = 0.4$, $\varpi_2 = 0.8$, $s_1 = s_2 = 200$, $k_1 = [4,4]$, $k_2 = [10,120]$, $R_1 = R_2 = 40$, $\iota_1 = \iota_2 = 0.001$, $\rho_1 = 0.2 + \sin^2(t)$, $\rho_2 = 0.1 + |\sin(t)|$.

Fig. 1 shows the switching signal. Fig. 2 illustrates the tracking performance of the system in response to the reference signal y_d. From the results, the output signal rapidly converges to the reference signal within a brief time interval. Depending on Fig. 3, the tracking error converges to near zero within a short period, and the tracking error fluctuation during fault occurrence is also small, indicating that the designed fault-tolerant controller effectively achieves the desired control objectives. From Fig. 4, it seems that the system state $\mathcal{X}_2$ is bounded. The control inputs ν_1 and ν_2 of subsystems 1 and 2, respectively, are displayed in Figs. 5 and 6. The adaptive update laws $\hat{\vartheta}_1$ and $\hat{\vartheta}_2$ are shown in Fig. 7, which can maintain the bounded stability in finite time. From them, the simulation results validate the feasibility and efficiency of the proposed method.

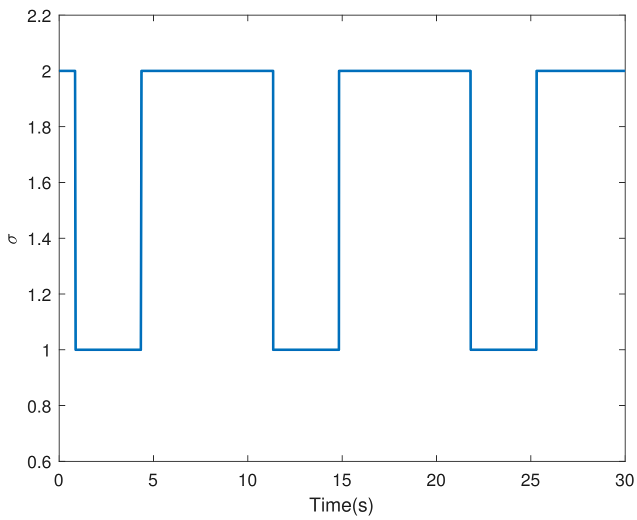

Fig. 1. The switching signal $\sigma(t)$.

VI. Conclusion

In this study, we investigate the finite-time fault-tolerant control problem for switching stochastic nonlinear systems, with a focus on addressing both loss of effectiveness and bias faults in the actuators. In order to address the uncertainty of the system, the unmeasured states are estimated via the observer, while the unknown functions are approximated using FLS. The command filtering technique employed effectively avoids the complexity of differential computation. The designed controller avoids the problem of off-line calculation of the upper and lower bounds of virtual control laws in DSC method, and reduces the computational burden. Maintaining bounded signals in the system under arbitrary switching rules is proved in the stability analysis, and the system tracking error converges to a tight set. Due to the choice of arbitrary switching laws, it is necessary to ensure that all subsystems must be stable. Future research will focus on the control of switched systems with unstable subsystems.

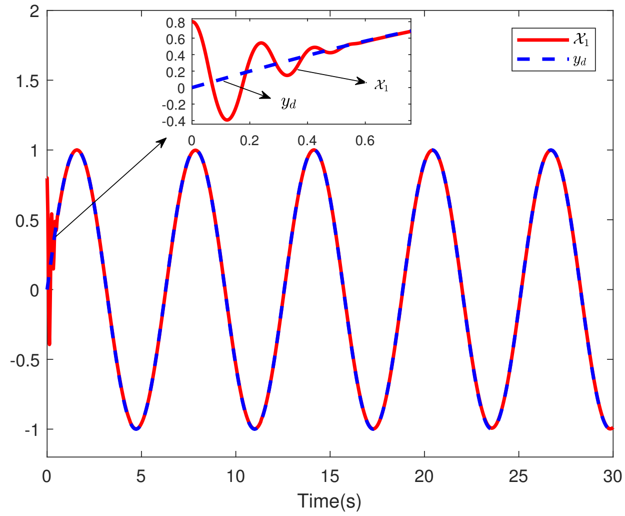

Fig. 2. Output $y = \mathcal{X}_1$ and y_d.

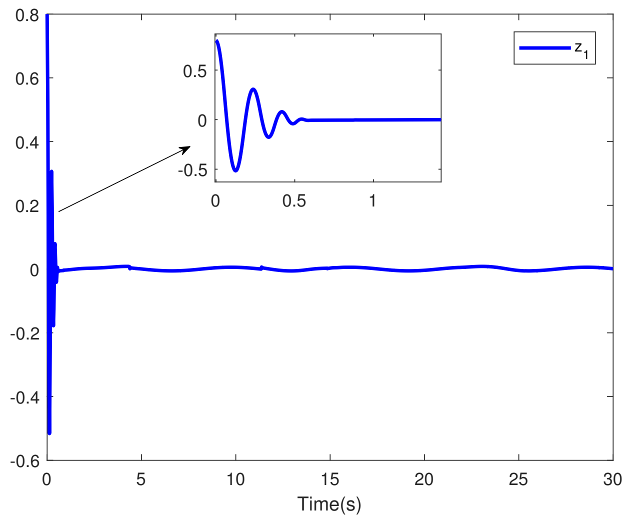

Fig. 3. Tracking error z_1.

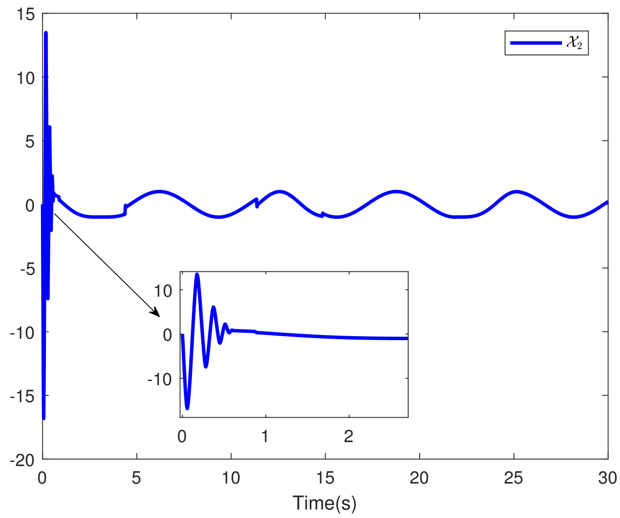

Fig. 4. System state $\mathcal{X}_2$.

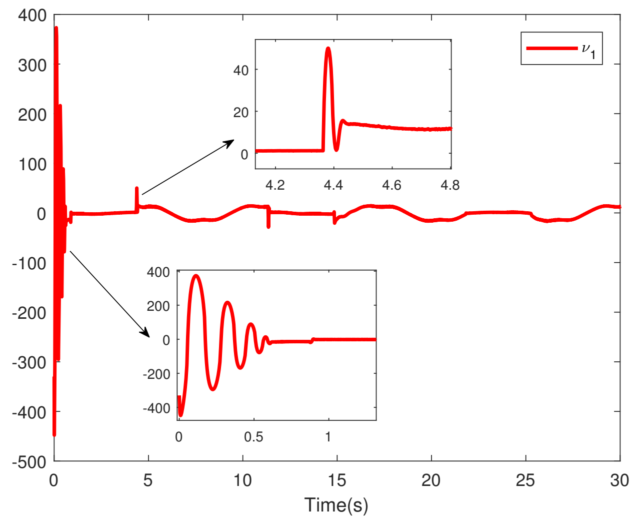

Fig. 5. Control input ν_1.

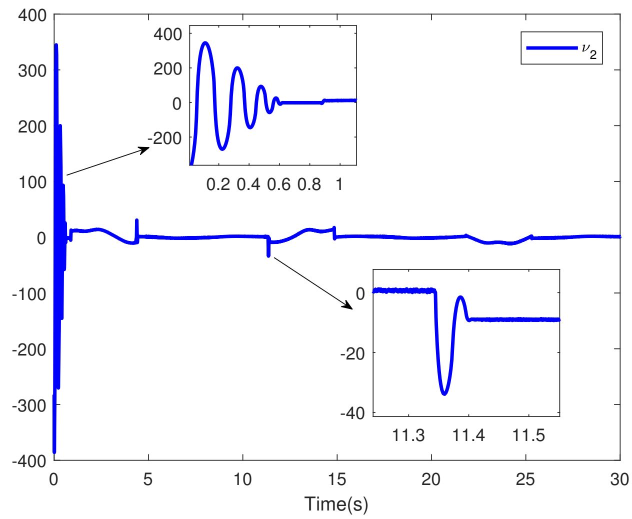

Fig. 6. Control input ν_2.

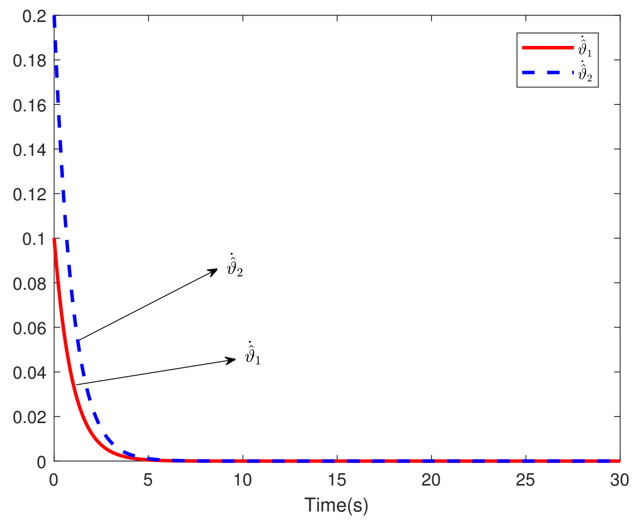

Fig. 7. Adaptive update law of Example.

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