Pricing Compound Ratchet-Type Equity-Indexed Annuities in the Indonesian Market: Comparing Constant and Stochastic Interest Rates

Jovan Brian Tanujaya, Ferry Jaya Permana, Jonathan Hoseana

***Abstract*—We study the pricing of compound Ratchet-type equity-indexed annuities (EIAs) under three different interest-rate assumptions: constant, stochastic based on the Vasicek model, and stochastic based on the Cox-Ingersoll-Ross (CIR) model. We assume that the asset price follows a geometric Brownian motion, and estimate the involved parameters using historical data from the Indonesian bond market and stock index. The results show that the constant interest-rate assumption leads to higher EIA prices compared to those obtained using the stochastic models, reflecting the simplified nature of ignoring interest rate variability. The stochastic Vasicek and CIR models lead to nearly identical prices, suggesting that the additional complexity of the CIR model has limited impact under the market conditions represented by the utilised data. Furthermore, a sensitivity analysis in the stochastic cases reveals that the interest rate's long-term mean is the parameter upon which the EIA price depends most sensitively.**

***Index Terms*—equity-indexed annuity, Ratchet, geometric Brownian motion, Vasicek model, Cox-Ingersoll-Ross model**

I. Introduction

THE pricing of financial instruments has been the subject of extensive research for decades, with numerous mathematical models and numerical methods being developed to handle the complexities of real-world market dynamics and asset behaviours. The pricing of bonds, for instance, has been studied by Wang et al. [39] using the so-called binomial tree method, and by Zhang et al. [43] using the Cox-Ingersoll-Ross (CIR) stochastic model with the incorporation of a certain threshold setting to enhance accuracy. In the realm of derivatives, Otani and Imai [32] have examined systematic factors in the pricing of credit derivatives, while Ma and He [27] have employed a fast Monte Carlo method for pricing covariance swaps under correlated stochastic volatility models. Studies on the pricing of options have also been abundant, with Du et al. [11] focusing on the acceleration of the Monte Carlo method for pricing multi-asset options, Liu et al. [26] discussing the potentiality of control variate methods for pricing options, and Wijayanti et al. [38] employing the finite element method as a numerical method for pricing options. More recently, Siswanah et al. [34] have conducted a comparative study of two numerical methods, the Newton-Raphson and Monte Carlo methods, for the pricing of American options. Besides in the above investment-based financial instruments, pricing problems also arise in other financial products, particularly those which integrate market-driven returns with protective guarantees. Choi [8], for instance, has studied the pricing of an equity-linked life insurance, using the so-called indifference pricing theory. While equity-linked life insurances focus on providing death benefits influenced by equity performance, a related class of financial products, equity-indexed annuities (EIAs), is designed to offer retirement income with returns linked to an equity index while ensuring a minimum payout.

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Jovan Brian Tanujaya is a graduate of the Mathematics Programme, Faculty of Science, Parahyangan Catholic University, Bandung, Indonesia (e-mail: jovanbrian611@gmail.com).

Ferry Jaya Permana is a researcher at the Center for Mathematics and Society, Faculty of Science, Parahyangan Catholic University, Bandung, Indonesia (e-mail: ferryjp@unpar.ac.id).

Jonathan Hoseana is a researcher at the Center for Mathematics and Society, Faculty of Science, Parahyangan Catholic University, Bandung, Indonesia (corresponding author, e-mail: j.hoseana@unpar.ac.id).

EIAs have drawn interest from financial markets since their establishment in the mid-1990s. An EIA is a long-term agreement with an insurance company that provides a variable rate of return determined by the behaviour of a stock market index, with a guaranteed minimum return. Customers may be drawn to EIA products for several reasons, such as the guaranteed minimum interest rate, which mitigates risks of loss, and the ability to grow with the stock market as returns are determined by the performance of the stock index. There are numerous types of EIA contracts, such as point-to-point, Ratchet, and look-back [12]. In a point-to-point-type EIA, the return on assets is determined by the change in the index between two time points. On the other hand, in a Ratchet-type EIA, the return on assets is determined annually, by comparing the year-beginning and year-ending index values. Finally, in a look-back-type EIA, the return on assets is determined using the highest index value at the time of the EIA contract. In the present study, we shall consider Ratchet-type EIAs.

Due to variable market conditions, pricing EIAs is not straightforward. Researchers such as Lin and Tan [25], Lee [24], Jaimungal [15], Kijima and Wong [23], Boyle and Tian [6] have confirmed the suitability of modelling the equity index using a geometric Brownian motion model, which relies on the assumption that the logarithmic value of the return on assets is normally distributed. Traditional approaches for evaluating such complex financial products often assume constant interest rates, which simplifies mathematical modelling but disregards the inherent risk and uncertainty associated with long-term interest rate variations [13]. Since most EIAs have a maturity duration of one to ten years, a constant interest-rate assumption may not lead to a realistic modelling. EIA pricing with stochastic interest rates has been studied by Lin and Tan [25], Kijima and Wong [23], and Qian et al. [33], using models such as the Vasicek and Cox-Ingersoll-Ross (CIR) models. These models and their variants have appeared in studies with various focuses, such as the pricing of bonds [43], the refinancing of mortgages [44], sensitivity analysis [36], and parameter estimation [40], [41], [42]. In the present paper, we employ both models for the pricing of compound Ratchet-type EIA products in the context of the Indonesian financial market.

Our pricing is built upon several assumptions. First, we assume that the market is complete and risk-neutral. In addition, we assume that the underlying asset price evolves over time following a geometric Brownian motion. Furthermore, we assume that there are no transaction costs, and that the policy does not lapse or expire before it matures. We determine the interest rate using historical data from 10-year Indonesian government bond yields, based on the assumption that Indonesian government bonds are risk-neutral. Specifically, the utilised dataset is the daily dataset for 10 years, from March 13th, 2014 to March 13th, 2024, taken from the Investing.com website [22]. We also employ some data provided by the Jakarta Stock Exchange Composite, namely the daily index dataset for the same 10-year period, also taken from the Investing.com website [21], as all stocks listed on the Indonesia Stock Exchange are included in the Jakarta Stock Exchange Composite stock market index. Both datasets are used to estimate the parameters of the Vasicek, CIR, and geometric Brownian motion models.

The rest of the paper is organised as follows. In the upcoming section II, we introduce the models employed in our study, including the constant, Vasicek, and CIR models for interest rates as well as a geometric Brownian motion model for asset prices. Subsequently, in section III, we explain the pricing method for compound Ratchet-type EIAs under the three different interest-rate assumptions. In section IV, we derive estimates for the values of the parameters involved in our model. Using the aforementioned Indonesian data, we carry out numerical simulations to illustrate the application of the models and to compare the EIA prices across different assumptions, the results of which are described in section V. In section VI, we conduct a sensitivity analysis in the stochastic interest-rate cases, determining the parameters upon which the EIA price depends most sensitively. In the final section VII, we state our conclusions and suggest future research directions.

II. The models

Interest rate and asset price are two important factors in the valuation of an EIA. In the present research, we assume that the asset price is a stochastic variable following a geometric Brownian motion, and consider not only the case of a constant interest rate but also that of a stochastic interest rate. Indeed, a constant interest-rate assumption may be reasonable only for short-term assets such as treasury bills, commercial papers, and certificates of deposit, whose maturities are typically less than a year. Typically, EIAs have maturities ranging from one to ten years, and it could be undesirable to assume no motion in interest rates over such a long period. Accordingly, in this paper we consider three different interest-rate assumptions: constant, stochastic following the Vasicek model, and stochastic following the CIR model. In Table I we summarise the parameters used in the Vasicek, CIR, and geometric Brownian motion models.

TABLE I
Summary of parameters.

Parameters	Description	Unit
κ	the interest rate's mean-reversion rate: the speed at which the interest rate reverts to its long-term mean	time^{-1}
θ	the interest rate's long-term mean: the long-term average level to which the interest rate reverts	$\% \cdot \text{time}^{-1}$
σ	the interest rate's volatility: the standard deviation of the interest rate changes	$\% \cdot \sqrt{\text{time}}$
μ	the asset price's drift rate: the expected rate of return of the asset	time^{-1}
ψ	the asset price's volatility: the standard deviation of the asset's returns	$\% \cdot \sqrt{\text{time}}$

To let the models reflect the economic situation in Indonesia, in our numerical simulations (section V) we shall utilise the historical data from Indonesian government bond yields and the Jakarta Stock Exchange Composite index to estimate the values of these parameters. In this section, we first discuss the models themselves.

A. The constant interest rate model

Under the constant interest-rate assumption, the price at time t of a bond maturing at time T is given by

$$P(t,T) = \mathrm{e}^{-r(T-t)}; \tag{1}$$

see [5, equation (1.7)]. This formula represents the present value of a unit of cash to be received at a future time T, discounted to the present time t using a constant interest rate r. The exponential discount factor captures the effects of time and the constant rate on the bond's price, i.e., future cash flows are diminished relative to their present value.

B. The Vasicek model

Let us now turn our attention to stochastic interest rate models. Let $r(t)$ be the short interest rate at time $t \in [0,T]$, where T is the maturity of the EIA. We say that the short rate $r(t)$ follows the Vasicek model if its time-evolution is governed by the stochastic differential equation

$$\mathrm{d}r(t) = -\kappa \left(r(t) - \theta\right) \mathrm{d}t + \sigma \, \mathrm{d}W_r(t), \tag{2}$$

where κ, θ, and σ are positive parameters, and $\{W_r(t)\}$ is the standard Brownian motion correlated with $\{W_s(t)\}$ with correlation coefficient ρ, i.e.,

$$\operatorname{corr}\left(W_r(t), W_s(t)\right) = \rho.$$

It can be shown [5, equation (3.6)] that a closed-form solution of the Vasicek model (2) is given by

$$r(t) = r(0)\,\mathrm{e}^{-\kappa t} + \theta - \theta\,\mathrm{e}^{-\kappa t} + \sigma \mathrm{e}^{-\kappa t} \int_0^t \mathrm{e}^{\kappa u}\,\mathrm{d}W(u). \tag{3}$$

A notable drawback of the Vasicek model is that it may generate negative values of the short rate $r(t)$, while in reality, the probability of a negative short rate is very low. However, the model's simplicity leads to its continued wide use. For a detailed discussion on the Vasicek model, the reader is referred to [16]. In the present study, we employ the formula derived in [9] for the price at time t of a zero-coupon bond with maturity T:

$$P(t,T) = E\left(\mathrm{e}^{-\int_t^T r(u)\,\mathrm{d}u} \middle| \mathcal{F}_t^r \right),$$

under the assumption that $r(t)$ follows the Vasicek model, namely,

$$P(t,T) = A(t,T)\,\mathrm{e}^{-r(t)\,B(t,T)}, \tag{4}$$

where

$$A(t,T) = \exp\left(\left(B(t,T) - (T-t)\right)\left(\theta - \frac{\sigma^2}{2\kappa^2}\right) - \frac{\sigma^2}{4\kappa}\left(B(t,T)\right)^2 \right)$$

and

$$B(t,T) = \frac{1 - \mathrm{e}^{-\kappa(T-t)}}{\kappa}.$$

C. The CIR model

The CIR model, being an improvement of the Vasicek model, is given by the stochastic differential equation

$$\mathrm{d}r(t) = -\kappa\left(r(t) - \theta\right)\mathrm{d}t + \sigma\sqrt{r(t)}\,\mathrm{d}W_r(t), \tag{5}$$

where, as before, $r(t)$ denotes the short rate at time $t \in [0,T]$, with T being the EIA's maturity. The parameters κ, θ, and σ are positive, and $\{W_r(t)\}$ is the standard Brownian motion correlated with $\{W_s(t)\}$ with correlation coefficient ρ, i.e.,

$$\operatorname{corr}\left(W_r(t), W_s(t)\right) = \rho.$$

Applying the method used to derive the closed-form solution (3) of the Vasicek model (2), one obtains the following closed-form solution of the CIR model (5):

$$r(t) = r(0)\,\mathrm{e}^{-\kappa t} + \theta - \theta\mathrm{e}^{-\kappa t} + \sigma\mathrm{e}^{-\kappa t}\int_0^t \mathrm{e}^{\kappa u}\sqrt{r(u)}\,\mathrm{d}W(u).$$

The presence of $\sqrt{r(t)}$ in the CIR model (5) prevents negative values of the short rate $r(t)$, thereby improving the Vasicek model [31]. In the present study, we employ the formula derived in [9] for the price at time t of a zero-coupon bond with maturity T:

$$P(t,T) = E\left(\mathrm{e}^{-\int_t^T r(u)\,\mathrm{d}u} \middle| \mathcal{F}_t^r \right),$$

under the assumption that $r(t)$ follows the CIR model, namely,

$$P(t,T) = A(t,T)\mathrm{e}^{-r(t)\,B(t,T)}, \tag{6}$$

where

$$A(t,T) = \left(\frac{2\gamma\mathrm{e}^{(\gamma+\kappa)(T-t)/2}}{C(T,t)} \right)^{2\kappa\theta/\sigma^2},$$
$$B(t,T) = 2\frac{\mathrm{e}^{\gamma(T-t)} - 1}{C(t,T)},$$
$$C(t,T) = 2\gamma + (\kappa+\gamma)\left(\mathrm{e}^{\gamma(T-t)} - 1\right),$$

with $\gamma = \sqrt{\kappa^2 + 2\sigma^2}$.

D. Geometric Brownian motion

In this paper, we assume that the asset price, i.e., the equity index level at time t, denoted by S_t, follows a geometric Brownian motion, i.e., that its time-evolution is governed by the stochastic differential equation

$$\mathrm{d}S_t = \mu S_t\,\mathrm{d}t + \psi S_t\,\mathrm{d}W(t),$$

where μ and ψ are positive parameters [28]. Letting $P_t = S_t/S_{t-1}$, one obtains, after discretisation, that $\ln P_t$ is normally distributed with parameters $\left(\mu - \psi^2/2\right)\Delta t$ and $\psi^2\Delta t$ [28].

III. Pricing the EIA

Equity-indexed annuities (EIAs) are designed with features that address investment risks and uncertainties. One key characteristic is the guaranteed minimum interest rate, which ensures that the investment value does not fall below the initial amount, providing protection against losses when asset values decline. Conversely, the guaranteed maximum interest rate limits returns during periods of significant stock index growth, thereby managing the potential liability for the insurance provider. Additionally, EIAs incorporate a participation rate, which specifies the proportion of index-linked returns allocated to the investor. Higher participation rates allow for a greater share of returns, depending on the terms of the contract.

In a Ratchet-type EIA [23], [2], the interest rate is calculated annually and compounded in subsequent periods for the duration of the contract. This enables the investor's funds, along with accrued interest, to grow progressively over time. Ratchet-type EIAs are further classified into two categories: simple and compound. In a simple Ratchet, profits are calculated independently for each period based on a fixed total payment amount. By contrast, a compound Ratchet carries forward the profits from previous periods, adding them to the principal for compounding in the subsequent period. In this study, we shall price a compound Ratchet-type EIA.

Suppose that an investor makes an initial investment of 1. Letting T be the contract's maturity and S_t be the asset price in year $t \in \{0, 1, 2, \ldots, T\}$, the investment's rate of return in year t is given by

$$P_t = \frac{S_t}{S_{t-1}}.$$

Thus, compared to the previous year's price, the present asset price increases if $P_t > 1$, decreases if $P_t < 1$, and remains unchanged if $P_t = 1$. Next, if the investor invests funds with participation rate α, then their profit is initially calculated as $\alpha\left(P_t - 1\right)$. However, to protect against drastic increases in asset prices that could result in significant company losses, suppose that the company imposes a maximum interest rate c. Taking this into account, the investor's profit is limited to $\min\left\{\alpha\left(P_t - 1\right), c\right\}$. On the other hand, when asset prices decrease, the investor's profit may fall below an acceptable threshold. To mitigate this, suppose that the company guarantees a minimum interest rate f. Taking this into account, the profit received by the investor is given by $\max\left\{\min\left\{\alpha\left(P_t - 1\right), c\right\}, f\right\}$. Consequently, the effective annual return of the EIA contract at the end of year t is given by

$$\begin{aligned}\tilde{P}_t &= 1+\max\left\{\min\left\{\alpha\left(P_t-1\right),c\right\},f\right\}\\ &= \begin{cases}1+\alpha\left(P_t-1\right), & \text{if } f_\alpha \leqslant P_t < c_\alpha;\\ 1+c, & \text{if } f_\alpha \leqslant c_\alpha \leqslant P_t;\\ 1+f, & \text{if } P_t < f_\alpha < c_\alpha,\end{cases}\end{aligned}$$

where $f_\alpha = 1+f/\alpha$ and $c_\alpha = 1+c/\alpha$. The expected value of $\tilde{P}_t$ can be computed as follows:

$$\begin{aligned}E(\tilde{P}_t) &= \int_0^\infty \tilde{p}_t f_{P_t}\left(p_t\right)\mathrm{d}p_t\\ &= \int_0^{f_\alpha}\left(1+f\right)f_{P_t}\left(p_t\right)\mathrm{d}p_t\\ &\quad+\int_{f_\alpha}^{c_\alpha}\left(1+\alpha\left(p_t-1\right)\right)f_{P_t}\left(p_t\right)\mathrm{d}p_t\\ &\quad+\int_{c_\alpha}^{\infty}\left(1+c\right)f_{P_t}\left(p_t\right)\mathrm{d}p_t\\ &= \mathcal{I}_1+\mathcal{I}_2+\mathcal{I}_3+\mathcal{I}_4,\end{aligned}$$

where

$$\begin{aligned}\mathcal{I}_1 &= \int_0^{f_\alpha}\left(1+f\right)f_{P_t}\left(p_t\right)\mathrm{d}p_t \qquad (7)\\ &= \left(1+f\right)\Pr\left(P_t\leqslant f_\alpha\right)\\ &= \left(1+f\right)\,\Phi\left(\frac{\ln\left(1+f/\alpha\right)-\left(\mu-\psi^2/2\right)}{\psi}\right),\end{aligned}$$

$$\begin{aligned}\mathcal{I}_2 &= \int_{f_\alpha}^{c_\alpha}(1-\alpha)f_{P_t}\left(p_t\right)\mathrm{d}p_t\\ &= \left(1-\alpha\right)\left(\int_0^{c_\alpha}f_{P_t}\left(p_t\right)\mathrm{d}p_t-\int_0^{f_\alpha}f_{P_t}\left(p_t\right)\mathrm{d}p_t\right)\\ &= \left(1-\alpha\right)\left(\Pr\left(P_t\leqslant c_\alpha\right)-\Pr(P_t\leqslant f_\alpha)\right)\\ &= \left(1-\alpha\right)\left(\Phi\left(\frac{\ln\left(1+c/\alpha\right)-\left(\mu-\psi^2/2\right)}{\psi}\right)\right.\\ &\qquad\left.-\Phi\left(\frac{\ln\left(1+f/\alpha\right)-\left(\mu-\psi^2/2\right)}{\psi}\right)\right),\end{aligned}$$

$$\begin{aligned}\mathcal{I}_3 &= \int_{f_\alpha}^{c_\alpha}\alpha P_t f_{P_t}\left(p_t\right)\mathrm{d}p_t\\ &= \alpha\int_{\ln f_\alpha}^{\ln c_\alpha}\frac{1}{\sqrt{2\pi\psi^2}}\exp\left(-\frac{\left(s-\left(\mu-\psi^2/2\right)\right)^2}{2\psi^2}\right)\exp\left(s\right)\mathrm{d}s\\ &= \alpha\,\mathrm{e}^{\mu}\int_{\ln f_\alpha}^{\ln c_\alpha}\frac{1}{\sqrt{2\pi\psi^2}}\exp\left(-\frac{1}{2\psi^2}\left(s-\left(\mu+\frac{\psi^2}{2}\right)\right)^2\right)\mathrm{d}s \quad (8)\\ &= \alpha\,\mathrm{e}^{\mu}\left(\Phi\left(\frac{\ln\left(1+c/\alpha\right)-\left(\mu+\psi^2/2\right)}{\psi}\right)\right.\\ &\qquad\left.-\Phi\left(\frac{\ln\left(1+f/\alpha\right)-\left(\mu+\psi^2/2\right)}{\psi}\right)\right),\end{aligned}$$

$$\begin{aligned}\mathcal{I}_4 &= \left(1+c\right)\left(1-\int_{-\infty}^{c_\alpha}f_{P_t}\left(p_t\right)\mathrm{d}p_t\right)\\ &= \left(1+c\right)\left(1-\Pr\left(P_t\leqslant c_\alpha\right)\right)\\ &= \left(1+c\right)\left(1-\Phi\left(\frac{\ln\left(1+c/\alpha\right)-\left(\mu-\psi^2/2\right)}{\psi}\right)\right).\end{aligned}$$

The symbol Φ denotes the cumulative distribution function of the standard normal distribution. Notice that we have used the fact that the integrand in (8) is the probability density function of the normal distribution with parameters $\mu+\psi^2/2$ and ψ^2.

Therefore, the expected value of $\tilde{P}_t$ is given by

$$\begin{aligned}E\left(\tilde{P}_t\right) &= \left(1+f\right)\Phi\left(d_1\right)+\left(1-\alpha\right)\left[\Phi\left(d_2\right)-\Phi\left(d_1\right)\right]\\ &\quad+\alpha\mathrm{e}^{\mu}[\Phi(d_3)-\Phi(d_4)]+(1+c)(1-\Phi\left(d_2\right)), \quad (9)\end{aligned}$$

where

$$\begin{aligned}d_1 &= \frac{\ln\left(1+f/\alpha\right)-\left(\mu-\psi^2/2\right)}{\psi},\\ d_2 &= \frac{\ln\left(1+c/\alpha\right)-\left(\mu-\psi^2/2\right)}{\psi},\\ d_3 &= \frac{\ln\left(1+c/\alpha\right)-\left(\mu+\psi^2/2\right)}{\psi},\\ d_4 &= \frac{\ln\left(1+f/\alpha\right)-\left(\mu+\psi^2/2\right)}{\psi}.\end{aligned}$$

If, at the time of purchasing the EIA, the investor invests an amount of R, then the compound Ratchet-type EIA investment value at the contract's maturity is given by

$$P_{\text{cr}} = R\prod_{t=1}^{T}\tilde{P}_t;$$

see [18]. Therefore, the compound Ratchet-type EIA price at time 0 with maturity T is given by

$$\begin{aligned}V(T) &= E\left(P(0,T)\cdot P_{\text{cr}}\right)\\ &= P(0,T)\cdot R\cdot E\left(\prod_{t=1}^{T}\tilde{P}_t\right)\\ &= P(0,T)\cdot R\cdot\left(E\left(\tilde{P}_t\right)\right)^T, \qquad (10)\end{aligned}$$

where $P(0,T)$ denotes the price at time 0 of a zero-coupon bond with maturity T. In the cases of the interest rate being constant, stochastic following the Vasicek model, and stochastic following the CIR model, we shall apply the formulae (1), (4), and (6), respectively, to compute $P(0,T)$ in the EIA price formula (10).

IV. Parameter estimation

As previously mentioned, in our numerical simulations (section V) we shall estimate the values of the parameters in Table I using the data representing the Indonesian market situation. In this section, we first derive formulae to be used to estimate these parameters.

A. Vasicek Model

To estimate the parameters involved in the Vasicek model (as also in the CIR model; see next subsection), we first apply Itô's Lemma [17] to determine the conditional expectation and variance of $r(t)$. These are given by

$$\begin{aligned}
&E\left[r(T)\,|\,r(t)\right]\\
&= E\left[r(t)\mathrm{e}^{-\kappa(T-t)} + \theta - \theta\mathrm{e}^{-\kappa(T-t)} + \sigma\mathrm{e}^{-\kappa T}\int_t^T \mathrm{e}^{\kappa u}\mathrm{d}W(u)\,\middle|\, r(t)\right]\\
&= r(t)\mathrm{e}^{-\kappa(T-t)} + \theta\left(1 - \mathrm{e}^{-\kappa(T-t)}\right)\\
&\qquad + \sigma\mathrm{e}^{-\kappa T} E\left[\int_t^T \mathrm{e}^{\kappa u}\,\mathrm{d}W(u)\,\middle|\, r(t)\right]\\
&= r(t)\mathrm{e}^{-\kappa(T-t)} + \theta\left(1 - \mathrm{e}^{-\kappa(T-t)}\right)
\end{aligned}$$

and

$$\begin{aligned}
&\mathrm{Var}\left[r(T)\,|\,r(t)\right]\\
&= \mathrm{Var}\left[r(t)\mathrm{e}^{-\kappa(T-t)} + \theta - \theta\mathrm{e}^{-\kappa(T-t)} + \sigma\mathrm{e}^{-\kappa T}\int_t^T \mathrm{e}^{\kappa u}\mathrm{d}W(u)\,\middle|\, r(t)\right]\\
&= E\left[\left(\sigma\mathrm{e}^{-\kappa T}\right)^2\left(\int_t^T \mathrm{e}^{\kappa u}\,\mathrm{d}W(u)\right)^2 \,\middle|\, r(t)\right]\\
&= \frac{\sigma^2}{2\kappa}\left(1 - \mathrm{e}^{-2\kappa(T-t)}\right).
\end{aligned}$$

Therefore,

$$\begin{aligned}
E\left[r_{j+1}\,|\,r_j\right] &= r_j\mathrm{e}^{-\kappa\Delta t} + \theta\left(1 - \mathrm{e}^{-\kappa\Delta t}\right),\\
\mathrm{Var}\left[r_{j+1}\,|\,r_j\right] &= \frac{\sigma^2}{2\kappa}\left(1 - \mathrm{e}^{-2\kappa\Delta t}\right).
\end{aligned}$$

Given that the interest rate $r(t)$ in the Vasicek model is normally distributed [5, Table 3.1], the parameters κ, θ, and σ can be estimated using the log-likelihood function of the normal distribution (see [14, equation (6.1.1)]):

$$\begin{aligned}
&\mathcal{L}(E\left[r_{j+1}\,|\,r_j\right], \mathrm{Var}\left[r_{j+1}\,|\,r_j\right])\\
&= -\frac{n}{2}\ln\left(\mathrm{Var}\left[r_{j+1}\,|\,r_j\right]\right)\\
&\quad - \frac{1}{2\mathrm{Var}\left[r_{j+1}\,|\,r_j\right]}\sum_{j=1}^{n}\left(r_{j+1} - E\left[r_{j+1}\,|\,r_j\right]\right)^2\\
&\quad - \frac{n}{2}\ln 2\pi.
\end{aligned}$$

Therefore,

$$\begin{aligned}
&\mathcal{L}(\kappa, \theta, \sigma)\\
&= -\frac{n}{2}\ln\left(2\pi\right) - \frac{n}{2}\ln\left(\frac{\sigma^3}{2\kappa}\left(1 - \mathrm{e}^{-2\kappa\Delta t}\right)\right)\\
&\quad - \frac{\sum_{j=1}^{n}\left(r_{j+1} - \left(r_j\mathrm{e}^{-\kappa\Delta t} + \theta\left(1 - \mathrm{e}^{-\kappa\Delta t}\right)\right)\right)^2}{\left(\sigma^2/\kappa\right)\left(1 - \mathrm{e}^{-2\kappa\Delta t}\right)}.
\end{aligned}$$

Estimates for κ, θ, and σ, which satisfy

$$\frac{\partial\mathcal{L}}{\partial\kappa} = \frac{\partial\mathcal{L}}{\partial\theta} = \frac{\partial\mathcal{L}}{\partial\sigma^2} = 0,$$

are given by

$$\begin{aligned}
\hat{\kappa} &= -\frac{1}{\Delta t}\ln\left(\frac{n\sum_{j=1}^{n} r_{j+1}r_j - \sum_{j=1}^{n} r_{j+1}\sum_{j=1}^{n} r_j}{n\sum_{j=1}^{n} r_j^2 - \left(\sum_{j=1}^{n} r_j\right)^2}\right),\\
\hat{\theta} &= \frac{1}{n\left(1 - \mathrm{e}^{-\hat{\kappa}\Delta t}\right)}\sum_{j=1}^{n}\left(r_{j+1} - r_j\mathrm{e}^{-\hat{\kappa}\Delta t}\right),\\
\hat{\sigma} &= \sqrt{\frac{2\hat{\kappa}\sum_{j=1}^{n}\left(r_{j+1} - r_j\mathrm{e}^{-\hat{\kappa}\Delta t} - \hat{\theta}\left(1 - \mathrm{e}^{-\hat{\kappa}\Delta t}\right)\right)^2}{n\left(1 - \mathrm{e}^{-2\hat{\kappa}\Delta t}\right)}}.
\end{aligned}$$

B. CIR Model

Applying Itô's Lemma, the conditional expectation and variance of $r(t)$ in the case of the CIR model are given by

$$\begin{aligned}
&E\left[r(T)\,|\,r(t)\right]\\
&= E\left[r(t)\mathrm{e}^{-\kappa(T-t)} + \theta - \theta\mathrm{e}^{-\kappa(T-t)}\right.\\
&\qquad \left. + \sigma\mathrm{e}^{-\kappa T}\int_t^T \mathrm{e}^{\kappa u}\sqrt{r(u)}\,\mathrm{d}W(u)\,\middle|\, r(t)\right]\\
&= r(t)\mathrm{e}^{-\kappa(T-t)} + \theta\left(1 - \mathrm{e}^{-\kappa(T-t)}\right)\\
&\qquad + \sigma\mathrm{e}^{-\kappa T} E\left[\int_t^T \mathrm{e}^{\kappa u}\sqrt{r(u)}\,\mathrm{d}W(u)\,\middle|\, r(t)\right]\\
&= r(t)\mathrm{e}^{-\kappa(T-t)} + \theta\left(1 - \mathrm{e}^{-\kappa(T-t)}\right),
\end{aligned}$$

and

$$\begin{aligned}
&\mathrm{Var}\left[r(T)\,|\,r(t)\right]\\
&= \mathrm{Var}\left[r(t)\mathrm{e}^{-\kappa(T-t)} + \theta - \theta\mathrm{e}^{-\kappa(T-t)}\right.\\
&\qquad \left. + \sigma e^{-\kappa T}\int_t^T \mathrm{e}^{\kappa u}\sqrt{r(u)}\,\mathrm{d}W(u)\,\middle|\, r(t)\right]\\
&= \mathrm{Var}\left[\sigma\mathrm{e}^{-\kappa T}\int_t^T \mathrm{e}^{\kappa u}\sqrt{r(u)}\,\mathrm{d}W(u)\,\middle|\, r(t)\right]\\
&= \sigma^2\left[\frac{r(t)}{\kappa}\left(\mathrm{e}^{-\kappa(T-t)} - \mathrm{e}^{-2\kappa(T-t)}\right) + \frac{\theta}{2\kappa}\left(1 - \mathrm{e}^{-\kappa(T-t)}\right)^2\right].
\end{aligned}$$

Therefore,

$$\begin{aligned}
E\left[r_j\,|\,r_{j-1}\right] &= r_{j-1}\mathrm{e}^{-\kappa\Delta t} + \theta\left(1 - \mathrm{e}^{-\kappa\Delta t}\right),\\
\mathrm{Var}[r_j\,|\,r_{j-1}] &= \sigma^2\left[\frac{r_{j-1}}{\kappa}\left(\mathrm{e}^{-\kappa\Delta t} - \mathrm{e}^{-2\kappa\Delta t}\right)\right.\\
&\qquad \left. + \frac{\theta}{2\kappa}\left(1 - \mathrm{e}^{-\kappa\Delta t}\right)^2\right].
\end{aligned}$$

In the case of the CIR model, since the interest rate $r(t)$ is not normally distributed [5, Table 3.1], to estimate κ, θ, and σ we could not use the normal distribution's log-likelihood function. Instead, we apply the least-squares estimation method [1]. Notice that in the CIR model,

$$r_j = r_{j-1}\mathrm{e}^{-\kappa\Delta t} + \theta\left(1-\mathrm{e}^{-\kappa\Delta t}\right) + \int_{j-1}^{j} \sigma \mathrm{e}^{-\kappa\Delta t}\sqrt{r(u)}\,\mathrm{d}W(u)$$

$$= E\left(r_j \,|\, r_{j-1}\right) + \int_{j-1}^{j} \sigma \mathrm{e}^{-\kappa\Delta t}\sqrt{r(u)}\,\mathrm{d}W(u).$$

Letting

$$\varepsilon_j = \int_{j-1}^{j} \sigma \mathrm{e}^{-\kappa\Delta t}\sqrt{r(u)}\,\mathrm{d}W(u),$$

we obtain

$$\sum_{j=1}^{n} \varepsilon_j^2 = \sum_{j=1}^{n} \left(r_j - E\left(r_j \,|\, r_{j-1}\right)\right)^2.$$

Estimates for κ, θ, and σ, which satisfy

$$\frac{\partial}{\partial\kappa}\left(\sum_{j=1}^{n}\varepsilon_t^2\right) = \frac{\partial}{\partial\theta}\left(\sum_{j=1}^{n}\varepsilon_t^2\right) = \frac{\partial}{\partial\sigma^2}\left(\sum_{j=1}^{n}\varepsilon_t^2\right) = 0,$$

are given by

$$\hat{\kappa} = -\frac{1}{\Delta t}\ln\left(\frac{n\sum_{j=1}^{n} r_j r_{j-1} - \sum_{j=1}^{n} r_j \sum_{j=1}^{n} r_{j-1}}{n\sum_{j=1}^{n}(r_{j-1})^2 - \left(\sum_{j=1}^{n} r_{j-1}\right)^2}\right),$$

$$\hat{\theta} = \frac{\sum_{j=1}^{n} r_j - \mathrm{e}^{-\hat{\kappa}\Delta t}\sum_{j=1}^{n} r_{j-1}}{n\left(1-\mathrm{e}^{-\hat{\kappa}\Delta t}\right)},$$

$$\hat{\sigma} = \sqrt{\frac{(1/n)\sum_{j=1}^{n}\left(r_j - r_{j-1}\mathrm{e}^{-\hat{\kappa}\Delta t} - \hat{\theta}\left(1-\mathrm{e}^{-\hat{\kappa}\Delta t}\right)\right)^2}{(1/\hat{\kappa}n)(\mathrm{e}^{-\hat{\kappa}\Delta t} - \mathrm{e}^{-2\hat{\kappa}\Delta t})\sum_{j=1}^{n} r_{j-1} + \left(\hat{\theta}/2\hat{\kappa}\right)\left(1-\mathrm{e}^{-\hat{\kappa}\Delta t}\right)^2}}.$$

C. EIA

Suppose that the EIA follows a geometric Brownian motion, as given by equation (7). Since $\ln P_t$ is normally distributed with parameters $\left(\mu - \psi^2/2\right)\Delta t$ and $\psi^2\Delta t$, we have

$$E(\ln P_t) = \left(\mu - \frac{\psi^2}{2}\right)\Delta t \quad \text{and} \quad \mathrm{Var}(\ln P_t) = \psi^2\Delta t.$$

Therefore, the parameters μ and ψ can be estimated by

$$\hat{\psi} = \sqrt{\frac{\mathrm{Var}(\ln P_t)}{\Delta t}} \quad \text{and} \quad \hat{\mu} = \frac{E(\ln P_t)}{\Delta t} + \frac{\hat{\psi}^2}{2}. \qquad (11)$$

V. Numerical simulation

In this section, we present the results of our numerical simulations, to demonstrate the application of the models discussed earlier to the pricing of a compound Ratchet-type EIA. The pricing is carried out under the three different interest-rate assumptions: constant, stochastic following the Vasicek model, and stochastic following the CIR model.

Suppose that an investor wishes to purchase a 10-year compound Ratchet-type EIA contract with an initial investment of $R = 100$, which features a minimum and maximum interest rates of $f = 6\%$ and $c = 11\%$, respectively. Suppose that the investment is made with a participation rate of $\alpha = 90\%$. Applying the formulae (11) to the 10-year daily dataset of the Jakarta Stock Exchange Composite index from March 13th, 2014 to March 13th, 2024, we obtain the estimates $\hat{\psi} \approx 0.1478$ and $\hat{\mu} \approx 0.0529$.

TABLE II
The estimated values of parameters.

	$\hat{\kappa}$	$\hat{\theta}$	$\hat{\sigma}$
Vasicek	0.9261	0.0711	0.0107
CIR	0.9253	0.0711	0.0396

	$\hat{\psi}$	$\hat{\mu}$
EIA	0.1478	0.0529

TABLE III
EIA price under three different interest-rate assumptions.

Interest-rate assumption	EIA price
Constant	107.2870
Vasicek	103.7411
CIR	103.7356

On the other hand, the parameters in the Vasicek and CIR models are estimated using the 10-year daily dataset of the Indonesian bond yields from March 13th, 2014 to March 13th, 2024. Applying the formulae derived in subsections IV-A and IV-B, one obtains the parameter values presented in Table II. For the constant interest rate case, we shall use as the interest rate the average of the long-term means θ in the cases of Vasicek and CIR models.

Using the parameter values presented in Table II along with equations (4), (9), and (10), we can now calculate the price of the EIA under three different assumptions on the interest rate: constant, stochastic based on the Vasicek model, and stochastic based on the CIR model. The results are shown in Table III.

Table III shows that the highest EIA price is obtained under the constant interest-rate assumption. This is unsurprising since the assumption of no interest rate fluctuations eliminates the possibility of rate decreases that could reduce the annuity value, thereby leading to a higher EIA price. On the other hand, while the CIR model accounts for interest rate volatility, its price is nearly identical to that of the simpler Vasicek model. This indicates that, under the market conditions represented by the utilised data, the additional volatility captured by the CIR model has little impact on the valuation compared to the mean-reverting nature of the Vasicek model.

VI. Sensitivity analysis

Our next aim is to analyse the sensitivity of the EIA prices obtained in section V with respect to each parameter involved in the models. We shall conduct this sensitivity analysis only in the two stochastic interest rate cases. To quantify the sensitivity of the EIA price $V(T)$ with respect to a parameter p upon which the price depends differentiably, we shall utilise the so-called sensitivity index of $V(T)$ with respect to p [7]:

$$\Upsilon_p^{V(T)} = \frac{\partial V(T)}{\partial p}\cdot\frac{p}{V(T)} \approx \frac{\Delta V(T)/V(T)}{\Delta p/p},$$

TABLE IV
THE SENSITIVITY INDICES OF THE EIA PRICE $V(T)$ WITH RESPECT TO κ, θ, σ, μ, AND ψ IN THE TWO STOCHASTIC INTEREST RATE CASES.

Sensitivity index	Interest rate assumption	
	Vasicek	CIR
$\Upsilon_{\kappa}^{V(T)}$	0.0090	0.0091
$\Upsilon_{\theta}^{V(T)}$	-0.6341	-0.6335
$\Upsilon_{\sigma}^{V(T)}$	0.0011	0.0003
$\Upsilon_{\mu}^{V(T)}$	0.0000	0.0000
$\Upsilon_{\psi}^{V(T)}$	0.0000	0.0000

which provides an estimate for the ratio of a relative change in the EIA price $V(T)$ with respect to a relative change in the parameter α. If $\Upsilon_p^{V(T)} > 0$, then an increase of 1% of p leads to an increase of $\Upsilon_p^{V(T)}\%$ of $V(T)$. If $\Upsilon_p^{V(T)} < 0$, then an increase of 1% of p leads to a decrease of $\Upsilon_p^{V(T)}\%$ of $V(T)$.

In both of the stochastic interest rate cases, i.e., Vasicek and CIR, the formula (10) enables us to compute analytically $\partial V(T)/\partial p$, and hence the index $\Upsilon_p^{V(T)}$, for each $p \in \{\kappa, \theta, \sigma, \mu, \psi\}$. Evaluating these indices at the parameter values used in our numerical simulation (section V), one obtains the values presented in Table IV.

Table II shows that in both of the stochastic interest rate cases, the calculated EIA price $V(T)$ depends most sensitively upon the interest rate's long-term mean θ, with a 1% increase in θ leading to 0.6341% and 0.6335% decreases in $V(T)$ in the Vasicek and CIR cases, respectively. By contrast, the EIA price $V(T)$ is fairly insensitive upon the interest rate's mean-reversion rate κ and the interest rate's volatility σ, and is entirely insensitive to the asset price's drift rate μ and the asset price's volatility ψ. This suggests that adjustments to these parameters have no discernible impact on the price, potentially due to the Ratchet design's structure, which mitigates downside risk by guaranteeing minimum returns.

VII. CONCLUSIONS AND FUTURE RESEARCH

We have studied the pricing of compound Ratchet-type equity-indexed annuities (EIAs) under three different interest-rate assumptions: constant, stochastic using the Vasicek model, and stochastic using the Cox-Ingersoll-Ross (CIR) model, with the price of the associated asset assumed to follow a geometric Brownian motion, the parameters being estimated using the historical data from Indonesian government bond yields and the Jakarta Stock Exchange Composite index. The results revealed that the constant interest-rate assumption yields the highest EIA price, as it eliminates the risk of interest rate declines, ensuring more predictable cash flows. The EIA prices calculated under the two stochastic interest-rate assumptions are not only lower but also nearly identical, showing that the additional complexity introduced in the CIR model has minimal influence under the market conditions represented by the utilised data. Under these assumptions, we have further shown through a sensitivity analysis that the calculated EIA price depends most sensitively upon the interest rate's long-term mean, with a 1% increase in the long-term mean leading to around 0.6% increase in the EIA price. Changes in other parameters, by contrast, lead to very little to no change in the calculated EIA price.

This research is extendible in a number of ways, primarily by choosing more flexible models for both stochastic interest rates and asset price movements, thereby enhancing the accuracy and applicability of the EIA pricing. For interest rates, alternatives such as the Hull-White model [20], the Dothan [10], and the Black-Derman-Toy model [3] offer greater adaptability to complex market conditions. Regarding asset prices, moving beyond the geometric Brownian motion assumption, future studies could consider regime-switching models [4] to account for abrupt volatility changes or Lévy processes [19] for modelling discontinuous price movements.

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