Optimization of Flexible Train Timetable with Passenger Travel Information by Demand Responsive Model

Songliang Zhang, and Dewei Li

***Abstract*—For railway system, significant information asymmetry exists between passenger and operating company, leading to overcrowding or insufficient capacity utilization during train operation. The demand responsive service model provides a potential solution to this issue. To address this, a two-stage optimization model has been proposed, aiming to respond to passenger travel demand with flexible train timetable. The first stage optimizes the passenger boarding and the second stage focuses on the time deviation for passenger departure and operating costs. This model enables scheduling with flexibility, determining variable actual arrival and departure time for each train for different dates within an available time range based on passenger travel demand information. The results show that, under the same optimal solution, the extension of available time range on the flexible timetable has a greater effect on reducing operating costs, with a maximum reduction of 17.65%. This approach can provide feedback for passenger demand. By arranging train operation with travel demand, demand responsive service enhances passenger satisfaction, providing a data-driven solution for human-centered railway system in potential.**

***Index Terms*—train timetable; demand responsive service; passenger demand; optimization model**

I. Introduction

FOR a long time, railway passenger transportation followed the planned organization mode, which predicts passenger demand for a period of time and formulates train operation plan based on this. Nowadays, the temporal and spatial evolution of travel demand presents uncertainties. What's more, the relationship between behavioral decision-making and operational mechanisms is highly complex. Due to instability and randomness of passenger demand, railway operating companies are difficult to grasp dynamic demand accurately in time.

For this issue, China Railway has implemented some strategies to balance the supply side and demand side. With the supply side, the daily train timetabling has put forward, which means the train timetables vary across different operating days to reflect fluctuations in passenger demand. In practice, this strategy comprises the basic timetable and the daily timetable. The former is revised quarterly and the later makes minor adjustments from the basic timetable based on the actual demand of each day. This approach balances the planned requirements of railway operation with the variability of passenger demand, allowing train operations adapt to demand fluctuations dynamically. With the demand side, as a platform for collecting passenger demand, the ticketing system launched a pilot of ticket reservation during the Spring Festival travel rush. With a 15-day ticket pre-sale period, passengers can submit their ticket reservation demands at least 17 days before departure. Tickets can be fulfilled on the following day and passengers will receive notice at the same time. This mechanism facilitates bidirectional information exchange between passengers and railway operating company, breaking the information barrier between supply and demand. Beyond fulfilling ticket for passengers, the collected demand information enables operating company to optimize train timetable, supporting demand responsive service for passenger in this way.

Based on this, demand responsive service model for railway industry has emerged, which facilitates close connection between the supply side and demand side. In this model, reservation is a key segment and passengers need to submit their travel information. The human-centered features of demand responsive service are reflected primarily in the processing of passenger reservation demands. With the demand from large amounts of passengers, railway operating company can offer advisory feedback on passenger travel, including travel time, train timetable and other information. The entire process requires railway operating company to respond demand with the reservation information provided by passengers, while considering line, station and train as fundamental conditions. Ultimately, reasonable train operation plan can be arranged according to passenger demand by optimization. With the demand responsive service model, the personalized travel demand can be satisfied, enhancing the attraction of railway system.

As a result, the train timetable will be influenced by passenger travel information, and this will change with reservation demands. This is referred to as a flexible train timetable. The flexible train timetable can implement variable departure times for each train on different operating days, specifying operation by responding passenger travel information, thereby optimizing the allocation of transportation resources. For passengers, both travel time costs and efficiency will be optimized simultaneously. Additionally, it can improve the competitiveness of railway and increase the benefits of operating company. This means that the theories and methods for train timetable are in need of profound revolution to support the development of demand responsive service in the future.

Manuscript received January 3, 2025; revised May 6, 2025.

This work was supported by the National Natural Science Foundation of China (No. 72471023).

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II. Literature Review

With the development of mobile Internet and the improvement of computing capabilities, the collaborative optimization between the supply side and demand side has become an important issue, especially in the field of service optimization with demand responsive model. In public transportation system, there are numerous applications with demand responsive services, particularly with the emergence of customized bus and Dial-a-Ride service in recent years, all of which represent demand responsive transportation (DRT). There is a solid research foundation around these fields. For example, customized bus, as a supplement to traditional public transportation, have enhanced service quality for passengers. Shu [1] proposed an innovative method for planning customized bus line and station, entirely based on passenger travel demand, meeting diverse travel needs. With regard to Dial-a-Ride service, Molenbruch [2] investigated the operational consequent of cooperation among Dial-a-Ride service providers, which will be benefit to minimize the cost. In addition, changes in service quality will also have an impact on operating costs and different combinations of service parameters can measure them [3]. In the context of the comprehensive transportation system, Liu [4] proposed an integrated mobility service system, consisting of a local DRT component and a fixed-route transportation network component. Traditional transportation service and demand responsive service both have their own advantages. Zhao [5] optimized the total travel time and fleet size in a hybrid network, concluding that the performance is influenced by passenger preferences and DRT service model. Posada [6] proposed that the door-to-door system should be composed by traditional buses and demand responsive vehicles. Part of the journey can be completed by the former and the latter can make passenger wait at transfer point. For potential transfers that may occur, Schönberger [7] considered possible transfer problems during reservation and added transfer scheduling constraints to prevent long travel times caused by frequent transfers. With cultural genetic algorithms, the robustness of the reservation system is improved.

Besides that, demand responsive service can improve the flexibility of transport operation. Ronald [8] compared the advantages and disadvantages of three simulation packages in evaluating DRT, and provided recommendations for different modeling approaches. Herminghaus [9] considered the dynamics of demand responsive ride pooling systems. For bus transportation agencies, Boyer [10] discussed the flexible scheduling of vehicle and crew. A mixed integer linear programming model was proposed, which can be solved by variable neighborhood search algorithm. In order to minimize the system cost, Shen [11] analyzed the vehicle routing operation in demand responsive connector system from the perspectives of service providers and passengers. For urban rail transit, Huan [12] proposed a passenger control strategy based on demand responsive model in oversaturated networks, showing significant performance in peak cutting and load balancing.

As a means to collect passenger demand accurately, reservation within demand responsive service can provide reliable decision-making basis, optimizing service quality better and realizing effective allocation of resources. Pimenta [13] proposed an integer linear programming model minimizing the number of stops for a customized service system. They adopted a heuristic approach with insertion mechanisms, which is well fitted to actual demand. As for total travel time, Tian [14] considered the through operation scenario between urban rail transit and suburban railway, increasing revenues of operating company and saving passengers travel time. In addition, collaborative optimization of line planning and train timetabling with multidimensional travel demands of passengers can minimize the cost of train operation and passenger travel costs, meeting the diverse travel demands of passengers at the same time [15]. Typically, the solution scale of such optimization problems is considerable, requiring efficient solution algorithms. Mohamed [16] proposed a hybrid genetic algorithm, which constructed heuristics, crossover operations and local search techniques for customized services. After that, 92 basic requirements and 40 maneuvering requirements were tested to proof the effectiveness of the algorithm.

When exploring the model of demand responsive, current research tends to adopt the combination of static and dynamic to add or insert new supply based on flexible demand assuming fixed supply. Mehran [17] proposed a form of organization which combines rigidity and flexibility, which can meet the passenger demand better. The improved solution enhances vehicle utilization and reduces operating costs. In addition, operating costs are closely related to vehicle service time. Viana [18] abstracted the reservation service as a multi-objective ride-hailing problem, optimizing both the supply and demand sides and establishing the model with the time window and capacity constraints. For railway transportation service, it cannot adapt to passenger demand very well due to its rigidity. Dong [19] focused on the rescheduling of train timetable. The multi-objective optimization model was committed to enhancing passenger satisfaction. Mo [20] proposed a flexible optimization model for train timetable and utilization with uneven passenger demand in two directions. Trains operation of different types with various capacities could be realized by the objective of minimizing energy costs and passenger waiting time. Kroon [21] put forward the concept of flexible connections between rolling stock and passenger based on the periodic event-scheduling problem (PESP). The example of three intercity lines in Dutch railway system was illustrated. Cats [22] concentrated on the direct non-stop service by determining the capacity of an on-demand rail-bound transit system. An optimization model was designed to minimize passenger, infrastructure and operating costs. Gao [23] proposed a flexible scheduling framework to get out the problem of complicated high-speed railway timetabling with overtaking. Moreover, an alternating direction method of multipliers (ADMM) was used to solve, ensuring the effectiveness and availability of this solution. An [24] described the train timetable with a directed space-time network. To enhance the quality of the optimal solution, Lagrangian relaxation algorithm based on fuzzy sub-gradient optimization was proved to be effective.

Due to the differences in passenger organization and train operation management, existing research cannot be applicable to actual operation of high-speed railway directly.

Furthermore, research on demand responsive service for train operation in railway system remains underdeveloped. At the same time, the imbalance between supply and demand also exists in the field of train operation for railway, which can be solved by reservation before passenger travel.

It is certain that demand responsive services will be widely applied in railways. However, the specific framework of this model remains unclear at present. Therefore, this study aims to design a feasible process to realize demand responsive service for railway system with flexible train timetable.

III. Mathematical Model of Flexible Timetable

A. Problem Description

With reservation before travel, passenger can input their origin, destination and expected departure time. Additionally, unlike current railway ticketing systems, the acceptable range for departure time adjustment also needs to be specified by passenger. Based on this, the flexible train timetable can be generated from the proposed model, with the times of arrival and departure for each train at each station presented in the form of time ranges. After that, the actual arrival and departure time for each train on a particular day will be selected within the time range by railway operating company, considering the travel information of passenger travel demand on that day. Therefore, the arrival and departure time for each train may be variable on different dates.

Based on the above process, it can be seen that the optimization of flexible train timetable can be divided into two stages. The first stage will determine the feasible time ranges for the arrival and departure time of each train at each station, which will occupy a portion of space-time area on the flexible timetable. The second stage will select the reasonable times for train operations on a particular day within the possible time ranges, which can be carried out with the result of the first stage. To represent the flexible train timetable visually, the results of the two stages need to be presented, as shown in Fig. 1.

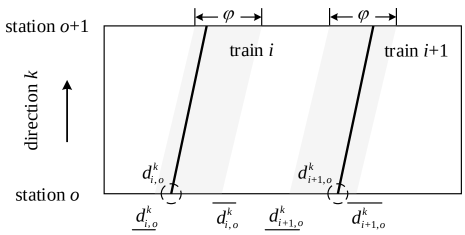

Fig. 1. Description of the flexible train timetable.

In Fig. 1, the result of the first stage can be described as a parallelogram shadow in the flexible train timetable. The horizontal width corresponds to the length of the feasible time range, which is denoted as φ in this model. The result of the second stage is a line, which must lie within the parallelogram obtained from the first stage. Any point on the line represents the operating time of each train at anywhere.

For each operating day, there is an organization cycle corresponding to it, which consists of three periods: reservation period, decision-making period and train operation period. As shown in Fig. 2, for any cycle w, the reservation period is used for passengers to input the travel information for date A. Within the decision-making period, operating company can determine the specific operating time for each train within date A and provide feedback on the responsive to passengers with their demand. During the train operation period, each train will service on the railway line based on demand responsive solutions.

B. Assumption

To simplify the modeling, the assumptions are made as follows.

(1) The stop plan of each train is the same. For the convenience of processing, it is assumed that each train will stop at each station and overtaking is not allowed at any station.

(2) The rolling stock is enough for passenger transportation on this railway line.

(3) All the passengers have considered carefully before they submit their travel information, which can reflect the actual demand for each passenger.

C. Model for Compiling Flexible Timetable

As mentioned before, this model should include two stages. The first stage determines the possible time range for each train and the second stage selects a reasonable specific moment within time range.

The sets, indexes, parameters and variables involved in the model are listed and explained in Table I.

TABLE I
List of sets, indexes, parameters and variables of the model

Sets and indexes	
$\boldsymbol{S}$	Station set: $\boldsymbol{S}$={o, d, s | o, d, s = 1, 2, ⋯ , S}.
$\boldsymbol{M}^{od}$	Passenger set: $\boldsymbol{M}^{od}$={ m | m = 1, 2, ⋯ , M}.
$\boldsymbol{K}$	Direction set: $\boldsymbol{K}$ ={ k | k = 1, 2}. For a single railway line, only two directions are included.
$\boldsymbol{I}^K$	Train set: $\boldsymbol{I}^K$ ={ i | i = 1, 2, ⋯ , I}.
Parameters	
$\tau_{m,o,d}^k$	Expected departure time for passenger m from station o to station d in direction k.
$\overline{\tau_{m,o,d}^k}$	Upper bound of expected departure time for passenger m from station o to station d in direction k.
$\underline{\tau_{m,o,d}^k}$	Lower bound of expected departure time for passenger m from station o to station d in direction k.
$r_{i,o}^k$	Operating time for train i from station o to station o+1 in direction k.
$t_{i,o}^k$	Dwell time for train i at station o in direction k.
$h_{\min}$	Minimum headway between two trains.
$h_{\max}$	Maximum headway between two trains.
c	Capacity of each train unit.
M	A large positive integer.
φ	Time range on the flexible train timetable.
η	Operating cost per unit.
α, β	Weight coefficients of different objectives.
Variables	
$\overline{d_{i,o}^k}$	Upper bound of departure time from station o for train i in direction k.
$\underline{d_{i,o}^k}$	Lower bound of departure time from station o for train i in direction k.
$d_{i,o}^k$	Actual departure time from station o for train i in direction k.
$\overline{a_{i,o}^k}$	Upper bound of arrival time to station o for train i in direction k.
$\underline{a_{i,o}^k}$	Lower bound of arrival time to station o for train i in direction k.
$a_{i,o}^k$	Actual arrival time from station o for train i in direction k.
$l_{i,m,o,d}^k$	Allocation of passenger boarding. If passenger m from station o to station d in direction k can take train i, it is 1; otherwise, it is 0.
f_i^k	Number of units for train i in direction k.

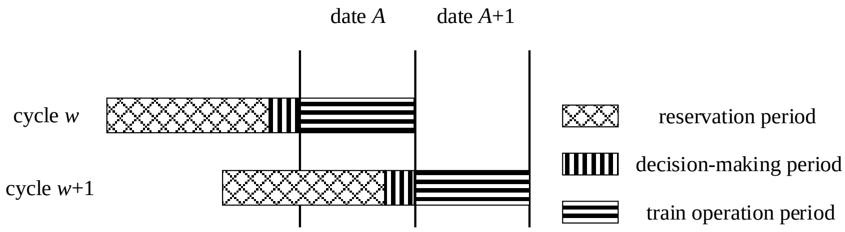

Fig. 2. Organization cycle of flexible train timetable with demand responsive service.

In the first stage of this model, the objective is to make sure that the number of passenger board the train successfully as much as possible. Therefore, it can be represented by Eq. (1).

$$\max Z_M = \sum_{k}^{\boldsymbol{K}} \sum_{i}^{\boldsymbol{I}^K} \sum_{m}^{\boldsymbol{M}^{od}} \sum_{o}^{s} \sum_{d=s+1}^{\boldsymbol{S}} l_{i,m,o,d}^k \tag{1}$$

The model of first stage includes three sets of constraints, chronological relationship, train operation and passenger service.

Chronological Relationship Constraints

φ is the time range, which represents the feasible range of operating time for each train. There are several φ on the flexible timetable because each φ corresponds to a train service. Trains may operate at different time every day, and the actual operating time for one train in a particular day will be determined with passenger travel information on that day. And the operating time must be within the time range φ. In this way, train timetable will be flexible. The flexible timetable requires the passengers to submit their expected departure time and acceptable range for departure time adjustment in advance. Since the length of time range φ on flexible timetable is fixed, the upper and lower bounds of train departure time need to satisfy the Eq. (2). As a parameter, the value of φ can be determined with actual conditions of the railway line.

$$\overline{d_{i,o}^k} = \underline{d_{i,o}^k} + \varphi \quad \forall i \in \boldsymbol{I}^K, o \in \boldsymbol{S}, k \in \boldsymbol{K} \tag{2}$$

Passenger m from station o to station d in direction k need to meet the chronological priority. If passenger can board successfully, $l_{i,m,o,d}^k = 1$ and $\underline{d_{i,o}^k} - \overline{\tau_{m,o,d}^k} \le 0$, so $\underline{d_{i,o}^k} \le \overline{\tau_{m,o,d}^k}$. At the same time, $\underline{\tau_{m,o,d}^k} - \overline{d_{i,o}^k} \le 0$, which equals $\underline{\tau_{m,o,d}^k} \le \overline{d_{i,o}^k}$. If the passenger cannot board, $l_{i,m,o,d}^k = 0$. Therefore, (3) and (4) can be established.

$$\underline{d_{i,o}^k} - \overline{\tau_{m,o,d}^k} \le M \times (1 - l_{i,m,o,d}^k) \quad \forall i \in \boldsymbol{I}^K, o, d \in \boldsymbol{S}, m \in \boldsymbol{M}^{od}, k \in \boldsymbol{K} \tag{3}$$

$$\underline{\tau_{m,o,d}^k} - \overline{d_{i,o}^k} \le M \times (1 - l_{i,m,o,d}^k) \quad \forall i \in \boldsymbol{I}^K, o, d \in \boldsymbol{S}, m \in \boldsymbol{M}^{od}, k \in \boldsymbol{K} \tag{4}$$

Train Operation Constraints

The departure time and arrival time of trains should satisfy the train operation constraints. Generally, the arrival and departure time have a relationship, which can be shown in (5) and (6). Meanwhile, the arrival time and departure time between different trains should subject to the restriction of minimum headway and maximum headway. The minimum headway $h_{\min}$ can guarantee the operation safety, separating any two consecutive trains. The maximum headway $h_{\max}$ may ensure the frequency of train service, preventing no trains available for passengers boarding for a long time. The corresponding two constraints are shown in (7) and (8).

$$\underline{d_{i,o}^k} \ge \underline{a_{i,o}^k} + t_{i,o}^k \quad \forall i \in \boldsymbol{I}^K, o \in \boldsymbol{S} / \{1\}, k \in \boldsymbol{K} \tag{5}$$

$$\underline{a_{i,o+1}^k} \le \underline{d_{i,o}^k} + r_{i,o}^k \quad \forall i \in \boldsymbol{I}^K, o \in \boldsymbol{S} / \{S\}, k \in \boldsymbol{K} \tag{6}$$

$$h_{\min} \le \underline{d_{i,o}^k} - \underline{d_{i-1,o}^k} \le h_{\max} \quad \forall i \in \boldsymbol{I}^K / \{1\}, o \in \boldsymbol{S}, k \in \boldsymbol{K} \tag{7}$$

$$h_{\min} \le \underline{a_{i,s}^k} - \underline{a_{i-1,s}^k} \le h_{\max} \quad \forall i \in \boldsymbol{I}^K / \{1\}, o \in \boldsymbol{S}, k \in \boldsymbol{K} \tag{8}$$

Passenger Service Constraints

Passengers m from station o to station d in direction k should be served by only one train, which can be presented by (9).

$$\sum_{i=1}^{\boldsymbol{I}^K} l_{i,m,o,d}^k \le 1 \quad \forall o, d \in \boldsymbol{S}, m \in \boldsymbol{M}^{od}, k \in \boldsymbol{K} \tag{9}$$

After the first stage of this model, possible operating time range for each train can be obtained. Each train can determine a specific moment within the corresponding time range. The second stage is to select the actual departure and arrival time of each train at each station.

In the second stage, the objective should be considered from two aspects: (1) the deviation between actual departure time and expected departure time for all passengers, as shown in Eq. (10), (2) the operating costs, determined by the number of units used in total, as shown in Eq. (11). The above two objectives ought to be minimized as much as possible, so they can be combined into Eq. (12) with weight coefficients.

$$\min Z_B = \sum_{k}^{\boldsymbol{K}} \sum_{i}^{\boldsymbol{I}^K} \sum_{m}^{\boldsymbol{M}^{od}} \sum_{o}^{s} \sum_{d=s+1}^{\boldsymbol{S}} \left| \tau_{m,o,d}^k \times l_{i,m,o,d}^k - d_{i,o}^k \right| \tag{10}$$

$$\min Z_F = \sum_{k \in \boldsymbol{K}} \sum_{i \in \boldsymbol{I}^k} \eta \cdot f_i^k \tag{11}$$

$$\min Z = \alpha \cdot \sum_{k \in \boldsymbol{K}} \sum_{i \in \boldsymbol{I}^k} \eta \cdot f_i^k + \beta \cdot \sum_{k}^{\boldsymbol{K}} \sum_{i}^{\boldsymbol{I}^K} \sum_{m}^{\boldsymbol{M}^{od}} \sum_{o}^{s} \sum_{d=s+1}^{\boldsymbol{S}} \left| \tau_{m,o,d}^k \times l_{i,m,o,d}^k - d_{i,o}^k \right| \tag{12}$$

For the constraints in the second stage, the actual arrival and departure time constraints need to satisfy the train operation constraints, as shown in (13) and (14).

$$d_{i,o}^k \ge a_{i,o}^k + t_{i,o}^k \quad \forall i \in \boldsymbol{I}^K, o \in \boldsymbol{S} / \{1\}, k \in \boldsymbol{K} \tag{13}$$

$$a_{i,o+1}^k \le d_{i,o}^k + r_{i,o}^k \quad \forall i \in \boldsymbol{I}^K, o \in \boldsymbol{S} / \{S\}, k \in \boldsymbol{K} \tag{14}$$

In addition, they should be in the time range, that is, within the upper bound and lower bound, as shown in (15) and (16).

$$\underline{d_{i,o}^k} \le d_{i,o}^k \le \overline{d_{i,o}^k} \quad \forall i \in \boldsymbol{I}^K, o \in \boldsymbol{S}, k \in \boldsymbol{K} \tag{15}$$

$$\underline{a_{i,o}^k} \le a_{i,o}^k \le \overline{a_{i,o}^k} \quad \forall i \in \boldsymbol{I}^K, o \in \boldsymbol{S}, k \in \boldsymbol{K} \tag{16}$$

Moreover, (17) represents the flexible train timetable should satisfy capacity constraints.

$$\sum_{m=1}^{\boldsymbol{M}^{od}} \sum_{o=1}^{s} \sum_{d=s+1}^{\boldsymbol{S}} l_{i,m,o,d}^k \le c \times f_i^k \quad \forall i \in \boldsymbol{I}^K, k \in \boldsymbol{K} \tag{17}$$

IV. Numerical Experiment of Proposed Model

A. Data Processing

Considering that demand responsive services have not been applied in railway system yet, the data required for proposed model cannot be obtained from historical demand of any railway line in real world. In order to ensure the rationality of travel demand information, an online survey in form of a questionnaire was regarded as an equivalent to the process of reservation and demand submission, which is the critical segment of demand responsive service in railway. A fictive scenario was set in the questionnaire, including 4 stations (labeled station 1, station 2, station 3 and station 4 respectively) and 3 sections, as shown in Fig. 3. Passengers submit their travel demand based on this fictive scenario and the collected data are used for the numerical experiment. When passengers fill in the demand information, they have to choose expected departure time and acceptable range for departure time adjustment from a number of alternative options based on their travel behavior. Additionally, the origin station and destination station, as the routine information of passenger demand, must also be specified. In this way, the upper and lower bounds of expected departure time for each passenger could be obtained. The alternative expected departure time and acceptable range for departure time adjustment options in the questionnaire are shown in Table II.

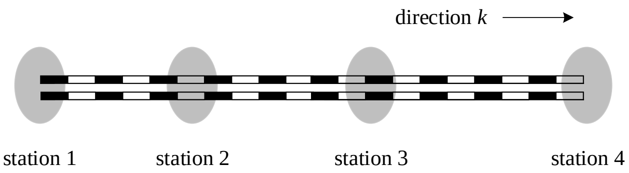

Fig. 3. Fictive scenario of the numerical experiment.

TABLE II
ALTERNATIVE OPTIONS FOR PASSENGERS

Parameters	Options	
Expected departure time	☐ 9:00	☐ 10:00
	☐ 11:00	☐ 12:00
	☐ 13:00	☐ 14:00
Acceptable range for departure time adjustment	☐ 10min	☐ 30min
	☐ 1h	☐ 2h

With the online survey, travel demands can be collected from passengers. A total of 217 valid travel demands were considered in this research. The allowable range of departure time for each passenger was represented by a line segment. The terminals of line segment represent the upper and lower bound of expected departure time for each passenger. The longer the acceptable range of departure time, the longer the corresponding line segment. The allowable range of departure time of all passengers are shown in Fig. 4, which contains 217 line segments of different lengths, representing the allowable range of departure time for 217 travel demands.

However, Fig. 4 cannot reflect the origin and destination stations for each passenger. In this case, 6 origin-destination (OD) combinations are included in one direction. The number of passengers for each OD combination is shown in Table III.

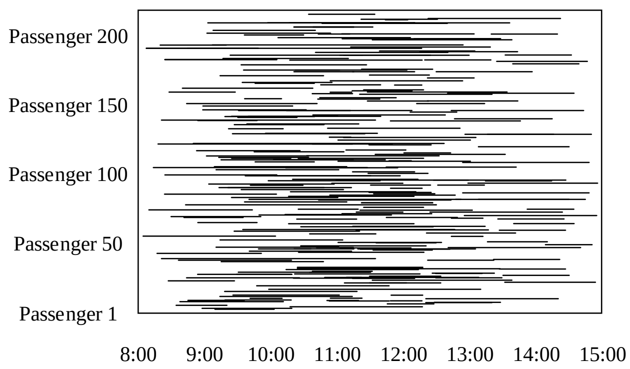

Fig. 4. Distribution of allowable range of departure time for passenger demand.

TABLE III
DISTRIBUTION OF PASSENGER TRAVEL ORIGIN-DESTINATION

O-D	Number of passengers
Station 1-2	46
Station 1-3	30
Station 1-4	21
Station 2-3	49
Station 2-4	28
Station 3-4	43

After obtaining the passenger travel information, the values of corresponding parameters in proposed model need to be clarified, as shown in Table IV. The dwell time for each train at each station for both directions is 3 min.

TABLE IV
VALUES OF PARAMETERS

Notation	Value	Notation	Value
$r_{i,1}^{k}$	22 min	$h_{\min}$	5 min
$r_{i,2}^{k}$	31 min	$h_{\max}$	90 min
$r_{i,3}^{k}$	35 min	c	2
φ	20 min	η	40
α, β	1, 1	f_i^k	8 or 16

B. Optimal Solution of Flexible Train Timetable

With passenger travel information and train operation parameters, the optimized flexible train timetable is shown in Fig. 5. Each train corresponds to a parallelogram shadow, indicating the available departure time for each train. The first stage of this model determines the position of each shadow, which defines the potential departure time window based on the values of upper bound and lower bound of departure time for each train. In the second stage, the model selects the specific operating time within the time window to optimize time deviation and operating costs, thereby determining the operation line. The optimal solution can satisfy passenger demand, which involves 10 trains, including two types of train with 8 units and 16 units, using a total of 112 units. During the peak demand period from 9:00 to 12:00, train service frequency is increased relatively to ensure that passenger can departure timely. In addition, the use of short formation (8 units) for urgent and intensive demand is also conducive to control operating costs more effective. This approach guarantees the service frequency, also meets passenger travel demands promptly.

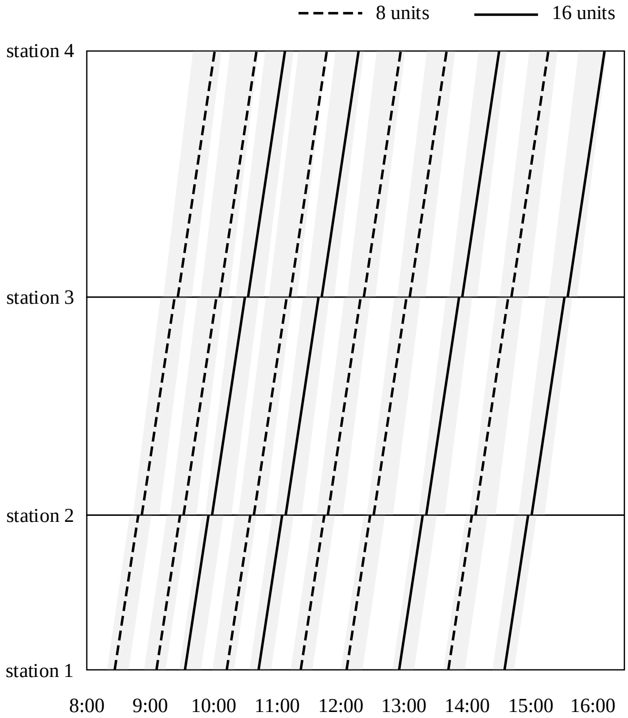

Fig. 5. Flexible train timetable with demand responsive service.

The operating cost per unit is denoted by η in the proposed model, which is not a fixed value in practice but depends on the type of electric multiple units (EMU) and maintenance schedules. To account for this, the optimization results under different values of η need to be analyzed, focusing on the number of units used, deviation for passengers, and the total objective value, as shown in Fig. 6. As η increases, the number of operating units used decreases consistently. To improve service quality, the operating company tends to deploy more capacity only when operating costs are low. As η increases, the operating company may reduce the supply of train capacity to control overall expenses. Under such condition, operating company may focus greater emphasis on ensuring whether passengers can realize their travel or not rather than minimizing deviation. As a result, deviation for passengers tends to increase with η rising. The passenger total demand is relatively low, which means surplus capacity in this case study. As a consequence, the optimization result is influenced by the supply side primarily. The total objective value is more significantly affected by the number of operating units used and the operating cost per unit.

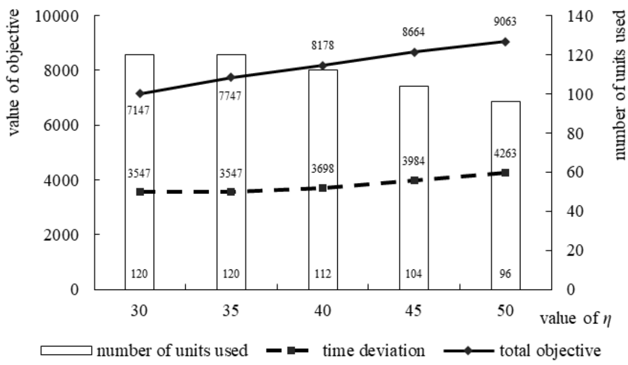

Fig. 6. Impact of operating cost per unit on optimization results.

C. Load Rate of Each Train with Different Numbers of Unit

The load rates for each train with the optimal solution in Fig. 5 are shown in Fig. 7. It can be observed that the average load rate is not too high, and there is a noticeable disparity among different trains. Due to the model objective including the deviation between actual departure time and expected departure time for passengers, the operating costs increased with the unnecessary operations of so many trains that could have been cancelled. In order to align actual departure times with their expected departure times for each passenger as closely as possible, more trains are scheduled to operate in order to ensure passenger departure as soon as possible. However, this makes operating costs increases, sacrificing the interests of operating company. In practice, the costs can be controlled by compressing train formations or reducing the number of trains operation. For this purpose, the weight coefficients of two objectives regarding passengers and operating costs in equation (12) are adjusted to amplify the impact of operating costs on the overall objective gradually. The results are shown in Fig. 8. It can be seen that as the operating costs decreases, the deviation between actual departure time and expected departure time for all passengers increases gradually. Lower operating costs means the reduce of trains operation. Thus, the number of trains that match passenger time expectation will reduce in turn. The potential times available for passengers to board at each station decrease, making the deviation between actual departure time and expected departure time lengthen.

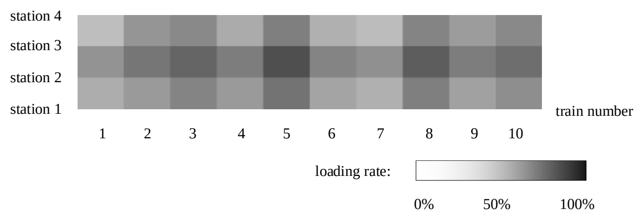

Fig. 7. Load rates for each train.

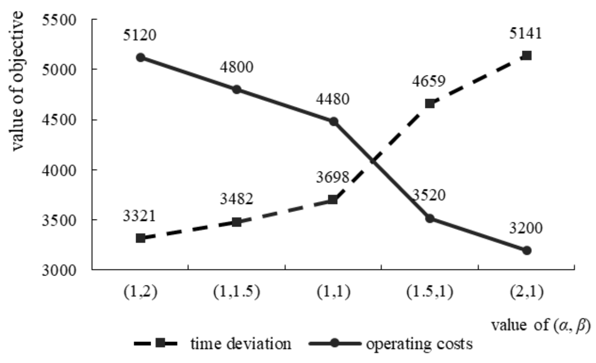

Fig. 8. Value of objective with different weight coefficients.

With the different number of units in service, the load rates of each train with different solutions can be obtained, as shown in Fig. 9. It is obvious that when the number of units used in total is around 80, the load rates in the busy section (station 2- station 3) generally reach 80% approximately. At this time, the deviation between actual departure time and expected departure time for all passengers has increased by 39.02% compared to the solution in Fig. 5. The demand for minimizing deviation for passengers is in conflict with operating costs. Therefore, the operating company can select appropriate strategy based on its profitability.

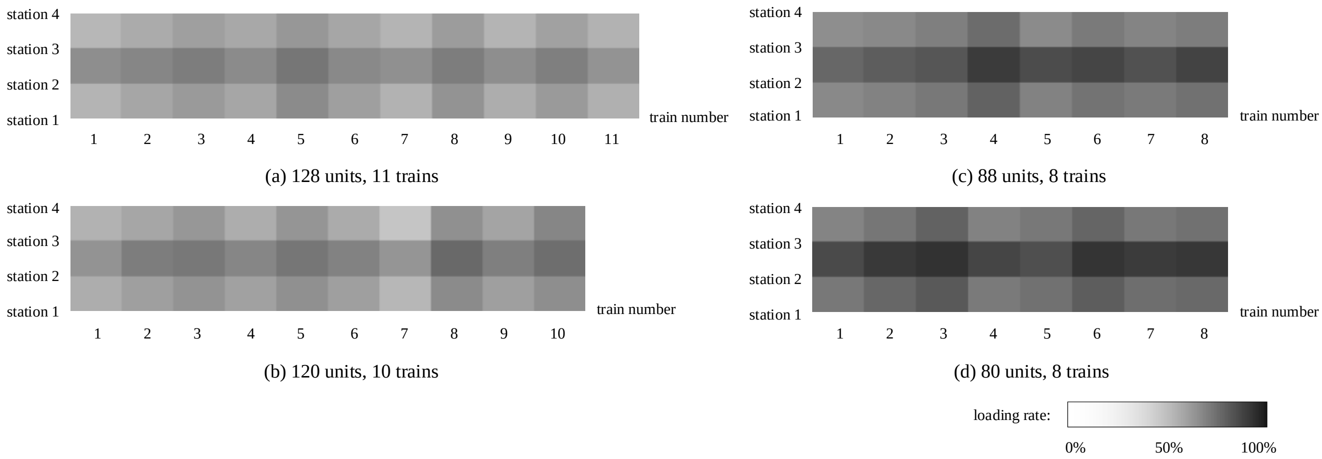

Fig. 9. Load rates of each train with different units used.

D. Discussion on the Minimum Units in Service

With different weight coefficients combinations of two objectives, the number of units used changes. What's more, the optimization performance is influenced by the supply side primarily. Therefore, the minimum number of units in service needs to be analyzed.

Based on the passenger demand in Table III, the section passenger volumes along this railway line can be calculated in Table V. The busy section of station 2- station 3 has the peak volume of 128 passengers. In practice, it is sufficient for operating company to provide capacity to meet the most intensive demand. According to given parameters, the minimum number of operating units required is 64. An extra constraint is added in the model, fixing the total units used to 64, as shown in Eq. (18). The optimized train timetable is presented in Table VI.

$$\sum_{k\in \boldsymbol{K}}\sum_{i\in \boldsymbol{I}^k} f_i^k = 64 \tag{18}$$

TABLE V
SECTION PASSENGER VOLUMES ALONG RAILWAY LINE

Section	Section passenger volumes
Station 1-2	97
Station 2-3	128
Station 3-4	92

TABLE VI
SECTION PASSENGER VOLUMES ALONG RAILWAY LINE

Train ID	Departure time from station 1	Formation
Train 1	8:42	small-formation
Train 2	9:34	long-formation
Train 3	10:28	small -formation
Train 4	11:46	long-formation
Train 5	13:08	small -formation
Train 6	14:26	small -formation

The solution in Table VI includes 6 trains in total, including 4 trains with short formations (8 units) and 2 trains with long formations (16 units). Notably, both long-formation trains are served during the peak period from 9:00 to 12:00. In this case, as the service capacity equals to the maximum section demand, load rate for all trains in the busy section maintain 100%. Compared with the solution in Fig. 5, the operating cost is reduced by 42.86%, while the time deviation increases by 74.23%.

In fact, there are multiple feasible combinations of train services with the use of 64 units. Fixing the number of units implies that the operating cost remains constant, while the number of trains service can be adjusted by changing train formations. However, compared to the solution in Table VI, increasing the number of trains with more short-formation train leads to a further increase in time deviation. This is because more passengers are unable to board during the peak period and are forced to wait for subsequent trains. Similarly, increasing the number of long-formation train may also increase time deviation. Although this helps alleviate passenger congestion, it reduces the number of trains operated, leading to lower service frequency. Passengers may be more difficult to board a train around their desired departure time.

E. Impact of Different Time Ranges on Optimal Solutions

φ is a self-defined parameter within the model that indicates the time range, i.e. the horizontal width of parallelogram shadow in the flexible train timetable. It can affect the available range in determining the actual time of departure and arrival for each train in the second stage. To analyze the impact of φ on optimal solution, the value of parameter φ is adjusted to analyze the changes in objectives. Based on the solution in Fig. 5, different values of time range are tested, revealing the impact on both time deviation for passenger departure and operating costs. As shown in Fig. 10, as φ increases, the time deviation for passenger departure tends to increase, but not significantly, while the operating costs decrease more than the increasing trend of time deviation. With operating costs decreased by 17.65%, the time deviation for passenger departure only increased by 3.71%. This is because with the horizontal width of time range increases, the feasible range of train departure and arrival time will be wider. As a result, departure and arrival time of each train may deviate from the passenger expected departure times. Furthermore, wider time range may allow it to cover a larger range of passenger demands in the time dimension, serving a larger number of passengers in potential. The number of passengers served by each train tends to converge as the time range expands. Each train will tend to operate short formation when capacity is surplus, which shortens operating costs effectively.

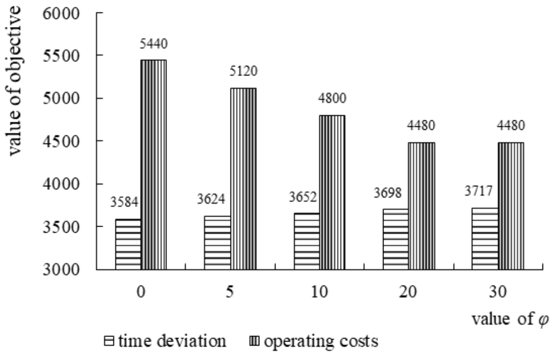

Fig. 10. Value of objective with the value of parameter φ based on optimal solution.

It should be specifically noted that when $\varphi = 0$, the width of time range is 0, the time ranges become operation lines in the timetable. At this time, there will be no available space for time adjustment and the departure and arrival times will be determined by the passenger demand on that day entirely, resulting in the minimum departure time deviation. However, during peak hours of intensive demands, trains need to operate with larger formation in order to serve more passengers as quickly as possible.

V. POSSIBLE CHALLENGES FOR THE FLEXIBLE TIMETABLE

Despite demand responsive model has been applied in public transportation systems (e.g. customized buses, etc.), flexible train timetable still faces great challenges to promote this pattern in railway system. Compared with customized bus, the difference between them is mainly reflected in adjustments. In the process of flexible train timetable decision, once the train operation is determined, it cannot be adjusted again. However, customized bus can be adjusted at any time even if it is operating. In addition, before traveling by railway, passengers need enough time to prepare and go to station. Once the travel reservation fails, they will face the risk of not being able to travel.

For this proposed model, the number of trains to operate in different days is not variable. Only the operating times for each train are adjustable, which reflects flexible in this way. However, allowing the cancel or insert of trains can be considered a better approach. This research only presents a flexible timetable optimization method which allows the arrival and departure time for each train to vary on different dates. Ignoring possible change for the number of trains operation is the limitation of this model.

Moreover, the promotion of flexible train timetable needs corresponding technical support in the future. Specifically, the travel reservation system should be developed, which can collect the travel demand information from passengers effectively. Secondly, autonomous driving technology can address the uncertainty in crew scheduling. Moreover, timely feedback on passenger travel demand is required to reduce the time to wait for responsive results. At the same time, the technology of virtual coupling, a hotspot of scientific research, can also provide the possibility for the application of flexible train timetable. With virtual coupling, it is possible to reduce the minimum passenger capacity of each train and provide more train service for customized railway travel.

VI. POTENTIAL APPLICATION OF FLEXIBLE TIMETABLE

In the future, if flexible train timetable is applied in practice, its promotion may have a limitation and suitable for specific situations. Considering the variability of flexible timetable, it may not be suitable for a railway line with excessive passenger travel demand, which requires train operation service with high frequency. For this situation, the solution is to operate trains as many as possible. In addition, due to the high density of train operating time, there is limited time space for adjustments. On the contrary, the flexible train timetable may be more applicable to the railway lines with lower travel demand and daily total amount of travel is stable. Since there are fewer train operations within a day, the adjustment range of operating time is larger, allowing for more possibility to match the train operation with passenger travel demand information.

Additionally, there is also another situation that may be applicable. For a new constructed railway line with no operating experience, how to determine a reasonable train timetable that satisfies passenger demand is an important issue. The flexible train timetable may be a possible option, even though demand forecasting has been made in the early stage of construction. It may be a feasible approach to guide the determination of timetable with travel information submitted by passengers with reservation. This could help to establish a stable and reliable train timetable for the future services.

VII. CONCLUSION

Flexibility can be reflected by variable train operating time within a fixed time range on different operating days. The flexible train timetable proposed in this paper can be implemented in two stages with demand responsive service model. The first stage mainly considers passenger demand and the second stage focuses on the control of operating costs on the basis of satisfying passenger demand. This offers a new approach to optimize the departure and arrival time of each train more precisely. Demand responsive service model enables the railway system to adjust train operating times according to passenger demand. This can accommodate temporal fluctuations in passenger demand better by flexible train timetable. It can balance the interests of passenger and operating company, while passenger interests can be measured by departure time deviation and operating company interests can be measured by operating costs. Moreover, with travel reservation, the railway transportation system can form an information loop between train services and passenger demand, making services align with demand better.

In future research, the uneven spatiotemporal distribution of passenger demand may impact the flexible train timetable, which can be deeply analyzed. What’s more, it may also meaningful to address passenger transfer in the condition of multiple railway lines or networks. Furthermore, potential reservation cancels could be considered, expanding the organization model of flexible train timetable with demand responsive service in railway system by more complicated scenario.

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