A Novel Reliable Path Planning Framework Using Linear Moments and Reliability Indices

Chang-Xi Ma, Jing-Qun Zhou*

Abstract—**This paper presents a comprehensive review and analysis of existing reliable path planning models, including their underlying theories and solution methods. To address identified limitations in current approaches, the following contributions are made: First, based on the definition of path reliability, a dimensionless coefficient β is introduced as the reliability index to more effectively measure reliability. Next, linear moments are used to characterize the travel time distribution, which not only provides a more comprehensive and accurate representation but also leverages the additivity of linear moments. Through linear moments, a mapping relationship between any random variable and a standard normal random variable can be established, making it possible to solve for the reliability index. Finally, a reliable path planning model and algorithm based on linear moments and the reliability index are proposed. The validity of the proposed reliability index β was rigorously evaluated using the National Performance Management Research Data Set through both Pearson's product-moment and Spearman's rank correlation analyses. Results confirm that the index meets all criterion validity requirements and serves as an effective tool for travel time reliability assessment. Experimental validation on the Nguyen-Dupuis network further demonstrates the correctness and practical effectiveness of our proposed model and algorithm. Future research could further explore the precise mapping relationship between the path reliability index and reliability while validating and improving the model and algorithm through more complex and realistic networks.**

Index Terms—**transportation engineering, reliable path planning, linear moments, reliability index**

I. INTRODUCTION

TRAVEL Time Reliability (TTR) is a crucial metric for evaluating the performance of transportation networks. Research indicates that when travelers plan their trips, the importance of TTR is equal to or even exceeds that of travel time itself, making it a key factor influencing decision-making [1]. Travelers use travel time reliability as a criterion for determining their travel routes, aiming to plan the most reliable path between their origin and destination, thereby giving rise to the reliable path planning problem [2].

In recent years, the proposed reliable path planning models mainly fall into four categories: mean-variance models [3], minimax optimization models [4], most reliable models, and α-reliable models [5]. The mean-variance model seeks the optimal path by minimizing a combination of the mean and variance, balancing efficiency and stability. The minimax optimization model selects the robust path with the shortest travel time under the most extreme scenarios of link travel time conditions. The most reliable model and the α-reliable model are probability-based models, the most reliable model selects the path that maximizes the probability of the traveler reaching the destination within the given travel time budget, the α-reliable path is defined as the route that ensures an on-time arrival probability of at least α while minimizing the necessary travel time budget. The travel time reliability metric used in the aforementioned model does not possess the property of additivity. As a result, it is impossible to derive the path travel time reliability metric through linear operations on the link travel time reliability metrics. Therefore, deterministic shortest path algorithms, such as the Dijkstra or the K-shortest path algorithm, cannot be applied to solve the model above. This makes solving the aforementioned model a challenging problem. Huo, Zhang, and other scholars have demonstrated that the mean-variance model can be mathematically formulated as a quadratic programming problem, solvable through equation-solving methods [6]. Pan investigated the constrained mean-variance model by solving its dual problem with gradient descent, which yielded upper and lower bounds for the original problem's optimal solution. He then proposed an iterative approximation strategy to progressively narrow the solution space, ultimately converging to a near-optimal solution [7]. Song reformulated the mean-variance shortest path problem as a mixed-integer conic quadratic program and proposed a generalized Benders decomposition method to solve it [8]. Boutilier proposed an exact mixed-integer programming reformulation for the minimax optimization model. By exploiting the generalized additive structure of utility functions, this formulation enables efficient computation via intelligent optimization algorithms such as genetic algorithms [9]. Zhang and Song derived an equivalent dual formulation for the minimax optimization model, and designed tight lower/upper bound approximation methods based on scenario approximation and semidefinite programming, respectively [10]. Tu and Cheng investigated travelers' optimal path selection behavior in multi-point-of-interest trip scenarios, proposing a label-correcting algorithm to solve the most reliable path planning problem [11]. Zeng and Miwa developed a solution framework based on Lagrangian relaxation, which

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decomposes the original problem into computationally tractable subproblems. The path-finding performance was subsequently validated on real-world road networks [12]. Lee et al. proposed a pseudo-polynomial time exact algorithm for the most reliable model, along with special cases solvable in polynomial time. They also developed a fully polynomial time approximation scheme that iteratively solves the deterministic shortest path problem [13]. Chen and colleagues innovatively developed a multi-criteria label-setting algorithm and an A* algorithm for solving the most reliable path problem. By establishing strict path dominance conditions, these algorithms effectively reduce the number of non-dominated paths requiring processing during the search process. This enhancement significantly improves computational efficiency, enabling the algorithms to address large-scale network optimization problems. Furthermore, the authors validated the approach using real-world data from Advanced Traveler Information Systems, demonstrating that the proposed algorithms exhibit excellent applicability and reliability in practical scenarios [14].

However, these solution theories and methods are complex, and some approaches struggle to yield deterministic solutions. While they may perform well in specific scenarios, they lack universality. Another issue worth considering is that the aforementioned model characterizes travel time distributions based on single or mixed distributions. This characterization method requires pre-assumptions about the type of travel time distribution, failing to capture the heterogeneity of travel times [15]. Is it reasonable and effective to compute travel time reliability metrics based on this characterization method? Empirical studies have shown that travel time distributions are often heavily right-skewed and long-tailed, with the kurtosis and skewness of the distribution also influencing path travel time reliability and travelers' choices of reliable paths [16]. The reliability evaluation metrics used in the aforementioned model only consider the mean and variance of the travel time distribution. Could this lead to insufficient accuracy?

To overcome the above problems, this paper first reviews the concepts related to travel time reliability and introduces a dimensionless coefficient β as the reliability index. The definition of travel time reliability is expressed in the form of a standard normal distribution concerning the reliability index. Using probability theory, the relationship between the value of the reliability index and path reliability is analyzed. Next, this paper innovatively employs linear moments to characterize the travel time distribution. Through linear moments, a mapping relationship is established between the random variable of path travel time for any distribution and the random variable of the standard normal distribution. Leveraging the favorable properties of the standard normal distribution, the reliability index is solved, and reliable paths are planned based on this index. The research is conducted according to the logical framework shown in Figure 1. The innovations are reflected in two aspects: first, the use of linear moments to characterize the travel time distribution, and second, a reliability index is introduced to evaluate path reliability, and optimal paths are determined by solving for this index. Figure 1 is placed at the end of the content and before the reference part.

II. Linear Moments

A. Superiority

In 1990, Hosking [17] pointed out in his paper that any random variable with a finite mean can be analyzed and estimated through linear combinations of its ordered statistics. Among these, linear moments refer to the expectations of certain linear combinations of order statistics and have been widely applied in various research fields such as finance, engineering [18], meteorology [19], hydrology [20], and oceanography. The theory of linear moments parallels that of conventional moments. Using linear moments to fit travel time distributions does not require prior assumptions about the distribution type and takes into account the skewness and kurtosis coefficients of the distribution [21], thereby capturing the heterogeneity of travel times. A series of applications of linear moments has demonstrated their advantages over conventional moments. Linear moments, being linear functions of the statistics of random variables, are less affected by sampling variability and are less sensitive to outliers in the data compared to conventional moments. They enable more reliable and robust inference of the underlying probability distribution from smaller samples [22]. Additionally, linear moments are linear functions of expected statistics, and the linear moments of a finite number of random variables are additive. Through (13) in this paper, the first four linear moments of a random variable can be used to establish a mapping relationship between any distribution and the standard normal distribution, leveraging the favorable properties of the standard normal distribution to solve problems. Furthermore, linear moments offer strong interpretability, simple computational principles, and concise calculation processes.

B. Definitions and Computational Methods

Given a random variable X with real-valued outcomes, its cumulative distribution function and quantile function are denoted as F(X) and X(F), respectively. From N ordered statistics $X_{1,N} \le X_{2,N} \le ... \le X_{N,N}$ randomly drawn from the random variable X, the linear moments of X are defined as shown in (1).

$$\lambda_r = r^{-1}\sum_{k=0}^{r-1}(-1)^{k}\binom{r-1}{k}EX_{r-k\,r} \qquad r=1,2,.... \tag{1}$$

From (1), it can be seen that linear moments are linear functions of the expected order statistics. Here, $EX_{j:r}$ represents the expectation of n ordered statistics, and the computational process is shown in (2).

$$EX_{j:r} = \frac{r!}{(j-1)!(r-j)!}\times\int X\{F(X)\}^{j-1}\{1-F(X)\}^{r-j}\,dF(X) \tag{2}$$

By substituting (2) into (1), performing a binomial expansion on F(X), and summing the coefficients of the powers of F(X), a more general expression for the linear moments is obtained as shown in (3).

$$\lambda_r = \int_0^1 X(F)P^*_{r-1}(F)\,dF \qquad r=1,2,... \tag{3}$$

Where $P_*^r(F)$ represents the r-th shifted Legendre polynomial, and its relationship with the general Legendre polynomial $P_r(u)$ is $P^*_r(u) = P_r(2u-1)$. The expression for $P_*^r(F)$ is given by (4), while the expression for $\mathrm{P_r}(u)$ is provided in (5).

$$P_{\mathrm{r}}^*(F)=\sum_{k=0}^{r}p_{r,k}^*F^k \tag{4}$$

$$p_{r,k}^*=(-1)^{r-k}\binom{r}{k}\binom{r+k}{k} \tag{5}$$

In practical applications, when the distribution is known, the computational expressions for the first four linear moments of the random variable can be derived based on (1) to (2), as shown in (6) to (9).

$$\lambda_{1X}=EX=\int_0^1 X(F)dF(X) \tag{6}$$

$$\lambda_{2X}=\frac{1}{2}E(X_{2:2}-X_{1:2})=\int_0^1 X(F)[2F(X)-1]dF(X) \tag{7}$$

$$\begin{aligned}\lambda_{3\mathrm{X}}&=\frac{1}{3}E(X_{3:3}-2X_{2:3}+X_{1:3})\\&=\int_0^1 \mathrm{X}(F)[6F^2(\mathrm{X})-6F(\mathrm{X})+1]dF(\mathrm{X})\end{aligned} \tag{8}$$

$$\begin{aligned}\lambda_{4\mathrm{X}}&=\frac{1}{4}E(X_{4:4}-3X_{3:4}+3X_{2:4}-X_{1:4})\\&=\int_0^1 \mathrm{X}(\mathrm{F})[20F^3(\mathrm{X})-30F^2(\mathrm{X})+12F(\mathrm{X})-1]dF(\mathrm{X})\end{aligned} \tag{9}$$

If obtaining the probability distribution of the variable is difficult but samples of the variable are readily available, the linear moments can be estimated from the samples. Assuming $X_{1,N}\le X_{2,N}\le\ldots\le X_{N,N}$ represents N ordered random samples arranged from smallest to largest, the first four linear moments can be calculated using (10) to (13).

$$\lambda_{1X}=\frac{1}{N}\sum_{i=1}^{N}X_{i:N} \tag{10}$$

$$\lambda_{2X}=\frac{2}{N}\sum_{i=2}^{N}\frac{(\mathrm{i}-1)}{(N-1)}X_{i:N}-\frac{1}{N}\sum_{i=1}^{N}X_{i:N} \tag{11}$$

$$\begin{aligned}\lambda_{3X}&=\frac{6}{N}\sum_{i=3}^{N}\frac{(\mathrm{i}-1)(\mathrm{i}-2)}{(N-1)(N-2)}X_{i:N}\\&-\frac{6}{N}\sum_{i=2}^{N}\frac{(\mathrm{i}-1)}{(N-1)}X_{i:N}+\frac{1}{N}\sum_{i=1}^{N}X_{i:N}\end{aligned} \tag{12}$$

$$\begin{aligned}\lambda_{4X}&=\frac{20}{N}\sum_{i=4}^{N}\frac{(\mathrm{i}-1)(\mathrm{i}-2)(\mathrm{i}-3)}{(N-1)(N-2)(N-3)}X_{i:N}\\&-\frac{30}{N}\sum_{i=3}^{N}\frac{(\mathrm{i}-1)(\mathrm{i}-2)}{(N-1)(N-2)}X_{i:N}\\&+\frac{12}{N}\sum_{i=2}^{N}\frac{(\mathrm{i}-1)}{(N-1)}X_{i:N}-\frac{1}{N}\sum_{i=1}^{N}X_{i:N}\end{aligned} \tag{13}$$

The skewness and kurtosis of the random variable X are described using the linear skewness coefficient τ_3 and the linear kurtosis coefficient τ_4, respectively. The expressions for τ_3 and τ_4 are given by (14).

$$\tau_{\mathrm{r}}=[\lambda_r(X)]/[\lambda_2(X)],\mathrm{r}=3,4 \tag{14}$$

where $\lambda_2(X)$ represents the second-order linear moment, and $\lambda_r(X)$ denotes the r-th order linear moment. When r=3 or 4, it refers to the third-order and fourth-order linear moments, respectively.

C. Standard Normal Distribution Transformation

When the first four linear moments of a random variable X, which follows a certain distribution (normal or non-normal), are known, its distribution can be expressed using a cubic polynomial [23], as shown in (15).

$$X=S(N)=aN^3+\mathrm{b}N^2+cN+d \tag{15}$$

Here, N is a random variable following the standard normal distribution (with a mean of 0 and a standard deviation of 1); S(N) is a cubic function of N; and a, b, c, and d are polynomial coefficients determined by (16) to (19) [24].

$$a=-0.19309293\lambda_{2X}+1.574961\lambda_{4X} \tag{16}$$

$$b=1.81379937\lambda_{3X} \tag{17}$$

$$c=2.25518617\lambda_{2X}-3.93740250\lambda_{4X} \tag{18}$$

$$d=\lambda_{1X}-1.81379937\lambda_{3X} \tag{19}$$

Equation (15) establishes a mapping relationship between a random variable following the standard normal distribution and a random variable following any arbitrary distribution. When the first four linear moments of a random variable are known, equation (15) can be used to express it as a function of a standard normal random variable. In practice, data for a random variable can be obtained through experiments, and the first four linear moments of the variable can then be derived from the sample experimental data using (10) to (13). Therefore, in cases where the distribution is unknown, equation (15), combined with the linear moments of the random variable, eliminates the need for assumptions about the distribution of the random variable.

D. Additivity

X and Y are known to be arbitrary random variables.

$$\lambda_r(X+Y)=r^{-1}\sum_{k=0}^{r-1}(-1)^{\mathrm{k}}\binom{r-1}{k}E(X+Y)_{\mathrm{r}-k:r}\qquad r=1,2,\ldots.$$

Prove that $\lambda_r(X+Y)=\lambda_r(X)+\lambda_r(Y)$. Since $EX_{\mathrm{j}:r}$ denotes the expectation of n ordinal statistics, the expectation of any random variable is additive, hence $E(X+Y)_{\mathrm{r}-k:r}=E(X)_{\mathrm{r}-k:r}+E(Y)_{\mathrm{r}-k:r}$. The proof process is as follows.

$$\begin{aligned}&\lambda_r(X+Y)\\&=r^{-1}\sum_{k=0}^{r-1}(-1)^{\mathrm{k}}\binom{r-1}{k}E(X+Y)_{\mathrm{r}-k:r}\\&=r^{-1}\sum_{k=0}^{r-1}(-1)^{\mathrm{k}}\binom{r-1}{k}\left[E(X)_{\mathrm{r}-k:r}+E(Y)_{\mathrm{r}-k:r}\right]\\&=r^{-1}\sum_{k=0}^{r-1}(-1)^{\mathrm{k}}\binom{r-1}{k}E(X)_{\mathrm{r}-k:r}+r^{-1}\sum_{k=0}^{r-1}(-1)^{\mathrm{k}}\binom{r-1}{k}E(Y)_{\mathrm{r}-k:r}\\&=\lambda_X+\lambda_Y\qquad r=1,2,\ldots.\end{aligned} \tag{20}$$

III. Reliability Index

A. Concepts of Reliability

Travel time reliability refers to the ability of travelers in a transportation system to complete a specified trip along a predetermined route within a preset travel time under specific traffic conditions. The probability of completing the specified trip is called reliability, denoted by R. This preset time refers to the travel time budget, which is the upper limit of travel time acceptable to travelers and consists of the expected travel time and the buffer travel time. The link travel time budget and the path travel time budget are given by (21) and (22), respectively.

$$\overline{T}_{\partial} = E_{\partial} + S_{\partial} \tag{21}$$

$$\overline{T}_{P} = E_{p} + S_{p} \tag{22}$$

Where $\overline{T}_{\partial}$, E_{∂}, and S_{∂} represent the link travel time budget, expected travel time, and buffer travel time, respectively, while $\overline{T}_{P}$, E_p, and S_p denote the total path travel time budget, average total travel time, and total buffer travel time, respectively.

In this paper, the travel time budget is determined using (23).

$$\overline{T} = \mathrm{T}_0\left(1 \pm \xi\right) \tag{23}$$

Where T_0 represents the free-flow travel time, and ξ denotes the relative delay threshold. The relative delay threshold is not fixed, as the same delay time can have different impacts depending on the travel time budget. For example, if the travel time budget for a link is 50 minutes, a 5-minute delay may seem negligible. Conversely, if the travel time budget for a link is 15 minutes, a 5-minute delay becomes significant. This paper considers the free-flow travel time of the link itself and uses a proportional method to determine the relative delay threshold, as shown in (24).

$$\xi_{od_i} = \frac{T_{od_i}}{\min T_{od}} \times \xi_o \tag{24}$$

Where ξ_o is the base threshold, set to 1.3, ξ_{od_i} is the relative delay threshold for path i from origin o to destination d, and T_{od_i} is the free-flow travel time for path i from origin o to destination d.

Since travel time is a continuous random variable, its reliability can be described using (25), where $\mathrm{f}\left(t\right)$ is the probability density function of travel time.

$$\begin{aligned} R\left(\overline{T}\right) &= P\left(T \le \overline{T}\right) = P\left(T \le E + S\right) \\ &= \int_{-\infty}^{\overline{T}} \mathrm{f}\left(t\right)dt = \int_{-\infty}^{E+S} \mathrm{f}\left(t\right)dt = F\left(\overline{T}\right) \end{aligned} \tag{25}$$

B. Reliability Index

In practical applications, obtaining the probability density function of travel time distribution is often challenging, and the computation of the probability density function and its integration is complex. To facilitate the calculation of travel time reliability, the concept of the reliability index β is introduced. To concisely and clearly explain the significance of β, assume that travel time T follows a normal distribution with $T \sim N(\mu_t, \sigma_t^2)$. By using $Y = \frac{t-\mu_t}{\sigma_t}$, T is transformed into a random variable $Y \sim N\left(0,1\right)$) that follows the standard normal distribution. Simplifying (25), the relationship between travel time reliability and the standard normal distribution is established, as shown in (26). Since $\overline{T}$ is a constant, let $\overline{T} = 0$. At this point, for ease of expression and computation, the symbol β is introduced, with $\beta = \frac{u_t}{\sigma_t}$. β is a dimensionless coefficient that has a one-to-one correspondence with reliability and is referred to as the reliability index. The relationship between reliability and the reliability index is illustrated in Figure 2. The larger β is, the higher the travel time reliability.

$$\begin{aligned} R\left(\overline{T}\right) &= \int_{-\infty}^{\overline{T}} f\left(t\right)dt = \int_{-\infty}^{\overline{T}} \frac{1}{\sqrt{2\pi}\sigma_t} \exp\left[-\frac{\left(t-u_t\right)^2}{2\sigma_t^2}\right]dt \\ &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\frac{\overline{T}-u_t}{\sigma_t}} \exp\left(-\frac{y^2}{2}\right)dy = \Phi\left[\frac{\overline{T}-u_t}{\sigma_t}\right] \end{aligned} \tag{26}$$

$$R(\overline{T}) = \Phi\left(-\beta\right) \tag{27}$$

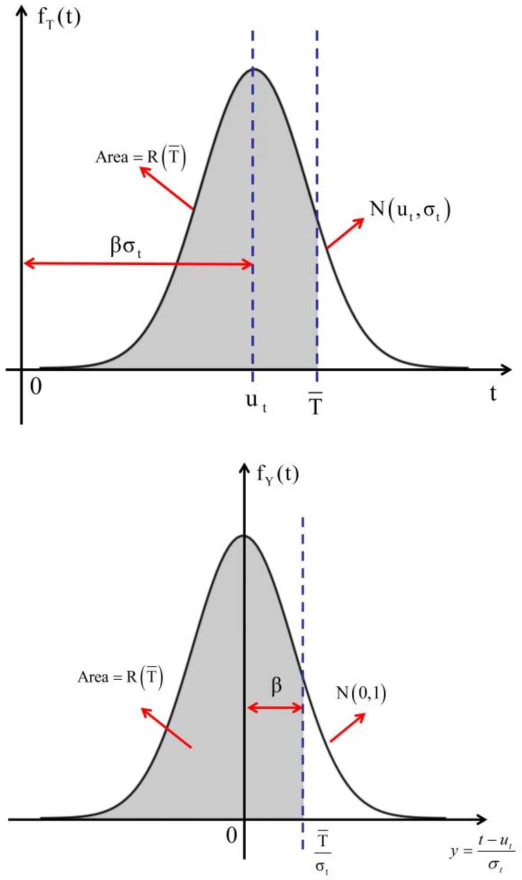


Fig. 2 Relationship between reliability and reliability index

C. Reliability Index Calculation

If the first four linear moments of the random variable travel time T are known, according to (15), T can be expressed as a cubic polynomial, as shown in (28).

$$T = S(N) = aN^3 + bN^2 + cN + d \tag{28}$$

The travel time reliability expression can be further simplified as shown in (29).

$$\begin{aligned} R\left(\overline{T}\right) &= P\left\{T \le \overline{T}\right\} \\ &= \mathrm{P}\left\{aN^3 + bN^2 + cN + d \le \overline{T}\right\} \\ &= \mathrm{P}\left\{aN^3 + bN^2 + cN + d - \overline{T} \le 0\right\} \end{aligned} \tag{29}$$

Based on the equal probability transformation, it is calculated that

$$R\left(\overline{T}\right)=\Phi\left(N\right)=\Phi\left[S^{-1}\left(T,M\right)\right] \tag{30}$$

where N is a random variable following a normal distribution, $\Phi\left(N\right)$ is the cumulative distribution function of the standard normal random variable; $S^{-1}\left(T,M\right)$ is the inverse function of (29); and M is the vector of the first four linear moments, $M=\left[\lambda_{1t}\lambda_{2t}\lambda_{3t}\lambda_{4t}\right]$.

The reliability index β can be expressed in (31).

$$\beta=-\Phi^{-1}\left(R\right)=-S^{-1}\left(T,M\right) \tag{31}$$

To solve for $S^{-1}\left(0,M\right)$, which involves finding the root N of the cubic equation $S(N)=aN^3+bN^2+cN+d-\overline{T}=0$, the Cardano formula is avoided due to its complexity involving cube roots and complex numbers. Instead, this paper employs the Shengjin formula. The multiple root discriminant of the Shengjin formula is given by (32).

$$\begin{cases} A=B^2-3ac \\ B=bc-9ad \\ C=c^2-3bd \end{cases} \tag{32}$$

When the discriminant $\Delta=B^2-4AC>0$, the equation has one real root and a pair of conjugate complex roots. The real root of the equation is calculated according to (33) and (34).

$$N=\frac{-b-\sqrt{Y_1}-\sqrt{Y_2}}{3a} \tag{33}$$

$$Y_{1,2}=Ab+3a\left(\frac{-B\pm\sqrt{B^2-4AC}}{2}\right) \tag{34}$$

At this point, the reliability index is given by (35).

$$\beta=-S^{-1}\left(T,M\right)=-N=\frac{b+\sqrt[3]{Y_1}+\sqrt[3]{Y_2}}{3a} \tag{35}$$

When the discriminant $\Delta=B^2-4AC<0$, the equation has three distinct real roots, and the expression for the result N is given by (36) and (37).

$$N=\frac{b-\sqrt{A}\left(\cos\frac{\theta}{3}+\sqrt{3}\sin\frac{\theta}{3}\right)}{3a} \tag{36}$$

$$\theta=\arccos\gamma\ ,\gamma=\frac{2Ab-3ab}{2\sqrt{A^3}}(A>0,-1<\gamma<1) \tag{37}$$

Similarly, the reliability index β at this point is given by (38).

$$\beta=-S^{-1}(T,M)=-N=\frac{b-\sqrt{A}(\cos\frac{\theta}{3}-\sqrt{3}\sin\frac{\theta}{3})}{3a} \tag{38}$$

IV. Reliable Path Planning Model

The reliable path planning model proposed in this paper is described as follows: G(N, A) represents a stochastic network, where N is the set of nodes, A is the set of links, o is the origin, and d is the destination; β^{od} is the reliability index for the path from origin o to destination d; λ_r^{od} is the r-th moment of travel time for the path from origin o to destination d; $\lambda_{r,ij}^{od}$ is the r-th moment of travel time for the link from node i to node j, where this link belongs to the path from origin o to destination d; $j\in SCS\left(i\right)$ and $k\in PDS\left(i\right)$ are the sets of successor nodes and predecessor nodes of node i, respectively. x_{ij}^{od} is the decision variable, where $x_{ij}^{od}=1$ indicates that link a_{ij} is included in the path from origin o to destination d, and $x_{ij}^{od}=0$ indicates that link a_{ij} is not included in the path; $F\left(\bullet\right)$ represents the operation of calculating the reliability index from the linear moments.

$$\max\beta^{od}=F\left(\lambda_r^{od}\right)=F\left(\sum_{a_{ij}\in A}x_{ij}^{od}\lambda_{r,ij}^{od}\right) \tag{39}$$

$$s.t.\ \sum_{j\in SCS(i)}x_{ij}^{od}-\sum_{k\in PDS(i)}x_{ki}^{od}=\begin{cases}1 & \forall i=o\\ 0 & \forall i\neq o,i\neq d\\ -1 & \forall i=d\end{cases} \tag{40}$$

$$x_{ij}^{od}\in\{0,1\},\ \forall a_{ij}\in A,\ \forall i,j,k\in N,\ r=1,2,3,4 \tag{41}$$

V. Experiment

A. Reliability Index Validity Analysis

This study validates the effectiveness of β as a travel time reliability evaluation metric by examining its statistical correlation with the widely recognized travel time reliability indicator, the Buffer Time Index (BTI). The BTI quantifies the additional time buffer required by travelers to mitigate delays by comparing the difference between the 95th percentile travel time and the median travel time. A higher BTI value indicates greater travel time variability and consequently lower road segment reliability, whereas a higher β value corresponds to improved reliability. Thus, a negative correlation is theoretically expected between BTI and β.

The experimental data were sourced from the National Performance Management Research Data Set, which comprehensively documents the traffic operating conditions of Alaska's highway network throughout 2019. The original dataset contains 2,622 unique road segments, each assigned a distinct identification code. Using a random sampling method, 200 segments were selected as study subjects, with each sample segment containing approximately 15,000 actual travel time records on average. The travel time data were collected across different dates throughout the year and various time periods within each day, capturing the full spectrum of traffic conditions from free-flow to congested states. The dataset encompasses highways of different classes, diverse road conditions, and varying traffic flow patterns, ensuring the samples possess high spatiotemporal representativeness and statistical reliability.

The Interquartile Range method was applied to identify and remove outliers falling outside 1.5 times the interquartile range, thereby mitigating the influence of extreme values on data analysis. Figure 3 presents a comparative visualization of the dataset before and after cleaning, demonstrating the substantial improvement in data quality achieved through this outlier treatment process.

In the specific calculation of BTI, the following procedure was implemented: First, the mean, variance, and key percentiles were computed for each road segment based on its observed travel time data. Subsequently, the BTI for each segment was derived from these percentiles, while β was calculated using the mean and variance values. The statistical association between BTI and β was then quantitatively analyzed through dual testing methods: Pearson's product-moment correlation coefficient and Spearman's rank correlation coefficient. All computational procedures were implemented using the SciPy statistical module in Python 3.8.

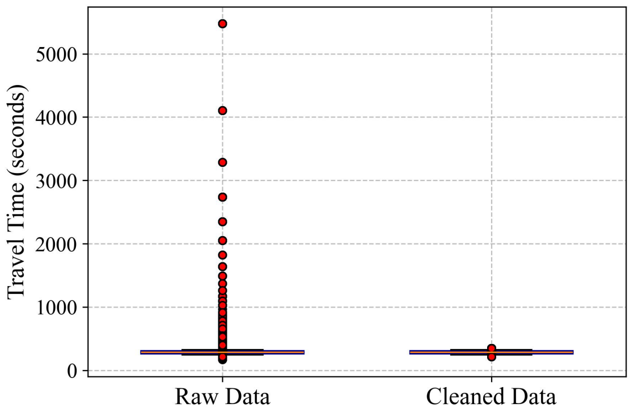


Fig. 3 Data Cleaning Results Visualization

The analysis yielded the following results: a Pearson correlation coefficient of -0.43 ($p < 0.001$) and a Spearman's rank correlation coefficient of -0.74 ($p < 0.001$). The p-values, displayed as 0.000, indicate that the actual values were below the computational precision threshold of the statistical software. These findings are visually presented in Figure 4.

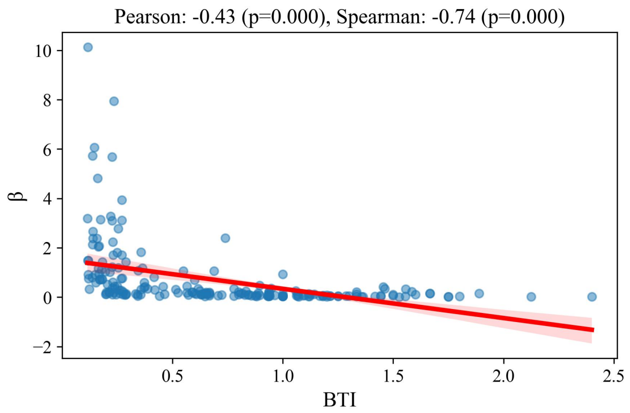


Fig. 4 Comparative correlation analysis of BTI and β

According to Cohen's criteria for correlation strength interpretation, the Pearson coefficient indicates a moderate-strength linear negative correlation, demonstrating a statistically significant inverse linear relationship between BTI and β. The Spearman coefficient reveals a stronger monotonic negative correlation, suggesting the potential existence of a nonlinear yet more pronounced monotonically decreasing relationship — specifically, as BTI increases, β consistently decreases, though the rate of decrease may vary. Both correlations exhibit highly statistically significant p-values ($p<0.001$). The notable discrepancy between the Spearman and Pearson coefficients (|-0.7426|>|-0.4305|) implies that the association between BTI and β may incorporate nonlinear components. While this potential nonlinearity may exert some influence on subsequent analytical interpretations, the current data characteristics and statistical testing results suggest that such effects are expected to remain relatively limited in magnitude.

The study conclusively demonstrates a statistically significant negative association between the reliability index β and the BTI. The reliability index satisfies the criterion validity requirements and can be effectively employed as an evaluation tool for travel time reliability.

B. Reliable Path Planning

The reliable path planning algorithm proposed in this paper is tested on the Nguyen-Dupuis (N-D) network [25]. The N-D network consists of 13 nodes, 19 links, and 4 OD pairs. Its network topology is shown in Figure 5, and the free-flow travel times T_0 and link capacities $\overline{C_a}$ are provided in Table I. Table II lists all feasible paths between each OD pair in the network. The link travel time is calculated using the BPR function proposed by the U.S. Federal Highway Administration, with the parameters ∂ and β set to 0.15 and 4, respectively. Factors such as traffic control and traffic environment are not considered. It is assumed that the link capacity C_a follows a uniform distribution $C_a \sim U\left(0.5\overline{C_a}, \overline{C_a}\right)$ and the standard deviation of link flow $\sigma_a = 0.3t_a^0$. If the OD flow follows a normal distribution, the link flow also follows a normal distribution [26].

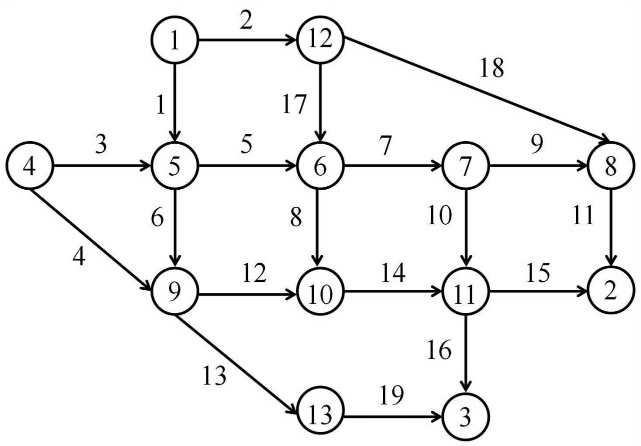


Fig. 5 Nguyen-Dupuis network topological structure

TABLE I
THE FREE-FLOW TRAVEL TIME AND CAPACITY OF THE NETWORK LINKS

a	T_0	$\overline{C_a}$	a	T_0	$\overline{C_a}$
1-5	7	900	8-2	10	700
1-12	8	700	9-10	10	700
4-5	9	700	9-13	9	600
4-9	14	900	10-11	8	700
5-6	5	800	11-2	9	700
5-9	9	600	11-3	8	700
6-7	5	900	12-6	7	300
6-10	13	500	12-8	15	700
7-8	5	300	13-3	11	700
7-11	9	400			

Under stochastic supply and demand conditions, the specific process of using the proposed model to plan reliable paths is as follows:

Step 1: Set relevant parameters, perform stochastic equilibrium assignment on the network to obtain the flow distribution, and use the Monte Carlo method to randomly generate link travel time samples.

Step 2: Calculate the linear moments of link travel time from the generated samples, with the results shown in Figure

6. Based on the additivity of linear moments, derive the linear moments of path travel time from those of link travel time, with the results shown in Figure 7. Figure 6 and Figure 7 are placed at the end of the content and before the reference part.

TABLE II
THE REACHABLE PATHS AND THEIR NUMBERS

Serial Number	OD	Sequence of Nodes	Serial Number	OD	Sequence of Nodes
1	1-2	1-12-8-2	15	4-2	4-9-10-11-2
2		1-5-6-7-8-2	16		4-5-6-7-8-2
3		1-5-6-7-11-2	17		4-5-6-7-11-2
4		1-5-6-10-11-2	18		4-5-6-10-11-2
5		1-5-9-10-11-2	19		4-5-9-10-11-2
6		1-12-6-7-8-2	20	4-3	4-9-13-3
7		1-12-6-7-11-2	21		4-9-10-11-3
8		1-12-6-10-11-2	22		4-5-9-13-3
9	1-3	1-5-9-13-3	23		4-5-6-7-11-3
10		1-5-6-7-11-3	24		4-5-6-10-11-3
11		1-5-6-10-11-3	25		4-5-9-10-11-3
12		1-5-9-10-11-3			
13		1-12-6-7-11-3			
14		1-12-6-10-11-3			

Step 3: Determine the relative delay threshold ξ and calculate the travel time budget $\overline{T}$. Compute the coefficients a, b, c, and d of the cubic equation using the fourth-order linear moments of path travel time, as shown in Table III. Table III is placed at the end of the content and before the reference part.

Step 4: Solve the cubic equation to obtain the reliability index β, and evaluate the reliability of link travel time using β. The results are shown in Table IV. Table IV is placed at the end of the content and before the reference part.

Step 5: Use the Monte Carlo method to validate the correctness of the proposed algorithm. Set the same travel time budget value $\overline{T}$. Let Q represent the total number of Monte Carlo simulation experiments, and let event A denote the simulated travel time being less than $\overline{T}$. If event A occurs, the state variable $x_i = 1$. Search for all feasible paths within the network interval and repeat the simulation experiments. Each path is simulated 30,000 times, and the travel time reliability is calculated according to (40). The calculation results are shown in Table IV.

$$R\left(\overline{T}\right) = P\left\{T_p < \overline{T}\right\} = f\left(A\right) = \frac{\sum_{i}^{N} X_i}{N} \tag{42}$$

Based on the reliability values obtained from the reliability index method and the Monte Carlo method, the paths are ranked in descending order of reliability, and Figure 8 is plotted to compare the differences in the results of the two methods. Figure 8 is placed at the end of the content and before the reference part. As shown in Table IV and Figure 8, the most reliable paths between each OD pair identified by both methods are the same. The reliability ranking results for the paths between OD pairs 1-2, 1-3, and 4-2 are consistent. For OD pair 4-3, only the reliability rankings of the 4th and 6th paths differ, but the reliability values calculated by the two methods for these paths are very close, with small errors. The accuracy rate of the reliability index method in solving reliable paths is 92%. Therefore, although the principle of the reliability index method for planning reliable paths differs from existing methods, it is correct and effective.

VI. CONCLUSION

Validity analysis was conducted on the proposed reliability index β, with experimental results demonstrating a statistically significant negative correlation between β and the BTI. These findings confirm that β satisfies criterion validity requirements and can effectively serve as an assessment tool for travel time reliability. Testing performed on the Nguyen-Dupuis network showed that the proposed reliable path planning algorithm and model achieved 92% accuracy, verifying their correctness and effectiveness.

The innovative adoption of linear moments for deriving the reliability index eliminates the need for prior assumptions about travel time distribution types, while effectively capturing the heterogeneity of travel times. The path planning algorithm developed based on this approach can directly compute path-level linear moments from link-level travel time linear moments. This methodology represents a fundamental departure from existing approaches while maintaining computational simplicity, precision, and high efficiency.

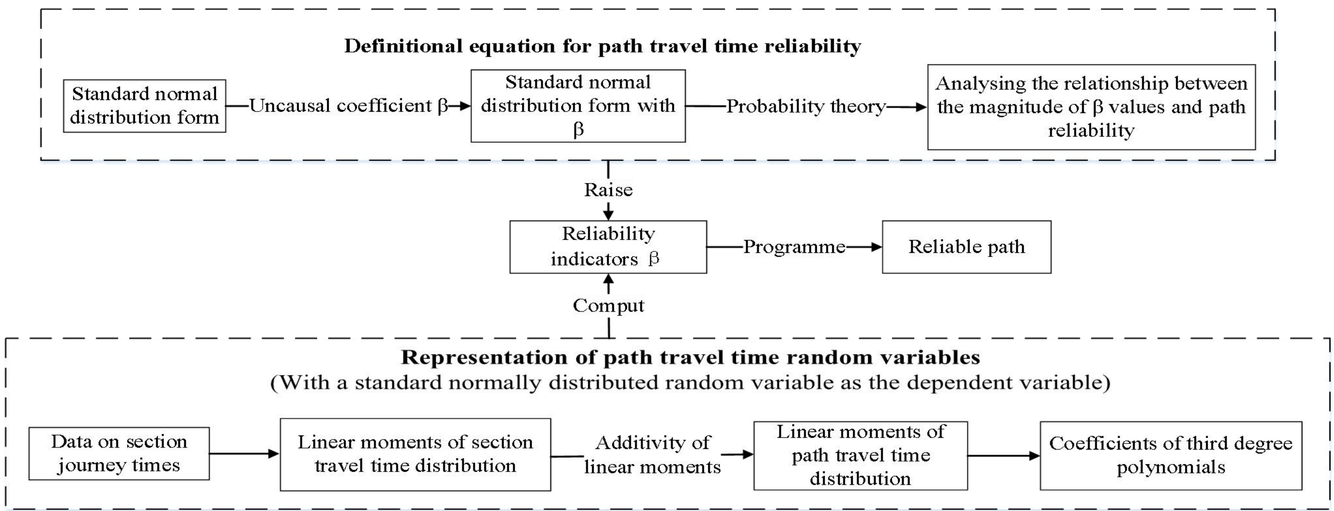


Fig. 1. Research logical framework.

TABLE III
THE CALCULATION RESULTS OF THE EQUATION COEFFICIENTS

OD	Serial number	Sequence of Nodes	T_0	ξ	$\overline{T}$	a	b	c	d
1-2	1	1-12-8-2	38	1.3	49.4	-0.1400	2.1833	6.7419	44.2341
	2	1-5-6-7-8-2	47	1.415	66.5	-0.1747	2.7049	8.3416	54.6936
	3	1-5-6-7-11-2	44	1.325	58.3	-0.1675	2.5325	7.8251	51.2097
	4	1-5-6-10-11-2	48	1.445	69.4	-0.1738	2.7662	8.5136	55.8633
	5	1-5-9-10-11-2	44	1.325	58.3	-0.1604	2.5371	7.8204	51.2241
	6	1-12-6-7-8-2	52	1.565	81.4	-0.1920	2.9872	9.2203	60.5209
	7	1-12-6-7-11-2	49	1.475	72.3	-0.1849	2.8147	8.7039	57.0370
	8	1-12-6-10-11-2	53	1.596	84.6	-0.1911	3.0485	9.3923	61.6906
1=3	9	1-5-9-13-3	35	1.3	45.5	-0.1337	2.0144	6.2331	40.7480
	10	1-5-6-7-11-3	45	1.471	66.2	-0.1706	2.5926	8.0031	52.3725
	11	1-5-6-10-11-3	49	1.602	78.5	-0.1768	2.8264	8.6916	57.0261
	12	1-5-9-10-11-3	45	1.471	66.2	-0.1634	2.5973	7.9983	52.3869
	13	1-12-6-7-11-3	50	1.634	81.7	-0.1879	2.8748	8.8818	58.1998
	14	1-12-6-10-11-3	54	1.765	95.3	-0.1942	3.1086	9.5703	62.8534
4-2	15	4-9-10-11-2	46	1.3	59.8	-0.1692	2.6486	8.1731	53.5480
	16	4-5-6-7-8-2	54	1.404	75.8	-0.2027	3.1066	9.5862	62.8337
	17	4-5-6-7-11-2	51	1.326	67.6	-0.1956	2.9341	9.0697	59.3498
	18	4-5-6-10-11-2	55	1.43	78.7	-0.2018	3.1679	9.7582	64.0034
	19	4-5-9-10-11-2	51	1.326	67.6	-0.1885	2.9388	9.0649	59.3642
4-3	20	4-9-13-3	37	1.266	46.8	-0.1425	2.1259	6.5858	43.0719
	21	4-9-10-11-3	47	1.415	66.5	-0.1723	2.7087	8.3511	54.7108
	22	4-5-9-13-3	42	1.264	53.1	-0.1617	2.4161	7.4777	48.8881
	23	4-5-6-7-11-3	52	1.565	81.4	-0.1986	2.9943	9.2477	60.5126
	24	4-5-6-10-11-3	56	1.686	94.4	-0.2049	3.2281	9.9361	65.1662
	25	4-5-9-10-11-3	52	1.565	81.4	-0.1915	2.9990	9.2429	60.5270

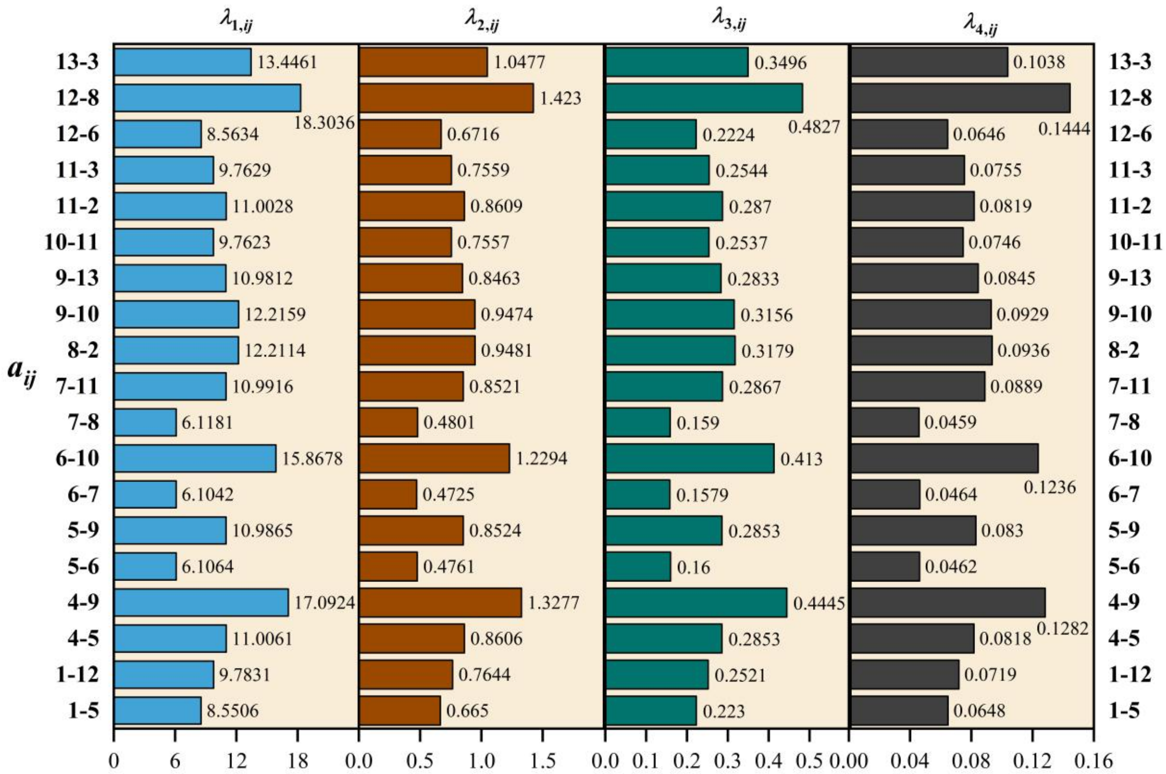


Fig. 6 Linear moments of link travel time

TABLE IV
PATH RELIABILITY CALCULATION RESULTS

OD	Serial Number	Sequence of Nodes	N	$\beta = -N$	Path Reliability (Monte Carlo)	Path Reliability Ranking: Reliability Index	Path Reliability Ranking: Monte Carlo
1-2	1	1-12-8-2	-3.1821	3.1821	0.7386	8	8
	2	1-5-6-7-8-2	-3.5220	3.5220	0.8377	5	5
	3	1-5-6-7-11-2	-3.2516	3.2516	0.7639	7	7
	4	1-5-6-10-11-2	-3.6125	3.6125	0.8582	4	4
	5	1-5-9-10-11-2	-3.2636	3.2636	0.7644	6	6
	6	1-12-6-7-8-2	-3.9006	3.9006	0.9241	2	2
	7	1-12-6-7-11-2	-3.6781	3.6781	0.8758	3	3
	8	1-12-6-10-11-2	-3.9780	3.9780	0.9379	1	1
1-3	1	1-5-9-13-3	-3.1710	3.1710	0.7387	6	6
	2	1-5-6-7-11-3	-3.6615	3.6615	0.8728	5	5
	3	1-5-6-10-11-3	-3.9866	3.9866	0.94	3	3
	4	1-5-9-10-11-3	-3.6759	3.6759	0.8731	4	4
	5	1-12-6-7-11-3	-4.0430	4.0430	0.9542	2	2
	6	1-12-6-10-11-3	-4.3243	4.3243	0.9977	1	1
4-2	1	4-9-10-11-2	-3.1812	3.1812	0.7389	5	5
	2	4-5-6-7-8-2	-3.4893	3.4893	0.8301	2	2
	3	4-5-6-7-11-2	-3.2532	3.2532	0.7651	4	4
	4	4-5-6-10-11-2	-3.5664	3.5664	0.8476	1	1
	5	4-5-9-10-11-2	-3.2635	3.2635	0.7654	3	3
4-3	1	4-9-13-3	-3.0525	3.0525	0.6988	5	5
	2	4-9-10-11-3	-3.5264	3.5264	0.8372	4	4
	3	4-5-9-13-3	-3.0471	3.0471	0.6977	6	6
	4	4-5-6-7-11-3	-3.8839	3.8839	0.9238	3	2
	5	4-5-6-10-11-3	-4.1587	4.1587	0.9743	1	1
	6	4-5-9-10-11-3	-3.8977	3.8977	0.9232	2	3

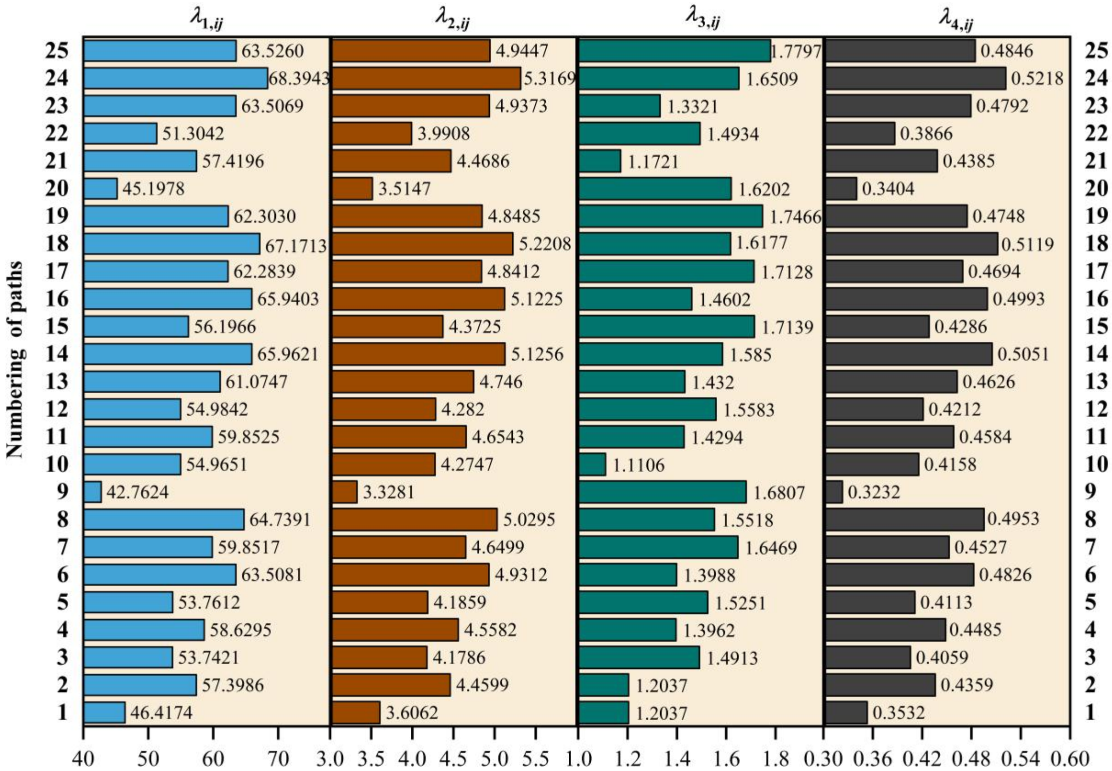
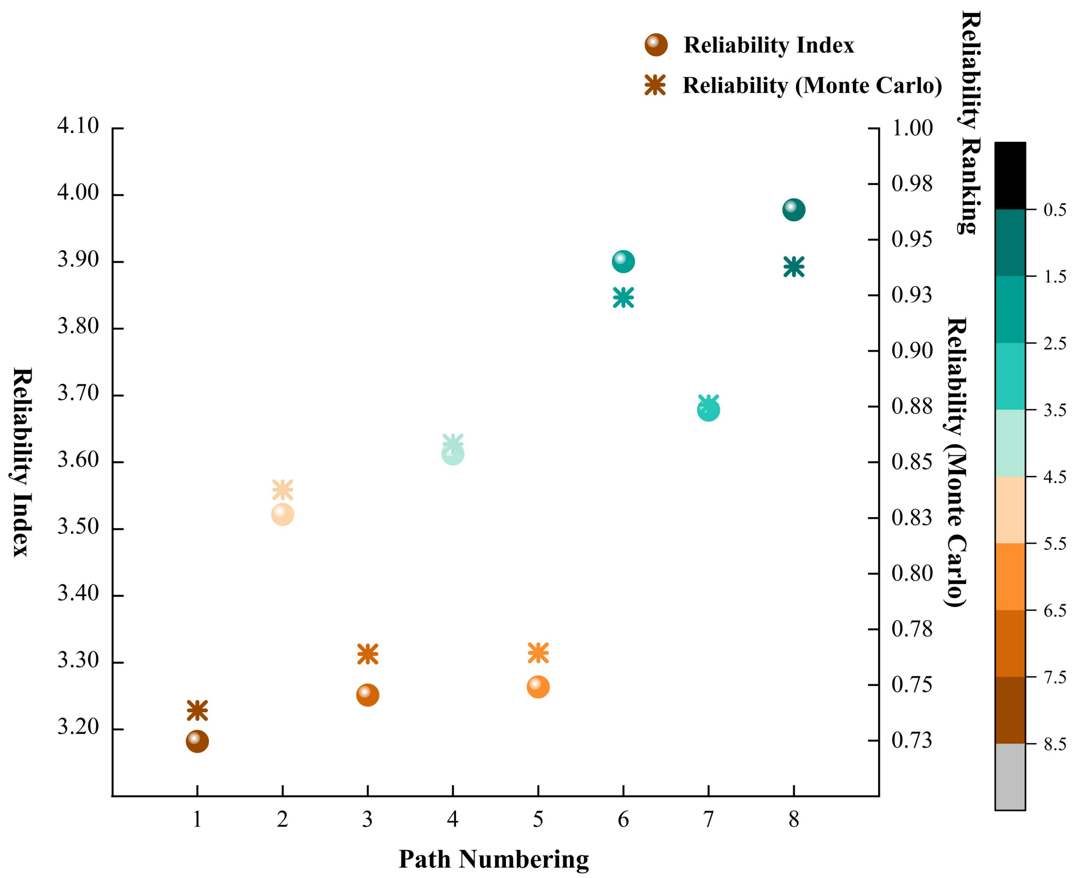
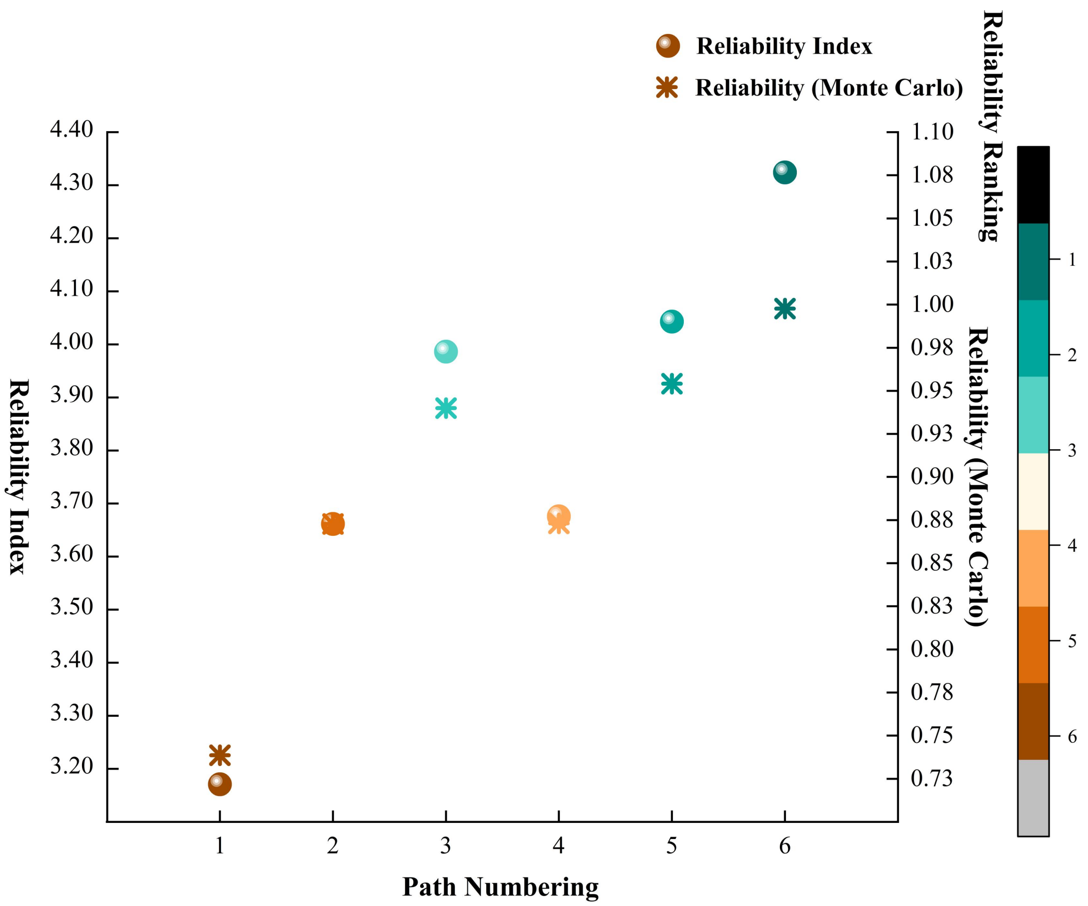


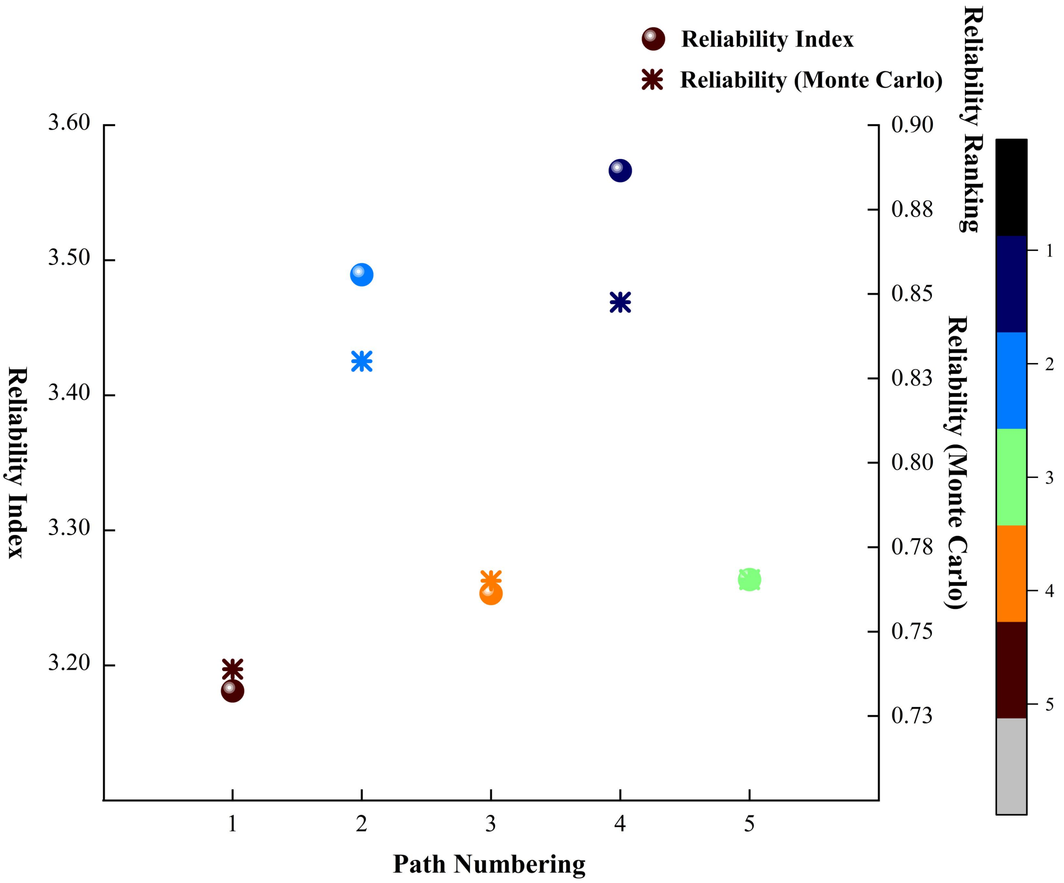
Fig. 7 Linear moments of path travel time


Reliability Index
Reliability (Monte Carlo)
Reliability Ranking
Reliability Index
Reliability (Monte Carlo)
Path Numbering
4.10
4.00
3.90
3.80
3.70
3.60
3.50
3.40
3.30
3.20
1.00
0.98
0.95
0.93
0.90
0.88
0.85
0.83
0.80
0.78
0.75
0.73
1
2
3
4
5
6
7
8
0.5
1.5
2.5
3.5
4.5
5.5
6.5
7.5
8.5

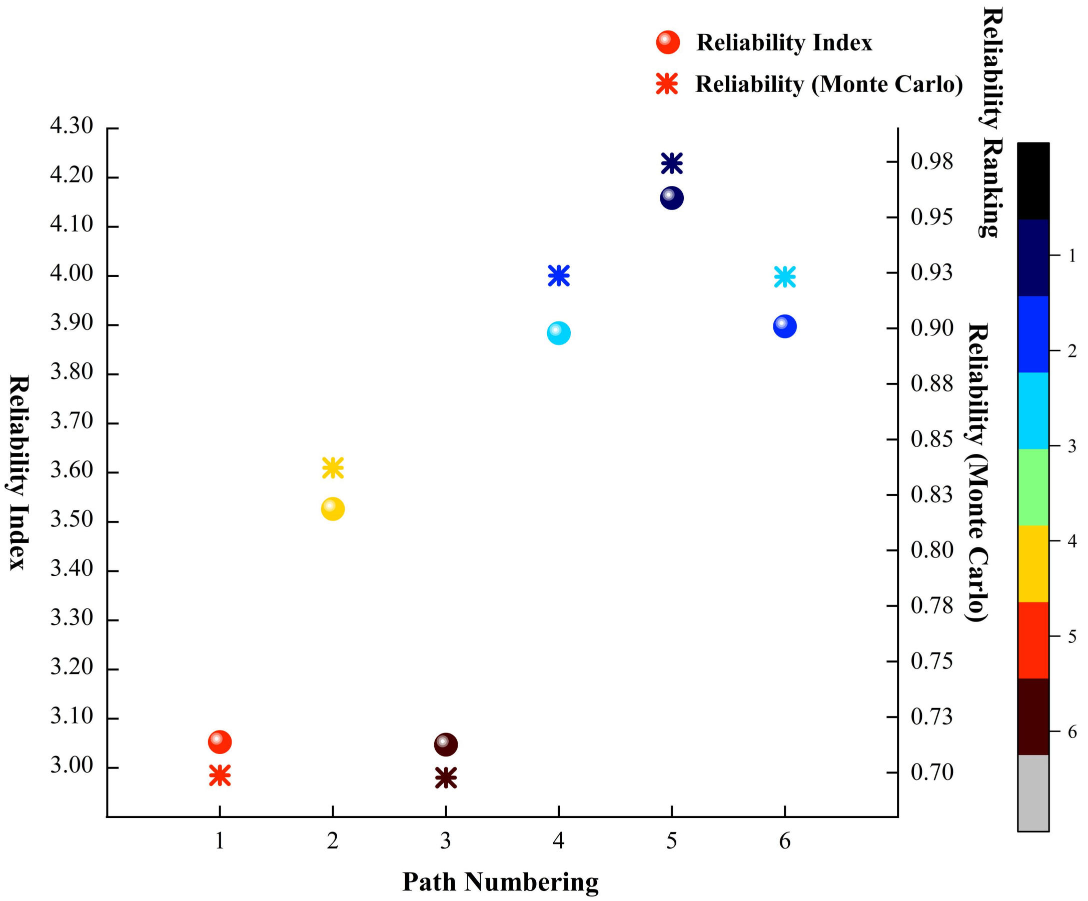
(a)


Reliability Index
Reliability (Monte Carlo)
Reliability Ranking
Reliability Index
Reliability (Monte Carlo)
Path Numbering
4.40
4.30
4.20
4.10
4.00
3.90
3.80
3.70
3.60
3.50
3.40
3.30
3.20
1.10
1.08
1.05
1.03
1.00
0.98
0.95
0.93
0.90
0.88
0.85
0.83
0.80
0.78
0.75
0.73
1
2
3
4
5
6
1
2
3
4
5
6

(b)



(c)



(d)

Fig. 8 Comparison chart of path reliability calculation results (a) Origin=1, Destination=2 (b) Origin=1, Destination=3 (c) Origin=4, Destination=2 (d) Origin=4, Destination=3

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