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Dynamic Behaviors of a Two-Species Amensalism Model with Wind-Dependent Refuge

Shiyu Cai, Zuzhi Cai and Fengde Chen

***Abstract*—This paper investigates a two-species amensalism model with a wind-dependent refuge. In classical amensalism models, the refuge (or cover) is typically treated as a fixed parameter. However, in natural environments, the effectiveness of the refuge may be influenced by environmental factors such as wind. To more accurately reflect the dynamic behaviors in natural settings, we introduce a wind-dependent refuge parameter $k(w)$, assuming that the size of the refuge decreases as wind speed increases. By analyzing the equilibrium points and their stability, we explore the impact of wind speed on the coexistence of the two species. Numerical simulations demonstrate that increasing wind speed reduces the effectiveness of the refuge, thereby affecting the coexistence state of the species. This study provides new insights into the influence of environmental factors on ecosystems and offers theoretical support for the conservation of endangered species.**

***Index Terms*—Amensalism model, refuge, wind speed, stability analysis, numerical simulation**

I. INTRODUCTION

In ecology, **amensalism** represents an asymmetric interaction between two species, where one species is negatively affected while the other remains unaffected. This relationship is prevalent in natural ecosystems, exemplified by certain plants releasing allelochemicals that inhibit the growth of neighboring plants without affecting themselves [1]. Such interactions play a crucial role in shaping community structure and ecosystem dynamics. Over the past few decades, the study of amensalism has gained significant attention, resulting in a wealth of findings across continuous [2]-[25], discrete [26]-[36], and hybrid modeling frameworks. Key advances include investigations into functional responses [3], Allee effects [3], [7], refuge dynamics [15], [17], [33], fear effect [32], delay[8] and human harvesting impacts [6], [9], [30].

A **refuge**—a protected area that reduces a species' exposure to predation or competition—is a critical factor in regulating species dynamics. In classical ecological models, the refuge is often treated as a static parameter, assuming its effectiveness remains constant over time [17]. However, in natural environments, the effectiveness of refuges is highly dynamic and can be influenced by various environmental factors, such as **wind**, **temperature**, and **humidity** [43]. Among these, wind is a particularly pervasive factor that not only affects vegetation structure and distribution but also alters microclimate conditions within refuges, indirectly impacting species survival and reproduction [44] -[49]. Recently, some scholars [37]-[57] have started attempting to study the impact of wind on the dynamic behavior of predator-prey systems using mathematical modeling.

Wind, as a significant environmental factor, influences refuge effectiveness through multiple mechanisms:

(1) Changes in vegetation structure: Strong winds can cause physical damage to vegetation, such as breaking branches or uprooting plants, leading to a reduction in vegetation density and coverage [45]. This directly diminishes the effectiveness of refuges, as species are more exposed to predation or competition.

(2) Alteration of microclimate conditions: Wind can also modify microclimate conditions within refuges, such as temperature and humidity. For instance, increased wind speed can lower temperatures within the refuge, which may adversely affect ectothermic species like insects and reptiles [43]. Additionally, wind can accelerate evaporation, reducing humidity levels and impacting species that rely on moist environments for survival and reproduction.

(3) Behavioral responses of species: Wind can influence the behavior of species, particularly their foraging and reproductive activities. For example, strong winds may hinder the flight of insects, affecting their ability to find food or mates. Similarly, wind can alter the dispersal of seeds or pollen, impacting plant reproduction and the availability of resources for other species.

(4) Interaction with other environmental factors: Wind often interacts with other environmental factors, such as temperature and precipitation, to further influence refuge effectiveness. For example, in hot climates, strong winds can exacerbate water loss through evaporation, leading to drier conditions within the refuge. Conversely, in cooler climates, wind may lower temperatures, creating harsher conditions for species that are sensitive to cold.

A rather interesting observation is that, to the author's knowledge, no scholar has yet explored the impact of wind on refuge from a mathematical modeling perspective. To address the dynamic nature of refuge effectiveness, we propose a **two-species amensalism model** that incorporates a **wind-dependent refuge parameter**, $k(w)$, which decreases with increasing wind speed. This approach allows us to more accurately reflect the dynamic behaviors of species in natural environments, where wind speed can vary significantly over time and space. By analyzing the equilibrium points and their stability, we aim to explore how wind speed affects the coexistence of two species in an amensalistic relationship. Our numerical simulations demonstrate that increasing wind speed reduces the effectiveness of the refuge, thereby influencing the coexistence state of the species.

This study contributes to the growing body of literature on the impact of environmental factors on ecosystem dynamics.

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S. Y. Cai is a postgraduate student of College of Mathematics and Statistics, Fuzhou University, Fuzhou, CHINA (e-mail:1326514571@qq.com).

Z. Z. Cai is a postgraduate student of College of Mathematics and Statistics, Fuzhou University, Fuzhou, CHINA (e-mail: 1103130482@qq.com).

F. D. Chen is a professor of College of Mathematics and Statistics, Fuzhou University, Fuzhou, CHINA (e-mail: fdchen@fzu.edu.cn).

By extending the classical amensalism model to include wind-dependent refuge effects, we provide new insights into how environmental variability can regulate species interactions. Our findings have important implications for the conservation of endangered species, particularly in wind-prone regions where refuge effectiveness may be compromised. For more works on amensalism species, one could refer to [2]-[36] and the references therein. For more works on wind effect, one could refer to [37]-[57].

II. MODEL CONSTRUCTION

A. Ecological background

In ecosystems, **amensalism** is a common interspecific relationship where one species is negatively affected while the other remains unaffected. A classic example is the release of allelochemicals by certain plants, which inhibit the growth of neighboring plants without affecting themselves [1]. Refuges (or covers) play a crucial role in regulating species dynamics by providing protection against predation or competition [42]. However, in natural environments, the effectiveness of refuges is not static but can be influenced by various environmental factors, particularly **wind** [43].

Wind is a pervasive environmental factor that affects vegetation structure, microclimate conditions, and species behavior [44]. For example, strong winds can reduce the density of vegetation cover, thereby diminishing the effectiveness of refuges [45]. This dynamic interaction between wind and refuge effectiveness has significant implications for species survival and coexistence, yet it has been largely overlooked in classical ecological models.

B. Model dynamic equations

In the classical two-species amensalism model[17], the refuge is typically treated as a fixed parameter k, representing the size or effectiveness of the refuge. However, to more realistically reflect dynamic behaviors in natural environments, we introduce a **wind-dependent refuge parameter** $k(w)$, where w denotes wind speed. Assuming the refuge size decreases with increasing wind speed, the specific form is given by:

$$k(w) = k_0 e^{-\alpha w},$$

where:

- k_0 $(0 < k_0 < 1)$ is the refuge size in windless conditions,
- α is a positive parameter indicating the strength of wind speed's impact on the refuge.

Now let's consider the following example: take $k_0 = 0.5$ and consider $k(w) = 0.5e^{-\alpha w}$, Fig.1 shows that with the increasing of α, $k(w)$ approaches zero more rapidly, consequently, Correspondingly, the role of the shelter diminishes more rapidly.

The model is described by the following system of differential equations:

$$\begin{aligned} \frac{dx}{dt} &= a_1 x(t) - b_1 x^2(t) - c_1(1-k(w))x(t)y(t), \\ \frac{dy}{dt} &= a_2 y(t) - b_2 y^2(t), \end{aligned} \tag{1}$$

where:

- $x(t)$ and $y(t)$ represent the population densities of species x and y at time t, respectively,
- a_1 and a_2 are the intrinsic growth rates of species x and y, respectively,
- b_1 and b_2 are the intraspecific competition coefficients of species x and y, respectively,
- c_1 is the negative impact coefficient of species y on species x,
- $k(w) = k_0 e^{-\alpha w}$ is the wind-dependent refuge parameter, representing the effect of wind speed w on the size of the refuge.

The refuge parameter $k(w)$ is modeled as an exponential decay function of wind speed w, where k_0 is the maximum refuge effectiveness (at zero wind speed) and α is a decay constant that determines how rapidly the refuge effectiveness decreases with increasing wind speed. This functional form is supported by empirical studies showing that wind can significantly reduce vegetation cover and alter microclimate conditions [44], [45].

C. Justification of model assumptions

The assumption that refuge effectiveness decreases with wind speed is based on empirical evidence. For instance, studies have shown that wind can cause physical damage to vegetation, reducing its density and coverage [44]. Additionally, wind can alter microclimate conditions within refuges, such as temperature and humidity, further impacting their effectiveness [43]. The exponential decay function $k(w) = k_0 e^{-\alpha w}$ is chosen to reflect the nonlinear relationship between wind speed and refuge effectiveness, as observed in field studies [45].

It must be pointed out that many biologists have already observed the effects of wind on predator-prey populations, as discussed in the review literature[50]. In recent years, several researchers[51]-[57] have used mathematical modeling to explore the impact of wind speed on the dynamical behavior of predator-prey systems. However, to date, no scholar has considered the influence of wind on the dynamical behavior of an amensalism population model.

III. LOCAL STABILITY ANALYSIS OF EQUILIBRIUM POINTS

In this section, we analyze the equilibrium points of the model and determine their local stability by computing the eigenvalues of the Jacobian matrix.

A. Equilibrium Points

The equilibrium points are the steady-state solutions of the system, satisfying $\frac{dx}{dt} = 0$ and $\frac{dy}{dt} = 0$. We solve the following system of equations:

$$\begin{aligned} &a_1 x - b_1 x^2 - c_1(1-k(w))xy = 0, \\ &a_2 y - b_2 y^2 = 0. \end{aligned} \tag{2}$$

By solving the above system, we obtain the following equilibrium points:

- $E_0(0,0)$: - This is the trivial equilibrium point, representing the extinction of both species.

- $E_1\left(\frac{a_1}{b_1},0\right)$: - This is the equilibrium point where only species x exists, representing the extinction of species y.
- $E_2\left(0,\frac{a_2}{b_2}\right)$: - This is the equilibrium point where only species y exists, representing the extinction of species x.
- $E_3(x^*,y^*)$: - This is the coexistence equilibrium point, where:

  $$x^*=\frac{a_1b_2-a_2c_1(1-k(w))}{b_1b_2},\quad y^*=\frac{a_2}{b_2}.$$

  The condition for the existence of this equilibrium point is $a_1b_2>a_2c_1(1-k(w))$.

Concerned with the local stability property of the above four equilibria, we have

Theorem 3.1. *$E_0(0,0)$ and $E_1(\frac{a_1}{b_1},0)$ are unstable; If $a_1b_2<a_2c_1(1-k(w))$, then $E_2(0,\frac{a_2}{b_2})$ is stable and if $a_1b_2>a_2c_1(1-k(w))$, then $E_2(0,\frac{a_2}{b_2})$ is unstable; If $a_1b_2>a_2c_1(1-k(w))$ hold, $E_3(x^*,y^*)$ is stable.*

Proof. To analyze the local stability of the equilibrium points, we compute the Jacobian matrix of the system. The Jacobian matrix $J(x,y)$ is defined as:

$$J(x,y)=\begin{pmatrix}\frac{\partial f}{\partial x} & \frac{\partial f}{\partial y}\\ \frac{\partial g}{\partial x} & \frac{\partial g}{\partial y}\end{pmatrix},$$

where $f(x,y)=a_1x-b_1x^2-c_1(1-k(w))xy$ and $g(x,y)=a_2y-b_2y^2$. Thus, the Jacobian matrix is:

$$J(x,y)=\begin{pmatrix}A & -c_1(1-k(w))x\\ 0 & a_2-2b_2y\end{pmatrix}.$$

where

$$A(x,y)=a_1-2b_1x-c_1(1-k(w))y.$$

We analyze the local stability of each equilibrium point.

- **Equilibrium point** $E_0(0,0)$:
  The Jacobian matrix is:

  $$J(0,0)=\begin{pmatrix}a_1 & 0\\ 0 & a_2\end{pmatrix}.$$

  The eigenvalues are $\lambda_1=a_1$ and $\lambda_2=a_2$, both of which are positive. Therefore, $E_0(0,0)$ is an unstable equilibrium point.
- **Equilibrium point** $E_1\left(\frac{a_1}{b_1},0\right)$:
  The Jacobian matrix is:

  $$J\left(\frac{a_1}{b_1},0\right)=\begin{pmatrix}-a_1 & -\frac{c_1(1-k(w))a_1}{b_1}\\ 0 & a_2\end{pmatrix}.$$

  The eigenvalues are $\lambda_1=-a_1$ and $\lambda_2=a_2$, where $\lambda_2>0$. Therefore, $E_1\left(\frac{a_1}{b_1},0\right)$ is an unstable equilibrium point.
- **Equilibrium point** $E_2\left(0,\frac{a_2}{b_2}\right)$:
  The Jacobian matrix is:

  $$J\left(0,\frac{a_2}{b_2}\right)=\begin{pmatrix}a_1-\frac{c_1(1-k(w))a_2}{b_2} & 0\\ 0 & -a_2\end{pmatrix}.$$

  The eigenvalues are $\lambda_1=a_1-\frac{c_1(1-k(w))a_2}{b_2}$ and $\lambda_2=-a_2$. If $a_1<\frac{c_1(1-k(w))a_2}{b_2}$, which is equivalent to $a_1b_2<a_2c_1(1-k(w))$, then $\lambda_1<0$, and E_2 is locally stable. If $a_1>\frac{c_1(1-k(w))a_2}{b_2}$, which is equivalent to $a_1b_2>a_2c_1(1-k(w))$, then $\lambda_1>0$, and E_2 is unstable.
- **Equilibrium point** $E_3(x^*,y^*)$:
  The Jacobian matrix is:

  $$J(x^*,y^*)=\begin{pmatrix}A(x^*,y^*) & -c_1(1-k(w))x^*\\ 0 & a_2-2b_2y^*\end{pmatrix}.$$

  where

  $$A(x^*,y^*)=a_1-2b_1x^*-c_1(1-k(w))y^*.$$

  Noting that $x^*=\frac{a_1b_2-a_2c_1(1-k(w))}{b_1b_2}$ and $y^*=\frac{a_2}{b_2}$, we can finally obtain:

  $$\begin{aligned}&J(x^*,y^*)\\ &=\begin{pmatrix}-\frac{a_1b_2-a_2c_1(1-k(w))}{b_2} & B\\ 0 & -a_2\end{pmatrix}.\end{aligned}\tag{3}$$

  where

  $$B=-\frac{c_1(1-k(w))(a_1b_2-a_2c_1(1-k(w)))}{b_1b_2}.$$

  The eigenvalues are $\lambda_1=-\frac{a_1b_2-a_2c_1(1-k(w))}{b_2}$ and $\lambda_2=-a_2$. Since $a_1b_2>a_2c_1(1-k(w))$ (the condition for the existence of the equilibrium point), $\lambda_1<0$ and $\lambda_2<0$. Therefore, $E_3(x^*,y^*)$ is a locally stable equilibrium point.

Remark 3.1. Biological interpretation of Theorem 2.1:

Stability of boundary equilibrium $E_2(0,\frac{a_2}{b_2})$

- If $a_1b_2<a_2c_1(1-k(w))$, E_2 is stable, indicating that:
  - Species x goes extinct due to the wind-induced decline in refuge effectiveness ($k(w)$)
  - Species y reaches its carrying capacity $\frac{a_2}{b_2}$
- **Ecological implication**: Strong wind compromises the refuge, leading to the exclusion of species x via amensalism..

Stability of coexistence equilibrium $E_3(x^*,y^*)$

- If $a_1b_2>a_2c_1(1-k(w))$, E_3 is stable, implying that:
  - The net growth rate of species x (a_1) overcomes the negative effect from species y (scaled by c_1 and refuge loss $1-k(w)$)
  - Both species coexist at densities (x^*,y^*)
- **Ecological implication**: Under low wind speeds, refuge effectiveness ($k(w)$) remains sufficient to mediate niche partitioning, enabling coexistence.

IV. GLOBAL STABILITY ANALYSIS OF EQUILIBRIUM POINTS

In this section, we investigate the global stability of the equilibrium points. Global stability implies that the solutions of the system converge to a specific equilibrium point regardless of the initial conditions. We analyze the global stability using Lyapunov functions and the Dulac criterion. First of all, we have the following result.

Lemma 4.1 *System*

$$\frac{dy}{dt}=a_2y(t)-b_2y^2(t),\tag{4}$$

has a unique globally attractive positive equilibrium $y^*=\frac{a_2}{b_2}$.

A. Global Stability of Boundary Equilibrium $E_2(0, \frac{a_2}{b_2})$

When $a_1 b_2 < a_2 c_1(1-k(w))$, the boundary equilibrium $E_2(0, \frac{a_2}{b_2})$ is locally stable. We will show that indeed it is globally stable.

Theorem 4.1.*If $a_1 b_2 < a_2 c_1(1-k(w))$ hold, then $E_2(0, \frac{a_2}{b_2})$ is globally stable.*

Proof. $a_1 b_2 < a_2 c_1(1-k(w))$ is equivalent to

$$a_1 - c_1(1-k(w))\frac{a_2}{b_2} < 0.$$

Hence, one could choose $\varepsilon > 0$ small enough such that

$$a_1 - c_1(1-k(w))(\frac{a_2}{b_2} - \varepsilon) < 0 \tag{5}$$

holds. For this ε, it follows from Lemma 4.1 that there exists a $T > 0$, such that every positive solution $y(t)$ of (1) satisfies

$$\frac{a_2}{b_2} - \varepsilon < y(t) < \frac{a_2}{b_2} + \varepsilon. \tag{6}$$

Now let us consider the following Lyapunov function:

$$V(x,y) = x + y - y^* - y^* \ln \frac{y}{y^*},$$

where $y^* = \frac{a_2}{b_2}$. This function attains its minimum value $V(0, y^*) = 0$ at $E_2(0, y^*)$.

The derivative of $V(x,y)$ along the system trajectories is:

$$\frac{dV}{dt} = \frac{\partial V}{\partial x}\frac{dx}{dt} + \frac{\partial V}{\partial y}\frac{dy}{dt}.$$

Substituting the system dynamics:

$$\begin{aligned} \frac{dV}{dt} &= (a_1 x - b_1 x^2 - c_1(1-k(w))xy) \\ &\quad + \left(1 - \frac{y^*}{y}\right) \cdot (a_2 y - b_2 y^2). \end{aligned} \tag{7}$$

Simplifying, we obtain:

$$\frac{dV}{dt} = -b_1 x^2 + (a_1 - c_1(1-k(w))y)x - \frac{b_2}{y}(y - y^*)^2.$$

Since $k(w) < 1 - \frac{a_1 b_2}{a_2 c_1}$, we have $a_1 - c_1(1-k(w))y^* < 0$. For $t > T$,

$$a_1 - c_1(1-k(w))(\frac{a_2}{b_2} - \varepsilon) < 0 \tag{8}$$

holds. Thus:

$$\frac{dV}{dt} \le -b_1 x^2 + (a_1 - c_1(1-k(w))(y^* - \varepsilon))x - \frac{b_2}{y}(y - y^*)^2.$$

Since $a_1 - c_1(1-k(w))(y^* - \varepsilon) < 0$, it follows that $\frac{dV}{dt} < 0$ for all $x > 0$ and $y > 0$, except at $x = 0$ and $y = y^*$. By the Lyapunov stability theorem, the boundary equilibrium $E_2(0, \frac{a_2}{b_2})$ is globally stable.

This ends the proof of Theorem 4.1.

B. Global stability of coexistence equilibrium $E_3(x^*, y^*)$

When $a_1 b_2 > a_2 c_1(1-k(w))$, the coexistence equilibrium $E_3(x^*, y^*)$ is locally stable. Concerned with the global stability of E_3, we have the following result.

Theorem 4.2. *If $a_1 b_2 > a_2 c_1(1-k(w))$ hold, then $E_3\big(x^*, y^*\big)$ is globally stable.*

Proof. Firstly we proof that every solution of system (1) that starts in R_+^2 is uniformly bounded. From the first equation of (1) one has

$$\frac{dx}{dt} \le a_1 x - b_1 x^2. \tag{9}$$

By using the differential inequality, we obtain

$$\limsup_{t \to +\infty} x(t) \le \frac{a_1}{b_1}. \tag{10}$$

From (6) and (10), there exists a $\varepsilon > 0$ such that for all $t > T$

$$x(t) < \frac{a_1}{b_2} + \varepsilon, \;\; y(t) < \frac{a_2}{b_2} + \varepsilon. \tag{11}$$

Let $B = \{(x,y)| \in R_+^2 : x < \frac{a_1}{b_2} + \varepsilon, \;\; y < \frac{a_2}{b_2} + \varepsilon.\}$. Then every solution of system (1) starts in R_+^2 is uniformly bounded on B. Also, from Theorem 3.1 there is an unique local stable positive equilibrium $E_3(x^*, y^*)$. The Dulac criterion states that if there exists a continuously differentiable function $B(x,y)$ in a region D such that:

$$\frac{\partial(BP)}{\partial x} + \frac{\partial(BQ)}{\partial y} < 0,$$

where

$$P(x,y) = a_1 x - b_1 x^2 - c_1(1-k(w))xy$$

and

$$Q(x,y) = a_2 y - b_2 y^2,$$

then the system has no periodic solutions in D, and all trajectories converge to an equilibrium point.

We choose $B(x,y) = \dfrac{1}{xy}$. Then:

$$\begin{aligned} &\frac{\partial(BP)}{\partial x} + \frac{\partial(BQ)}{\partial y} \\ &= \frac{\partial}{\partial x}\left(\frac{a_1 - b_1 x - c_1(1-k(w))y}{y}\right) \\ &+ \frac{\partial}{\partial y}\left(\frac{a_2 - b_2 y}{x}\right). \end{aligned} \tag{12}$$

After computation, we obtain:

$$\frac{\partial(BP)}{\partial x} + \frac{\partial(BQ)}{\partial y} = -\frac{b_1}{y} - \frac{b_2}{x} < 0.$$

Since $\frac{\partial(BP)}{\partial x} + \frac{\partial(BQ)}{\partial y} < 0$ for all $x > 0$ and $y > 0$, by the Dulac criterion, the system has no periodic solutions in the first quadrant, and all trajectories converge to an equilibrium point. Since the coexistence equilibrium $E_3(x^*, y^*)$ is locally stable and there are no periodic solutions in the first quadrant, $E_3(x^*, y^*)$ is globally stable.

The proof of Theorem 4.2 is finished.

Remark 4.1
Through the above analysis, we conclude the following:

- When $k(w) < 1 - \frac{a_1 b_2}{a_2 c_1}$, the boundary equilibrium $E_2(0, \frac{a_2}{b_2})$ is globally stable.
- When $k(w) > 1 - \frac{a_1 b_2}{a_2 c_1}$, the coexistence equilibrium $E_3(x^*, y^*)$ is globally stable.

V. NUMERIC SIMULATIONS

In this section, numerical simulations are carried out to verify the feasibility of the results we have obtained.

Example 5.1.

Set the following parameters:

$$\begin{aligned} &a_1 = 1, \quad b_1 = 0.5, c_1 = 2, \\ &a_2 = 2, \quad b_2 = 1, k_0 = 0.5, \alpha = 0.1, w = 10. \end{aligned} \tag{13}$$

The wind-dependent refuge parameter is calculated as:

$$k(w) = k_0 e^{-\alpha w} = 0.5 \cdot e^{-1} \approx 0.1839.$$

In this case, one has:

$$a_1 b_2 = 1 < a_2 c_1 (1 - k(w)) \approx 2 \cdot 2 \cdot (1 - 0.1839) \approx 3.264.$$

That is, the condition of Theorem 3.1 is satisfied, it follows from Theorem 3.1 that the boundary equilibrium $(0, 2)$ is globally asymptotically stable. Fig.2 and 3 also support this assertion.

Through numerical simulations covering multiple sets of initial conditions, all trajectories converge to the boundary equilibrium point $E_2(0, 2)$.

This indicates that under the wind-related shelter effect, species x will eventually go extinct, and species y stabilizes at its carrying capacity $y^* = \frac{b_2}{a_2}$.

Example 5.2.

Set the following parameters:

$$\begin{aligned} &a_1 = 3, \quad b_1 = 0.5, \quad c_1 = 1, \quad a_2 = 2, \\ &b_2 = 1, \quad k_0 = 0.5, \quad \alpha = 0.1, \quad w = 5. \end{aligned} \tag{14}$$

The wind-dependent refuge parameter $k(w)$ is calculated as:

$$k(w) = k_0 e^{-\alpha w} = 0.5 \cdot e^{-0.1 \cdot 5} \approx 0.3033.$$

Then

$$a_1 b_2 = 3 \cdot 1 = 3,$$

$$a_2 c_1 (1 - k(w)) = 2 \cdot 1 \cdot (1 - 0.3033) \approx 1.3934.$$

Since $3 > 1.3934$, the condition $a_1 b_2 > a_2 c_1 (1 - k(w))$ is satisfied, the condition of Theorem 4.2 is satisfied, and the coexistence equilibrium $E_3(x^*, y^*) = (3.2132, 2)$ is globally stable. Fig.3 and 4 also support this assertion.

Indeed, Fig.4 shows that all trajectories from different initial conditions converge to the coexistence equilibrium $E_3(x^*, y^*)$. Fig.5 shows that the population densities $x(t)$ and $y(t)$ stabilize over time, confirming the global stability of $E_3(x^*, y^*)$.

Example 5.3.

Set the following parameters:

$$\begin{aligned} &a_1 = 3, \quad b_1 = 0.5, \quad c_1 = 1, \quad a_2 = 2, \\ &b_2 = 1, \quad k_0 = 0.5, \quad \alpha = 0.1. \end{aligned} \tag{15}$$

Now let's vary w, to observe the influence of wind effect, from Fig.6 one could see that with the increasing of wind, the density of the first species decreasing.

Example 5.4.

Set the following parameters:

$$\begin{aligned} &a_1 = 3, \quad b_1 = 0.5, \quad c_1 = 1, \quad a_2 = 2, \\ &b_2 = 1, \quad k_0 = 0.5, \quad w = 2. \end{aligned} \tag{16}$$

Now let's vary α, to observe the influence of parameter α, from Fig.7 one could see that with the increasing of α, the density of the first species decreasing.

VI. CONCLUSION AND DISCUSSION

A. *Summary of findings*

This study investigates a two-species amensalism model with a wind-dependent refuge by introducing the refuge parameter $k(w) = k_0 e^{-\alpha w}$. The main findings are as follows:

1) When $k(w) < 1 - \frac{a_1 b_2}{a_2 c_1}$, the boundary equilibrium $E_2(0, \frac{a_2}{b_2})$ is globally stable, leading to the extinction of species x.
2) When $k(w) > 1 - \frac{a_1 b_2}{a_2 c_1}$, the coexistence equilibrium $E_3(x^*, y^*)$ is globally stable, allowing both species x and y to coexist.
3) Increasing wind speed reduces the effectiveness of the refuge, thereby influencing the coexistence state of the species.

B. *Ecological implications*

Wind speed not only affects the physical structure of refuges but may also indirectly impact species survival and reproduction by altering microclimate conditions (e.g., temperature, humidity). The findings demonstrate that wind speed plays a significant role in regulating species coexistence by influencing refuge effectiveness. This provides new insights into how environmental factors shape ecosystem dynamics.

C. *Limitations of the model*

1) The model assumes a linear relationship between wind speed and refuge effectiveness ($k(w) = k_0 e^{-\alpha w}$). However, in real ecosystems, this relationship may be nonlinear. Future studies could consider more complex functional forms, such as piecewise or sigmoid functions.
2) The model only considers amensalism between two species and does not account for other interspecific relationships (e.g., competition, mutualism). Future research could extend the model to include multiple species and complex interactions.

D. *Conservation and management implications*

1) In regions with high wind speeds, planting wind-resistant vegetation or constructing windbreaks could mitigate the destructive effects of wind on refuges, thereby protecting endangered species.
2) In ecological restoration projects, the impact of wind on refuge effectiveness should be considered, and vegetation cover and microclimate conditions should be designed to enhance species survival.

E. Future research directions

1) Investigate the combined effects of wind speed and other environmental factors (e.g., temperature, humidity) on refuge effectiveness and species dynamics.
2) Extend the model to include multiple species and complex interspecific relationships, exploring how wind speed influences ecosystem stability.
3) Validate the model predictions through field experiments or laboratory simulations and refine the model parameters accordingly.

F. Conclusion

By introducing a wind-dependent refuge parameter, this study reveals how wind speed regulates species coexistence by influencing refuge effectiveness. The findings provide new theoretical insights into the impact of environmental factors on ecosystems and offer practical guidance for ecological conservation and management.

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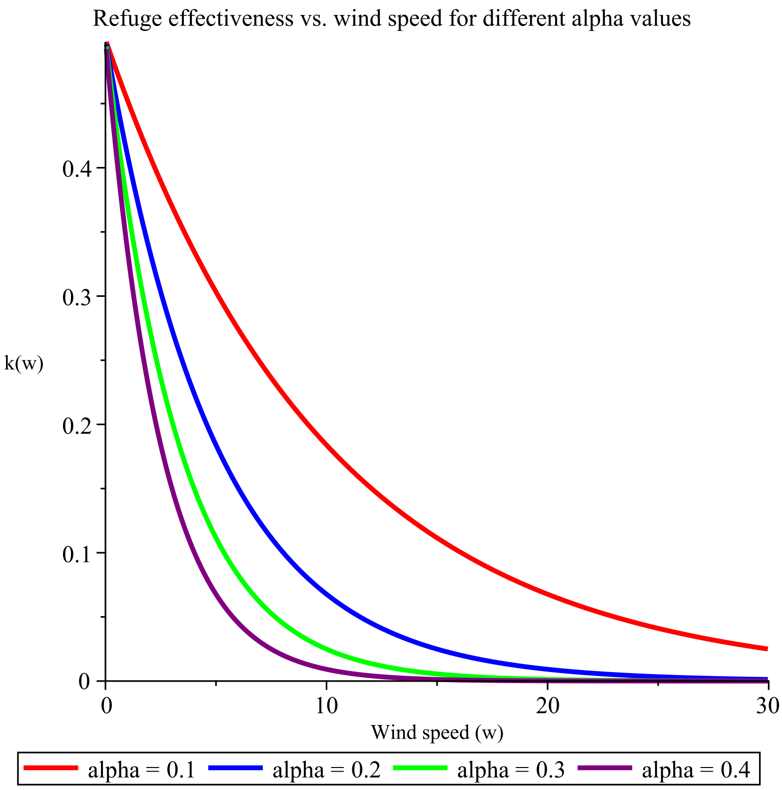

Fig. 1. Refuge effectiveness vs. wind speed for different alpha values.

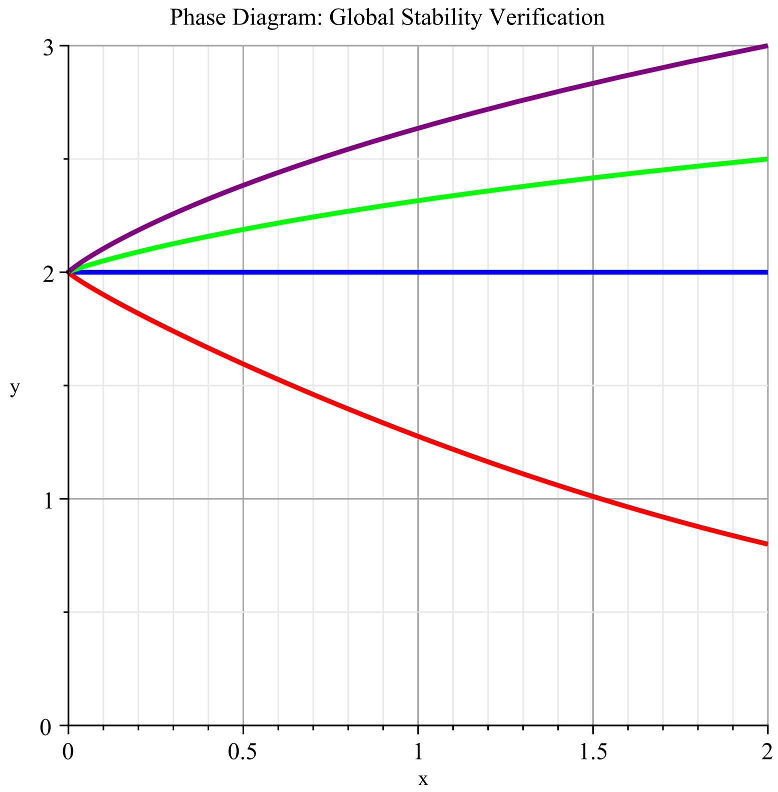

Fig. 2. Phase diagram:The trajectories under different initial conditions all converge to $E_2(0,2)$.

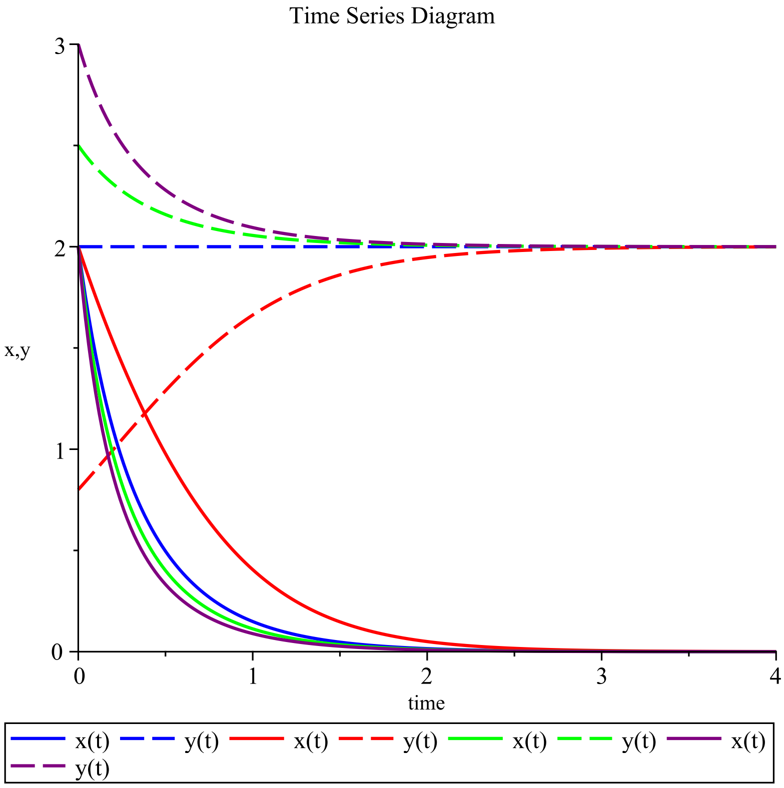

Fig. 3. Time series: The species $x(t)$ goes extinct, and the species $y(t)$ approaches $y^* = 2$.

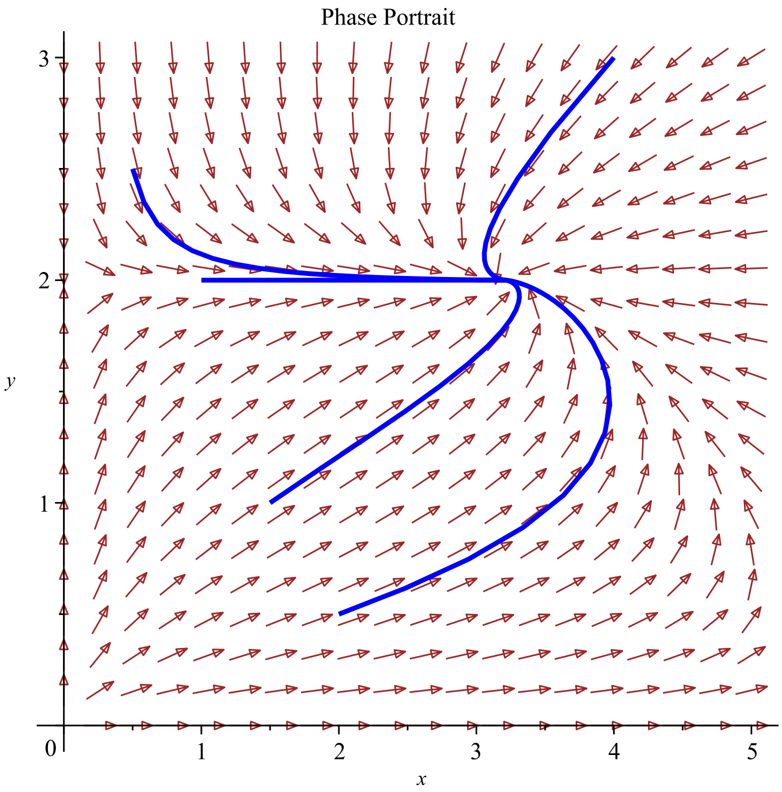

Fig. 4. Phase diagram:The trajectories under different initial conditions all converge to $E_3(x^*, y^*) = (3.2132, 2)$.

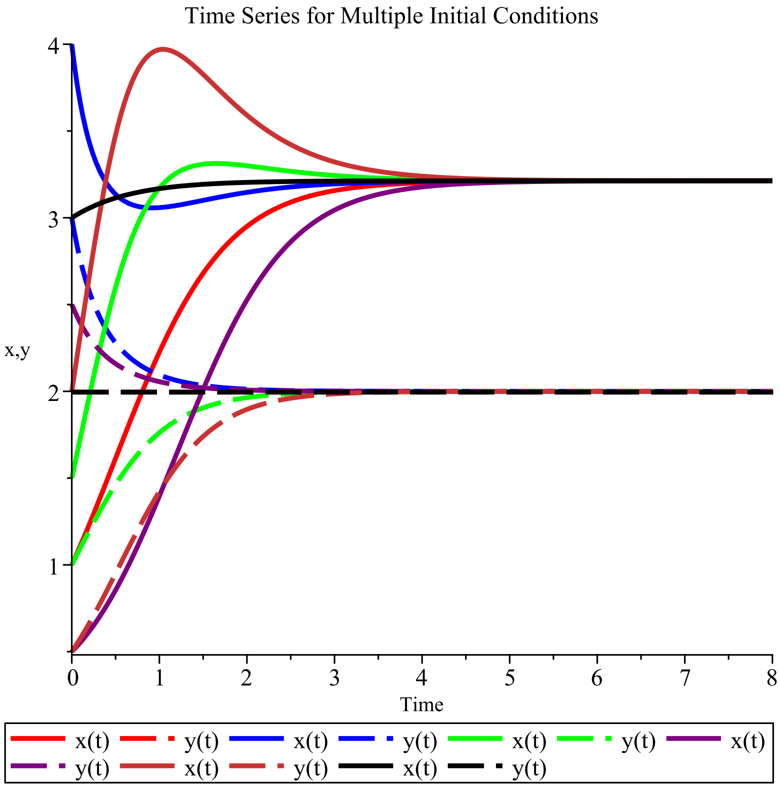

Fig. 5. Time series: The species $x(t)$ approaches $x^* = 3.2132$, and the species $y(t)$ approaches $y^* = 2$.

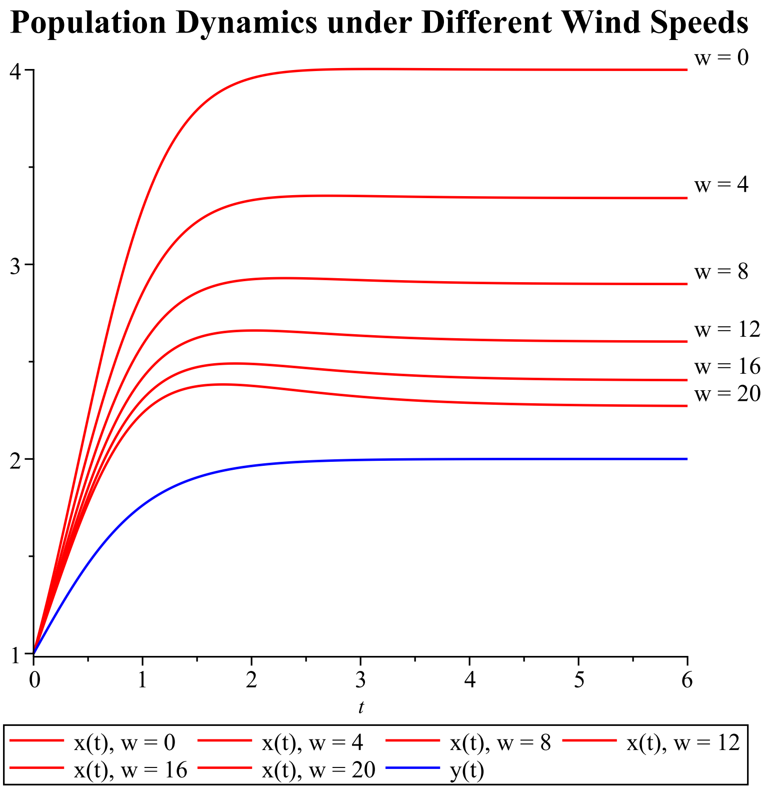

Fig. 6. $x(t)$ decreasing with the increasing of w.

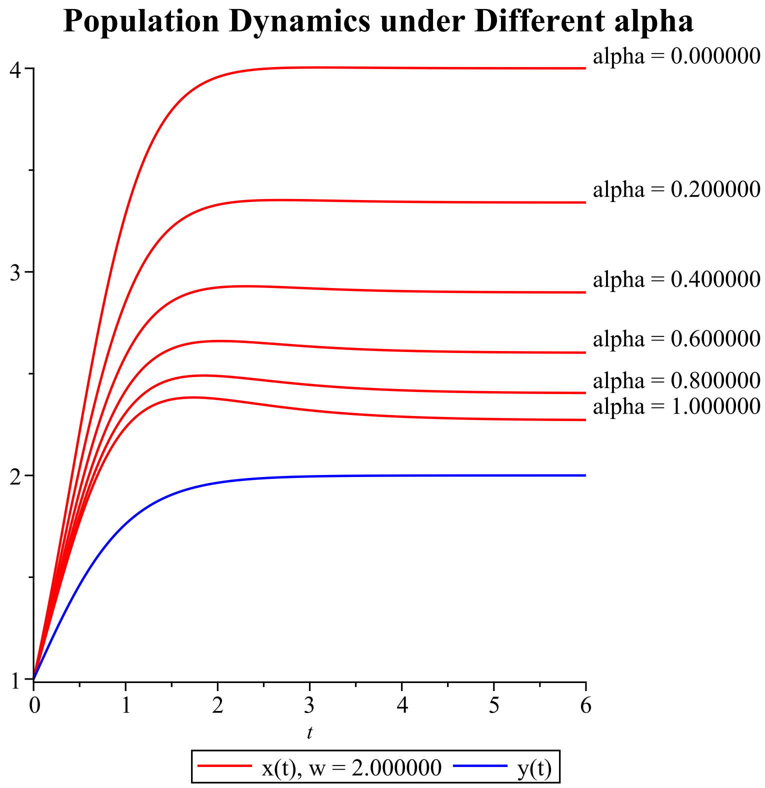

Fig. 7. $x(t)$ decreasing with the increasing of α.