The Steady-state Indexes of Single Machine Repairable System with Two Unreliable Servers

You Lyu, *Member, IAENG,* Tingyu Yan, Suran Kong, Wenguo Jiang

Abstract—**This paper analyzes a single-machine repairable system comprising two unreliable servers and one repairman. The machine can break down at any time, and the two unreliable servers are responsible for servicing it. The two servers can fail at any time, with different failure rates during idle and busy periods. A reliable repairman is tasked with repairing server failures. The time distributions are assumed to follow an exponential distribution. Using quasi-birth-death (QBD) process theory, the steady-state indices of the system are derived. Case analyses and numerical illustrations are provided to visualize the effects of system parameters on performance indices.**

Index Terms—**machine repairable system, reliability, availability, failure frequency**

I. Introduction

THE machine repairable system is a complex framework in which machines or equipment can break down but can be restored to a functional state through repair or maintenance actions. The study of machine repairable systems is essential across various fields, including manufacturing, industrial engineering, and operations research. This study aids in understanding and predicting machinery performance and reliability over time. Key indices in a machine repairable system include machine availability, server availability, and their respective failure rates. Failure times for machines and unreliable servers typically follow specific statistical patterns, such as the exponential distribution. These time distribution patterns are utilized to estimate the likelihood of failure occurring within a specified time frame. Failure times can also vary based on the operational state of the machine or server, whether functional or non-functional. Additionally, repair times can vary based on fault complexity and the availability of servers for repair. Reduced repair times enhance system availability and productivity. Effective management of machine repairable systems entails implementing preventive maintenance strategies to decrease failure frequency, optimizing spare parts inventory to minimize downtime, and training skilled repair personnel for efficient, timely repairs. Consequently, the analysis and design of machine repairable systems aim to maximize system performance, minimize costs related to failures and repairs, and ensure smooth operation of equipment and industrial processes .

Early literatures on repairable machine system are presented in a survey article[1]. The machine repairable system can be viewed as a queueing system with a finite customer source, encompassing both multi-server and single-server configurations. The single-server models are explored in references [2], [3], [4], [5], [6], [7], [8]. Research on multi-server systems is addressed in references [9], [10], [11], [12], [13], [14], [15], [16]. Chen et al.[17] analyzed the system reliability of a retrial machine repair system with warm standby units and a single repairman under the N-policy, deriving the reliability function and mean time to failure. Ke et al. [18] investigated a machine repairable system with standbys, in which multiple servers supervise the machines and implement a synchronous vacation policy. In their research, most scholars focus on steady-state characteristics, while some investigate transient-state indices [14], [19]. Ke et al. [20] presented optimization results as applications of their study.

In many real systems, it is common for machines to break down, necessitating the use of servers or maintenance equipment to address these failures. Machines are restored to their original condition after service and can resume operation. Furthermore, the servers responsible for maintaining the machines may also experience failures. When a server fails, a repairman will fix the faulty server, returning it to like-new condition after repair. Some researchers have examined models in which the server is unreliable, assuming that the server's failure rate is a constant value[9], [13]. However, in many cases, system parameters are not fixed due to varying working conditions [7], making it reasonable to assume that the server failure rate is variable in real systems. Yen et al.[21] studied the reliability and sensitivity analysis of a retrial machine repair problem involving working breakdowns under the F-policy. The server was subject to breakdown only when at least one machine in the system had failed, and it operated at a slow rate during these breakdowns. Performance measures, such as system reliability and mean time to system failure (MTTF), were derived using the Laplace transform technique. Meena et al. [22] studied a model in which the server takes a vacation when there are no failed machines in the system. The steady-state queue size distribution was established, and the Laplace-Stieltjes transform, along with recursive and supplementary variable approaches, was employed to derive the mean queue length, machine availability, system availability, and operational utilization.

In actual production, single-machine systems are common. If a machine may break down, a server is needed to service the breakdown. Furthermore, if the server is unreliable, it makes sense to have more than one server in

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the system. Therefore, we consider a repairable system with one machine, two unreliable servers, and one reliable repairman. The machine may break down at any time and will be serviced immediately if at least one server is available; after service, the machine will continue to operate. Furthermore, each server may fail at any time, and the failure rate is variable, adapting to whether the server is busy or not [8]. One reliable repairman is responsible for repairing server failures.

II. Model description

The system consists of one repairable machine, two unreliable servers, and one reliable repairman. The machine performs the system's function and may break down according to an independent Poisson process with a rate of λ. When the machine breaks down, it is immediately serviced by a server if there is at least one server is available; otherwise, the breakdown machine must wait till one server is available after repairing. The service time for the server attending to the breakdown machine follows an exponential distribution with parameter μ. The servers may fail at any time, with the time to failure following an exponential distribution characterized by different failure rates: ξ_1 during server idle time and ξ_2 during server busy time. The first failed server will be repaired by the repairman immediately. The repairman can repair only one failed server at a time and completes the repair in one go; the repair time follows an exponential distribution with a repair rate of η. The breakdown machine and the failed server will be as good as new after service and repair, respectively. All time distributions are mutually independent.

Let $N(t)$ denote the number of operational machines and $S(t)$ denote the number of available servers at time t, then $\{N(t), S(t)\}$ is a QBD process, as the time distributions are exponentially distributed. The system is in state (i, j) at time t if $N(t) = i$ and $S(t) = j$, the state space, arranged in lexicographic order, is as follows:

$$\Omega = \{(0,0),(0,1),(0,2),(1,0),(1,1),(1,2)\}.$$

We denote $P\{N(t) = i, S(t) = j\}$ as $P_{i,j}(t)$. Since the state space is finite and irreducible, the steady-state probability of the system state exists clearly[18]. We denote the steady-state probability of the system being in state (i, j) as $P_{i,j}$. Therefore, we have

$$P_{i,j} = \begin{cases} \lim_{t\to\infty} P_{(i,j)}(t), & i = 0,1, \quad j = 0,1,2, \\ 0, & \text{other.} \end{cases}$$

Since all the time distributions are exponential and mutually independent, the transitions of the system states form a QBD process [23]. By applying QBD process theory, we obtain the steady-state transition rate matrix $\bar{Q}$ as follows[24]:

$$\bar{Q} = \begin{bmatrix} A & B \\ C & D \end{bmatrix},$$

where

$$A = \begin{bmatrix} -\eta & \eta & 0 \\ \xi_2 & -\xi_2-\eta-\mu & \eta \\ 0 & \xi_1+\xi_2 & -\xi_1-\xi_2-\mu \end{bmatrix},$$

$$B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \mu & 0 \\ 0 & 0 & \mu \end{bmatrix}, \quad C = \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix},$$

$$D = \begin{bmatrix} -\eta-\lambda & \eta & 0 \\ \xi_1 & -\xi_1-\eta-\lambda & \eta \\ 0 & 2\xi_1 & -2\xi_1-\lambda \end{bmatrix}.$$

III. System analysis

A. Steady-state Probability of System State

The differential equations for the instantaneous state probabilities are as follows:

$$\begin{cases} P'_{0,0}(t) = -\eta P_{0,0}(t) + \lambda P_{1,0}(t) + \xi_2 P_{0,1}(t), \\ P'_{0,1}(t) = \eta P_{0,0}(t) - (\mu + \xi_2 - \eta) P_{0,1}(t) + \\ \qquad (\xi_1 + \xi_2) P_{0,2}(t) + \lambda P_{1,1}(t), \\ P'_{0,2}(t) = \xi_2 P_{0,1}(t) - (\eta + \xi_1 + \xi_2) P_{0,2}(t) + \lambda P_{1,2}(t), \\ P'_{1,0}(t) = -(\eta + \lambda) P_{1,0}(t) + \xi_1 P_{1,1}(t), \\ P'_{1,1}(t) = \mu P_{0,1}(t) + \eta P_{1,0}(t) - \\ \qquad (\eta + \lambda + \xi_1) P_{1,1}(t) + 2\xi_1 P_{1,2}(t), \\ P'_{1,2}(t) = \eta P_{1,1}(t) - (\lambda + 2\xi_1) P_{0,2}(t) + \mu P_{0,2}(t). \end{cases} \tag{1}$$

From Eq. (1), letting $t \to \infty$ we obtain the following steady-state probability equations:

$$\begin{cases} -\eta P_{0,0} + \xi_2 P_{0,1} + \lambda P_{1,0} = 0, \\ \eta P_{0,0} - (\mu + \xi_2 - \eta) P_{0,1} + (\xi_1 + \xi_2) P_{0,2} + \lambda P_{1,1} = 0, \\ \eta P_{0,1} - (\mu + \xi_1 + \xi_2) P_{0,2} + \lambda P_{1,2} = 0, \\ -(\eta + \lambda) P_{1,0} + \xi_1 P_{1,1} = 0, \\ \mu P_{0,1} + \eta P_{1,0} - (\eta + \lambda + \xi_1) P_{1,1} + 2\xi_1 P_{1,2} = 0, \\ \mu P_{0,2} + \eta P_{1,1} - (\lambda + 2\xi_1) P_{1,2} = 0, \\ P_{0,0} + P_{0,1} + P_{0,2} + P_{1,0} + P_{1,1} + P_{1,2} = 1. \end{cases} \tag{2}$$

Letting

$$\begin{aligned} \Phi = {} & 2\xi_1^3 \left[\eta\lambda + (\eta+\lambda)\mu + \lambda\xi_2\right] + \\ & (\eta+\lambda) \Big\{ \eta^2 (\lambda+\mu)(\eta+\lambda+\mu) + \\ & \quad (\eta+\lambda)\xi_2 \left[\eta(\lambda+\mu) + \lambda\xi_2\right] \Big\} + \\ & \xi_1^2 \Big\{ 4\eta^2(\lambda+\mu) + \lambda\mu(\lambda+2\mu) + \\ & \quad \eta(3\lambda+\mu)(\lambda+2\mu) + \\ & \quad \xi_2 \left(4\eta\lambda + 3\lambda^2 + 2\eta\mu + 4\lambda\mu + 2\lambda\xi_2\right) \Big\} + \\ & \xi_1 \Big\{ \eta(\lambda+\mu)[3\eta^2 + 5\eta\lambda + \lambda^2 + 2(\eta+\lambda)\mu] + \\ & \quad \xi_2[5\eta\lambda(\lambda+\mu) + \eta^2(3\lambda+2\mu) + \\ & \quad \lambda^2(\lambda+3\mu) + \lambda(2\eta+3\lambda)\xi_2] \Big\}, \end{aligned}$$

then the steady-state probabilities of the system as the solutions of Eq. (2) can be expressed as follows:

$$P_{0,0}=\lambda\Big\{\mu\xi_1^2(2\eta+\lambda+2\mu+2\xi_1)+\xi_1\Big[(\eta+\lambda)^2+(2\eta+3\lambda)\mu+\xi_1(2\eta+3\lambda+4\mu+2\xi_1)\Big]\xi_2+\Big[(\eta+\lambda)^2+\xi_1(2\eta+3\lambda+2\xi_1)\Big]\xi_2^2\Big\}\Phi^{-1},$$

$$P_{0,1}=\eta\lambda\Big\{\xi_1\Big[(\eta+\lambda)(\eta+\lambda+2\mu)+\xi_1(2\eta+3\lambda+2\mu+2\xi_1)\Big]+\Big[(\eta+\lambda)^2+\xi_1(2\eta+3\lambda+2\xi_1)\Big]\xi_2\Big\}\Phi^{-1},$$

$$P_{0,2}=\eta^2\lambda\Big[(\eta+\lambda)(\eta+\lambda+\mu)+\xi_1(2\eta+3\lambda+2\xi_1)\Big]\Phi^{-1},$$

$$P_{1,0}=\eta\mu\xi_1\Big[\xi_1(2\eta+\lambda+2\mu+2\xi_1)+(\lambda+2\xi_1)\xi_2\Big]\Phi^{-1},$$

$$P_{1,1}=\eta(\eta+\lambda)\mu\Big[\xi_1(2\eta+\lambda+2\mu+2\xi_1)+(\lambda+2\xi_1)\xi_2\Big]\Phi^{-1},$$

$$P_{1,2}=\eta^2\mu\Big[(\eta+2\lambda)\xi_1+(\eta+\lambda)(\eta+\lambda+\mu+\xi_2)\Big]\Phi^{-1}.$$

B. Steady-state Indexes of System Performance

Utilizing the steady-state probabilities mentioned above, we can derive the significant indices of the system as follows:

◆The steady-state availability of the machine is

$$\begin{aligned}AM&=P_{1,0}+P_{1,1}+P_{1,2}\\&=\eta\mu\Big\{\eta(\eta+\lambda)(\eta+\lambda+\mu)+\xi_1\Big[3\eta^2+5\eta\lambda+\lambda^2+2(\eta+\lambda)\mu+\xi_1(4\eta+3\lambda+2\mu+2\xi_1)\Big]+\Big[(\eta+\lambda)^2+\xi_1(2\eta+3\lambda+2\xi_1)\Big]\xi_2\Big\}\Phi^{-1},\end{aligned}$$

◆The steady-state availability of the server is

$$\begin{aligned}AS&=P_{0,1}+P_{0,2}+P_{1,1}+P_{1,2}\\&=\eta\Big\{2\lambda\xi_1^3+\xi_1^2\left(4\eta\lambda+3\lambda^2+2\eta\mu+4\lambda\mu+2\lambda\xi_2\right)+(\eta+\lambda)(\lambda+\mu)\Big[\eta(\eta+\lambda+\mu)+(\eta+\lambda)\xi_2\Big]+\xi_1\Big\{(\lambda+\mu)\Big[3\eta^2+5\eta\lambda+\lambda^2+2(\eta+\lambda)\mu\Big]+\Big[\lambda(2\eta+3\lambda)+2(\eta+\lambda)\mu\Big]\xi_2\Big\}\Big\}\Phi^{-1},\end{aligned}$$

◆The steady-state probability of the repairman being busy is

$$\begin{aligned}RB&=1-P_{0,2}-P_{1,2}\\&=1-\eta^2\Big\{(\eta+\lambda)(\lambda+\mu)(\eta+\lambda+\mu)+\xi_1\Big[\eta(2\lambda+\mu)+\lambda(3\lambda+2\mu)+2\lambda\xi_1\Big]+(\eta+\lambda)\mu\xi_2\Big\}\Phi^{-1},\end{aligned}$$

◆The steady-state breakdown frequency of the machine is

$$\begin{aligned}BFM&=\lambda AM=\lambda(P_{1,0}+P_{1,1}+P_{1,2})\\&=\lambda\eta\mu\Big\{\eta(\eta+\lambda)(\eta+\lambda+\mu)+\xi_1\Big[3\eta^2+5\eta\lambda+\lambda^2+2(\eta+\lambda)\mu+\xi_1(4\eta+3\lambda+2\mu+2\xi_1)\Big]+\Big[(\eta+\lambda)^2+\xi_1(2\eta+3\lambda+2\xi_1)\Big]\xi_2\Big\}\Phi^{-1},\end{aligned}$$

◆The steady-state failure frequency of the servers is

$$\begin{aligned}FFS&=\xi_2P_{0,1}+(\xi_1+\xi_2)P_{0,2}+\xi_1P_{1,1}+2\xi_1P_{1,2}\\&=\Big\{\eta^2\lambda\Big[(\eta+\lambda)(\eta+\lambda+\mu)+\xi_1(2\eta+3\lambda+2\xi_1)\Big](\xi_1+\xi_2)+\\&\eta(\eta+\lambda)\mu\xi_1\Big[\xi_1(2\eta+\lambda+2\mu+2\xi_1)+(\lambda+2\xi_1)\xi_2\Big]+\\&\eta\lambda\xi_2\Big[\xi_1\Big((\eta+\lambda)(\eta+\lambda+2\mu)+\xi_1(2\eta+3\lambda+2\mu+2\xi_1)\Big)+\Big((\eta+\lambda)^2+\xi_1(2\eta+3\lambda+2\xi_1)\Big)\xi_2\Big]+\\&2\eta^2\mu\xi_1\Big[(\eta+2\lambda)\xi_1+(\eta+\lambda)(\eta+\lambda+\mu+\xi_2)\Big]\Big\}\Phi^{-1}.\end{aligned}$$

IV. Special cases

A special case is letting $\eta\to\infty$ or $\xi_1=\xi_2=0$, meaning that the servers are always available. In this case, the steady-state probabilities are as follows:

$$P_{0,0}=0,\quad P_{0,1}=0,\quad P_{0,2}=\frac{\lambda}{\lambda+\mu},$$

$$P_{1,0}=0,\quad P_{1,1}=0,\quad P_{1,2}=\frac{\mu}{\lambda+\mu}.$$

Then the steady-state availability of the machine is

$$AM=P_{1,0}+P_{1,1}+P_{1,2}=\frac{\mu}{\lambda+\mu}.$$

The steady-state availability of the server is

$$AS=P_{0,1}+P_{0,2}+P_{1,1}+P_{1,2}=1.$$

The steady-state probability of the repairman being busy is

$$RB=1-P_{0,2}-P_{1,2}=0.$$

The steady-state breakdown frequency of the machine is

$$BFM=\lambda AM=\lambda(P_{1,0}+P_{1,1}+P_{1,2})=\frac{\lambda\mu}{\lambda+\mu}.$$

The steady-state failure frequency of the servers is

$$FFS=\xi_2P_{0,1}+(\xi_1+\xi_2)P_{0,2}+\xi_1P_{1,1}+2\xi_1P_{1,2}=0.$$

These results are consistent with the corresponding results in the reference [24].

V. Numerical experiments

A. Numerical example

Letting $\lambda = 1, \mu = 1.5, \xi_1 = 0.5, \xi_2 = 1, \eta = 2$, we obtain

$$P_{0,0} = 0.0902, P_{0,1} = 0.1507, P_{0,2} = 0.2132,$$

$$P_{1,0} = 0.0297, P_{1,1} = 0.1782, P_{1,2} = 0.3380.$$

Then the steady-state availability of the machine is

$$AM = P_{1,0} + P_{1,1} + P_{1,2} = 0.5459.$$

The steady-state availability of the server is

$$AS = P_{0,1} + P_{0,2} + P_{1,1} + P_{1,2} = 0.8801.$$

The steady-state probability of the repairman being busy is

$$RB = 1 - P_{0,2} - P_{1,2} = 0.4488.$$

The steady-state breakdown frequency of the machine is

$$BFM = \lambda AM = \lambda(P_{1,0} + P_{1,1} + P_{1,2}) = 05459.$$

The steady-state failure frequency of the servers is

$$FFS = \xi_2 P_{0,1} + (\xi_1 + \xi_2) P_{0,2} + \xi_1 P_{1,1} + 2\xi_1 P_{1,2} = 0.8976.$$

B. Parametric sensitivity

The purpose of this section is to examine the effects of the parameters on the steady-state availability of the machine(AM) and the steady-state availability of the server(AS).

In Fig. 1, we set $\xi_1 = 0.1, \xi_2 = 0.2$ and $\eta = 0.8$, and calculate the steady-state availability of the machine(AM) by varying the values of μ and λ. The results indicate that AM decreases as λ increases and μ decreases, with the influence of μ becoming significant as λ decreases. Furthermore, when the values of μ and λ are relatively low, their mutual influence is more pronounced, and $AM = 1$ when $\lambda = 0$, which is intuitively clear.

In Fig. 2, we set $\xi_1 = 0.1, \xi_2 = 0.2$ and $\eta = 0.8$, and calculate the steady-state availability of the servers(AS) by varying the values of μ and λ. The results indicate that AS decreases as λ increases and μ decreases. Additionally, the influence of μ becomes significant as λ decreases. Similarly, the influence of λ becomes significant as μ decreases. Given that the failure rate of the server is different between idle and busy times, it is evident that μ and λ have a significant impact on the steady-state availability of the server(AS).

In Fig. 3, we set $\lambda = 0.3, \mu = 0.5$ and $\eta = 1.2$, and calculate the steady-state availability of the machine(AM) by varying the values of ξ_1 and ξ_2. The results indicate that AM decreases as ξ_1 and ξ_2 increase. Additionally, AM is clearly less than 1 when $\xi_1 = 0$ and $\xi_2 = 0$. Given the relatively high value of η, the influence of ξ_1 and ξ_2 is less significant.

In Fig. 4, we set $\lambda = 0.3, \mu = 0.5$ and $\eta = 1.2$, and calculate the steady-state availability of the servers(AS) by varying the values of ξ_1 and ξ_2 . The results indicate that AS decreases as ξ_1 and ξ_2 increase. Additionally, $AS = 1$ when $\xi_1 = 0$ and $\xi_2 = 0$, which is intuitively clear.

In Fig. 5, we set $\xi_1 = 0.1, \xi_2 = 0.2$ and $\lambda = 0.1$, and calculate the steady-state availability of the machine(AM) by varying the values of μ and η. The results indicate that

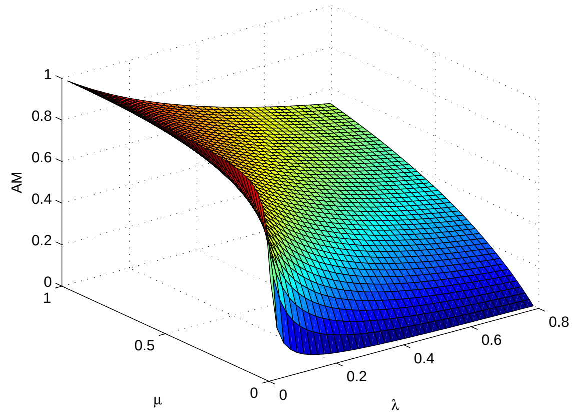

Fig. 1. The steady-state availability of the machine AM versus λ and $\mu(\xi_1 = 0.1, \xi_2 = 0.2, \eta = 0.8)$.

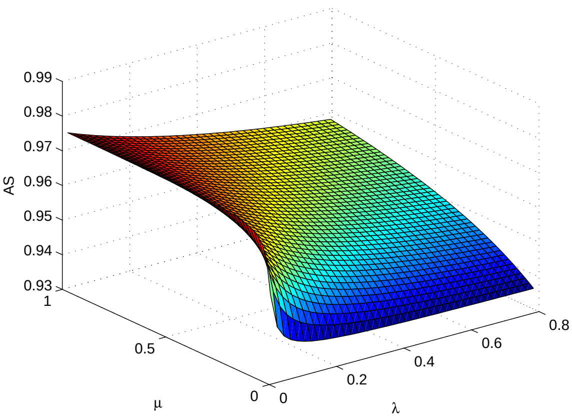

Fig. 2. The steady-state availability of the servers AS versus λ and $\mu(\xi_1 = 0.1, \xi_2 = 0.2, \eta = 0.8)$.

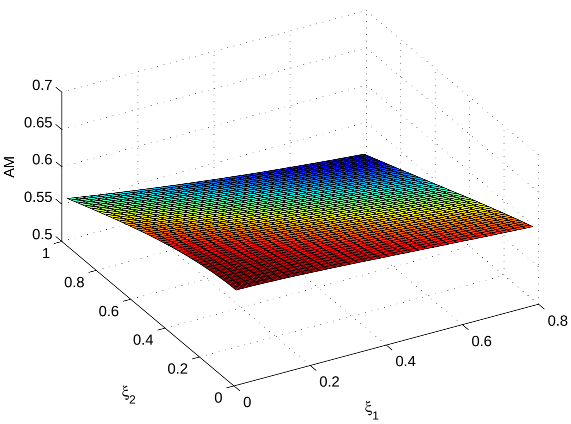

Fig. 3. The steady-state availability of the machine AM versus ξ_1 and $\xi_2(\lambda = 0.3, \mu = 0.5, \eta = 1.2)$.

AM decreases as η and μ decrease, and $AM = 0$ when $\mu = 0$ or $\eta = 0$. We also find that μ and η have a similar significance for the steady-state availability of the machine(AM), and when the values of μ and η are relatively low, their mutual influence is more pronounced.

In Fig. 6, we set $\xi_1 = 0.1, \xi_2 = 1$ and $\lambda = 0.1$, and calculate

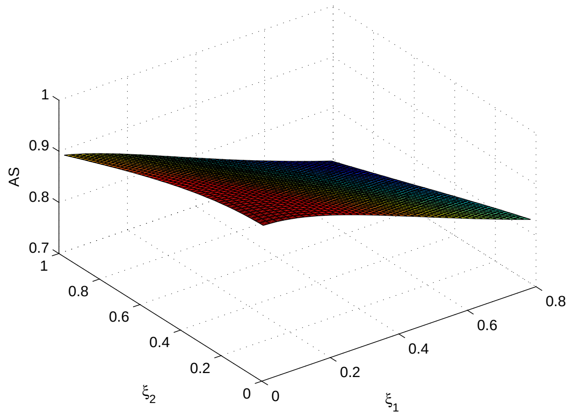

Fig. 4. The steady-state availability of the servers AS versus ξ_1 and $\xi_2(\lambda = 0.3, \mu = 0.5, \eta = 1.2)$.

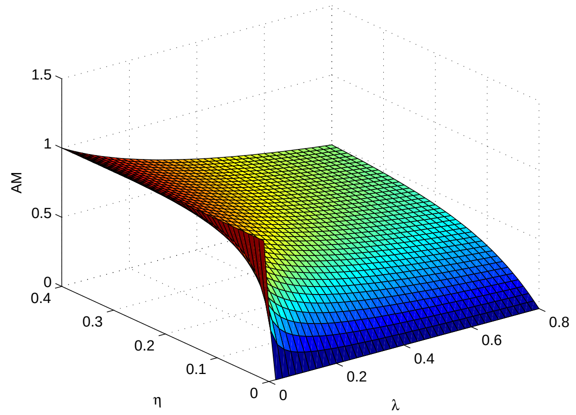

Fig. 7. The steady-state availability of the servers AM versus η and $\lambda(\xi_1 = 0.1, \xi_2 = 0.2, \mu = 1)$.

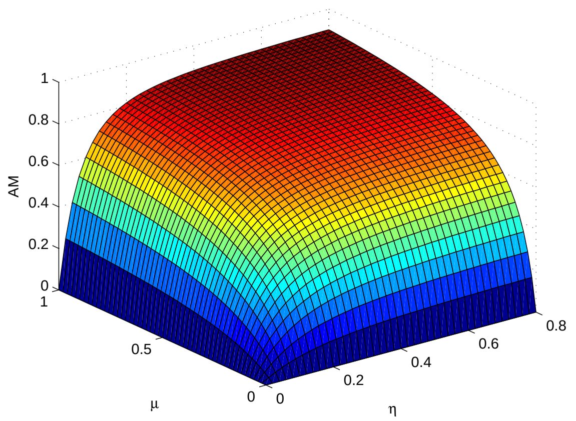

Fig. 5. The steady-state availability of the servers AM versus μ and $\eta(\lambda = 0.1, \xi_1 = 0.1, \xi_2 = 0.2)$.

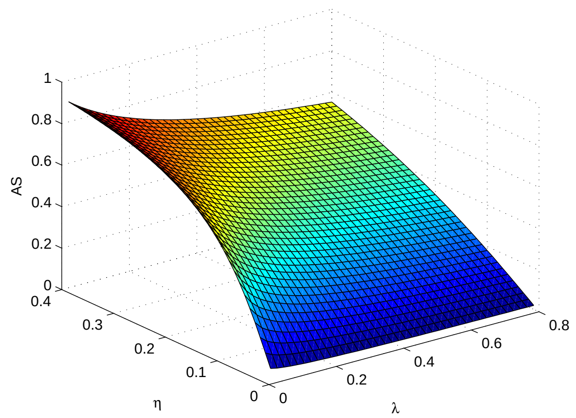

Fig. 8. The steady-state availability of the servers AS versus η and $\lambda(\xi_1 = 0.1, \xi_2 = 1, \mu = 1)$.

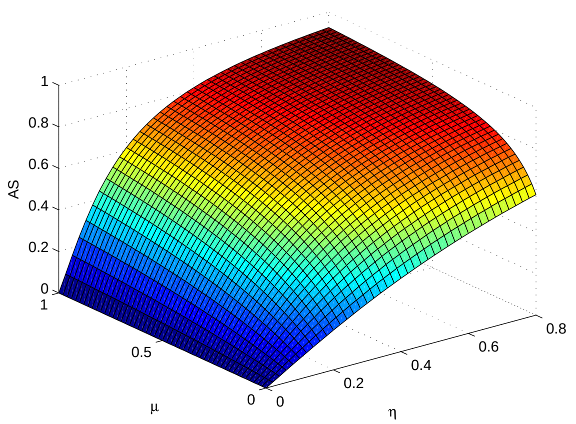

Fig. 6. The steady-state availability of the servers AS versus μ and $\eta(\lambda = 0.1, \xi_1 = 0.1, \xi_2 = 1)$.

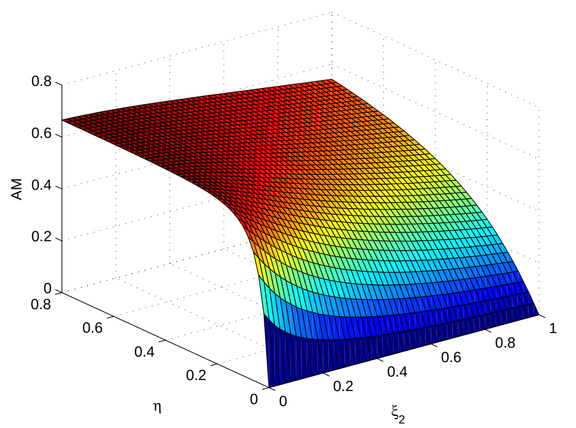

Fig. 9. The steady-state availability of the machine AM versus η and $\xi_2(\xi_1 = 0.1, \lambda = 0.1, \mu = 0.2)$.

the steady-state availability of the servers(AS) by varying the values of μ and η . The results indicate that AS decreases as η and μ decrease, and $AS = 0$ when $\eta = 0$, These observations align with our intuitive expectations. We also find that μ and η have a similar significance for the steady-state availability of the servers(AS).

In Fig. 7, we set $\xi_1 = 0.1, \xi_2 = 0.2$ and $\mu = 1$, and calculate the steady-state availability of the machine(AM) by varying the values of λ and η. The results indicate that AM decreases as η decreases and λ increases, $AM = 1$ when $\lambda = 0$, and $AM = 0$ when $\eta = 0$. The figure demonstrates that the combined influence of λ and η on the

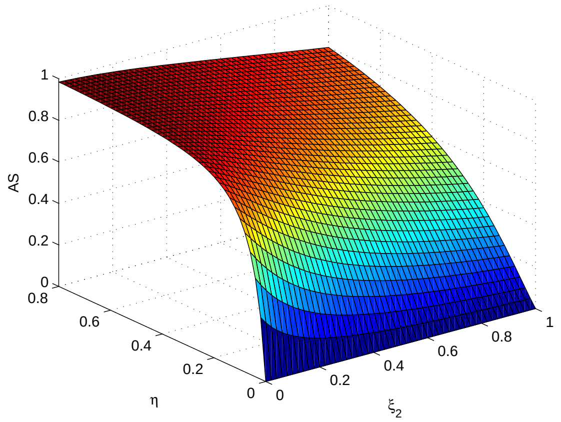

Fig. 10. The steady-state availability of the servers AS versus η and ξ_2(ξ_1 = 0.1, $\lambda = 0.1, \mu = 0.2$).

steady-state availability of the machine(AM) is significant.

In Fig. 8, we set $\xi_1 = 0.1, \xi_2 = 1$ and $\mu = 1$, and calculate the steady-state availability of the servers(AS) by varying the values of λ and η. The results indicate that AS decreases as η decreases and λ increases, and AS approaches 0 as η approaches 0. The influence of η increases as the value of λ decreases because ξ_2 is larger than ξ_1. Consequently, the value of AS will significantly increase with decreasing of λ. The figure demonstrates that the combined effect of λ and η on the steady-state availability of the servers(AS) is substantial.

In Fig. 9, we set $\xi_1 = 0.1, \lambda = 0.1$ and $\mu = 0.2$. We then calculate the steady-state availability of the machine(AM) by varying the values of ξ_2 and η. The results demonstrate that AM decreases with η decreases as η decreases and ξ_2 increases, reaching $AM = 0$ when $\eta = 0$. Our analysis indicates that when both η and ξ_2 are small, the system indicator AM is more sensitive to variations in these parameters. The figure illustrates that the combined influence of ξ_2 and η on the steady-state availability of the machine(AM) is significant.

In Fig. 10, we set $\xi_1 = 0.1, \lambda = 0.1$ and $\mu = 0.2$, We then calculate the steady-state availability of the server (AS) by varying the values of ξ_2 and η. The results indicate that AS decreases as η decreases and ξ_2 increases, reaching $AS = 0$ when $\eta = 0$. Conversely, AS approaches 1 as η increases and ξ_2 decreases. Our analysis reveals that when both ξ_2 and η are small, the system indicator AS is more sensitive to variations in these parameters. The figure illustrates that the combined effect of ξ_2 and η on the steady-state availability of the servers(AS) is significant.

We thoroughly analyze the previously mentioned numerical experimental results and identify a general rule: the changes in AM and AS show a positive correlation under identical conditions, aligning with our intuition. These numerical results illustrate the impacts and functional roles of system parameters on key system indicators. The experiments provide us with a robust understanding of the operational principles of the system, which are crucial for efficiently managing its operation.

VI. Conclusion

The model presented in this paper represents a common system in practice. It assumes that the server is unreliable, which is reasonable given that there are more servers than machines, as well as more repairmen in the system. Future work could involve analyzing a model with more machines and repairmen in the system. Furthermore, optimization design is a crucial aspect of the model. Cost-benefit analysis could be a significant focus for future work on this model.

References

[1] L. Haque and M. J. Armstrong, "A survey of the machine interference problem," *European Journal of Operational Research*, vol. 179, no. 2, pp. 469–482, 2007.

[2] U. C. Gupta and T. S. S. S. Rao, "On the M/G/1 machine interference model with spares," *European Journal of Operational Research*, vol. 89, no. 1, pp. 164–171, 1996.

[3] K. V. Vijayashree and A. Anjuka, "Stationary Analysis of an M/M/1 Driven Fluid Queue Subject to Catastrophes and Subsequent Repair," *Iaeng International Journal of Applied Mathematics*, vol. 43, no. 4, pp. 238–241, 2013.

[4] S. Lv, R. Wang, J. Li, and Y. Wu, "The M/M/1 Queueing System with Bernoulli Secondary Service and Variable Service Rate," *Engineering Letters*, vol. 30, no. 4, pp. 1253–1259, 2022.

[5] A. I. Shawky, "The single-server machine interference model with balking, reneging and an additional server for longer queues," *Microelectronics Reliability*, vol. 37, no. 2, pp. 355–357, 1997.

[6] M. Jain, "Optimal N-policy for single server Markovian queue with breakdown, repair and state dependent arrival rate," *International Journal of Management Systems*, vol. 13, no. 3, pp. 245–260, 1997.

[7] S. Ouaret, J.-P. Kenn, and A. Gharbi, "Joint Production and Replacement Planning for an Unreliable Manufacturing System Subject to Random Demand and Quality," *IFAC-PapersOnLine*, vol. 51, no. 11, pp. 951–956, 2018.

[8] F. A. Van, deDuyn Schouten and P. Wartenhorst, "A two-machine repair model with variable repair rate," *Naval Research Logistics (NRL)*, vol. 40, no. 4, pp. 495–523, 1993.

[9] Wang and K.-H, "Profit Analysis of the M/M/R Machine Repair Problem with Spares and Server Breakdowns," *Journal of the Operational Research Society*, vol. 45, no. 5, pp. 539–548, 1994.

[10] M. Jain, Rakhee, and S. Maheshwari, "N-policy for a machine repair system with spares and reneging," *Applied Mathematical Modelling*, vol. 28, no. 6, pp. 513–531, 2004.

[11] S. Sani, "The M/G/2 Queue with Heterogeneous Servers Under a Controlled Service Discipline: Stationary Performance Analysis," *IAENG International Journal of Applied Mathematics*, vol. 45, no. 1, pp. 31–40, 2015.

[12] S. Lv and T. Li, "Optimization study of M/M/c machine repairable system with two service rates," *Engineering Letters*, vol. 30, no. 4, pp. 1537–1542, 2022.

[13] M. Jain and R. P. Ghimire, "Machine repair queueing system with non-reliable service stations and heterogeneous service discipline," *J.MACT*, vol. 30, pp. 105–115, 1997.

[14] M. Jain and S. Maheshwari, "Transient analysis of a redundant system with additional repairmen," *American Journal of Mathematical and Management Sciences*, vol. 23, no. 3-4, pp. 347–382, 2003.

[15] C. H. Wu and J. C. Ke, "Multi-server machine repair problems under a (v,r) synchronous single vacation policy," *Applied Mathematical Modelling*, vol. 38, no. 7-8, pp. 2180–2189, 2014.

[16] D. Yue, W. Yue, and H. Qi, "Analysis of a machine repair system with warm spares and n-policy vacations," in *The 7 th International Symposium on Operations Research and Its Applications (ISORA08) China*, 2008.

[17] C. Wu-Lin and W. Kuo-Hsiung, "Reliability analysis of a retrial machine repair problem with warm standbys and a single server with n-policy," *Reliability Engineering System Safety*, vol. 180, no. DEC., pp. 476–486, 2018.

[18] J. C. Ke and C. H. Wu, "Multi-server machine repair model with standbys and synchronous multiple vacation," *Computers Industrial Engineering*, vol. 62, no. 1, pp. 296–305, 2012.

[19] Chen, Wu-Lin, Zheng-Han, Wen, and Chia-Hsien, "System reliability of a machine repair system with a multiple-vacation and unreliable server," *Journal of Testing and Evaluation: A Multidisciplinary Forum for Applied Sciences and Engineering*, vol. 44, no. 4, pp. 1745–1755, 2016.

[20] J. C. Ke, T. H. Liu, and D. Y. Yang, "Machine repairing systems with standby switching failure," *Computers Industrial Engineering*, vol. 99, no. sep., pp. 223–228, 2016.

[21] T. C. Y. A, K. H. W. B, and C. H. W. C, "Reliability-based measure of a retrial machine repair problem with working breakdowns under the f -policy," *Computers Industrial Engineering*, vol. 150, 2020.

[22] R. K. Meena, M. Jain, S. S. Sanga, and A. Assad, "Fuzzy modeling and harmony search optimization for machining system with general repair, standby support and vacation," *Applied Mathematics and Computation*, vol. 361, pp. 858–873, 2019.

[23] M. F. Neuts, *Matrix-Geometric solutions in stochastic models*. London: The Johns Hopkins University press Ltd, 1981, pp. 254–300.

[24] J. H. Cao, *Introduction to reliability mathematics*. Science Press, 2012, pp. 189–220.