Optimization of Low-carbon Multimodal Transportation Routing with Consideration of Government Subsidies

Tingyu Wang, Cunjie Dai, Jianghong Feng, Jian Zhang, Songlin Tian, Runyu Wu

Abstract—**Multimodal transportation is essential for enhancing efficiency and establishing a low-carbon transportation system. This research employs a bi-level programming model to elucidate the dynamic interplay between government subsidies and operators' routing decisions. The upper level aims to minimize subsidies and carbon emissions while considering total transportation time, the capacity of transfer nodes, and the volume of subsidies. The lower level, conversely, seeks to minimize transportation costs while considering network flow equilibrium and coupling interactions. Subsequently, uncertainty theory is employed to convert the upper-level model into a predictable structure. The KKT condition is subsequently employed to transform the lower-level model, therefore consolidating the model into a unified level. The numerical solution is executed using Gurobi software to validate the model and further investigate the impacts of subsidy, arrival time, and confidence level on the routing outcomes. The study's results indicate that the government subsidy policy positively contributes to the effective and sustainable growth of multimodal transportation.**

Index Terms—**Multimodal Transportation, Routing Optimization, Government Subsidy, Low Carbon, Bi-level Programming.**

I. Introduction

Multimodal transportation is typically defined as the conveyance of commodities from the origin to the destination using a single loading unit that integrates two or more modes of transport. The surge in international trade has resulted in an increasing need for transportation services.[1] However, this has coincided with an increase in energy consumption and heightened environmental degradation. Carbon emissions from the transportation sector constitute around 10% of total carbon emissions, presenting a significant threat to the global climate system. Consequently, the exploration and implementation of low-carbon transportation methods have emerged as a shared priority for the global community.

Multimodal transportation significantly reduces energy consumption and carbon emissions through optimized transport chains and minimized transfers and idle times. By effectively integrating rail, maritime, and other low-carbon transport modes, it demonstrates superior energy efficiency and emission reduction capabilities compared to single-mode road transportation. However, the inherent uncertainty in transportation duration, caused by factors such as extreme weather, traffic congestion, and equipment malfunctions, necessitates explicit consideration in multimodal transportation planning.

Capacity constraints across different transport modes significantly impact overall system efficiency, particularly at transfer nodes where infrastructure enhancement and optimization are crucial. These improvements not only increase transfer efficiency and service quality but also ensure seamless connectivity between transport modes. Government subsidies have played a pivotal role in advancing technology and equipment at transfer nodes, promoting greener technologies, upgrading facilities, and reducing pollution from transfer operations.

While numerous studies have investigated low-carbon multimodal transportation with capacity constraints and temporal uncertainty, limited research has addressed routing optimization that simultaneously considers government subsidies, capacity limitations, and time uncertainty. The allocation of government subsidies presents a strategic challenge: nodes with higher emissions require greater investment for transformation, while cleaner nodes need less. With limited subsidy budgets and ambitious dual-carbon goals, governments must carefully evaluate subsidy allocation to ensure policy effectiveness, equity, and sustainability, while preventing resource misallocation and free-riding behaviors. Simultaneously, operators focus on cost reduction, creating a complex interplay between government subsidy decisions and operator routing choices within established policy frameworks.

This structure organizes the rest of the paper. The

Manuscript received January 13, 2025; revised March 5,2025.

This research was supported by the Natural Science Foundation of Gansu Province(23JRRA858), Science and Technology Program (Joint Research Fund) Project of Gansu Province(24JRRA868), and Gansu Provincial Department of Education: Outstanding Graduate Students “Innovation Star” Program (2025CXZX-704).

Tingyu Wang is a postgraduate student at the School of Traffic and Transportation, Lanzhou Jiaotong University, Lanzhou 730070, China. (e-mail: 17855825504@163.com).

Cunjie Dai is an associate professor at the School of Traffic and Transportation, Lanzhou Jiaotong University, and the Key Laboratory of Railway Industry on Plateau Railway Transportation Intelligent Management and Control, Lanzhou 730070, China. (Corresponding author, phone: +86 13919467628, e-mail:daicunjie@mail.lzjtu.cn).

Jianghong Feng is a postgraduate student at the School of Traffic and Transportation, Lanzhou Jiaotong University, Lanzhou 730070, China. (e-mail: 2766172529@qq.com).

Jian Zhang is a postgraduate student at the School of Traffic and Transportation, Lanzhou Jiaotong University, Lanzhou 730070, China. (e-mail: 2959025682@qq.com).

Songlin Tian is a postgraduate student at the School of Traffic and Transportation, Lanzhou Jiaotong University, Lanzhou 730070, China. (e-mail: tiansonglin1999@163.com).

Runyu Wu is a postgraduate student at the School of Traffic and Transportation, Lanzhou Jiaotong University, Lanzhou 730070, China. (e-mail: wurunyu001@163.com).

"Literature Review" presents an examination of low-carbon multimodal transportation, government subsidies, and bi-level programming, along with the primary content. The "Problem Description" explains the study's purpose and gives an idea of the problem. The book "Modeling" creates a bi-level computing method. The "Case Study" identifies optimization paths utilizing various weight combinations for a multimodal transportation network comprising 35 nodes and examines the effects of subsidy constraints, arrival time, and confidence level on the optimization of multimodal transportation routes. The final section, titled "Conclusion", presents the paper's findings.

II. LITERATURE REVIEW

In the domain of sustainable multimodal transportation, the majority of research focuses on optimizing the transportation system to mitigate environmental effects. Emrah Demir et al.[2] developed and implemented an enhanced emission computation approach integrated with a bi-objective model for cost and carbon reduction, establishing a technical foundation for the advancement of efficient transportation management system software. Yan Sun et al.[3] identified the carbon emission component as a critical consideration, hence emphasizing the significance of environmental sustainability in transportation decision-making. Qing-Zhou Wang et al.[4] proposed a dynamic calculation method for intermodal transport line parameters to effectively reduce carbon emissions from multimodal transportation. They utilized Witness software simulation and validated their approach through a case study of the Central European railroad network. Luo-Jun Yang et al.[5] developed an optimization model that thoroughly addressed transit distance, time, and carbon emissions, employing an enhanced fuzzy adaptive evolutionary algorithm to resolve the issue. carbon emissions. Tom Binsfeld et al.[6] demonstrated a high level of precision in evaluating pollutant emissions, encompassing not only conventional emissions like carbon dioxide but also carbon dioxide equivalents, greenhouse gases, and energy consumption.

Government subsidies can immediately lower the operational expenses of transport firms, stimulate technical innovation and service enhancement, and efficiently facilitate the extension and refinement of the transportation network, thereby improving transportation efficiency and safety. Yun-Qiang Wu et al.[7] examine the issue of container transportation with government subsidies, calculating the subsidy amount for each heavy container per train from the government's perspective to optimize the volume of sea-rail intermodal transportation. Lingchunzi Li et al.[8] examine the positioning of multimodal transportation hubs within the framework of the "Belt and Road" initiative. Qing-Quan Tang et al.[9] conducted a study utilizing sample data from listed companies, revealing that government subsidies significantly enhance the social benefits of enterprises. Bruno F. Santos et al.[10] developed a multimodal transportation hub siting model grounded in European policy and hub siting theory, examining the influence of government subsidies on traffic volume to offer insights into how such subsidies facilitate the advancement of multimodal transportation.

Santos Maria Joao et al.[11] examined the cost-profit equilibrium between shippers and carriers in bi-level programming models, whereas Dung-Ying Lin et al.[12] focused on pricing and fee optimization between platform companies and shipping firms. In hazardous products transportation, Nishit Bhavsar et al.[13] examined governmental initiatives to mitigate transportation risks via subsidies or fee structures, while Assadipour Ghazal et al.[14] concentrated on the management of hazardous goods networks. Jun Yang et al.[17] developed a bi-level programming model for customer routing decisions by integrating multiple optimization methodologies (e.g., greedy search, genetic algorithm, stochastic simulation, and fuzzy simulation) to maximize the overall flow of service facilities while optimizing customer routing choices. Vinod Chandra S.S et al.[18] integrated a bi-level multi-objective optimization model with the intelligent traits of ant foraging behavior to propose a novel method for dynamic detection and optimal path estimate of road traffic measures. Sheng-Zhong Zhang et al.[19] created a new meta-heuristic algorithm that combines the particle swarm algorithm and the genetic algorithm to solve the bi-level optimization model in the lane reservation problem for transporting dangerous goods. This proved that bi-level programming works in difficult transportation problems. In the bi-level programming model, the majority convert the bi-level problem into a single-level problem utilizing the KKT conditions[12]、[13]、[15]、[16] whereas Fontaine Pirmin et al.[16] further decompose the solution model using Benders to enhance solving efficiency.

Despite the growing academic interest in the role of government subsidies in multimodal transportation development, existing research remains fragmented and limited compared to other disciplines. This study establishes a bi-level programming model to examine the strategic interaction between government and operators. The upper-level model aims to minimize both government subsidies and carbon emissions, incorporating constraints on subsidy allocation, transfer node capacities, and total transportation time. The lower-level model focuses on transportation cost minimization while maintaining interdependence with the upper-level objectives. Through uncertainty theory, the upper-level model is transformed into a deterministic formulation, while KKT conditions are applied to convert the lower-level model, enabling single-level optimization. The model's validity is verified through numerical analysis using Gurobi software, with comprehensive examination of subsidy levels, arrival times, and confidence levels in relation to routing outcomes.

III. PROBLEM DESCRIPTION

Government subsidies serve as a crucial external catalyst for advancing multimodal transportation systems and promoting green logistics development. The inherent complexity of multimodal transportation stems from uncertainties in transportation and transfer durations, which tend to accumulate throughout the logistics chain, potentially leading to significant quantitative and qualitative changes at subsequent nodes. These uncertainties, combined with the positive effects of government subsidies, demonstrate the diverse, complex, and efficient nature of multimodal transportation under variable temporal conditions.

The government-operator relationship in this context follows a dual decision-making framework: the government determines subsidy allocation based on operators' routing selections, while operators optimize their choices according to government policy guidance. This interaction reflects the government's dual objectives of minimizing subsidy expenditure while achieving emission reduction targets, contrasted with operators' primary focus on reducing total transportation costs per task. Based on these premises, the following fundamental assumptions are established:

(1) The cargo maintains its integrity throughout the transportation process, undergoing only necessary transfer operations at designated nodes;

(2) Potential alterations or damage to goods during transit are considered negligible;

(3) Total transportation costs encompass all transfer node expenses and related operational costs;

(4) While all nodes are potentially eligible for government subsidy programs, only specific nodes can activate these incentives.

IV. MODEL FORMULATION

A. Model Parameters

The multimodal transportation network, supported by government subsidies, is represented by $G=(N,S,A)$ and is characterized as a directed acyclic graph. N is the set of transportation nodes, S represents the set of transportation modes, and A comprises the set of all arc segments. The sets, parameters, and variables are delineated in Table I.

B. Mathematical Model

B.1 Upper level modal

To accelerate the achievement of "dual-carbon" objectives, government agencies have implemented various initiatives to improve transfer node infrastructure. These measures aim to promote the transition of the transportation sector toward a low-carbon, high-efficiency model within constrained budgetary parameters. While maintaining effective control over total subsidy allocations, the government strategically prioritizes the environmental benefits of carbon emission reduction, ensuring substantial decreases in overall emissions to support sustainable development goals.

$$Minf_1=\sum_i^N\sum_j^N\sum_m^S x_{ij}^m e_{ij}^m d_{ij}^m q+\sum_j^N y_j q(e_j-e_j^{reduce}) \quad (1)$$

$$Minf_2=\sum_j^N y_j c_j \quad (2)$$

Eq. (1) formulates the optimization objective of minimizing carbon emissions from both transportation routes and transfer nodes, aiming to comprehensively reduce the environmental impact of the entire logistics chain through route optimization and the reduction of energy consumption and emissions at transfer nodes during cargo handling operations. Eq. (2) establishes the objective of minimizing total government subsidies, reflecting the government's dual focus of providing essential financial support while optimizing resource allocation and preventing unnecessary expenditures.

Constraints:

$$C_{subsidy}^{lower}\le\sum_j^N y_j c_j\le C_{subsidy}^{upper} \quad (3)$$

$$qy_j\le Q_j \quad \forall j\in N \quad (4)$$

$$\sum_i^N\sum_j^N\sum_m^S x_{ij}^m\tilde{t}_{mij}+\sum_j^N y_j\tilde{t}_j\le T^* \quad (5)$$

$$y_j\in\{0,1\} \quad \forall j\in N\setminus\{o,d\} \quad (6)$$

Eq. (3) delineates the constraint on the aggregate government subsidy to guarantee that total investment remains above the minimum threshold $c_{subsidy}^{lower}$ and does not surpass the maximum limit $c_{subsidy}^{upper}$. This configuration seeks to reconcile the projec's fiscal requirements with the government's capacity to offer financial assistance, guaranteeing that the project obtains essential funding while preventing the misallocation or squandering of resources due to excessive dependence on governmental subsidies. Eq. (4) represents the capacity constraint of the transfer node, Eq. (5) delineates the overall transportation time constraint, and Eq. (6) signifies the decision variable, with x_{ij}^m derived by resolving the lower-level model.

B.2 Lower level modal

For multimodal transportation operators, the primary purpose in formulating transportation plans and planning routings is to effectively minimize transportation costs. Consequently, the lower-level model focuses on minimizing transportation expenses.

$$Minf_3=\sum_m^S\sum_i^N\sum_j^N x_{ij}^m c_{ij}^m d_{ij}^m q \quad (7)$$

Constraints:

$$\sum_m^S\sum_j^N x_{ij}^m-\sum_m^S\sum_j^N x_{jh}^m=\begin{cases}1 & i=o\\-1 & i=d\\0 & i=N\setminus\{o,d\}\end{cases} \quad (8)$$

$$\sum_i^N\sum_m^S x_{ij}^m=y_j \quad \forall i\in N, j\in N \quad (9)$$

$$\sum_h^N\sum_m^S x_{jh}^m=y_j \quad \forall h\in N, j\in N \quad (10)$$

$$x_{ij}^m\in\{0,1\} \quad \forall(i,j)\in A,\forall m\in S \quad (11)$$

Eq. (8) represents the flow equilibrium, stipulating that the fluxes at both the initial and terminal points are equal to 1, while the intermediate nodes must maintain an incoming flow that matches the outgoing flow. Eqs. (9) and (10) provide compatibility requirements; specifically, arc segments traversing a node are deemed legitimate pathways solely if a site is chosen and supported by the government for retrofitting and enhancement. Eq. (11) represents the decision variable, with y_j being defined by the upper-level model.

TABLE I
SYMBOL DESCRIPTION

Symbol	Description
Set	
N	The set of nodes, $N=\{1,2,\cdots,N\}$, $i,j,h\in N$, o is the original node, d is the destination node, and $N\setminus\{o,d\}$ is the transition node.
S	The set of transportation modes, $S=\{1,2\}$,1 for railway, 2 for road, $m,n\in S$
A	The set of arcs, $(i,j)\in A$
Parameters	
q	Number of containers(unit: carton)
Q_j	Transfer capacity at the node j (unit: carton)
c_{ij}^m	Transportation costs for transportation m by mode from node i to node j (unit: CNY/(carton·kilometer))
c_j	Government subsidies at the node j (unit: CNY)
$c_{subsidy}^{upper}$	The upper limit of government subsidies at the node j (unit: CNY)
$c_{subsidy}^{lower}$	The lower limit of government subsidies at the node j (unit: CNY)
d_{ij}^m	Transportation distance from node i to node j by transportation mode m (unit: kilometer)
$\tilde{t}_{mij}$	Uncertain transportation time from node i to node j by mode m of transportation(unit: minute)
$\tilde{t}_j^{mn}$	Uncertain transfer time from transportation mode m to transportation mode n at the node j (unit: minute)
T^*	Arrival time
e_{ij}^m	Carbon emission for transportation m by mode from node i to node j (unit: kg/(carton·kilometer))
e_j	Carbon emission at the node j (unit: kg/carton)
e_j^{reduce}	Reduced carbon emissions at the node j after government subsidies (unit: kg/carton)
M	A very large number
Decision variables	
x_{ij}^m	0-1 decision variables, if from node i to node j by transportation mode m , $x_{ij}^m=1$, or $x_{ij}^m=0$
y_j	0-1 decision variables, if overnment subsidies invested at node j , $y_j=1$, or $y_j=0$

C. Model Processing

C.1 Upper level model processing

(1) Uncertain variable processing

Given that the upper-level model includes uncertain variables $\tilde{t}_{mij}$ and $\tilde{t}_j$, which complicates direct resolution, it is essential to convert them deterministically. This transformation is executed using the methods outlined in references.[20]、[23]、[24]

This research employs the optimistic value criterion to examine the complexities of transportation and transfer durations amid uncertainties in the transportation process, including variations in weather conditions. Decision-makers can employ two uncertainty variables [21]、[22] derived from limited historical data, personal experience, and the assessment of these uncertainties to accurately represent the uncertainty in transportation time and transfer time. The variables of uncertainty are denoted as follows:

Theorem 1[21]: if $\xi=\mathcal{L}(a,b)$ is a Liner uncertain variable, for any real numbers a,b ,and $a<b$, under the optimistic value criterion

$$\xi_{\sup}(\alpha)=\alpha a+(1-\alpha)b \quad 0<\alpha\le 1 \tag{12}$$

Theorem 2[22]: if $\xi=\mathcal{Z}(a,b,c)$ is a Zigzag uncertain variable, for any real numbers a,b,c ,and $a<b<c$, under the optimistic value criterion

$$\xi_{\sup}(\alpha)=\begin{cases}(1-2\alpha)c+2\alpha b & 0<\alpha\le 0.5\\ 2(1-\alpha)b-(1-2\alpha)a & 0.5<\alpha\le 1\end{cases} \tag{13}$$

(2) Single-objective processing

Due to the differing quantitative categories of subsidies and emissions in the upper-level model, the linear weighting method is inapplicable un this situation. The ideal point method from reference[20] is employed to address this problem. If the optimal solutions for objectives k and X_{ij}^*, f_i^* are identical, then this constitutes the optimal solution for multi-objective planning, and the algorithm concludes; otherwise, a transformation is executed.

$$MinF=\sum_i^k \eta_i[(f_i(x)-f_i^*)/f_i^*] \quad \forall\eta_i\ge 0,\sum_i^k\eta_i=1 \tag{14}$$

C.2 Lower level model processing

(1) Lower level model relaxation

The model developed in this study exhibits a strong connection between upper and lower-level decisions, resulting in challenges when attempting to solve the model directly. Reference[16] indicates that Eq. (10) can be relaxed as follows:

$$x_{ij}^m\ge 0 \quad \forall(i,j)\in A,\forall m\in S \tag{15}$$

equivalent to

$$-x_{ij}^m\le 0 \quad \forall(i,j)\in A,\forall m\in S \tag{16}$$

Furthermore, Eqs. (9) and (10) ensure that $x_{ij}^m\le 1$, and hence, the previous relaxation does not affect the optimization.

(2) KKT condition transformation

In optimization models with inequality constraints, the KKT conditions are typically employed to determine the optimal value. Consequently, the KKT conditions relax the lower-level model, leading to the following formulation:

$$L_i(x_{ij}^m,\omega_i,\lambda_j,\theta_j,v_{ij}^m)=\sum_{m}^{S}\sum_{i}^{N}\sum_{j}^{N}x_{ij}^m c_{ij}^m q$$
$$+\omega_i(\sum_{m}^{S}\sum_{j}^{N}x_{ij}^m-\sum_{m}^{S}\sum_{j}^{N}x_{jh}^m) \tag{17}$$
$$+\lambda_j(\sum_{i}^{N}\sum_{m}^{S}x_{ij}^m-y_j)+\theta_j(\sum_{h}^{N}\sum_{m}^{S}x_{jh}^m-y_j)+v_{ij}^m(-x_{ij}^m)$$

Further the lower level model can be transformed into, Eq. (12) and

$$qc_{ij}^m-\omega_i+\omega_j+\lambda_j+\theta_j-v_{ij}^m=0 \quad \forall i\in N, j\in N, m\in S \tag{18}$$
$$v_{ij}^m(-x_{ij}^m)=0 \quad \forall (i,j)\in A, \forall m\in S \tag{19}$$
$$\lambda_j(\sum_{i}^{N}\sum_{m}^{S}x_{ij}^m-y_j)=0 \quad \forall j\in N \tag{20}$$
$$\theta_j(\sum_{h}^{N}\sum_{m}^{S}x_{jh}^m-y_j)=0 \quad \forall j\in N \tag{21}$$
$$\omega_i,\lambda_j,\theta_j\ free \quad v_{ij}^m\geq 0 \quad \forall i\in N, j\in N, m\in S \tag{22}$$

(3) Linear transformation

Eqs. (18) to (20) are identified as nonlinear constraints following the transformation of the KKT condition, which can be reformulated in accordance with references[16]、[25] as follows

$$v_{ij}^m\leq M(1-x_{ij}^m) \quad \forall (i,j)\in A, \forall m\in S \tag{23}$$
$$\lambda_j\leq M(1-(\sum_{i}^{N}\sum_{m}^{S}x_{ij}^m-y_j)) \quad \forall j\in N \tag{24}$$
$$\theta_j\leq M(1-(\sum_{h}^{N}\sum_{m}^{S}x_{jh}^m-y_j)) \quad \forall j\in N \tag{25}$$

D. Complete Model

The objective function is

$$MinF=\sum_{i}^{k}\eta_i[(f_i(x)-f_i^*)/f_i^*]$$
$$=\eta_1\frac{\sum_{i}^{N}\sum_{j}^{N}\sum_{m}^{S}x_{ij}^m e_{ij}^m q+\sum_{j}^{N}y_j q(e_j-e_j^{reduce})-f_1^*}{f_1^*}+\eta_2\frac{\sum_{j}^{N}y_j c_j-f_2^*}{f_2^*}$$
$$\forall \eta_i\geq 0, \eta_1+\eta_2=1 \tag{26}$$

Note that $\Phi_{mij}(\alpha)$ and $\Gamma_j(\alpha)$ represent the uncertainty distributions, whereas $\Phi_{mij}^{-1}(\alpha)$ and $\Gamma_j^{-1}(\alpha)$ denote the uncertainty inverse distributions. This research will examine them using the optimistic value criterion as follows:

$$\sum_{i}^{N}\sum_{j}^{N}\sum_{m}^{S}x_{ij}^m\Phi_{mij}^{-1}(1-\lambda)+\sum_{j}^{N}qy_j\Gamma_j^{-1}(1-\lambda)\leq T^* \tag{27}$$

Above, the constraints are Eq. (3), Eq. (4), Eq. (6), Eq. (15), Eq. (18), Eq. (22) ~ Eq. (25), and Eq. (27).

V. CASE STUDY

A. Parameters Setting

This research presents a multimodal transportation network arithmetic example with 35 nodes, with the objective of validating the efficacy of the developed model. The cargo originates at the initial node 0, sequentially traverses each intermediate node, and ultimately reaches the terminal node 34, as illustrated in Fig 1. To enhance the solution process, the time variables are discretized: the initiation of the transportation activity is designated as 8:00, recorded as 0 minutes; 10:45 is transformed into a 165-minute representation; and the time point of 9:00, extending to the subsequent day, is represented as 1500 minutes.

The subsidy amount and transfer duration of the multimodal transportation network's transfer nodes are presented in Table II. The carbon emissions associated with the transfer are 4.05 kg per carton, as referenced [27]. The enhancement of the transfer nodes following government subsidies can decrease carbon emissions by 0.81 kg per carton. The transfer costs and transfer time between identical transportation modalities are currently not taken into account.

Data necessary for the arithmetic example have been systematically gathered by examining the 95306 website and reference[26], as illustrated in Table III. The unit cost for rail and road transportation of a 20ft container is determined by summing the base price and the distance price, referred to as $c_{ij}^1=440+3.185d_{ij}^1$ and $c_{ij}^2=400+6d_{ij}^2$. This study employs zigzag uncertainty variables to thoroughly evaluate potential uncertainties in the transportation process. Furthermore, based on carbon emission data from reference[27], it establishes that the carbon emission per unit of railroad transportation is 0.06 kg/(carton·kilometer), while for road transportation, it is 0.24 kg/(carton·kilometer).

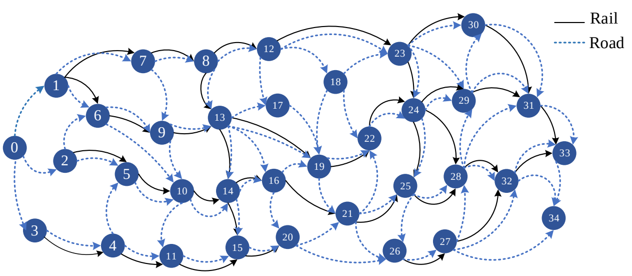


Fig 1. Multimodal transportation network

TABLE II
GOVERNMENT SUBSIDY and TRANSFER TIME at TRANSFER NODES

（Subsidy：CNY）/（Time：minute/carton）

Node	Rail-Road	Node	Rail-Road	Node	Rail-Road	Node	Rail-Road
0	75/[40,48]	9	39/[40,48]	18	37/[40,48]	27	71/[40,48]
1	62/[40,48]	10	50/[40,48]	19	38/[40,48]	28	34/[40,48]
2	54/[40,48]	11	74/[40,48]	20	74/[40,48]	29	44/[40,48]
3	77/[40,48]	12	64/[40,48]	21	49/[40,48]	30	62/[40,48]
4	78/[40,48]	13	34/[40,48]	22	36/[40,48]	31	64/[40,48]
5	44/[40,48]	14	45/[40,48]	23	63/[40,48]	32	47/[40,48]
6	49/[40,48]	15	75/[40,48]	24	39/[40,48]	33	63/[40,48]
7	61/[40,48]	16	41/[40,48]	25	50/[40,48]	34	74/[40,48]
8	62/[40,48]	17	45/[40,48]	26	72/[40,48]		

TABLE III
DISTANCE and TIME

Arc	Rail（km/minute）	Road（km/minute）	Arc	Rail（km/minute）	Road（km/minute）
(0,1)	-	110/[78,88,102]	(17,19)	-	67/[47,54,62]
(0,2)	-	134/[95,107,124]	(18,19)	-	110/[78,88,102]
(0,3)	-	75/[53,60,69]	(18,22)	-	76/[54,61,70]
(1,6)	161/[138,149,161]	148/[104,118,137]	(18,23)	-	115/[81,92,106]
(1,7)	133/[114,123,133]	120/[85,96,111]	(19,21)	-	119/[84,95,110]
(2,5)	90/[77,83,90]	103/[73,82,95]	(19,22)	114/[98,105,114]	125/[88,100,115]
(2,6)	-	85/[60,68,78]	(20,21)	-	95/[67,76,88]
(3,4)	125/[107,116,125]	117/[83,94,108]	(20,26)	-	110/[78,88,102]
(4,5)	-	75/[53,60,69]	(21,22)	-	73/[52,58,67]
(4,11)	166/[142,153,166]	150/[106,120,138]	(21,25)	89/[76,82,89]	111/[78,89,102]
(5,10)	86/[74,79,86]	101/[71,81,93]	(21,26)	-	65/[46,52,60]
(6,9)	167/[143,154,167]	144/[102,115,133]	(22,24)	115/[99,106,115]	133/[94,106,123]
(6,10)	-	133/[94,106,123]	(23,24)	135/[116,125,135]	133/[94,106,123]
(7,8)	119/[102,110,119]	128/[90,102,118]	(23,29)	-	150/[106,120,138]
(7,9)	-	105/[74,84,97]	(23,30)	145/[124,134,145]	135/[96,108,125]
(8,12)	113/[97,104,113]	112/[79,90,103]	(24,25)	98/[84,90,98]	94/[66,75,87]
(8,13)	112/[96,103,112]	113/[80,90,104]	(24,28)	138/[118,127,138]	-
(9,10)	-	126/[89,101,116]	(24,29)	136/[117,126,136]	146/[103,117,135]
(9,13)	149/[128,138,149]	135/[95,108,125]	(25,26)	-	73/[52,58,67]
(10,11)	-	88/[62,71,82]	(25,28)	82/[70,76,82]	107/[76,86,99]
(10,14)	136/[117,126,136]	120/[85,96,111]	(26,27)	120/[103,111,120]	132/[93,106,122]
(11,15)	130/[111,120,130]	90/[64,72,83]	(27,28)	-	137/[97,110,126]
(12,17)	-	55/[39,44,51]	(27,32)	104/[89,96,104]	110/[78,88,102]
(12,18)	-	103/[73,82,95]	(27,34)	-	117/[83,94,108]
(12,23)	138/[118,127,138]	128/[90,102,118]	(28,29)	-	76/[54,61,70]
(13,14)	146/[125,135,146]	143/[101,114,132]	(28,31)	-	130/[92,104,120]
(13,16)	-	63/[44,50,58]	(28,32)	96/[82,89,96]	104/[73,83,96]
(13,17)	-	107/[76,86,99]	(29,30)	-	141/[100,113,130]
(13,19)	136/[117,126,136]	130/[92,104,120]	(29,31)	140/[120,129,140]	129/[91,103,119]
(14,15)	87/[75,80,87]	103/[73,82,95]	(30,31)	89/[76,82,89]	95/[67,76,88]
(14,16)	141/[121,130,141]	132/[93,106,122]	(31,33)	106/[91,98,106]	106/[75,84.8,98]
(15,20)	150/[129,138,150]	100/[71,80,92]	(32,33)	125/[107,115,125]	120/[85,96,111]
(16,19)	-	129/[91,103,119]	(32,34)	-	117/[83,94,108]
(16,20)	-	81/[57,65,75]	(33,34)	-	108/[76,86.4,100]
(16,21)	80/[69,74,80]	-			

B. *Numerical Results*

This paper utilizes the optimistic value as a research example, establishing the initial transportation time at 0 minutes, with the upper time limit set at $T^* = 3000$ minutes, the lower subsidy limit at $c_{subsidy}^{lower} = 5$ million CNY, and the upper subsidy limit at $c_{subsidy}^{upper} = 7$ million CNY. The weight value ω_1 is varied from 0 to 1 in increments of 0.1, alongside the confidence level $\alpha = 0.6$. To validate the model, the Gurobi solver is employed using Python on a personal computer equipped with an APPLE M1 Pro CPU and 8GB RAM.

The computational results presented in Table IV, complemented by the visual representation in Fig 2, demonstrate the distribution patterns of subsidy allocations, carbon emissions, transportation duration, operational costs, and objective function values across different weight combinations. This comparative analysis facilitates a comprehensive understanding of the fundamental relationships among these variables and their implications for policy formulation and operational decision-making.

From a governmental perspective, the analysis primarily focuses on the correlation between subsidy allocation and carbon emissions. Plan 1, characterized by the highest subsidy allocation of 6.5 million CNY, simultaneously achieves the lowest carbon emission levels, demonstrating the effectiveness of subsidy policies in promoting emission reduction and underscoring the importance of policy incentives in fostering sustainable transportation development. In contrast, plans 5 and 6, with minimal subsidy allocations, exhibit significantly higher carbon emissions, establishing a positive correlation between subsidy investment and emission reduction while highlighting the potential environmental consequences of insufficient regulatory measures.

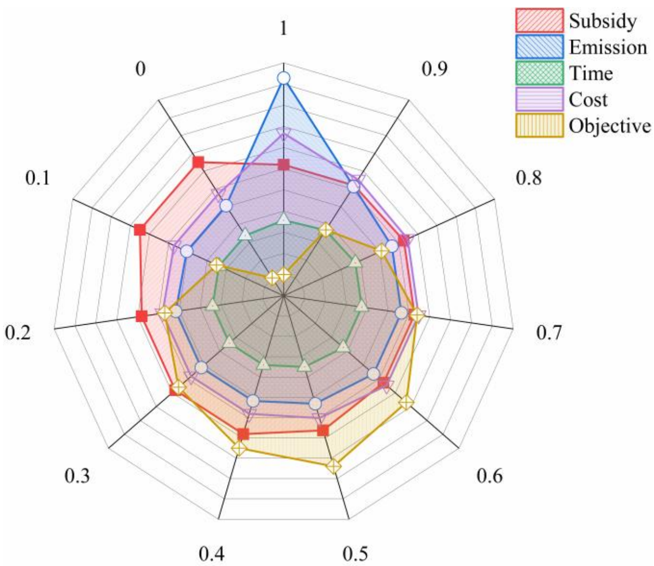


Fig 2 Distribution of subsidies, carbon emissions, transportation time, transportation costs and objective function values

Operators prioritize transportation economics. While road transport incurs higher costs and emissions than rail, it offers faster delivery and greater flexibility, enhancing customer satisfaction and enabling “door-to-door” service. Conversely, rail transport provides cost efficiency and lower emissions despite longer transit times, effectively balancing economic and environmental objectives. This is demonstrated in plans 5 and 6, where rail proves more cost-effective and environmentally friendly on identical routes.

These findings offer valuable insights for government subsidy allocation and environmental assessment, while guiding operators in selecting optimal transportation solutions. Governments should evaluate environmental performance when determining subsidies to balance economic and ecological goals, while operators should adapt transport modes based on operational needs and market conditions to maximize economic benefits.

C. *Sensitivity Analysis*

(1) Without government subsidies

The comparative analysis of optimization results reveals a significant 6% increase in carbon emissions in the non-subsidy scenario (Table V) compared to plan 1 in Table IV, demonstrating the critical role of government subsidies in facilitating emission reduction. These subsidies not only promote technological innovation and service improvement but also serve as a crucial mechanism for achieving substantial carbon emission mitigation in transportation systems.

(2) Sensitivity analysis of the subsidy lower bound

The experimental results demonstrate that increasing the minimum government subsidy threshold from 5 million CNY to 5.6 million CNY significantly influences system performance, as shown in Fig 3. The analysis reveals that 6 transportation alternatives are available at the 5 million CNY subsidy level, while only 3 options remain feasible when the threshold is raised to 5.2 million CNY. This variation in subsidy floor directly affects the diversity of transportation solutions, consequently altering the optimal combinations of carbon emissions, transit time, and operational costs. These findings provide policymakers and operators with multiple alternatives that can be strategically selected according to specific operational requirements and policy objectives.

(3) Sensitivity analysis of the subsidy upper bound

The analytical results demonstrate that reducing the maximum government subsidy from RMB 7 million to RMB 5.5 million significantly affects system performance, as illustrated in Fig 4. The data reveal that while 6 viable transportation solutions exist at the RMB 7 million subsidy level, only 2 alternatives remain feasible at the RMB 5.5 million threshold. This reduction in subsidy ceiling directly influences the diversity of available transportation options, consequently affecting the optimal combinations of carbon emissions, transportation time, and operational costs.

(4) Sensitivity analysis of arrival time

Maintaining constant parameters while adjusting cargo arrival time produced the results shown in Fig 5. The analysis indicates significant sensitivity of subsidies, carbon emissions, time, and costs to arrival time variations. This necessitates thorough temporal assessment in transportation planning. For flexible shipments, efficiency can be improved through optimized subsidies, reduced emissions, shorter times, and lower costs. For time-sensitive cargo, selection must be limited to ensure timely delivery despite potential trade-offs.

(5) Sensitivity analysis of confidence

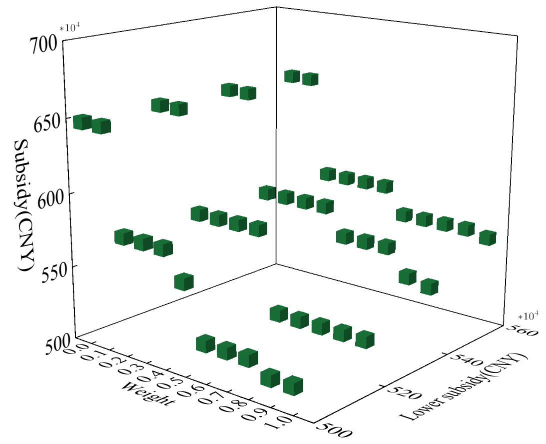
All parameters were held constant except for the confidence level α, which was systematically varied from 0 to 1 in increments of 0.1, yielding the results presented in Fig 6. The analysis demonstrates that variations in the confidence level significantly influence transportation planning outcomes, resulting in substantial fluctuations in subsidies, carbon emissions, time, and costs. These findings suggest that both governmental authorities and transportation operators should adopt a comprehensive approach to determine the optimal confidence level, thereby improving transportation efficiency, reducing costs, and developing scientifically sound transportation plans.

TABLE IV
OPTIMIZATION RESULTS of MULTIMODAL ROUTING BASED on OPTIMISTIC VALUE CRITERION

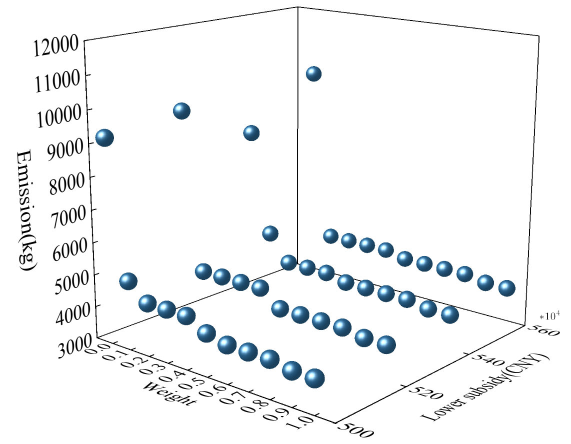
Plan	ω_1	Routing	Modes	Subsidy(CNY)	Emission(kg)	Time(minute)	Cost(CNY)
1	0、0.1	0-1-7-8-12-23-30-31-33-34	Road-rail-rail-rail-rail-rail-rail-rail-road	6500000	4059	2377	236189
2	0.2~0.4	0-1-7-8-12-23-24-28-32-34	Road-rail-rail-rail-rail-rai-rail--rail-road	5810000	4176	2411	240580
3	0.5	0-2-5-10-14-16-21-25-28-32-34	Road-rail-rail-rail-rail-rai-rail--rail-rail-road	5630000	4316	2508	251220
4	0.6~0.8	0-1-6-9-13-19-22-24-28-32-34	Road-rail-rail-rail-rail-rai-rail--rail-rail-road	5270000	4640	2740	273272
5	0.9	0-2-6-9-13-19-22-24-28-32-34	Road-road-rail-rail-rail -rail-rail-rail-rail-road	5190000	5135	2679	276308
6	1	0-2-6-9-13-19-22-24-28-32-34	Road-road-road-road-road-road-road-rail-road-road	5190000	9288	2574	333646

TABLE V
RESULTS of MULTIMODAL TRANSPORTATION ROUTING OPTIMIZATION without GOVERNMENT SUBSIDIES

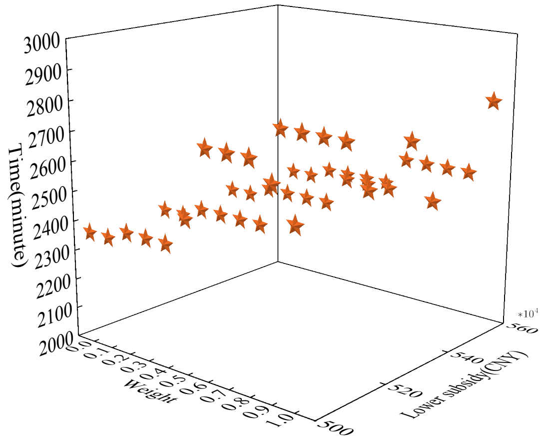
Plan	Objective	Routing	Modes	Emission(kg)	Time(minute)	Cost(CNY)
1	*Min* Emission	0-1-7-8-12-23-30-31-33-34	Road-rail-rail-rail-rail-rail-rail-rail-road	4302	2377	236189
2	*Min* Time	0-3-4-11-15-20-26-27-34	Road-road-road-road-road-road-road -road	7509	1992	256380
6	*Min* Cost	0-3-4-11-15-20-26-27-34	Road-rail-rail-rail-rail-road-rail-road	4512	2161	222385



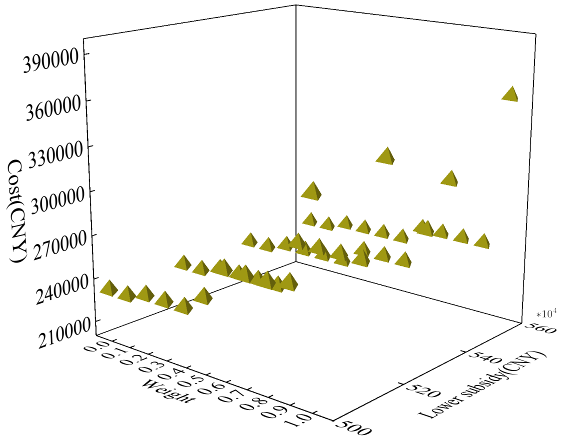
3(a) Subsidy



3(b) Emission

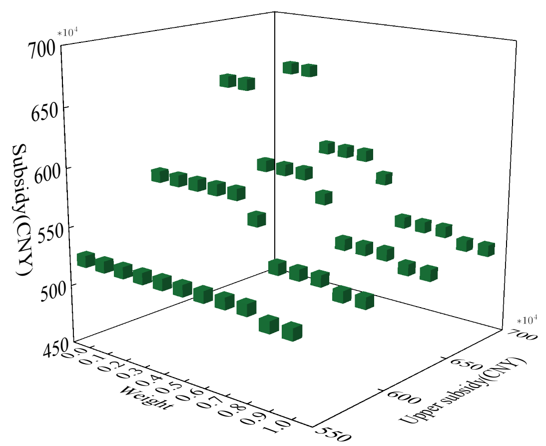


3(c) Time

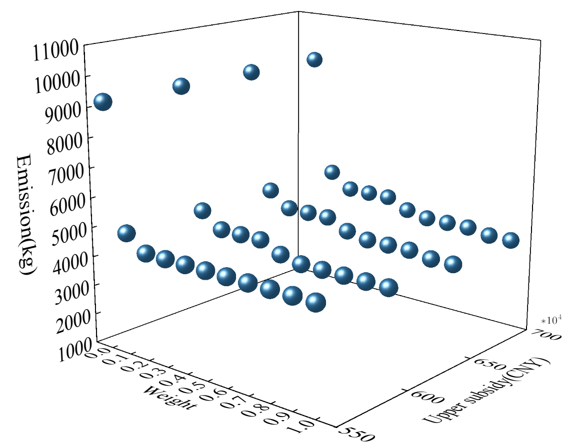


3(d) Cost

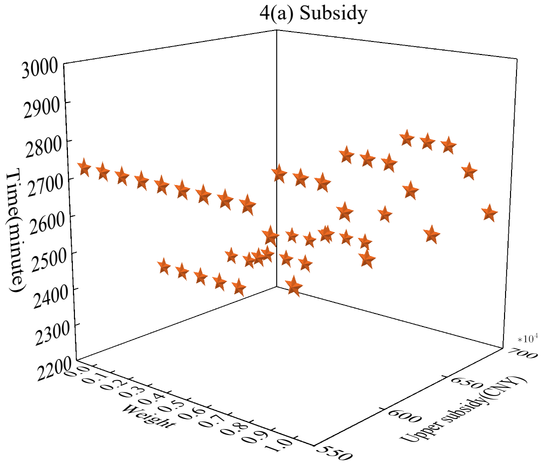
Fig.3 Sensitivity analysis of the subsidy lower bound



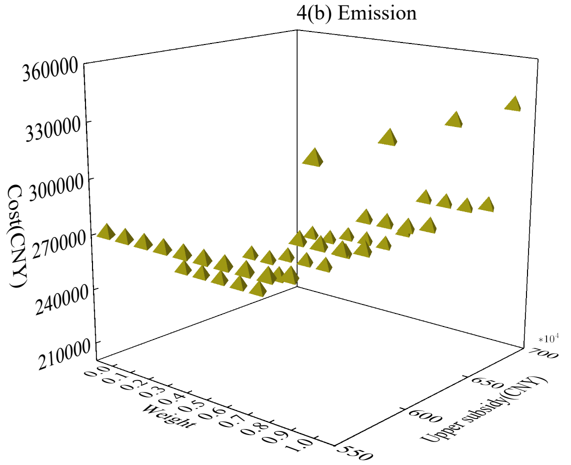
4(a) Subsidy



4(b) Emission

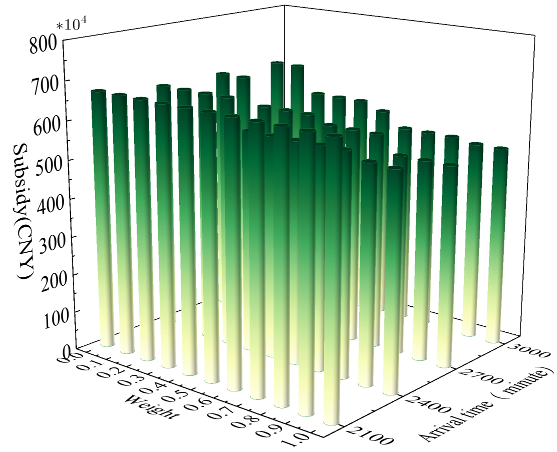


4(c) Time

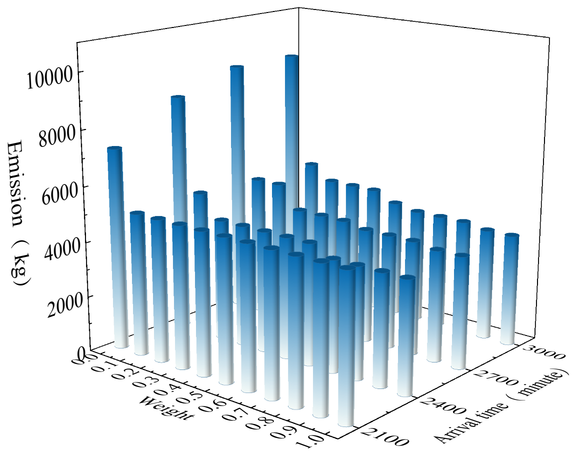


4(d) Cost

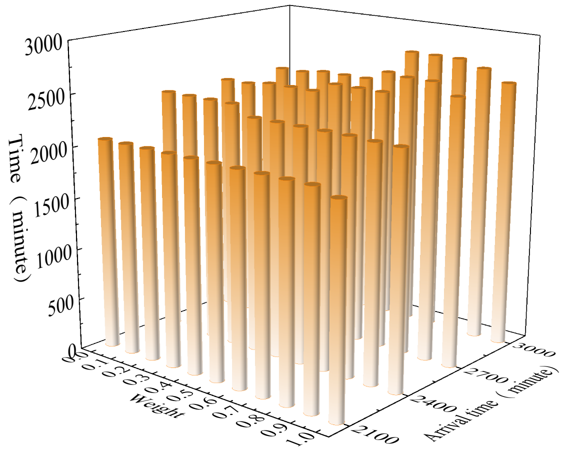
Fig.4 Sensitivity analysis of the subsidy upper bound



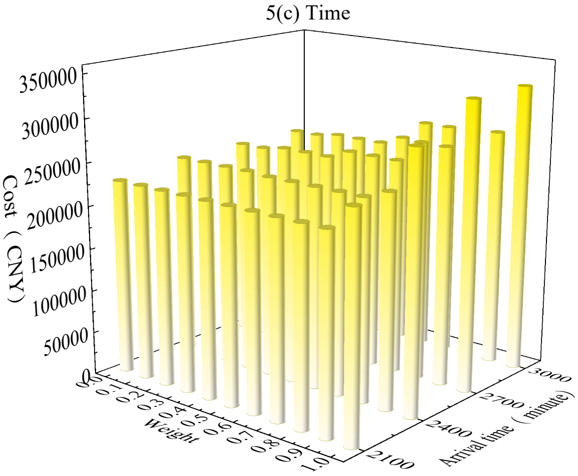
5(a) Subsidy



5(b) Emission

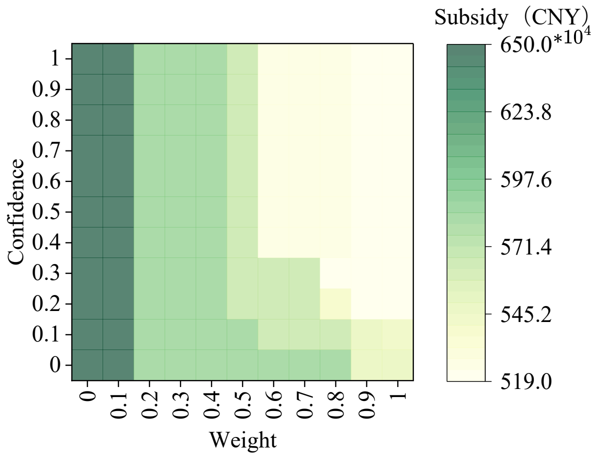


5(c) Time

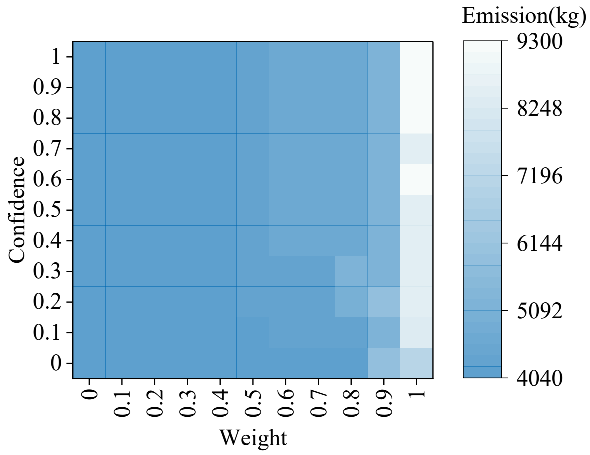


5(d) Cost

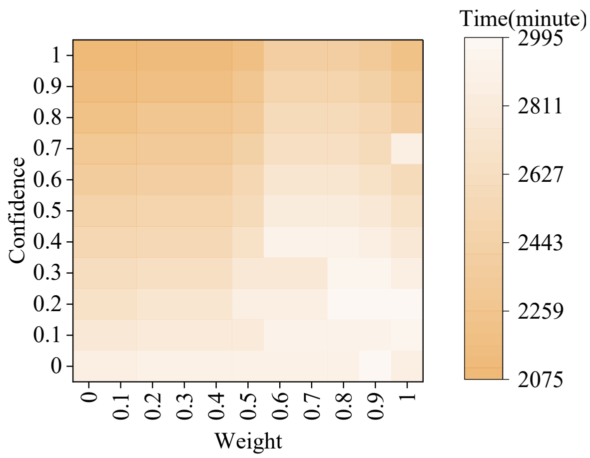
Fig.5 Sensitivity analysis of arrival time



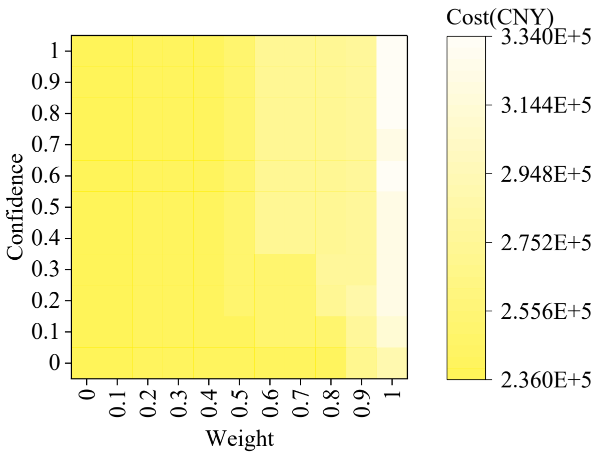
6(a) Subsidy



6(b) Emission



6(c) Time



6(d) Cost

Fig. 6. Sensitivity analysis of confidence

VI. Conclusion

The significance of multimodal transportation in enhancing transportation efficiency, optimizing resource allocation, reducing logistical costs, and promoting the development of an environmentally sustainable, low-carbon transportation system is increasingly acknowledged.

1) A bi-level programming model has been developed to elucidate the complex relationship between government subsidies and operator routing decisions.
2) Government subsidies play a crucial role in facilitating the efficient and sustainable development of multimodal transportation. Well-designed subsidies can guide operators to choose more environmentally friendly and efficient routing options, thereby reducing carbon emissions and improving overall transportation efficiency.
3) Key factors such as subsidies, arrival times, and confidence levels significantly influence routing decisions, providing a vital theoretical foundation and practical guidance for governments to formulate precise subsidy policies and optimize multimodal transportation routes.
4) Looking ahead, with continuous technological advancements, multimodal transportation is expected to be increasingly implemented and supported across a wider range of sectors.

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Tingyu Wang was born in Sichuan Province, China, in 1995. She is currently pursuing her postgraduate studies in Transportation Planning and Management at the School of Traffic and Transportation, Lanzhou Jiaotong University, China. Her research interests primarily focus on multimodal transportation systems.