

Model Reference Adaptive Control of a Mobile Robot Using Popov Hyperstability Theory

Haitao Zhang and Wenshao Bu

Abstract—After getting dynamic model of two wheels driven mobile robot, a model reference adaptive control system using Popov hyperstability theory is proposed. The model reference adaptive control system has simple structure, and can adapt to the systems affected by interference. Using Popov hyperstability criterion the mobile robot system can be designed and obtain same characteristics of dynamic response with reference model for same control input. Then by using Matlab/Simulink simulation software, the dynamic model based simulation control for a mobile robot is carried out, and the simulation result shows that the model reference adaptive control algorithm using hyperstability theory can effectively improve the adaptive ability of mobile robot system.

Index Terms—Adaptive control, Mobile robot systems, Dynamic model, Popov hyperstability

I. INTRODUCTION

Wheeled mobile robots have been widely used due to its fast speed, strong flexibility, high accuracy, high efficiency, and simple mechanical structure. A common structure is differential drive, with two independent drive wheels and one or two unpowered wheel for balancing. Currently, differential drive robots have been used in many fields, such as transportation, monitoring, automated vehicles, and so on [1-3].

For a differential drive mobile robot, an important issue is the design of its motion controller. Mai, etc. proposed an optimal fuzzy system to control the operations of two-wheeled balancing mobile robots [4]. Cui, etc. proposed an adaptive control strategy proposed for simultaneous tracking and stabilization of nonholonomic mobile robot with uncertainties [5]. Qin, etc. proposed a super-twisting fractional-order sliding mode fault-tolerant control method combined with a fault observe aiming at the actuator fault-tolerant trajectory tracking problem of two-wheeled differential-driven mobile robots [6]. Almomani, etc. first developed a discrete-time linear model for the mobile robot from the robot dynamics, then proposed a discrete optimal controller to track desired velocities of robot wheels [7]. Pang, etc. developed a hyperbolic tangent function-based adaptive sliding mode controller to remove the negative

impacts imposed by the nonlinear external disturbances on it based on the established dynamics model of a practical two wheels mobile robot [8].

The controller design of mobile robot system is mainly based on two kinds of model. One is kinematic model, the other is dynamic model. Because of the complexity of dynamic model, kinematic model is often used. However, for getting better velocity control, the control based on dynamic model for mobile robot is also researched.

A kind of model reference adaptive control algorithm using Popov hyperstability theory is presented in this paper. The model reference adaptive control is used so as to solve the problem that is generated by the inaccuracy of controlled object model. Model reference adaptive controller possesses the ability of self-adapting. It can automatically adjust the parameters of controller, and achieve the optimal control of mobile robot system. The performance of proposed algorithm is better than traditional PID controller. Furthermore, it shows excellent adaptability in the control of the nonlinear and complex object.

II. THE DYNAMICS MODEL OF MOBILE ROBOTS

In the following, we first establish the dynamic model of the wheeled mobile robot.

A. Robot structure

A differential drive mobile robot is shown as Fig. 1. There, v and ω are, respectively, the linear and angular velocities, M is the midpoint of the connecting line between the two traction wheels. M is with coordinates x and y in the xOy plane, φ is the robot orientation, and d is the distance between the center lines of the traction wheels [9].

The state of the robot is represented by the position and heading angle of the midpoint M of its two driving wheels in the coordinate system xOy

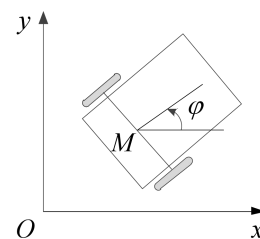


Fig. 1. The differential drive mobile robot

B. Dynamic equation of linear velocity for robot

In the absolute coordinate system xOy , a rigid mobile robot with independent dual rear wheel drive moves in the plane, the dynamic equations can be used to describe the

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dynamic characteristics of the robot [10].

For the whole vehicle body, according to the principle of torque balance, the rotational torque of the vehicle body is equal to the active torque of the right wheel minus the active torque of the left wheel, that is

$$I_M \ddot{\phi} = F_r l - F_l l \quad (1)$$

According to Newton's laws, there is

$$M \ddot{s} = F_r + F_l \quad (2)$$

where, I_M is the Moment of inertia around the point M of the robot, F_l and F_r are the driving forces of the left and right wheels respectively, l is the distance from the left and right wheels to the point M of the robot, ϕ is the pose angle of the robot, and s is the linear displacement the point M of the robot.

For two driving wheels, according to the principle of torque balance, the dynamic characteristics of the left and right wheels are represented by the following equation.

$$\begin{cases} I_w \ddot{\theta}_r + c \dot{\theta}_r = k u_r - d F_r \\ I_w \ddot{\theta}_l + c \dot{\theta}_l = k u_l - d F_l \end{cases} \quad (3)$$

In the above left and right wheel parameters, I_w is the moment of inertia of the wheel, c is the coefficient of viscous friction, k is the driving gain, d is the radius of the wheel, θ_l and θ_r are respectively the angle of the left and right wheels, and u_r and u_l are respectively the driving input of left and right wheels.

According to the principles of mobile robots, there are the following equations.

$$\dot{s} = \frac{\dot{s}_r + \dot{s}_l}{2} \quad (4)$$

$$l \dot{\phi} = \frac{\dot{s}_r - \dot{s}_l}{2} \quad (5)$$

where s_r and s_l are respectively the linear displacement of left and right wheels.

From equations (4) and (5), the following result can be gotten.

$$\begin{cases} \dot{s}_r = d \dot{\theta}_r = \dot{s} + l \dot{\phi} \\ \dot{s}_l = d \dot{\theta}_l = \dot{s} - l \dot{\phi} \end{cases} \quad (6)$$

Multiply both sides of equation (3) by d ,

$$\begin{cases} I_w d \ddot{\theta}_r + c d \dot{\theta}_r = k d u_r - d^2 F_r \\ I_w d \ddot{\theta}_l + c d \dot{\theta}_l = k d u_l - d^2 F_l \end{cases} \quad (7)$$

From the equation (7), the following result can be gotten.

$$I_w d (\ddot{\theta}_r + \ddot{\theta}_l) + c d (\dot{\theta}_r + \dot{\theta}_l) = k d (u_r + u_l) - d^2 (F_r + F_l) \quad (8)$$

From equations (4), the following result can be gotten.

$$d (\ddot{\theta}_r + \ddot{\theta}_l) = 2 \ddot{s} \quad (9)$$

Take the derivative on both sides of the above equation, and get the following equation.

$$d (\ddot{\theta}_r + \ddot{\theta}_l) = 2 \ddot{s} \quad (10)$$

Substitute equation (9) and (10) into equation (8), and get:

$$2 I_w \ddot{s} + 2 c \dot{s} = k d (u_r + u_l) - M d^2 \ddot{s} \quad (11)$$

From equation (11), we can get:

$$\ddot{s} = -\frac{2c}{M d^2 + 2 I_w} \dot{s} + \frac{k d}{M d^2 + 2 I_w} (u_r + u_l) \quad (12)$$

C. Dynamic equation of angular velocity of mobile robot

Multiply both sides of equation (1) by d^2 , and get:

$$I_M d^2 \ddot{\phi} = (F_r - F_l) d^2 l \quad (13)$$

From equation (3), the following result can be gotten.

$$I_w (\ddot{\theta}_r - \ddot{\theta}_l) + c (\dot{\theta}_r - \dot{\theta}_l) = k (u_r - u_l) - d (F_r - F_l) \quad (14)$$

Multiply both sides of equation (3) by dl ,

$$I_w (\ddot{\theta}_r - \ddot{\theta}_l) dl + c (\dot{\theta}_r - \dot{\theta}_l) dl = k (u_r - u_l) dl - (F_r - F_l) d l \quad (15)$$

From equations (6), the following result can be gotten.

$$d (\dot{\theta}_r - \dot{\theta}_l) = 2 l \dot{\phi} \quad (16)$$

Take the derivative on both sides of the above equation, and get the following equation.

$$d (\ddot{\theta}_r - \ddot{\theta}_l) = 2 l \ddot{\phi} \quad (17)$$

Substitute equation (16) and (17) into equation (15), and get:

$$I_w \cdot l \cdot 2 l \ddot{\phi} + c l \cdot 2 l \dot{\phi} = k (u_r - u_l) dl - (F_r - F_l) d^2 l \quad (18)$$

Substitute equation (13) into equation (18), and get:

$$I_w \cdot l \cdot 2 l \ddot{\phi} + c l \cdot 2 l \dot{\phi} = k (u_r - u_l) dl - I_M d^2 \ddot{\phi} \quad (19)$$

The above equation can be rewritten as:

$$I_w \cdot l \cdot 2 l \ddot{\phi} + I_M d^2 \ddot{\phi} = k (u_r - u_l) dl - c l \cdot 2 l \dot{\phi} \quad (20)$$

From equation (20), we can get:

$$\ddot{\phi} = \frac{-2 c l^2}{2 I_w l^2 + I_M d^2} \dot{\phi} + \frac{k d l}{2 I_w l^2 + I_M d^2} (u_r - u_l) \quad (21)$$

D. State space model of mobile robot

In the following, by defining state variables, we can obtain a state space model of dynamics equations of mobile robot.

Defining the state variable $x_p = [s \ \dot{s} \ \phi \ \dot{\phi}]^T$, the control input is $u_p = [u_r \ u_l]^T$, the output variable $y_p = [s \ \dot{s} \ \phi \ \dot{\phi}]^T$, the dynamic equation of robot can be gotten as follows:

$$\begin{cases} \dot{x}_p = A_p x_p + B_p u_p \\ y_p = C_p x_p \end{cases} \quad (22)$$

$$\text{where } A_p = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & a_1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & a_2 \end{bmatrix}, \quad B_p = \begin{bmatrix} 0 & 0 \\ b_1 & b_1 \\ 0 & 0 \\ b_2 & -b_2 \end{bmatrix},$$

$$C_p = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad a_1 = -\frac{2c}{M d^2 + 2 I_w},$$

$$a_2 = \frac{-2 c l^2}{2 I_w l^2 + I_M d^2}, \quad b_1 = \frac{k d}{M d^2 + 2 I_w},$$

$$b_2 = \frac{k d l}{2 I_w l^2 + I_M d^2}.$$

III. THE MODEL REFERENCE ADAPTIVE CONTROL USING POPOV HYPERSTABILITY THEORY

We design a model reference adaptive controller which makes the plant track the output of the reference model with the same reference input. In other words, the MRAC method best suits us here to obtain perfect tracking goal [11-12]. The target in our case study is tracking the desired movement under the given control input.

A. The Structure of Model Reference Adaptive Control

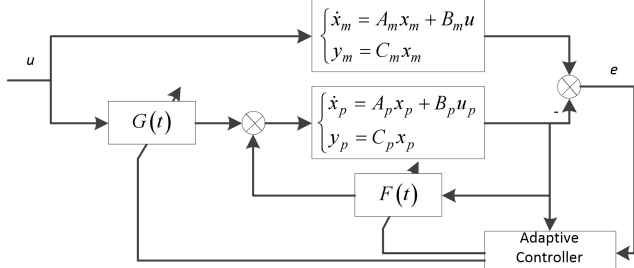


Fig. 2. The Model Reference Adaptive Control System

Fig. 2 is the model reference adaptive control system of mobile robot. In Fig.2, the state space model of the reference model is:

$$\begin{cases} \dot{x}_m = A_m x_m + B_m u \\ y_m = C_m x_m \end{cases} \quad (23)$$

where C_m is unit matrix I , and the design method of matrices A_m and B_m will be provided below.

The state space model of the controlled object is equation (22). $G(t)$ and $F(t)$ are respectively gain matrix and feedback matrix which need to be adjusted in real time by the adaptive controller. The controlled object, $G(t)$, and $F(t)$ form an adjustable system.

After $G(t)$ and $F(t)$ are introduced, it can be concluded that

$$u_p = G(t)u + F(t)x_p \quad (24)$$

Substitute equation (24) into equation (22), and get:

$$\dot{x}_p = [A_p + B_p F(t)]x_p + B_p G(t)u \quad (25)$$

From equation (25), it can be seen that by changing $G(t)$ and $F(t)$, the input matrix B_s and state matrix A_s of the adjustable system can be changed, then the output of the controlled object is changed.

In equation (25), let $A_p' = A_p + B_p F(t)$, $B_p' = B_p G(t)$, and get:

$$\dot{x}_p = A_p' x_p + B_p' u \quad (26)$$

Defining the generalized output vector error:

$$e = y_m - y_p \quad (27)$$

The adaptive controller adjusts $G(t)$ and $F(t)$ based on generalized output error e , so that the output of the control object tracks the output of the reference model

B. Popov Hyperstability Theory

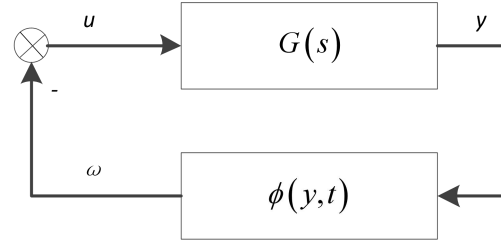


Fig. 3. The block diagram of a nonlinear system

As shown in Fig. 3, $G(s)$ is the forward block and $\phi(y, t)$ is the feedback block. In order for the closed-loop system to become a hyper stable system, $G(s)$ must satisfy positive real property and $\phi(y, t)$ must satisfy the Popov integral inequality.

In the following, first convert the system described in Fig. 2 to the system described in Fig. 3, and then apply the Popov hyperstability criterion.

Differentiate two sides of equation (27), and get the following equation.

$$\begin{aligned} \dot{e} &= \dot{y}_m - \dot{y}_p = \dot{x}_m - \dot{x}_p \\ &= A_m x_m + B_m u - A_p' x_p - B_p' u \\ &= A_m e + [A_m - A_p'] x_p + (B_m - B_p') u \end{aligned} \quad (28)$$

In order to ensure the stability of the system, Popov Hyperstability theory is applied. Thus, a linear compensator D that satisfies the following equation is introduced.

$$v = De \quad (29)$$

The following adaptive adjustment law is used for variables A_p' and B_p' .

$$\begin{cases} A_p' = \int_0^t \Phi_1(v, t, \tau) d\tau + \Phi_2(v, t) + A_p \\ B_p' = \int_0^t \Psi_1(v, t, \tau) d\tau + \Psi_2(v, t) + B_p \end{cases} \quad (30)$$

where $\Phi_1(v, t, \tau)$ and $\Phi_2(v, t, \tau)$ are both functions of variables v , t , and τ ($0 \leq \tau \leq t$).

Substitute equation (30) into equation (28), and get the following results.

$$\begin{cases} \dot{e} = A_m e + I(-\omega) \\ v = De \\ \omega = \left[\int_0^t \Phi_1(v, t, \tau) d\tau + \Phi_2(v, t, \tau) + A_p - A_m \right] x_p \\ \quad + \left[\int_0^t \Psi_1(v, t, \tau) d\tau + \Psi_2(v, t, \tau) + B_p - B_m \right] u \end{cases} \quad (31)$$

Equation (31) converts the system described in Fig. 2 to the system described in Fig. 3, and it includes two parts.

One is the forward block which is represented as follows:

$$\begin{cases} \dot{e} = A_m e + I(-\omega) \\ v = De \end{cases} \quad (32)$$

The other is feedback block which is represented as follows:

$$\omega = \left[\int_0^t \Phi_1(v, t, \tau) d\tau + \Phi_2(v, t, \tau) + A_p - A_m \right] x_p + \left[\int_0^t \psi_1(v, t, \tau) d\tau + \psi_2(v, t, \tau) + B_p - B_m \right] u \quad (33)$$

In order to make the system meet the Popov criterion, the variables of equation (32) and equation (33) need satisfy the following three conditions:

1) Determine variable $\Phi_1(v, t, \tau)$ and $\Phi_2(v, t)$

$$\begin{cases} \Phi_1(v, t, \tau) = F_a(t - \tau)v(\tau)x_p^T(\tau) \\ \Phi_2(v, t) = F_a'(t)v(t)x_p^T(t) \end{cases} \quad (34)$$

where $F_a(t)$ is the impulse response function of positive real function system $F_a(s)$, and $F_a'(t) \geq 0$.

2) Determine variable $\psi_1(v, t, \tau)$ and $\psi_2(v, t)$

$$\begin{cases} \psi_1(v, t, \tau) = F_b(t - \tau)v(\tau)u_p^T(\tau) \\ \psi_2(v, t) = F_b'(t)v(t)u_p^T(t) \end{cases} \quad (35)$$

where $F_b(t)$ is the impulse response function of positive real function system $F_b(s)$, and $F_b'(t) \geq 0$.

3) Determine variable D

$$\begin{cases} PA_m + A_m^T P = -Q \\ PI = D \end{cases} \quad (36)$$

where P and Q are always a Positive-definite matrix, and D is strictly positive real function matrix.

Substitute $A_p' = A_p + B_p F(t)$ and $B_p' = B_p G(t)$ into equation (30), and get:

$$\begin{cases} F(t) = B_p^+ \left[\int_0^t \Phi_1(v, t, \tau) d\tau + \Phi_2(v, t) \right] \\ G(t) = B_p^+ \left[\int_0^t \psi_1(v, t, \tau) d\tau + \psi_2(v, t) + B_p \right] \end{cases} \quad (37)$$

where B_p^+ is the generalized inverse of B_p .

C. The Design of Reference Model

In a model reference adaptive control system, the static and dynamic characteristics of the controlled object output will tend to be consistent with the static and dynamic characteristics of the reference model output as much as possible.

We first consider mobile robots as two decoupled second-order differential equations containing damping ratio ξ and undamped natural oscillation frequency ω_n :

$$\begin{cases} \ddot{s} + 2\xi_1\omega_{n1}\dot{s} + \omega_{n1}^2 s = \omega_{n1}^2 r \\ \ddot{\phi} + 2\xi_2\omega_{n2}\dot{\phi} + \omega_{n2}^2 \phi = \omega_{n2}^2 r \end{cases} \quad (38)$$

where ξ_1 and ξ_2 are both damping ratio, and ω_{n1} and ω_{n2} are both undamped natural oscillation frequency.

Then, we convert equation (38) into a state space model represented by equation (23), the relevant parameters are as follows:

$$A_m = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\omega_{n1}^2 & -2\xi_1\omega_{n1} & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -\omega_{n2}^2 & -2\xi_2\omega_{n2} \end{bmatrix}, \quad B_m = \begin{bmatrix} 0 & 0 \\ \omega_{n1}^2 & \omega_{n1}^2 \\ 0 & 0 \\ \omega_{n2}^2 & -\omega_{n2}^2 \end{bmatrix}, \quad C_m = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

IV. SIMULATION

A. Simulation Parameters

In order to verify the effectiveness of the proposed method, the mobile robot system is simulated in the Matlab/Simulink environment. The relative parameters of the mobile robot are described as follows:

$$a_1 = -0.05, a_2 = -0.09,$$

$$b_1 = 0.25, b_2 = 1.67$$

The relative parameters of the reference model are described as follows:

$$\xi_1 = 0.707, \omega_1 = 1,$$

$$\xi_2 = 0.707, \omega_2 = 2$$

Q is unit matrix,

$$D = \begin{bmatrix} 0.1450 & 0.5000 & 0 & 0 \\ 0.5000 & 0.6250 & 0 & 0 \\ 0 & 0 & 1.1813 & 0.1250 \\ 0 & 0 & 0.1250 & 0.1953 \end{bmatrix},$$

$$F_a(t - \tau) = F_a'(t) = F_b(t - \tau) = F_b'(t) = 1$$

B. Simulation Results without Interference

In order to verify the effectiveness of the proposed method, given $u = [u_l \ u_r]^T = [2 \ 1]^T$, the step response is shown in Fig. 4~Fig.7.

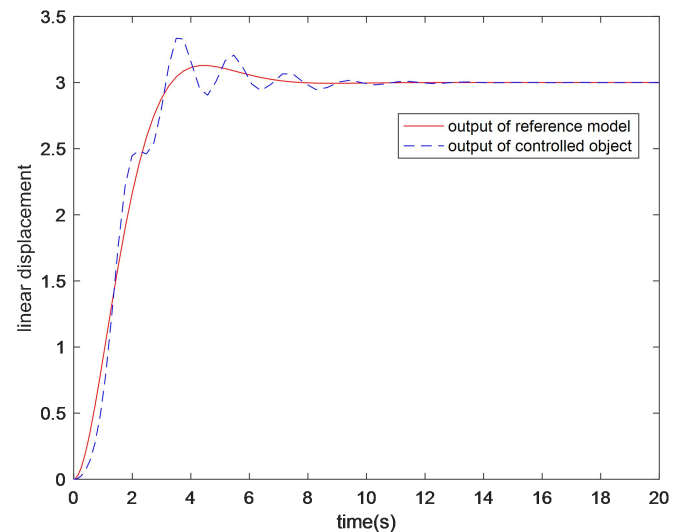


Fig. 4. The Line Displacement Tracking Diagram

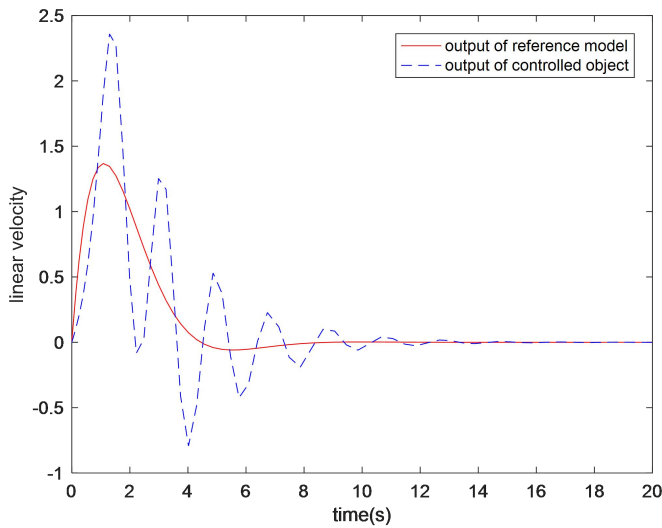


Fig. 5. The Line Velocity Tracking Diagram

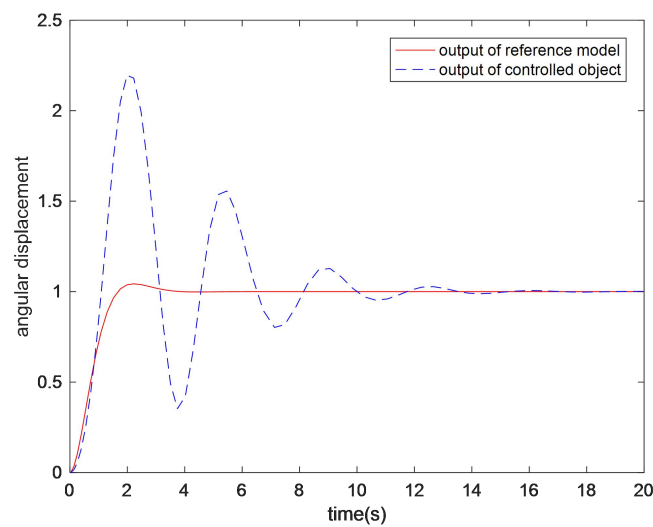


Fig. 6. The Angular Displacement Tracking Diagram

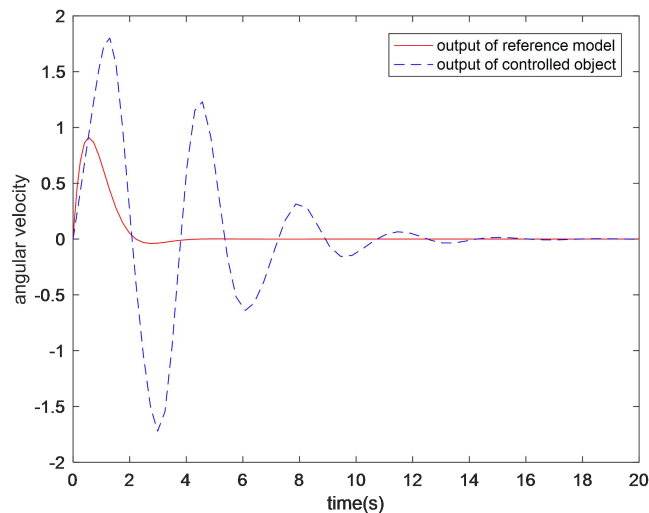


Fig. 7. The Angular Velocity Tracking Diagram

From Figures 2 to 4, it can be seen that the output of the controlled object has dynamic and static characteristics that are similar to the output of the reference model.

C. Simulation Results with Interference

In order to verify the adaptability of proposed algorithm,

the interference is added to the controller output at 10 second during the simulation. The Pulse generator chosen from the Sources module library is used to generate interference, and parameters of Pulse generator are: the Pulse height is 1, the Pulse period is 50 seconds, and the Pulse width is 2 seconds. The system simulation diagram with interference is shown in Fig. 8, Fig. 9, Fig. 10 and Fig.11.

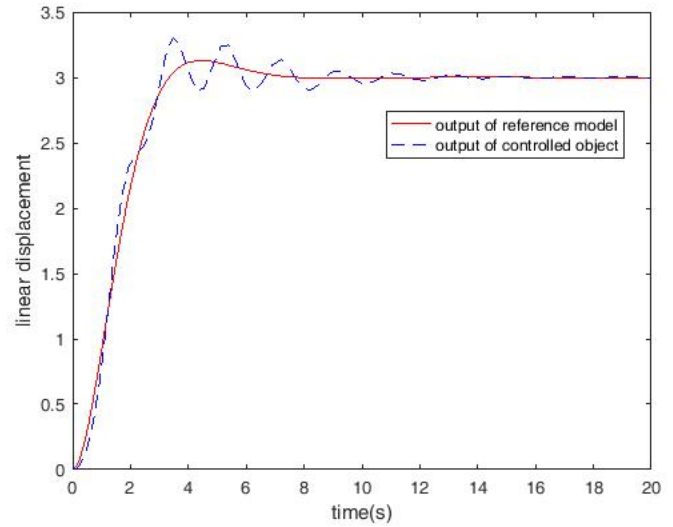


Fig. 8. The Line Displacement Tracking Diagram

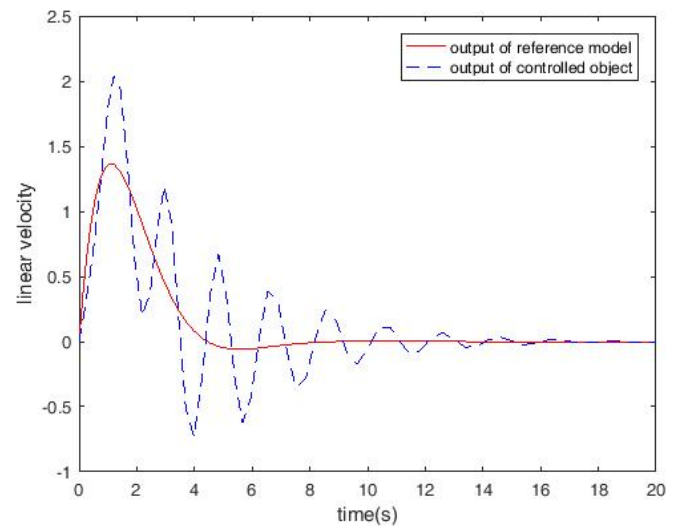


Fig. 9. The Line Velocity Tracking Diagram

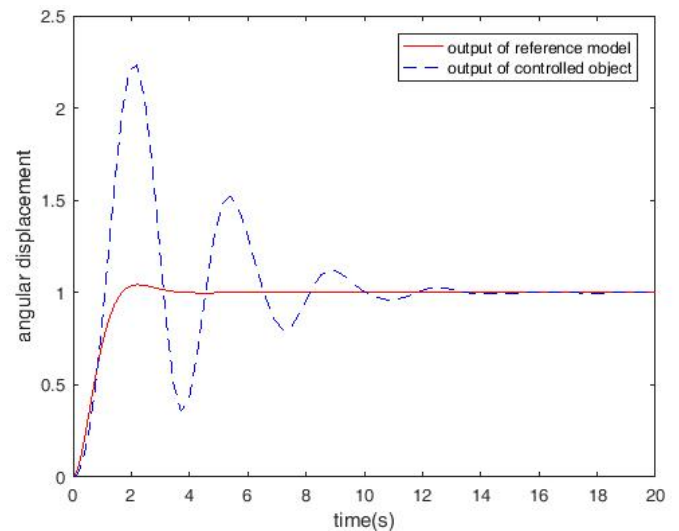


Fig. 10. The Angular Displacement Tracking Diagram

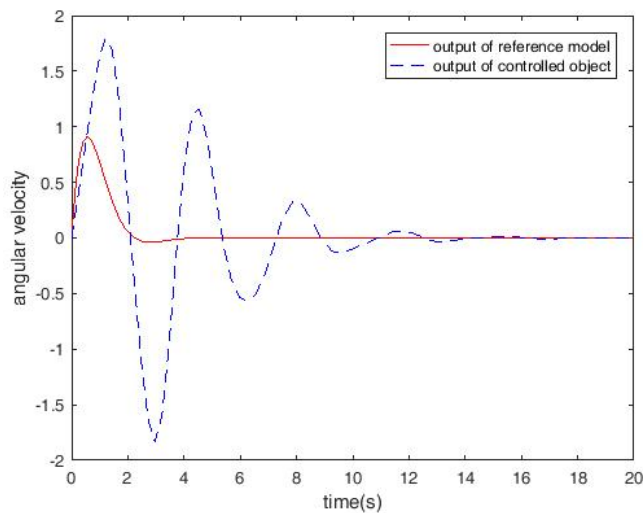


Fig. 11. The Angular Velocity Tracking Diagram

The simulation results show that the proposed algorithm can still well track the output of reference model for the system under interference influence.

V. CONCLUSION

We have presented a simulation model of mobile robot system, and a model reference adaptive control algorithm based on Popov Hyperstability theory for a mobile robot system. The simulation results shows that the proposed algorithm has better control effect for a mobile robot system.

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