Research on Structure and Vulnerability of Regional Comprehensive Transportation Network Based on Supernetwork Theory

Wei Liu, Ruhu Gao, Jiarui He, Jiaping Xi, Yunben Bai

Abstract—**As the transportation system progresses into a new era characterized by networking, interconnection, and multimodal transport, traditional research methods struggle to accurately capture the complex structure of multi-layered, heterogeneous transportation networks. Viewed through the lens of supernetworks, this paper explores the structural characteristics and vulnerabilities inherent in regional comprehensive transportation systems. Firstly, we introduce an innovative approach by establishing connections between heterogeneous sub-networks through hyperedges and incorporating various supernetwork topological features to analyze regional transportation networks. Building upon this foundation, we propose a dual attack strategy targeting both nodes and hyperedges, and we assess the network's vulnerability using three key indicators: network efficiency, network connectivity, and the size of the largest connected subgraph. To validate the efficacy of the proposed method, we conducted experiments on the transportation network of the Yangtze River Delta region. The findings reveal that the structural topological properties of the multi-layer supernetwork model outperform those of individual networks, demonstrating enhanced stability when subjected to attacks. This study holds significant value for optimizing the layout of regional transportation networks and fostering greater connectivity among cities in the region.**

Index terms—**comprehensive transportation, supernetwork theory, network vulnerability, topological characteristics**

I. Introduction

WITH the ongoing implementation of national policies and the continuous enhancement of regional transportation infrastructure, the transportation network system has become increasingly complex. The development of transportation has now entered a new stage characterized by networking, three-dimensionality, and multimodal integration. Consequently, to expedite the process of regional transportation integration and establish an efficient and orderly comprehensive transportation channel system, it is essential to conduct reasonable planning of the regional transportation layout.

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In the process of transportation integration, the identification of key nodes is crucial for optimizing the network layout. Yang and Ullah A proposed a novel method for identifying key nodes in complex networks based on global structure, which offers a new metric for accurate identification of these nodes[1][2]. Additionally, Yang et al. introduced a method for evaluating node importance based on neighboring connections and local network structures[3]. L WAN proposed a multi-layer heterogeneous network node importance identification method that effectively utilizes the correlation information between different types of nodes[4]. To identify influential nodes that facilitate faster and broader dissemination in complex networks, Sheng and Chen each proposed innovative methods for identifying these influential nodes[5][6].

Currently, numerous scholars have conducted research on the construction and characteristics of transportation network models based on complex networks. In accordance with the characteristics of urban agglomerations, Song et al. utilized urban flow intensity to assess the significance of nodes and employed the K-means clustering method to objectively categorize urban importance[7]. Feng et al. constructed a multi-layer network based on the China-Europe freight train transportation network and proposed a method for evaluating node importance in multi-layer networks, which integrates an improved TOPSIS method and grey correlation analysis[8]. Additionally, Jiao et al. developed an evaluation model for important nodes within the urban agglomeration network from the dual perspectives of high-speed railway flow and highway flow[9].

The study of network attack strategies facilitates the rational allocation of resources, optimization of network structure, and enhancement of network robustness. Utilizing complex network theory, Yuhao Yang assessed the robustness of the Beijing Metro against both random and deliberate attacks, employing spatial analysis techniques to evaluate the network's vulnerabilities to such attacks[10]. Li proposed a Fractional-order SS1IR model and introduced the concept of super-spreading nodes[11]. Ye, Wang, Ma et al. conducted an analysis of the topology of the urban transportation network and performed a cascading failure analysis on this network[12]-[14]. Xv investigated the role of interconnectedness in enhancing the resilience of Hong Kong's public transport system, revealing that interconnected transport systems improve resilience by reducing topological vulnerability, increasing attack tolerance, and enhancing the interoperability of bases[15].

As an emerging discipline, supernetwork theory offers a novel perspective for understanding and analyzing the intricate relationships among networks. It also introduces new methods and tools for studying the interactions and influences between these networks[16]. Hypernetwork theory has found extensive application in complex network systems, including supply chain management, logistics systems, travel behavior analysis, and transportation networks[17]-[19].

The complexity and multi-dimensionality of transportation networks have increasingly drawn scholarly attention to the application of hypernetwork theory within this field. Du Wenju utilized the theory of coupled complex networks to investigate the synchronization problem of an urban public transport supernetwork model, which integrates conventional bus transportation and urban rail transit systems[21]. Shanmukhappa T introduced the concept of supernodes, synthesizing nodes from various transportation modes within a specified distance into supernodes, thereby establishing a transfer hypernetwork model centered around subway stations[22]. To offer a more refined travel scheme for heterogeneous passengers, Yujian developed an air-rail composite hypernetwork model and employed a depth-first traversal algorithm to determine efficient and optimal combined travel paths within the network[23].

Due to the limited development of hypernetwork theory, current analyses of hypernetworks are conducted as extensions of complex networks. Suo Q employs hypernetwork theory to describe the evolutionary mechanisms of high-speed railway systems and analytically derives the node over-degree distribution, which exhibits a power-law distribution that adheres to a shift. This theory is validated using China's high-speed railway network as a case study[24]. Wei Y adopts a supernetwork approach, utilizing hyperedges to represent subway lines as subsystems and subsequently simplifying them into nodes[25]. This method allows for the application of complex network analysis techniques to new supernetwork models, measured by three metrics: network connection efficiency, maximum connection subgraph size, and average subgraph size[26]. These metrics are used to analyze the robustness of metro network nodes and the impact of hyperedge attacks on the Nanjing metro network. Wang J connects the bus network to the subway network through the concept of 'Hyperedge,' constructing a multi-layer hypernetwork with an interactive mechanism, and compares the topological characteristics of the hypernetwork with those of complex networks[27].

Upon reviewing the existing body of research, it becomes evident that much of the current literature predominantly centers on single modes of transportation or examines multiple modes within an urban setting, with little focus on the broader, integrated urban transportation network at the regional level. Traditional network theory struggles to accommodate the complexity of modeling three-dimensional and holistic regional transportation systems. This limitation leads to an incomplete representation of the regional city hub network, resulting in significant discrepancies in the findings. In this paper, we draw upon the supernetwork theory of multi-subnetwork composites to leverage its advantages in representing multi-layered networks. Based on this foundation, we define and construct a regional comprehensive transportation supernetwork model. The performance of this network is subsequently analyzed and evaluated, aiming to provide valuable insights for the strategic planning of regional transportation systems. This article makes the following contributions:

1) This paper introduces an innovative supernetwork modeling approach that preserves the heterogeneity of nodes across each layer of the transportation subnetwork while leveraging hyper-edge connections between subnetworks to analyze the regional comprehensive transportation network through the lens of supernetwork theory.

2) We extend the network topology characteristics from complex networks to supernetworks and examine the structural attributes of regional transportation networks from three dimensions: node, city, and region.

3) By integrating the proposed dual attack strategy focusing on both nodes and hyperedges, we analyze the supernetwork of integrated traffic and assess its vulnerabilities.

This article will elaborate on several key aspects. Section II introduces the supernetwork theory and the construction method for the integrated transportation supernetwork model, using the Yangtze River Delta region as a case study. Section III provides an analysis of the regional integrated transportation supernetwork model. Parts A, B, and C examine the topological characteristics of the comprehensive transportation network structure from three perspectives: nodes, cities, and regions. Part D assesses the network's vulnerability in the context of potential attacks. Section IV serves as the conclusion, summarizing the work and findings of the dissertation while offering prospects for future research.

II. Supernetwork Model Building

This section begins with a brief introduction to the theory of supernetworks and some assumptions that provide some basic understanding for the subsequent modeling of regional comprehensive transport networks.

A. Supernetwork Theory

In 1970, C. Berge first proposed the concept of hyper network, systematically established the theory of undirected hyper graph, and applied the hypergraph theory to conduct research in operations research[20]. When dealing with the interweaving of logistics networks, information networks, and capital networks, the American scientist Nagurney for the first time referred to multi-layer networks that surpass existing networks as supernetworks[18][19].

Supernetworks are often regarded as an advanced class of complex networks. Although the concept of supernetworks represents a specialized form of complex networks, it has been proposed relatively recently and remains in a developmental phase. Currently, there is no clear consensus on the definition and boundaries of supernetworks.

In this paper, we will briefly introduce two main aspects. The first aspect is a supernetwork composed of multiple subnets (as illustrated in Fig. 1.a). This structure can describe the intricate correlations between various types of sub-networks, providing a novel perspective and tool for analyzing the interactions among the components of the network layer. The second aspect is a hypernetwork based on hypergraph theory (as depicted in Fig. 1.b). This concept is grounded in the theory of hypergraphs, where each hypergraph consists of multiple nodes, and the interconnections among these nodes can be effectively represented by hypergraphs.

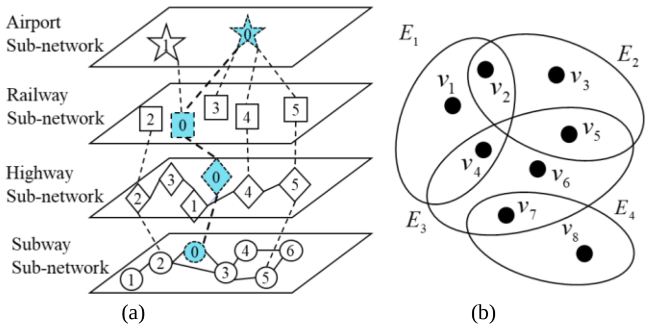

Fig. 1. Two manifestations of supernetworks. (a). supernetwork based on multi-subnet composite. (b). supernetwork based on the hypergraph theory.

B. Case Studies

The regional comprehensive transportation supernetwork is a complex and multi-dimensional concept that encompasses a wide array of interconnected sub-networks representing various modes of transportation, including high-speed rail, highways, and aviation. These sub-networks collectively facilitate the seamless movement of people and goods between cities within the region, each contributing its unique characteristics and advantages. The supernetwork operates as a cohesive system, where the interaction and integration of diverse transportation modes enable optimal efficiency and connectivity throughout the entire region.

To facilitate the depiction of this integrated transportation supernetwork, we propose the following assumptions:

Assumption 1. Given the high efficiency demands for intra-regional mobility, we focus exclusively on three transportation modes—high-speed rail, expressways, and aviation—as the subjects of our study.

Assumption 2. Transportation stations are treated as network nodes. When a route sequentially passes through two adjacent nodes, an edge is formed between them. There exists only one edge between any two nodes, ensuring no duplicate connections.

Assumption 3. In consideration of the transfer dynamics between different transportation subnetworks, if multiple transportation mode nodes coexist within a city, a transfer relationship is assumed, consolidating these nodes into a single hyperedge.

Assumption 4. The analysis disregards the distinction between uplink and downlink within the network, focusing solely on the structure of the regional transportation network. This approach constructs a non-directional, unweighted network.

Based on the supernetwork theory, the regional transportation network is divided into a number of different sub-networks according to the transportation mode, including the aviation layer, the railway layer, and the road layer, and then build a supernetwork model $M=(V^T, S^T, E)$, $T \in (High-speed\ Railway, Expressway, Aviation)$.

1) V^T is a collection of points where the number of nodes is n. When a city has multiple stations of the same mode of transport at the same time, it is usually considered to have only one node.

2) S^T is a set of edges, and the number of connected edges is m. The edge collection contains both intra-layer and inter-layer paths. The intra-layer path represents the path relationship between different nodes in the same transportation mode. Interlayer paths represent connections between different modes of transport in the same city.

3) $E=\{e_1, e_2, ..., e_p\}$ is the set of hyperedges, and p is the number of hyperedges. The hyperedge indicates that there are stations of different modes of transportation in a city. For any hyperedge e_i, it can be represented as a subset of the node set V^T.

The steps to build the supernetwork model are as follows:

Step 1: Mapping. Map different transit stations as nodes of the network and cities as hyperedges.

Step 2: Initialize. The adjacency matrix that defines the supernetwork is A, its size is $n \times n$, n is the number of nodes. Define the supernetwork association matrix as C, and its size is $n \times p$, p is the number of hyperedges.

$$A = |a_{i,j}|,$$

$$a_{i,j} = \begin{cases} 1, exists\ a\ line\ connection\ between\ i\ and\ j \\ 0, else \end{cases} \quad (1)$$

$$C = |c_{i,j}|$$

$$c_{i,j} = \begin{cases} 1, The\ hyperedge\ contains\ nodes\ i\ and\ j \\ 0, else \end{cases} \quad (2)$$

Step 3: Correlation of nodes and hyperedges. In this step, we will traverse through the traffic nodes of each layer. If an intra-layer path connection exists, the nodes will establish an edge connection. Additionally, we will examine each city to gather information regarding the various modes of transportation stations within it. These stations will be grouped together to form a hyperedge. For instance, if nodes A and B are located in the same city, then both nodes will be added to the corresponding hyperedge.

Step 4: Cyclic process. Repeat step 3 until all sites and cities have been retrieved.

Step 5: End.

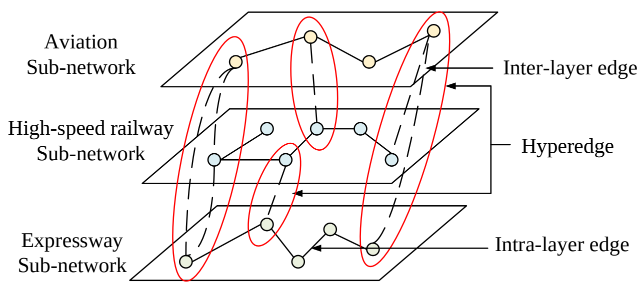

Fig. 2. Schematic diagram of the comprehensive transportation supernetwork

A schematic diagram of the multi-layer supernetwork model constructed using the aforementioned method is presented in Fig. 2. To validate the model's rationality, an empirical analysis was conducted on 107 traffic nodes and 283 connecting edges within the comprehensive transportation system of the Yangtze River Delta region. The data utilized for this analysis were sourced from the 'Comprehensive Development Plan for Higher Quality Transportation in the Yangtze River Delta Region.'

For the Yangtze River Delta comprehensive transportation network, the adjacency matrix A size is 107×107, which includes the aviation layer A_α (23×23), the high-speed railway layer A_β (42×42), and the expressway layer A_γ (42×42).

$$A=\begin{Bmatrix} A_\alpha & & \\ & A_\beta & \\ & & A_\gamma \end{Bmatrix}_{107\times107}$$

The correlation matrix C is 42×107, which contains 42 hyperedges and 107 nodes and their corresponding relationships.

$$C=\begin{matrix} e_1 \\ e_2 \\ \cdots \\ e_p \end{matrix}\begin{Bmatrix} c_{1,1} & c_{1,2} & \cdots & c_{1,107} \\ c_{2,1} & c_{2,2} & \cdots & c_{2,107} \\ \cdots & \cdots & \cdots & \cdots \\ c_{42,1} & c_{42,2} & \cdots & c_{42,107} \end{Bmatrix}_{42\times107}$$

Gephi drawing software is used to visualize the extracted comprehensive traffic sub-networks in the Yangtze River Delta region, as shown in Fig. 3.

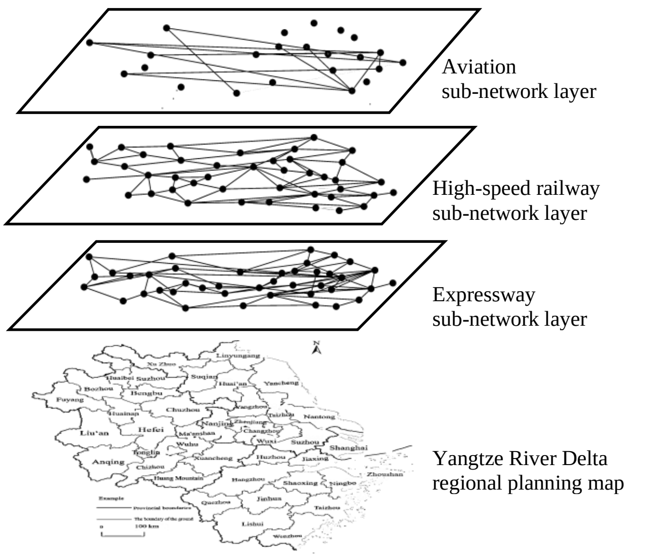

Fig. 3. Schematic diagram of the comprehensive transportation network in the Yangtze River Delta region. The Yangtze River Delta comprehensive transportation supernetwork includes high-speed railway(42 nodes), expressway(42 nodes), and aviation three-layer subnetwork(23 nodes).

III. Results and Analysis

In this section, we provide a detailed introduction to the structural topology characteristics of the regional transportation network. Using the Yangtze River Delta comprehensive transportation supernetwork model as a case study, we analyze and assess its vulnerability while validating the effectiveness of the proposed methodology.

A. Traffic Node Structure

a. Node degree

Node degree is a concept in complex networks, and it is the most concise, intuitive, and commonly used method for evaluating node importance. The degree of a node i is the total number of other nodes adjacent to the node i, denoted as $k(i)$.

$$k(i)=\sum_{j=1}^{n}a_{ij} \tag{3}$$

According to Equation (3), the degree value of each transportation network can be determined, and the statistical results of the degree distribution can be illustrated in Fig. 4. From Fig. 4, several key observations emerge. The degree distribution of the integrated transportation network is notably more concentrated, with an average degree of 5.29. This value is significantly higher than that of the individual transportation sub-networks, indicating that the integration of various transport modes substantially enhances regional transportation efficiency. Notably, the road and rail sub-networks exhibit relatively high degree values, reflecting China's robust infrastructure development. In contrast, the aviation sub-network demonstrates a lower degree, primarily due to the high costs and long distances associated with air travel.

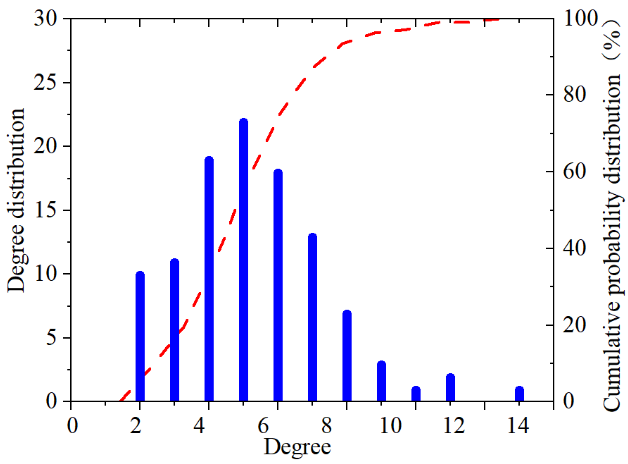

Fig. 4. The distribution and cumulative probability distribution of the comprehensive transportation network

b. Node betweenness

The betweenness centrality of nodes serves as a metric for assessing the significance of nodes within a comprehensive transportation network. It reflects a node's role in facilitating interactions among other nodes within the network's structure. Specifically, the betweenness of a node quantifies the proportion of edges that traverse through that node in the shortest paths connecting any pair of nodes in the network.

$$B_i=\sum_{i,\ j\in N, i\neq j}\frac{\lambda_{ij}(i)}{\lambda_{ij}} \tag{4}$$

Where B_i is the betweenness of the node i; λ_{ij} represents the number of shortest paths between node i and j; $\lambda_{ij}(i)$ Indicates the number of edges that pass through the node i in the shortest path.

Betweenness centrality denotes how central a node is in its connected neighboring nodes, which is important for identifying and securing critical resources, and betweenness centrality is denoted as BC.

$$BC_i = \frac{2B_i}{(N-1)(N-2)} \quad (5)$$

Thereinto, N is the total number of nodes in the network.

Table I presents the top ten nodes in the overall ranking of the integrated transportation network, indicating that rail and road transportation continue to dominate the region. Nanjing, recognized as one of the most vibrant cities in the Yangtze River Delta, occupies a pivotal position with substantial passenger flow, underscoring its status as a critical hub within the comprehensive transportation network. This reinforces Nanjing's essential role in transit, necessitating prioritized protection and development. Additionally, as illustrated in the distribution map, the betweenness centrality of most nodes ranges between 0 and 0.05. It is noteworthy that many aviation nodes remain isolated within the broader transportation network, contributing minimally to regional connectivity. This situation highlights the underdeveloped state of the region's airports, indicating that they have yet to reach their full potential.

TABLE I
RANKING OF THE NUMBER OF COMPREHENSIVE TRAFFIC NODES (TOP 10)

Rank	Node	B	BC
1	Nanjing (High-speed Railway)	982.02	0.1765
2	Hangzhou (High-speed Railway)	673.61	0.1210
3	Nanjing (Expressway)	666.05	0.1197
4	Hefei (High-speed Railway)	460.49	0.0827
5	Shanghai (Expressway)	442.44	0.0795
6	Wenzhou Longwan International Airport	386.59	0.0695
7	Hefei (Expressway)	373.65	0.0671
8	Pudong International Airport	348.86	0.0627
9	Fuyang Xiguan Airport	344.34	0.0619
10	Huai'an (high-speed railway)	324.72	0.0584

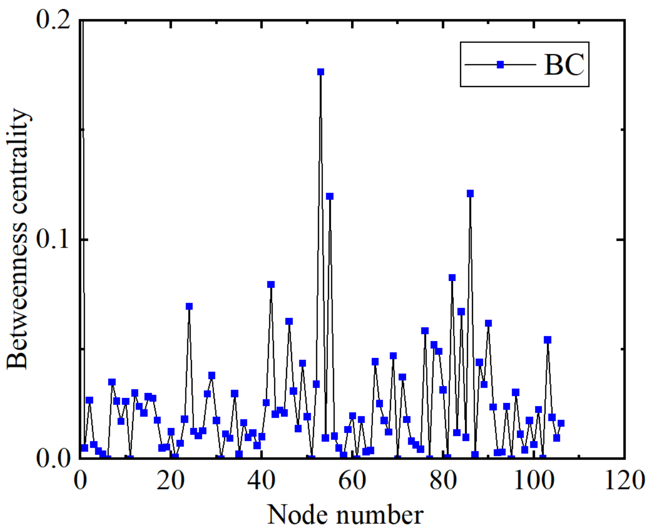

Fig. 5. Node betweenness centrality plot

B. Urban Hyperedge Structure

Next, we will extend the concepts of degree and betweenness centrality to supernetworks and conduct a detailed computational analysis. Before delving into the calculation of topological characteristics, we first integrate the nodes corresponding to different modes of transportation within the same city into a hyperedge, in accordance with the established transmission rules. This step effectively simplifies the complex transportation network into a more manageable urban transportation connection network, as illustrated in Fig. 6. By adopting a supernetwork perspective, we gain a deeper understanding of the strength and robustness of transportation links between cities within the region. This approach provides valuable insights that can significantly inform and guide regional urban transportation planning, optimizing infrastructure development and connectivity.

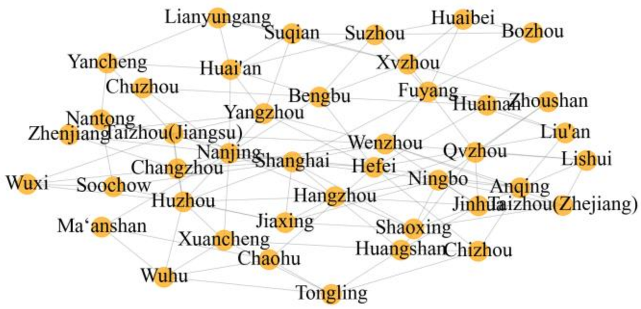

Fig. 6. Topology diagram of the hyperedge relationship of the comprehensive transportation supernetwork

c. Hyperedge hyperdegree

The hyperedge hyperdegree is the number of all nodes contained in a hyperedge. It can be used to represent the capacity of a hyperedge, and to evaluate the complexity of the hyperedge and its influence[27].

See Equation (6) for the calculation formula.

$$d(e_i) = \sum_{v_i, v_j \in e_i} e_i \quad (6)$$

where $d(e_i)$ represents the hyperdegree value of the hyperedge e_i. In the supernetwork model, the hyperedge hyperdegree represents the number of nodes of different sub-network modes in the same hyperedge.

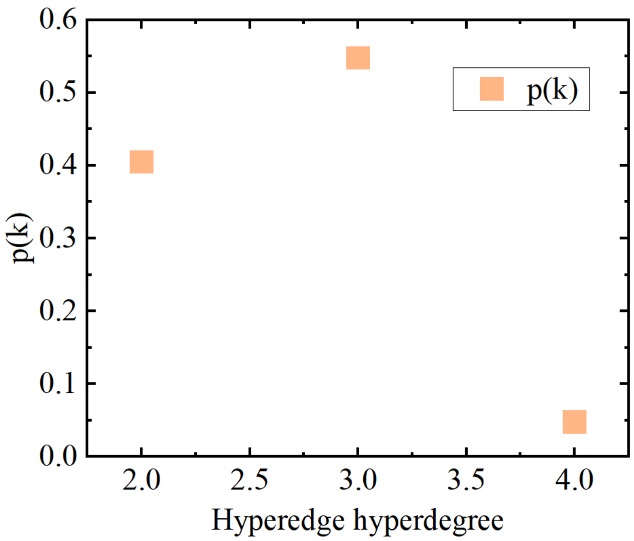

Fig. 7. Probability distribution of hyperedge hyperdegree of comprehensive traffic supernetwork

According to Equation (6), the average hyperedge hyperdegree of the integrated transportation supernetwork is 2.64, with the majority of hyperedges consisting of three nodes. Three modes of transportation are employed, and most of the cities involved function as significant hub cities within the region. The comprehensive transportation system is well-developed and adept at accommodating various transportation modes. The Yangtze River Delta region depends on an integrated network of high-speed rail and expressways to establish rapid transportation corridors, facilitating swift and direct intercity connections across the area.

d. Hyperedge degree

The hyperedge is the hyperedge that is adjacent to the hyperedge e_i through some node connection and is denoted as $k(e_i)$ [27]. See Equation (7) for the calculation method.

$$k(e_i)=\sum\{e \mid (v_i \cap v_j \neq \varnothing\},\ v_i \in e_i, v_j \in e_j \qquad (7)$$

From Equation (7), the average hyperedge degree of the comprehensive traffic supernetwork is calculated to be 5.86. The distribution of hyperedge degrees is illustrated in Fig. 8, exhibiting characteristics of a "peak fat tail thin" distribution. Most hyperedges are concentrated around a degree of 5, with only a limited number exhibiting significantly higher hyperedge degree values.

Nanjing (hyperedge) and Shanghai (hyperedge) exhibit degrees exceeding 10, indicating their connectivity with over a dozen cities and exceptional accessibility. As pivotal hub cities in the Yangtze River Delta region, both cities possess highly developed economies and substantial passenger flows. A strong degree of connectivity significantly facilitates intercity exchanges, fostering regional integration and collective development. Overall, the distribution of urban hyperedge degrees in the Yangtze River Delta region is uneven. Moving forward, efforts should focus on enhancing transportation infrastructure in smaller cities to alleviate transit pressure, promote a more balanced network, and improve the overall efficiency and stability of regional transportation systems.

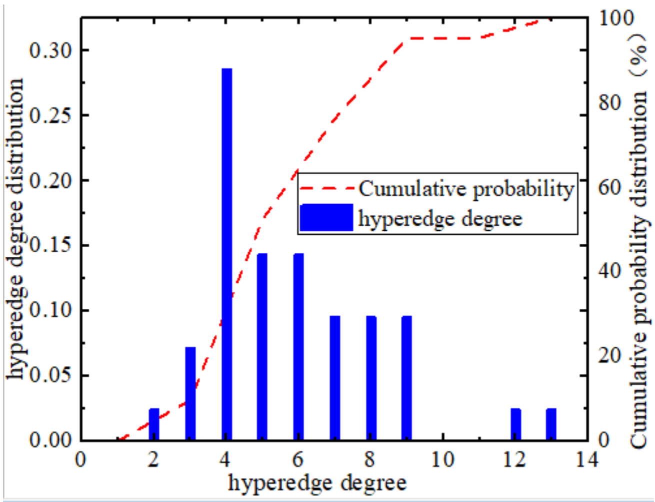

Fig. 8. The distribution of the hyperedge degree and cumulative probability of the comprehensive traffic supernetwork

e. Hyperedge betweenness

The hyperedge betweenness is an indicator that measures the importance of the interaction of the hyperedge in the whole supernetwork, which is the proportion of any shortest path in the supernetwork passing through the hyperedge. By analogy with the betweenness of nodes in a complex network, the formula for calculating the hyperedge betweenness is as follows.

$$B(e_i)=\sum_{i,\ j\in N, i\neq j}\frac{\lambda_{ij}(e_i)}{\lambda_{ij}} \qquad (8)$$

where $B(e_i)$ is the betweenness of the hyperedge; λ_{ij} represents the number of shortest paths between node i and j; $\lambda_{ij}(e_i)$ Indicates the number of edges that pass through the hyperedge e_i in the shortest path between node i and j.

In accordance with equation (8), the betweenness of each hyperedge is computed, and the corresponding betweenness distribution is illustrated in Fig. 9. The average hyperedge betweenness is found to be 29, with a small number of hyperedges exhibiting exceptionally high betweenness, while the majority display relatively low betweenness values.

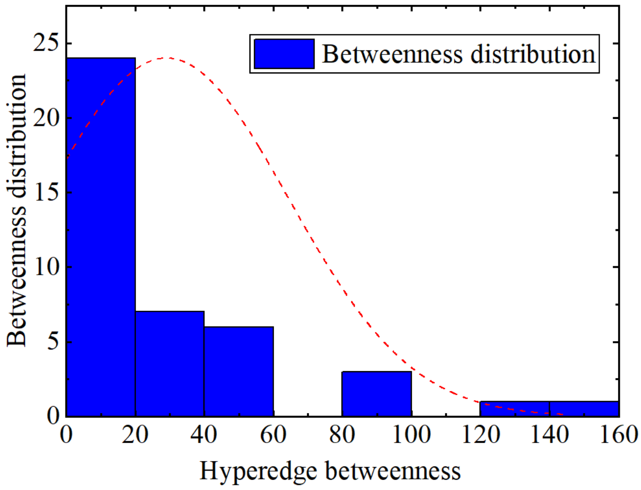

Fig. 9. Probability of hyperedge betweenness distribution interval of the comprehensive traffic supernetwork

A topological diagram (Fig. 10) is constructed based on the sizes of the hyperedge betweenness centers. This visualization provides a clearer and more intuitive representation, revealing that cities such as Nanjing, Shanghai, Hefei, Wenzhou, and others—characterized by high betweenness—serve as pivotal hyperedges within the region. These cities, functioning as major transportation hubs, bear a substantial share of transportation responsibilities. Their prominence underscores the critical role they play within the comprehensive transportation supernetwork. Consequently, the stability of these high-betweenness cities is essential for maintaining the connectivity and overall functionality of the entire transportation network, highlighting their strategic importance in ensuring network resilience and efficiency.

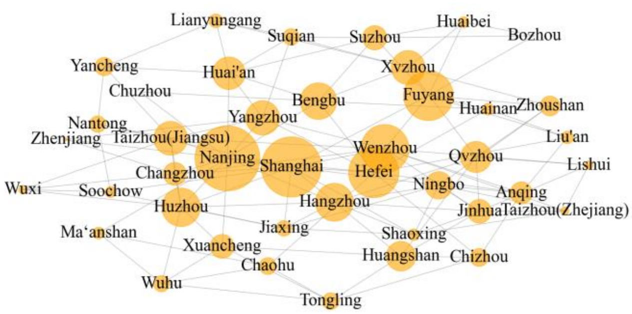

Fig. 10. Distribution of hyperedge betweenness

C. Regional Spatial Structure

f. Clustering coefficients and average path lengths

Similar to a complex network, the supernetwork also exhibits clustering coefficients and average path lengths among its hyperedges, which serve as indicators of the strength of the relationships between these hyperedges. The clustering coefficient and average path length collectively reflect the overall compactness of cyberspace, with their values being computable through equations (9) and (10), respectively. In modeling both the individual transportation network and the comprehensive transportation supernetwork, these two topological characteristics are compared in detail, as shown in Table II. Considering that aviation is limited to a select number of cities, the transportation network remains sparse with relatively loose spatial connections; consequently, it is excluded from the comparative analysis.

$$C_i = \frac{2M_i}{k_i\left(k_i - 1\right)} \tag{9}$$

$$L = \frac{1}{C_n^2}\sum_{i \neq j} d_{ij} \tag{10}$$

Thereinto, M_i Indicates the actual number of connected edges between neighboring nodes of node i . k_i indicates the degree of the node i . d_{ij} is the diameter of the network, which refers to the distance between any nodes in the network.

TABLE II
COMPARISON OF SPATIAL CHARACTERISTICS OF COMPREHENSIVE TRANSPORTATION NETWORK AND SINGLE TRANSPORTATION NETWORK

subnetwork layer	C	L	Diameter
High-speed rail subnetwork layer	0.37	3.07	7
Expressway subnetwork layer	0.27	3.28	8
Comprehensive transportation supernetwork	0.41	2.41	5

As illustrated in Table II, the clustering coefficient of the integrated transportation supernetwork is 0.41, which exceeds that of the individual transportation subnetworks but remains marginally below the median value of 0.5. This indicates that the connections within the region are not particularly dense, and the nodes (hyperedges) exhibit only moderate clustering. Furthermore, the propagation efficiency of the transportation network may be hindered by the absence of direct links between the nodes (hyperedges), necessitating the traversal of additional intermediary nodes (hyperedges).

The average path length of the integrated transportation network is 2.41, with a network diameter of 5—both values being smaller than those observed in the individual transportation networks. These findings imply that the integrated transportation system significantly enhances the efficiency of the regional transportation network, thereby promoting the high-quality, coordinated development of the region.

g. Urban community structure

The urban community structure is analyzed using the Girvan-Newman algorithm, which is a community detection method based on edge betweenness centrality. This algorithm reveals the community structure of the network by progressively removing edges with the highest betweenness centrality. The Girvan-Newman (GN) algorithm effectively partitions the transportation network into multiple communities, where nodes within each community are closely interconnected, while connections between different communities are relatively sparse. This partitioning aids in identifying natural zoning within the transportation network, thereby providing a foundation for transportation planning.

TABLE III
COMMUNITY HIERARCHY TABLE

Community level	Cities included
Community 0	Lianyungang, Bozhou, Suzhou, Fuyang, Huainan, Xvzhou, Chuzhou, Huaibei, Bengbu, Huai'an, Zhoushan, Suqian, Yancheng
Community 1:	Ma'anshan, Chaohu, Wuhu, Hefei, Tongling, Anqing, Chizhou, Liu'an
Community 2:	Yangzhou, Nanjing, Xuancheng Changzhou, Shanghai, Soochow', Huzhou, Wuxi, Nantong, Zhenjiang, Wenzhou, , Taizhou(Jiangsu)
Community 3	Qvzhou, Shaoxing, Taizhou(Zhejiang), Huangshan, Jinhua, Hangzhou, Ningbo, Jiaxing, Lishui

The community structure is illustrated in Table Ⅲ. It is apparent that the northern Jiangsu metropolitan area, centered around Xuzhou; the Anhui metropolitan area, with Hefei as its hub; the southern Jiangsu metropolitan area, led by Su-Xi-Chang; and the Zhejiang metropolitan area, dominated by Hangzhou, each exhibit distinct characteristics. The transportation networks of these four major metropolitan areas are designed to radiate outward from one or more central cities, thereby stimulating the economic growth of surrounding cities. By identifying key communities, priority can be assigned to infrastructure development in lower-tier communities, such as the construction of new roads and the expansion of transportation routes, to enhance overall traffic efficiency and ensure seamless traffic flow in critical regions.

h. Urban connectivity intensity

By weighting the supernetwork based on inter-city travel time, the strength of inter-city connections can be effectively determined. In analyzing the integrated transportation network, it is crucial to not only examine its overall structure and understand the relationship between traffic supply and demand but also to assess the efficiency of the network based on travel times.

After conducting the calculations, it is observed that the average travel time between nodes of two adjacent cities in the Yangtze River Delta region is 67.98 minutes, with the overall average path length reaching 124.16 minutes, and the network diameter measuring 261 minutes. The data suggests that the Yangtze River Delta region has effectively established a '123 travel traffic circle'—that is, a one-hour commute within metropolitan areas, two hours across urban agglomerations, and three hours between major cities. Our findings indicate that the development of a comprehensive, three-dimensional transportation network facilitates seamless access to major cities, ensures effective node coverage, and ultimately achieves intra-regional connectivity.

D. Vulnerability Analysis

Network vulnerability refers to the extent to which the overall performance of a network is compromised when specific nodes or edges are deliberately targeted or disrupted. The primary objective of vulnerability analysis is to identify critical nodes or edges whose failure would significantly impact the network’s functionality, thereby revealing the weak links that render the system susceptible to disruptions. This analysis is essential as it provides valuable insights into the structural weaknesses within the network, which can then be addressed to enhance resilience. The significance of studying network vulnerability cannot be overstated, as it directly informs emergency response strategies and risk management practices, enabling the development of proactive measures to mitigate potential disruptions. In the context of an integrated transportation system, where efficiency and connectivity are paramount, understanding and addressing vulnerabilities is not merely necessary but vital for ensuring the smooth and secure operation of the entire network, particularly during crises or unforeseen events.

In this paper, we propose a dual-layer attack strategy that simultaneously targets both individual traffic nodes and urban hyperedges, offering a comprehensive analysis of network vulnerability across these two levels, as illustrated below.

1) Node-Based Attack strategy(NBAs)

Node-based attack strategy(NBAs) are based on changes in network performance, which are assessed by removing a node and comparing the performance of the integrated transportation system before and after the removal. Once a node is removed, all edges connected to that node are deleted, while the remaining nodes and edges continue to operate normally, as illustrated in Fig. 11. Typically, node attacks are often triggered by small-scale random events, such as service interruptions caused by failures at airports or railway stations due to power outages, fires, and other incidents.

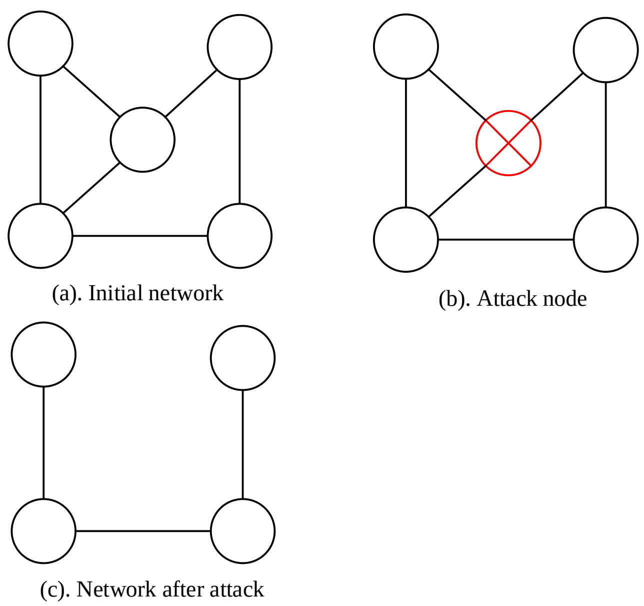

Fig. 11. Schematic diagram of NBAs

2) Hyperedge-Based Attack strategy(HBAs)

In a city with multiple traffic nodes, an attack on a single node will not necessarily result in a complete collapse of the transportation system. We conceptualize the urban public transport system as a complex network composed of multi-layered and interconnected elements. This paper provides a comprehensive analysis of how city-level traffic paralysis in extreme situations can affect this system and trigger a chain reaction. Consequently, we propose a novel attack method based on a hyper-network modeling strategy, termed the Hyper-edge-based Attack Strategy (HBAs). A hyperedge attack occurs when the traffic of a city within a network is disrupted by various factors, leading to a loss of its traffic functionality and a subsequent degradation of the overall system performance.

These attacks correspond to scenarios where a city is impacted by widespread events such as floods, snowstorms, or earthquakes, causing the failure of all transportation facilities within that city. This attack method is specifically manifested in the following ways: when a hyperedge is attacked, it is deleted from the supernetwork, and its internal nodes and connected edges are all invalidated. The attack schematic diagram is shown in Fig. 12.

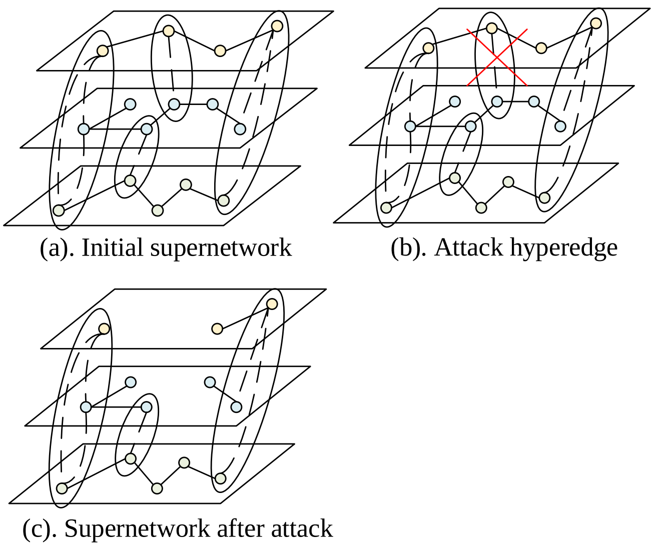

Fig. 12. Schematic diagram of HBAs

To assess the overall performance of a network following an attack, we have developed a comprehensive evaluation system that encompasses three fundamental metrics[26]: network efficiency (NE), the size of the maximum connected subgraph(N_{max}), and network connectivity (NC). These metrics are designed to evaluate the network's response to an attack across multiple dimensions, facilitating a nuanced understanding of the impact on the entire system.

Network Efficiency (NE) quantifies the effectiveness of connections between nodes within a network. It provides insights into a node's capacity to communicate and exchange information, serving as an indicator of the impact on travel efficiency within the network post-attack. A decline in network efficiency signifies a deterioration in the overall functionality of the network, particularly concerning accessibility and traffic flow. Conversely, the size of the maximum connected subgraph(N_{max}) and network connectivity(NC) are critical measures of the network's structural integrity and connectivity under attack conditions. These metrics reflect the network's ability to maintain connectivity despite disruptions caused by cyberattacks. The size of the maximum connected subgraph identifies the largest cohesive component that remains connected after an attack, while network connectivity evaluates the general degree of interconnection across the entire network.

Collectively, these metrics constitute a robust system for assessing the resilience and vulnerability of a network to various forms of attack.

$$\begin{cases} NE = \dfrac{1}{n(n-1)} \sum\limits_{1<i<j<N} \dfrac{1}{d_{ij}} \\ N_{max} = |V(G_{max})| \\ NC = \dfrac{\sum\limits_{i,j \in D} n_{ij}}{n} \end{cases} \quad (11)$$

Thereinto, d_{ij} is the diameter of the network. $V(G_{max})$ represents the set of nodes in the largest connected subgraph G_{max}, $|V(G_{max})|$ indicates the number of nodes in the node set. n_{ij} is the number of node pairs that can be connected normally in the network, n indicates the total number of nodes in the network after the attack.

In conjunction with the aforementioned attack strategies, we employ three distinct methods for comparative analysis: random attack (RA), deliberate degree attack (DA), and

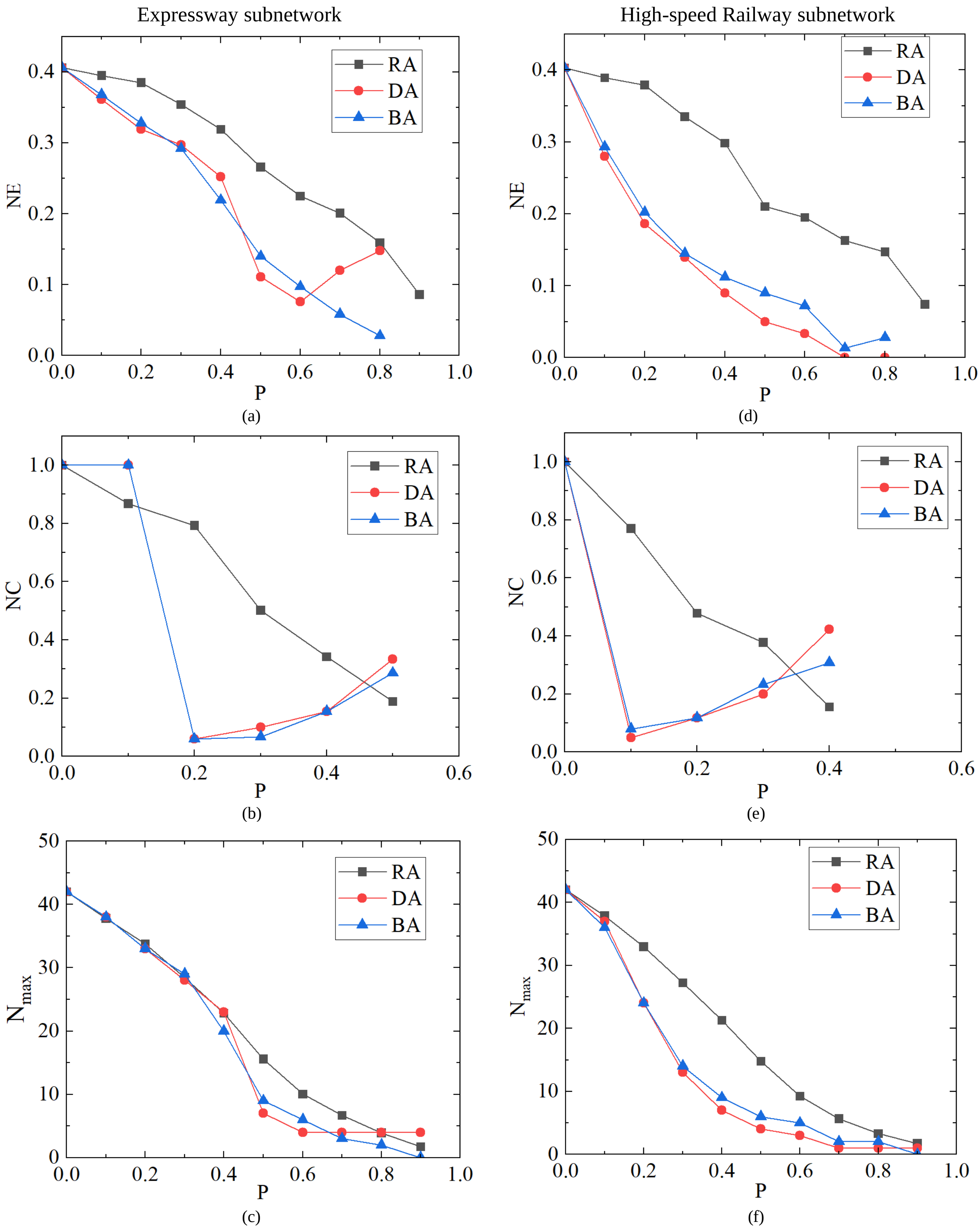

Fig. 13. Changes in network stability indicators for different sub-networks under NBAs. Expressway subnetwork (left) and High-speed Railway subnetwork (right)

deliberate between-number attack (BA). Python is utilized to simulate attacks on the integrated transportation network, which includes both random attacks with varying proportions and deliberate attacks based on node degree or inter-node connections. To ensure statistical reliability, random attacks are conducted 50 times and averaged. Key network performance metrics—such as network efficiency, network connectivity, and the size of the largest connected subgraph—are computed following Equation (11). The results are subsequently synthesized to generate performance curves for various indicators.

The Node-Based Attack strategy (NBAs) is specifically implemented for high-speed rail and highways, with the corresponding results presented in Fig. 13. Based on the analysis in Fig. 13(a) and 13(d), we can draw several conclusions. From the perspective of network efficiency, the efficiency of the railway subnetwork and the expressway subnetwork, when not subjected to attacks, is approximately 0.403 and 0.406, respectively, indicating a relatively similar performance. However, when subjected to different types of attacks, the efficiency of the networks fluctuates significantly. The network demonstrates a certain level of stability under random attacks; however, it exhibits considerable vulnerability under two types of deliberate attacks, resulting in a sharp decrease in network efficiency. It is important to note that the rail subnetwork exhibits a greater level of vulnerability in comparison to the road network, which we hypothesize is attributable to differences in accessibility.

From the perspective of network connectivity, transportation networks are more susceptible to deliberate attacks. When the ratio of the two deliberate attacks reaches 0.1, the transportation network collapses and loses full connectivity. In contrast, under random attacks, the network approaches a state of collapse only when the attack rate reaches 0.4. This observation is further supported by the variation in the size of the maximum connectivity subplot (Fig. 13(c) & 13(f)). Compared to the relatively stable curve observed under random attacks, the changes observed under the two deliberate attacks are more pronounced. Notably, when the degree attack rate reaches 0.2 and the intermediary attack rate reaches 0.3, the size of the maximum connectivity subgraph decreases sharply, significantly impairing the network's ability to maintain site connectivity. Therefore, to prevent the network from being paralyzed by deliberate attacks, it is crucial to balance the importance of network nodes and alleviate the pressure on critical nodes.

After merging the transferable nodes of different transportation modes within the same city into a hyperedge(See Fig. 6), a hyperedge attack is conducted on the comprehensive transportation supernetwork in the Yangtze River Delta region. The curves of various indicators are plotted, as illustrated in Fig. 14. Hyperedge-Based Attack strategy (HBAs) can identify key cities in a region from a macro perspective.

From Fig. 14(a), we can infer that in the absence of an attack, the initial efficiency of the supernetwork is 0.48, which is slightly higher than that of a single network. This suggests that the integration of multiple modes of transport within a region can significantly enhance the efficiency of intercity transportation. Under a random hyperedge attack, network efficiency exhibits volatility; however, it demonstrates a certain level of robustness compared to a single transportation network, with efficiency dropping below 0.10 when the attack rate reaches 0.9. Notably, during a deliberate attack on the hyperedge, network efficiency falls below 0.10% when the ratio of degree attacks to intermediate attacks approaches approximately 60%. This phenomenon indicates an increasing reliance of the network on critical hyperedges, whereby the failure of these critical hyperedges can easily precipitate network failures.

From a network connectivity perspective, as attacks intensify, the vulnerability of networks to deliberate attacks becomes increasingly apparent. As illustrated in Fig. 14(b), networks are more susceptible to crashing than to random attacks, and network connections deteriorate more rapidly under deliberate assaults, in descending order of degree and betweenness. Furthermore, the variation in the maximum connected subgraph indicates that when the rate of deliberate attacks on the hyperedge reaches 0.3, its size begins to decrease significantly. This observation suggests that traffic in this region is highly dependent on hyperedges with large degree values and betweenness. To ensure the stability of the entire network, it is crucial to focus on protecting the transportation functions of these key nodes. The results demonstrate that continuous attacks will cause the transportation network to fragment into multiple connected subgraphs or isolated nodes, leading to a sudden decline in network accessibility. Therefore, measures should be implemented promptly to avert this phenomenon.

In evaluating the vulnerability of a network to node and hyperedge attacks, we identify both similarities and distinctions in the system's responses to these two forms of disruption. A notable similarity is that deliberate attacks significantly heighten the network's fragility under both methods. This vulnerability is evidenced by a marked decline in key performance metrics, including network efficiency, connectivity, and overall stability. Both types of attacks illustrate how the system deteriorates when critical components are strategically compromised. Conversely, the differences between node and hyperedge attacks are equally pronounced. Node attacks primarily assess the impact of removing individual nodes, analyzing how their absence disrupts the network's overall functionality. In contrast, hyperedge attacks investigate the ramifications of transportation failures on a broader scale, focusing on how the breakdown of essential intercity connections can destabilize the entire system.

From the perspective of vulnerability, interconnected transportation supernetworks demonstrate greater resilience to random and unforeseen disruptions, indicating that this structure is more robust in the face of unpredictable failures. However, when subjected to deliberate and targeted attacks, transportation supernetworks that rely heavily on critical hyperedges become particularly susceptible to these attacks, revealing a specific vulnerability. In such instances, the stability of the network relies on the integrity of critical hyperedges. Should these hyperedges fail, the consequences could be catastrophic, potentially resulting in a complete network collapse. In order to mitigate network vulnerability, we recommend establishing a multi-center transportation network structure.

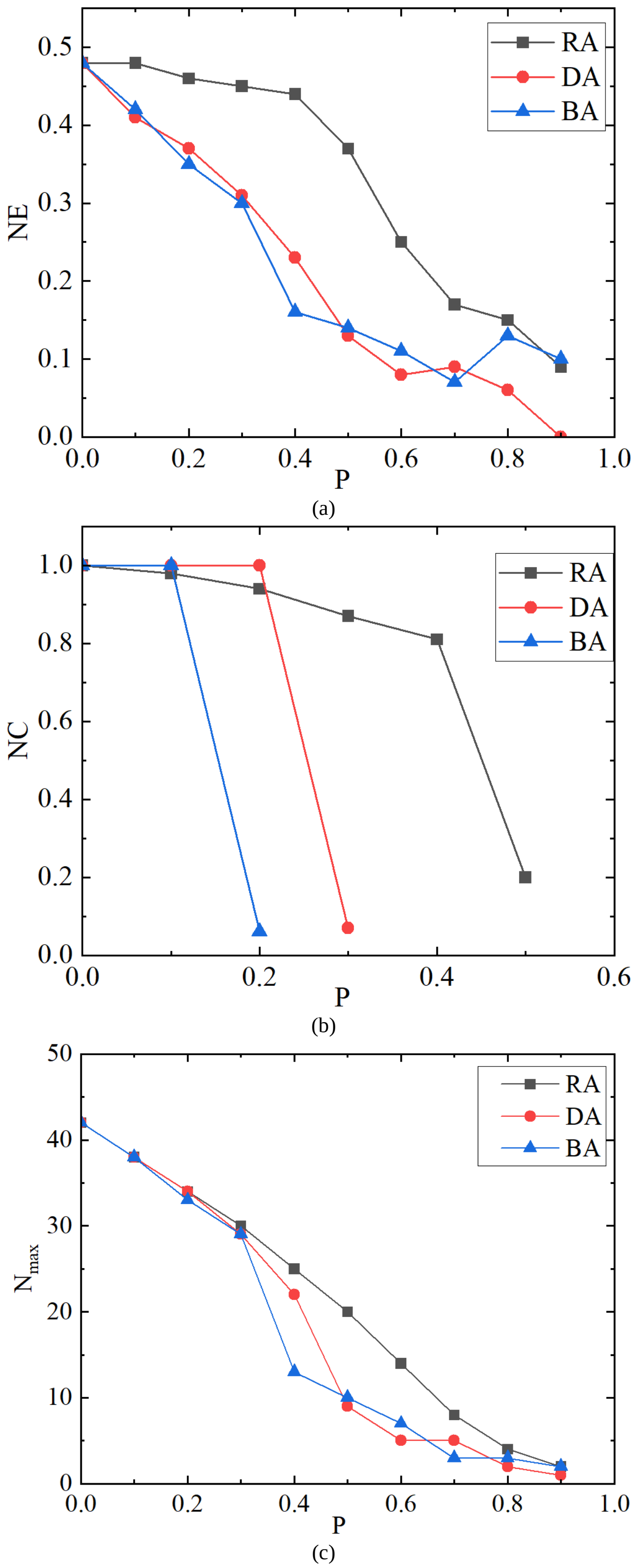

Fig.14. Changes in network stability indicators under HBAs. (a) The change of network efficiency. (b) The change of network connectivity. (c) The change of the size of the maximum connected subgraph.

IV. Conclusion

This paper discusses the structural topology characteristics of integrated transportation networks from an innovative perspective, examining network vulnerability. This innovation preserves node heterogeneity and facilitates the analysis of the global network structure. Utilizing data from 42 cities in the Yangtze River Delta region, we constructed a three-layer supernetwork comprising high-speed railways, expressways, and aviation. Based on this framework, we draw the following conclusions:

1). A more interconnected transport network enhances the efficiency of regional transport. Certain nodes, such as the Nanjing station, are particularly significant due to their excellent accessibility and pivotal role within the overall network.

2). Our analysis reveals that the performance of the integrated traffic supernetwork exhibits greater stability compared to that of individual traffic networks when subjected to various attacks, indicating a degree of robustness.

3). It is essential to emphasize that the integrated transport network fundamentally relies on major cities. This realization highlights the importance of enhancing the role of non-critical nodes in promoting network connectivity and stability.

Overall, our analysis provides unique insights into how interconnecting vulnerable systems can bolster overall network robustness. By employing network science to emphasize the system's topology, we offer planners a design perspective that aids in the planning of transportation partitions. Our findings suggest that enhanced connectivity can serve as a unique strategy for improving transportation resilience. These insights propose a potential solution for addressing the vulnerabilities within public transport networks. Transportation planners in other regions may consider exploring enhanced connectivity to tackle similar challenges.

In future research, we can focus on incorporating weighting factors into the network, such as current-carrying capacity. This approach will enable us to more accurately identify and analyze the functional structure of the transportation network, thereby enhancing the overall layout of the network.

Reference

[1] Y. Yang, X. Wang, Y. Chen and M. Hu, "Identifying Key Nodes in Complex Networks Based on Global Structure," *IEEE Access*, vol. 8, pp. 32904-32913, 2020,

[2] Ullah A, Wang B, Sheng J, Long J, Khan N, Sun Z, “Identification of nodes influence based on global structure model in complex networks,” *Sci Rep* 11, 2021.

[3] Jieming Yang, Jinghan Lu, Yun Wu, Tianyang Li, and Yuehua Yang. “ A Node Ranking Method Based on Local Structure Information in Complex Networks,” *Engineering Letters*, vol. 30, no. 1, pp. 161-167, 2022.

[4] L. Wan, M. Zhang, X. Li, L. Sun, X. Wang and K. Liu, "Identification of Important Nodes in Multilayer Heterogeneous Networks Incorporating Multirelational Information," in *IEEE Transactions on Computational Social Systems*, vol. 9, no. 6, pp. 1715-1724, Dec. 2022.

[5] Jinfang Sheng, Jinying Dai, Bin Wang, Guihua Duan, Jun Long, Junkai Zhang et al. “Identifying Influential Nodes in Complex Networks Based on Global and Local Structure,” *Physica A: Statistical Mechanics and Its Applications*, vol. 541, pp. 123262, 2020.

[6] Duanbing Chen, Linyuan Lü, Ming-Sheng Shang, Zhou, Tao. “Identifying influential nodes in complex networks,” *Physica A: Statistical Mechanics and its Applications*, vol. 391, no. 4, pp. 1777-1787, 2012.

[7] SONG Xin-Sheng, WANG Xiao-Xiao, LI Ai-Zeng, ZHANG Lei. “Node Importance Evaluation Method for Highway Network of Urban Agglomeration,” *Journal of Transportation Systems Engineering and Information Technology*, vol. 11, no. 2, pp. 84-90, April, 2011.

[8] FENG Fen-ling, CAI Ming-xu, JIA Jun-jie. “Key Node Identification of China Railway Express Transportation Network Based on Multi-layer Complex Network,” *Journal of Transportation Systems Engineering and Information Technology*, vol. 22, no. 6, pp. 191-200,

2022.
[9] JIAO Liudan, ZHANG Xinyu, HUO Xiaosen, ZHANG Yu. “Analysis of Important Nodes of Urban Agglomeration Network from the Perspective of High-Speed Traffic Flow,” *Journal of Chongqing Jiaotong University(Natural Science)*, vol. 43, no. 7, pp. 77-84, 2024.
[10] Yuhao Yang, Yongxue Liu, Minxi Zhou, Feixue Li, Chao Sun. “Robustness assessment of urban rail transit based on complex network theory: A case study of the Beijing Subway,” *Safety Science*, vol. 79, pp. 149-162, 2015.
[11] Linna Li, Yuze Li and Jianke Zhang. “Fractional-order SS1IR Model for Predicting Public Opinion Dissemination in Social Networks,” *Engineering Letters*, vol. 31, no. 1, pp.239-245, 2023.
[12] YE Yuling, LI Wenqing, ZHANG Jun. “Complex Characteristics and Propagation Dynamics of High Speed Railway Network,” *Journal of Tongji University (Natural Science)*, vol. 47, no. 5, pp. 655-662, 2019.
[13] Wang, Q.-L & Zhu, Z.-Y & Chen, H. & Sun, J. & Guan, W.-Y & Rivera-Durón, R.R. “Node Resistance Characteristic of Heterogeneous Multi-layer Transportation Networks Under Cascading Failure Based on Coupled Map Lattice,” *Zhongguo Gonglu Xuebao/China Journal of Highway and Transport*, vol. 35, pp. 263-274, 2022.
[14] Ma F, Shi W, Yuen K.F, Shi W.J, Sun Q.P, Xu X.B, Wang Y.J et al. “Exploring the robustness of public transportation for sustainable cities: A double-layered network perspective,” *Journal of Cleaner Production*, vol. *265*, pp.121747, 2020.
[15] Zizhen Xu & Shauhrat S. Chopra, “Interconnectedness enhances network resilience of multimodal public transportation systems for Safe-to-Fail urban mobility,” *Nature Communications*. Vol 14:4291, 2023.
[16] P. Guo and M. Zhang, "Influence of the connection between different subnetworks on the supernetwork reliability," *2011 6th International Conference on Pervasive Computing and Applications*, Port Elizabeth, South Africa, pp. 80-83, 2011.
[17] Q. Deng and Zhiping Wang, "Degree centrality in scientific collaboration supernetwork," *International Conference on Information Science and Technology*, Nanjing, pp. 259-262, 2011
[18] Nagurney A, Dong J. “Supernetworks: Decision－Making for the Information Age,” *Elgar, Edward Publishing, Incorporated*, vol. 4, pp. 368, 2002.
[19] Nagurney A, Wakolbinger T. “Supernetworks: An introduction to the concept and its applications with a specific focus on knowledge supernetworks,” *International Journal of Knowledge Culture and Change Managemen*t, vol. 4, pp. 1-16, 2005.
[20] BERGE C. “Graphs and hypergraphs,” *New York: Elsevier*, pp. 389-391, 1973.
[21] Du, Wen-ju and Zhang, Jiangang and An, Xin-lei and Qin, Shuang and Yu, Jian-ning. “Outer Synchronization between Two Coupled Complex Networks and Its Application in Public Traffic Supernetwork,” *Discrete Dynamics in Nature and Society*. pp. 1-8, 2016.
[22] T. Shanmukhappa, I. W. H. Ho, C. K. Tse, X. Wu and H. Dong, "Multi-layer Public Transport Network Analysis," *2018 IEEE International Symposium on Circuits and Systems (ISCAS)*, Florence, Italy, pp. 1-5, 2018.
[23] YU Jian, DONG Xiaojie, CHEN Yuxiu. “Rapid Combined Travel Path Planning for Passengers in Air and High-Speed Railway Compound Supernetwork,” *Computer Engineering and Applications*, vol. 59, no. 14, pp. 315-322, 2023.
[24] Suo Q, Guo J L. “The evolutionary mechanism of high-speed railway system based on hypernetwork theory,” *International Journal of Modern Physics B*, vol. 32, no. 15, pp. 1850182, 2018.
[25] Wei Y, Ning S. “Establishment and Analysis of the Supernetwork Model for Nanjing Metro Transportation System,” *Complexity*, 2018.
[26] W. Yu, T. Wang, Y. Zheng and J. Chen. “Parameter Selection and Evaluation of Robustness of Nanjing Metro Network Based on Supernetwork,” *IEEE Access*, vol. 2019, no. 7, pp. 70876-70890, 2019.
[27] J. Wang, J. Ren and X. Fu, "Research on Bus and Metro Transfer From Perspective of Hypernetwork—A Case Study of Xi’an, China (December 2020)," *IEEE Access*, vol. 8, pp. 227048-227063, 2020.