Error and Reduced-order Extended State Observer Based Full-State Constrained Tracking Control of Permanent Magnet Synchronous Motor under Multi-uncertainties

Yuhang Sun, Liang Xue, Zhihua Zhai, Meijuan Bai, Bingkuan Yin*, Qi Xiao*, Hua Geng and Kaiwen Chen

***Abstract*—High-precision and safe control of permanent magnet synchronous motor (PMSM) is of great significance. However, multiple uncertainties can impact control accuracy and even induce hazardous operational conditions. Conventional approaches for state-constrained control of multi-uncertain systems often encounter issues like differential explosion and overly complex controller structure. This paper proposes a novel barrier Lyapunov function based state-constrained control method integrated with reduced-order extended state observers of tracking error systems. The proposed method introduces a reduced-order extended state observer of the error system at each step of backstepping, achieving two fundamental improvements: (i) avoiding estimating the derivatives of the disturbances or virtual controllers and solving the differential explosion problem; (ii) more compact and concise controller structure. Its effectiveness is subsequently validated through simulation.**

***Index Terms*—PMSM; error system; reduced-order ESO; state constrained control; uncertainty.**

I. Introduction

PERMANENT magnet synchronous motor (PMSM) is widely used in various applications due to their superior dynamic response, large torque-to-torsion ratio, low-noise operation, impressive power density, and excellent efficiency [1]. PMSM requires strict compliance with voltage, current, and speed limits to avoid irreversible damage to the motor and power electronics, such as: overcurrent, overvoltage, overspeed [2], [3], [4]. Moreover, multiple uncertainties during PMSM operations, including parameter deterioration throughout the lifecycle, unknown loads, and unmodeled dynamics, etc., can induce severe performance degradation and even push critical system states (currents, rotational speed) to violate predefined safety boundaries, resulting in efficiency deterioration and reliability reduction [5], [6], [7]. Therefore, considering the impact of multi-uncertainties, it becomes crucial to develop a state-constrained controller for PMSM.

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Active disturbance rejection control (ADRC) [8], [9], [10] and disturbance observer-based control (DOBC) [11] are hot topics in modern control theory. Their core principle lies in the active estimation and compensation of total disturbances, aiming to eliminate the disturbances before they affect the system. When implemented, these methods offer more flexible and adaptive approaches to handle the multiple uncertainties, thus enhancing the comprehensive performance of the system [12], [13], [14], [15].

In the author's previous research [12], the mismatched uncertainties estimation and compensation were implemented through the generalized proportional integral observer (GPIO) framework. However, as the relative order of uncertainty with respect to the system output increases, so does the number of uncertainty derivatives requiring estimation, thus leading to the differential explosion problem. The differential explosion problem also exists in [16]. In [17], [18], [19], the command filter and sliding mode differentiator were utilized to address the issue of differential explosion respectively. However, these methods need the simultaneous construction of a disturbance observer and a command filter or sliding mode differentiator, thereby increasing the complexity of the control design. In recent years, error based ADRC [20], [21] has been developed to enhance controller compactness, and this method has potential in dealing with the differential explosion problem and simplifying the design process.

The state-constrained problem of uncertain systems have been extensively addressed in recent years. The first method is based on predictive control [22]. The state constrained problem of the system is characterized by inequality constraints, and the state constrained problem is tackled by solving the constrained indicator function. Nevertheless, this technique is associated with significant computational efforts. The second method leverages smooth, bounded nonlinear functions to transform the state-constrained system into an alternative system that inherently meets the state constraints [23], [24]. Following this transformation, a stable control strategy is designed for the transformed system, which guarantees that the original system will adhere to the constraints. However, the transformed system has strong nonlinearity, which brings great challenges to controller design. The third approach integrates the barrier Lyapunov function with the backstepping [12], [25], [26]. This framework provides a more direct and computationally efficient method to ensure the satisfaction of constraints. State-constrained control of uncertain systems has also been extensively studied in [12], [17], [18], [19], however, its differential explosion problem and controller architecture simplification still need further exploration and optimization.

This paper presents an innovative state-constrained controller for PMSM systems with multiple uncertainties. The controller is synthesized by integrating barrier Lyapunov functions with backstepping. The core contribution of this work is that the error-based reduced-order extended state observer is introduced into the design of barrier Lyapunov based backstepping state-constrained controller to simplify the controller structure and solve the multi-uncertainties and differential explosion problem.

The remainder of this paper is structured as follows: Section II addresses the system modeling and problem description. The design of a state-constrained controller is provided in Section III. Section IV evaluates the proposed method's performance through numerical simulation. Concluding remarks for the article are offered in Section V.

II. System Modeling and Problem statement

To facilitate simplified control and analysis, the PMSM model [27] in d - q axes is used, and the multi-channel uncertainties are introduced, which can be described as:

$$\begin{cases} \dot{\omega} = a_{11}\omega + a_{12}i_q + f_1 \\ \dot{i}_d = a_{21}i_d + a_{22}\omega i_q + b_1 u_d + f_2 \\ \dot{i}_q = a_{31}\omega + a_{32}i_q + a_{33}\omega i_d + b_2 u_q + f_3 \end{cases} \quad (1)$$

where ω denotes the rotor speed, i_d and i_q represent the stator currents of the d-q axes, u_d and u_q denote the stator voltages of the d-q axes. $a_{11} = -\frac{B}{J}$, $a_{12} = \frac{K_t}{J}$, $a_{21} = -\frac{R_s}{L_d}$, $a_{22} = n_p$, $b_1 = \frac{1}{L_d}$, $a_{31} = -\frac{n_p\phi_v}{L_q}$, $a_{32} = -\frac{R_s}{L_q}$, $a_{33} = -n_p$, $b_2 = \frac{1}{L_q}$. L_d and L_q represent the inductances of the d-q axes. R_s stands for the stator resistance, n_p represents the number of pole pairs, and ϕ_v represents the rotor flux linkage. K_t is defined as $3n_p\phi_v/2$. J represents the moment of inertia, and B denotes the viscous friction coefficient. $f_1 = \Delta_{a11}\omega + \Delta_{a12}i_q + b_0 T_L$, $b_0 = -\frac{1}{J}$, $f_2 = \Delta_{a21}i_d + \Delta_{a22}\omega i_q$ and $f_3 = \Delta_{a31}\omega + \Delta_{a32}i_q + \Delta_{a33}\omega i_d$ are the multi-uncertainties. T_L is the time-varying unknown load torque.

Assume the time derivatives of f_1, f_2 and f_3 are bounded.

The control objectives of this paper are:

(1) The rotor speed ω of the PMSM tracks a desired signal r_1. Suppose that r_1, $\dot{r}_1$ and $\ddot{r}_1$ are bounded;

(2) The state constrained conditions $\omega_{min} \le \omega \le \omega_{max}$ and $|i_q| \le i_{safe}$ are satisfied .

(3) The $\lim_{t\to\infty} i_d$ is bounded.

III. State Constrained Controller Design

In order to facilitate the subsequent derivations, define $\kappa_1^{\min} \le \omega - r_1 \le \kappa_1^{\max}$, $r = r_1 + \frac{\kappa_1^{\max}+\kappa_1^{\min}}{2}$; $e_1 = \omega - r_1 - \frac{\kappa_1^{\max}+\kappa_1^{\min}}{2}$; $\kappa_2^{\min} \le i_q - \alpha_1 \le \kappa_2^{\max}$, $e_2 = i_q - \alpha_1 - \frac{\kappa_2^{\max}+\kappa_2^{\min}}{2}$; $k_{b1} = \frac{\kappa_1^{\max}-\kappa_1^{\min}}{2}$, $k_{b2} = \frac{\kappa_2^{\max}-\kappa_2^{\min}}{2}$.

Remark 1: With $R_1^{\min} \le r_1 \le R_1^{\max}$ and $\alpha_1^{\min} \le \alpha_1 \le \alpha_1^{\max}$, the $\kappa_1^{\min}$, $\kappa_1^{\max}$ $\kappa_2^{\min}$ and $\kappa_2^{\max}$ require careful tuning to adhere to the state constraints: $\omega_{min} \le R_1^{\min} + \kappa_1^{\min} \le \omega \le R_1^{\max} + \kappa_1^{\max} \le \omega_{max}$, $-i_{safe} \le \alpha_1^{\min} + \kappa_2^{\min} \le i_q \le \alpha_1^{\max} + \kappa_2^{\max} \le i_{safe}$. The error e_1 and e_2 are introduced to meet $|e_1| \le \frac{\kappa_1^{\max}-\kappa_1^{\min}}{2} = k_{b1}$ and $|e_2| \le \frac{\kappa_2^{\max}-\kappa_2^{\min}}{2} = k_{b2}$.

Stpe 1: Define the first barrier Lyapunov function as:

$$V_1 = \frac{1}{2}\ln\frac{k_{b1}^2}{k_{b1}^2 - e_1^2}. \quad (2)$$

Define $\varepsilon_1 = \omega - r_1$. According to (1), the dynamic of the tracking error system ε_1 can be written as:

$$\dot{\varepsilon}_1 = \dot{\omega} - \dot{r}_1 = a_{11}\omega + a_{12}i_q + f_1 - \dot{r}_1 \quad (3)$$

Define the total disturbance $d_1 = f_1 - \dot{r}$. Subsequently, the $\dot{V}_1$ is calculated as:

$$\begin{aligned} \dot{V}_1 &= \frac{e_1\dot{e}_1}{k_{b1}^2-e_1^2} = \frac{e_1(a_{11}\omega+a_{12}i_q+d_1)}{k_{b1}^2-e_1^2} \\ &= \frac{e_1\left[a_{12}e_2+a_{12}\alpha_1+a_{12}\frac{\kappa_2^{\max}+\kappa_2^{\min}}{2}+a_{11}\omega+d_1\right]}{k_{b1}^2-e_1^2} \end{aligned} \quad (4)$$

To eliminate the need for estimating the known variable ε, the reduced-order extended state observer (ESO) of the tracking error system (3) is designed as follows:

$$\begin{cases} \dot{\theta}_1 = (0 - L_1)(\theta_1 + L_1\varepsilon_1) + (-L_1 X_1) \\ \hat{x}_{d_1} = \theta_1 + L_1\varepsilon_1 \end{cases} \quad (5)$$

where $\theta \in R$ is the state of the observer, $\hat{x}_{d_1}$ represents the estimated value of d_1, L_1 is the gain for the reduced ESO, and $X_1 = a_{11}\omega + a_{12}i_q$.

Remark 2: Define $x_{d1} = d_1$, then system (3) can be rewritten as $\begin{bmatrix} \dot{\varepsilon}_1 \\ \dot{x}_{d_1} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} \varepsilon_1 \\ x_{d_1} \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} X_1 + \begin{bmatrix} 0 \\ \dot{d}_1 \end{bmatrix}$. Accorting to [12], the reduced order ESO can be designed as (5). Define $e_{d_1} = x_{d_1} - \hat{x}_{d_1}$, the observation error system is $\dot{e}_{d_1} = (0 - L_1)e_{d_1} + \dot{d}_1$. The $\lim_{t\to\infty} e_{d1}$ is bounded. The detailed derivation and the determination method of parameter L_1 can be found in [12].

The virtual controller α_1 can be formulated as:

$$\alpha_1 = -\frac{1}{a_{12}}\left[k_1 e_1 + a_{12}\left(\frac{\kappa_2^{max} + \kappa_2^{min}}{2}\right) + a_{11}\omega + \hat{x}_{d_1}\right] \quad (6)$$

where $k_1 > 0$.

Substituting (6) into (4), we obtain

$$\dot{V}_1 = -\frac{k_1 e_1^2}{k_{b_1}^2 - e_1^2} + \frac{a_{12}e_1 e_2}{k_{b_1}^2 - e_1^2} + \frac{e_1 e_{d_1}}{k_{b_1}^2 - e_1^2} \quad (7)$$

Stpe 2: Define the second barrier Lyapunov function as:

$$V_2 = V_1 + \frac{1}{2}\ln\frac{k_{b_2}^2}{k_{b_2}^2 - e_2^2} \quad (8)$$

Define $\varepsilon_2 = i_q - \alpha_1$. According to (1), the dynamic of the tracking error system ε_2 can be written as:

$$\dot{\varepsilon}_2 = \dot{i}_q - \dot{\alpha}_1 = a_{31}\omega + a_{32}i_q + a_{33}\omega i_d + b_2 u_q + f_3 - \dot{\alpha}_1 \quad (9)$$

Remark 3: When constructing the controller u_q to the error system (9), the approach outlined in [12], [17], [18] necessitates the knowledge of the derivative of the virtual controller α_1. Alternatively, one can derive the derivative of α_1, using the system state model structure and the estimated information of multiple disturbances (For details, see Remark 11 of [12]). However, this process requires the knowledge of the derivatives of the uncertainty f_1, and the controller becomes very complicated. In addition, as the relative degree [16] from the uncertainty to control input of the system increases, it becomes essential to obtain higher-order derivatives of the uncertainty, which can lead to the phenomenon known as "differential explosion" and complex controller.

Define the total disturbance $d_2 = f_3 - \dot{\alpha_1}$.

The $\dot{V}_2$ is calculated as

$$\dot{V}_2 = \dot{V}_1 + \frac{e_2\dot{e}_2}{k_{b2}^2 - e_2^2} = \dot{V}_1 + \frac{e_2[a_{31}\omega + a_{32}i_q + a_{33}\omega i_d + b_2 u_q + d_2]}{k_{b2}^2 - e_2^2} \tag{10}$$

The reduced order ESO can be designed as follows:

$$\begin{cases} \dot{\theta_2} = (0 - L_2)(\theta_2 + L_2\varepsilon_2) + (-L_2 X_2) \\ \hat{x}_{d_2} = \theta_2 + L_2\varepsilon_2 \end{cases} \tag{11}$$

where $\theta_2 \in R$ is the state of the observer, $\hat{x}_{d_2}$ represents the estimated value of $x_{d_2} = d_2$, L_2 is the gain for the reduced ESO, and $X_2 = a_{31}\omega + a_{32}i_q + a_{33}\omega i_d + b_2 u_q$.

Remark 4: Accorting to [12], the error system (9) can be rewritten as $\begin{bmatrix} \dot{\varepsilon}_2 \\ \dot{x}_{d_2} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \varepsilon_2 \\ x_{d_2} \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} X_2 + \begin{bmatrix} 0 \\ \dot{d}_2 \end{bmatrix}$. The reduced order ESO can be designed as (11). Define $e_{d_2} = x_{d_2} - \hat{x}_{d_2}$, the observation error system is $\dot{e}_{d_2} = (0 - L_2) e_{d_2} + \dot{d}_2$. If $\dot{d}_2$ is bounded, the $\lim_{t\to\infty} e_{d_2}$ is bounded.

The controller u_q can be designed as

$$\begin{aligned} u_q = -\frac{1}{b_2} \Big[& k_2 e_2 + a_{31}\omega + a_{32}i_q + a_{33}\omega i_d \\ & + \frac{k_{b_2}^2 - e_2^2}{k_{b_1}^2 - e_1^2} a_{12} e_1 + \hat{x}_{d_2} \Big] \end{aligned} \tag{12}$$

where $k_2 > 0$.

Remark 5: The methods in [12], [17], [18] require the construction of observer to estimate the derivatives of the virtual controller α_1 or f_1. Compared with [12], [17], [18], the proposed method eliminates the necessity for constructing additional observers or filters to estimate the derivatives of the virtual controller α_1 or disturbance f_1. The derivatives of the set point r is also no need to be known. In general, the overall observer order is reduced, and the issue of differential explosion is tackled. Furthermore, the backstepping method is employed to establish reduced-order observers based on the error systems (3) and (9). This approach allows for the design of a state-constrained controller at each step, as detailed in (6) and (12), thereby greatly simplifying the overall controller structure.

Combining (12) with (10), we can get:

$$\dot{V}_2 = -\frac{k_1 e_1^2}{k_{b_1}^2 - e_1^2} - \frac{k_2 e_2^2}{k_{b2}^2 - e_2^2} + \frac{e_1 e_{d_1}}{k_{b_1}^2 - e_1^2} + \frac{e_2 e_{d_2}}{k_{b2}^2 - e_2^2} \tag{13}$$

The proof of stability is as follows:

Theorem 3.1: For the system (1), assume the multi-uncertainties are bounded, with the virtual controller (6) and controller (12), if the initial values of the system satisfy $|e_1(0)| < k_{b1}$ and $|e_2(0)| < k_{b2}$, and the error systems e_1 and e_2 are bounded and stable, then the system states meet the constrained conditions: $s_1^{\min} \le \omega \le s_1^{\max}$, $|i_q| \le i_{\text{safe}}$.

TABLE I
THE PARAMETER VALUES OF PMSM

Parameters	Value
Moment of Inertia J	0.0081 $[kg \cdot m^2]$
Friction Coefficient B	0.0015 $[N \cdot m \cdot s/rad]$
Number of Poles n_p	4
Magnetic Flux Linkage ϕ_v	0.2715 $[V \cdot s/rad]$
Inductance $L_q = L_d$	0.019 $[H]$
Stator Resistance R_s	0.17 $[\Omega]$

Proof of Theorem 1: With e_{d_1} and e_{d_2} are bounded, there exists E_d satisfying: $|e_{d_1}| < E_d$, $|e_{d_2}| < E_d$. Suppose $K < min(|k_1|, |k_2|)$, and the subsequent inequality is satisfied:

$$\dot{V}_2 \le -K \sum_{i=1}^{2} \frac{e_i^2}{k_{bi}^2 - e_i^2} + E_d \sum_{i=1}^{2} \frac{|e_i|}{k_{bi}^2 - e_i^2}. \tag{14}$$

For any $\sum_{i=1}^{2} \frac{e_i^2}{k_{bi}^2 - e_i^2} > \frac{E_d}{K} \sum_{i=1}^{2} \frac{|e_i|}{k_{bi}^2 - e_i^2}$, $\dot{V}_2 < 0$, therefore the error e_1 and e_2 are bounded.

The proof is completed. ■

Using the control strategy of $i_d^* = 0$, u_d is designed as

$$u_d = -L_d (a_{22}\omega i_q + a_{21} i_d + k_3 i_d) \tag{15}$$

Combining (15) with (1) yields

$$\dot{i}_d = f_2 - k_3 i_d \tag{16}$$

where $k_3 > 0$ is obtained.

IV. SIMULATION

A numerical simulation is performed to confirm the efficacy of the proposed control algorithm.

The parameters of the PMSM are shown in the Table I. Take $\Delta_{a_{11}} = \frac{\Delta_B}{J} = \frac{0.0005}{0.0081}$, $\Delta_{a_{12}} = 0$, $\Delta_{a_{21}} = \frac{\Delta_{R_s}}{L_d} = \frac{0.05}{0.019}$, $\Delta_{a_{22}} = 0$, $\Delta_{a_{31}} = 0$, $\Delta_{a_{32}} = \frac{\Delta_{R_s}}{L_q} = \frac{0.05}{0.019}$, $\Delta_{a_{33}} = 0$. The objective is to ensure v track $r = \begin{cases} 25, & 0 \le t \le 5 \\ 25 + 3 \cdot \frac{2}{\pi} \cdot \arctan[(50 \cdot (t-5)], & t > 5 \end{cases}$. Take $\omega^{min} = 22$, $\omega^{max} = 31$, $\kappa_1^{min} = -3$, $\kappa_1^{max} = 3$ to make $\omega^{min} \le \kappa_1^{min} + R_1^{min} \le \omega \le \kappa_1^{max} + R_1^{max} \le \omega^{max}$. Take $i_{\text{safe}} = 10$, $\kappa_2^{min} = -8$, $\kappa_2^{max} = 8$, $\alpha_1^{\min} = -2$ and $\alpha_1^{\max} = 2$ to make $-i_{\text{safe}} \le \alpha_1^{\min} + \kappa_2^{\min} \le i_q \le \alpha_1^{\max} + \kappa_2^{\max} \le i_{\text{safe}}$. $T_L = 0.1 \cdot sin(2\pi t)$.

The observer gains of the reduced ESO (5) and (11) are $L_1 = 15$, $L_2 = 20$. Take $k_1 = 14$, $k_2 = 16$, $k_3 = 20$. The initial values of the system (1), (5), (11) are set as $[25, 0, 0]^T$, $[0]^T$ and $[0]^T$.

To validate the proposed method, we compare it with the reduced-order GPIO-based state-constrained controller [12]. The compared reduced-order GPIO and controller are as

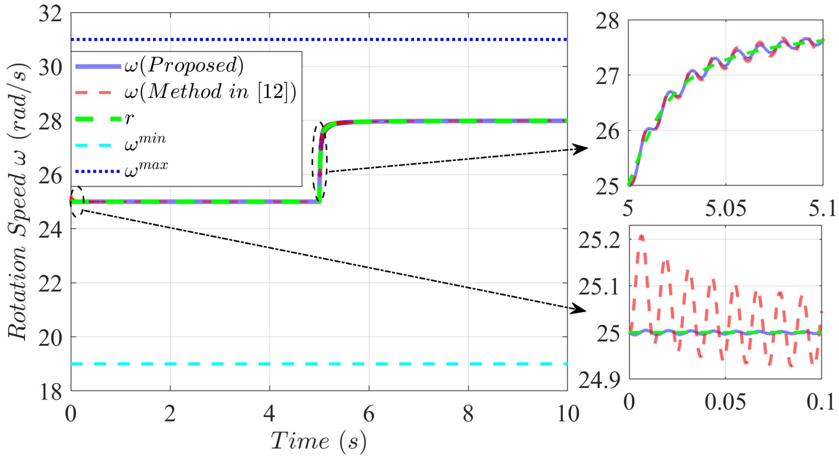

Fig. 1. The Rotation Speed ω of PMSM

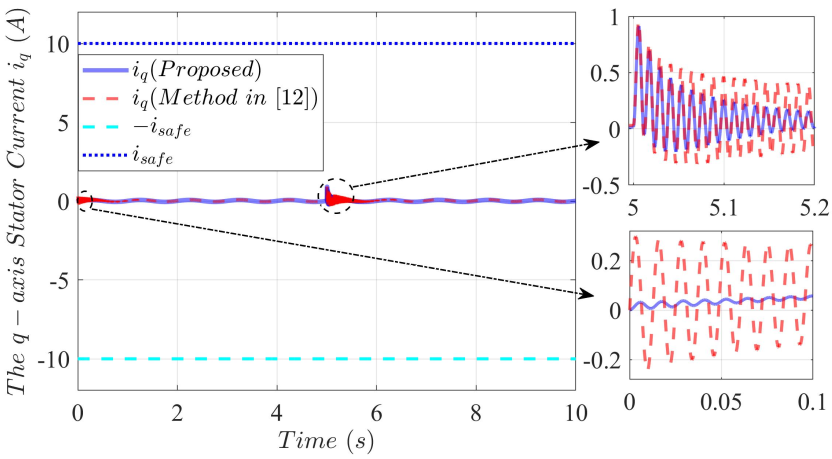

Fig. 2. The q-axis Stator Current i_q

follows:

$$\begin{cases} \dot{\xi}_1 = \begin{bmatrix} -l_{11} & 1 \\ -1_{12} & 0 \end{bmatrix} \left(\xi_1 + \begin{bmatrix} l_{11} \\ l_{12} \end{bmatrix} \omega \right) \\ \qquad - \begin{bmatrix} l_{11} \\ l_{12} \end{bmatrix} (a_{11}\omega + a_{12} i_q), \\ \dot{\xi}_3 = -l_3 (\xi_3 + l_3 i_q + a_{32} i_q + a_{31}\omega + a_{33}\omega i_d + b_2 u_q), \\ \begin{bmatrix} \hat{x}_{11} \\ \hat{x}_{12} \end{bmatrix} = \xi_1 + \begin{bmatrix} l_{11} \\ l_{12} \end{bmatrix} \omega, \\ \hat{x}_{31} = \xi_3 + l_3 i_q, \end{cases} \tag{17}$$

$$\begin{cases} \alpha_1 = -\dfrac{1}{a_{12}} (k_1 e_1 - \dot{r}) \\ \qquad + \dfrac{1}{a_{12}} (a_{11}\omega + \hat{x}_{11}) + \dfrac{\kappa_2^{\min} + \kappa_2^{\max}}{2}, \\ u_q = -L_q \Big[k_2 e_2 + a_{31}\omega + a_{32} i_q + a_{33}\omega i_d + \hat{x}_{31} \\ \qquad - \dot{\alpha}_1 + a_{12} \cdot \dfrac{k_{b2}^2 - e_2^2}{k_{b1}^2 - e_1^2} \cdot e_1 \Big] \end{cases} \tag{18}$$

The total order of the proposed reduced order ESO (5) and (11) is 2, however the reduced order GPIO (17) is 3. Therefore, the reduced-order ESOs of the error systems are introduced in the backstepping controller design, which reduces the total order of the observers, especially for high-order systems. In addition, according to [12], to approximate the first-order derivative of the virtual controller α_1 of (18) (For details, see Remark 11 of [12]), the controller (18) requires a more complex design process than the controllers (6) and (12).

For fair comparison, take the parameters of the reduced order GPIO: $l_{11} = 2L_1$, $l_{12} = L_1^2$, $l_3 = L_2$, so that the observer error system has the same characteristic roots as (5) and (11), and take the same controller parameters: $\kappa_1^{min} = -3$, $\kappa_1^{max} = 3$, $\kappa_2^{min} = -8$, $\kappa_2^{max} = 8$, $k_1 = 14$, $k_2 = 16$, $k_3 = 20$.

Fig 1 shows that the rotor speed ω of PMSM is regulated to maintain a specified reference value r, while simultaneously satisfying the state constraint: $\omega^{min} \leq \omega \leq \omega^{max}$. From Figure 1, it is evident that the proposed method offers improved dynamic control accuracy and a notably smaller fluctuation amplitude compared to the method in [12], therefore the proposed method uses a lower-order observer and a simpler controller structure, achieving better control performance. As illustrated in Figure 2, the current i_q satisfies the state constraint condition $-i_{\text{safe}} \leq i_q \leq i_{\text{safe}}$, and the proposed method has smaller fluctuation amplitude than the method proposed in [12]. The virtual control input α_1 and control input u_q are illustrated in the Figure 3 and Figure 4. Figure 5 shows that the current i_d is regulated to

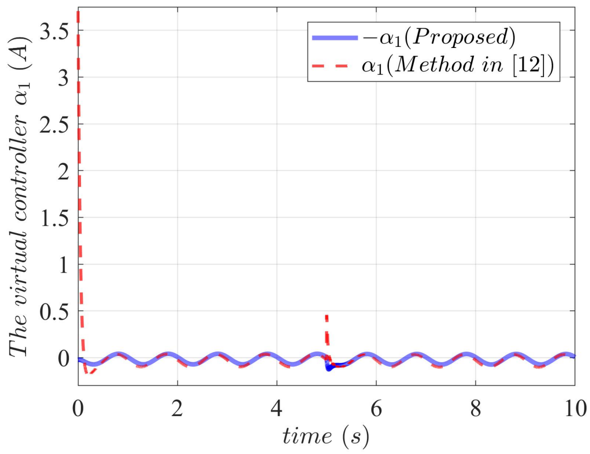

Fig. 3. The Virtual Controller α_1

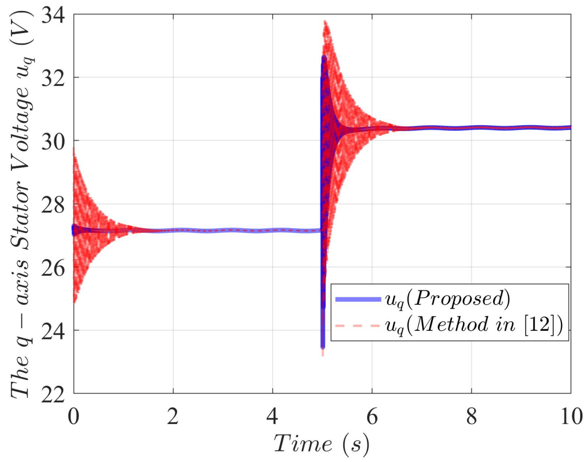

Fig. 4. The q-axis Stator Voltage u_q

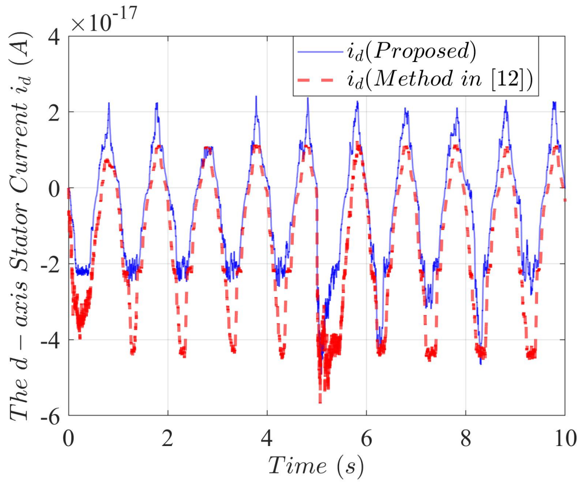

Fig. 5. The d-axis Stator Current i_d

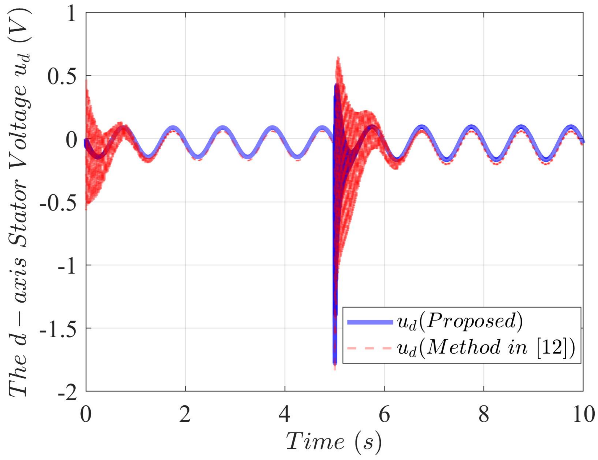

Fig. 6. The d-axis Stator Voltage u_d

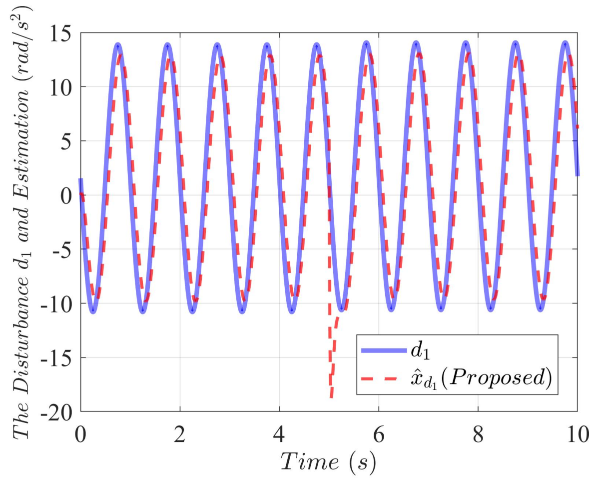

Fig. 7. The Total Disturbance d_1 and Estimation $\hat{x}_{d_1}$

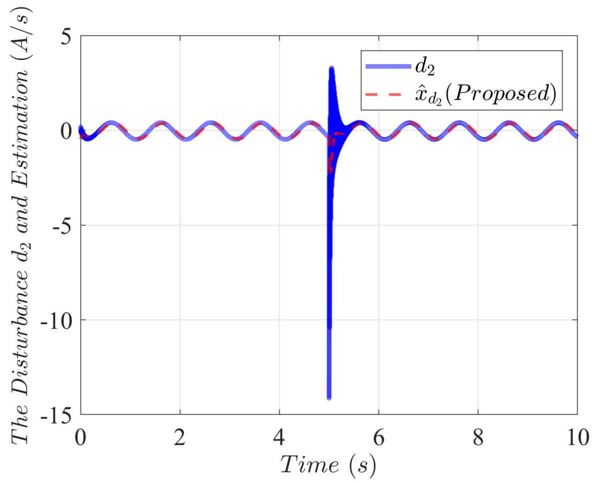

Fig. 8. The Total Disturbance d_2 and Estimation $\hat{x}_{d_2}$

a small neighborhood around 0, and Figure 6 shows that the proposed control input u_d has smaller fluctuation amplitude. Figure 7 and Figure 8 show that the reduced order ESO (5) and (11) can accurately estimate the disturbances. Figure 9 and Figure 10 show that the reduced order GPIO (17) can accurately estimate the disturbances.

The simulation results clearly indicate that estimation of multi-uncertainties, setpoint tracking and state constraint conditions are all realized, with better control effect. Compared with [16], [17], [18], there is no need to design additional observers to estimate derivatives of the multi-uncertainties or virtual controllers. Therefore, the proposed controller structure is simplified, and the differential explosion problem is well solved.

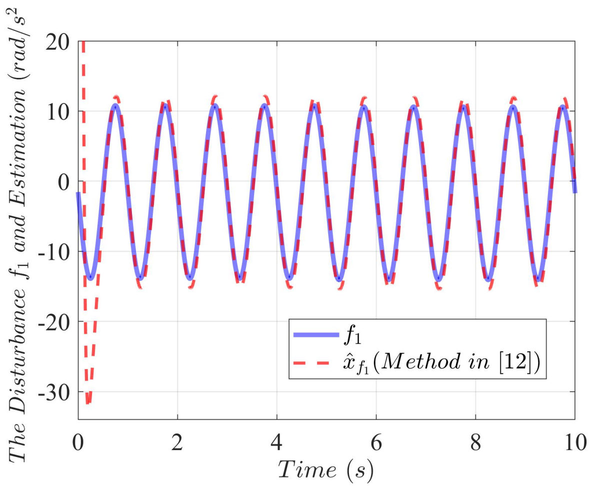

Fig. 9. The Total Disturbance f_1 and Estimation $\hat{x}_{f_1}$

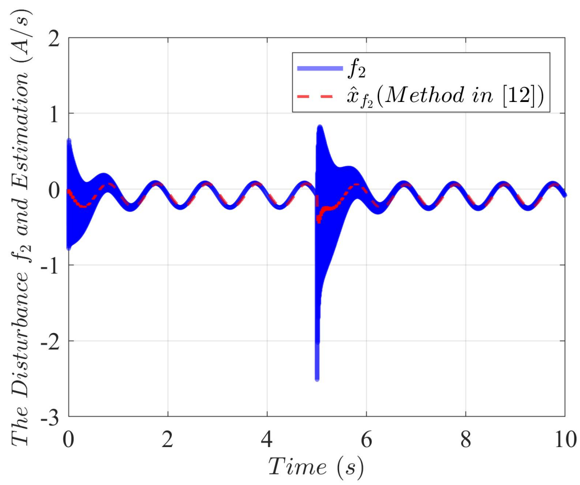

Fig. 10. The Total Disturbance f_2 and Estimation $\hat{x}_{f_2}$

V. Conclusion

In this paper, the error-based reduced-order observer is introduced into the barrier Lyapunov-based state-constrained controller design, which simplifies the complexity of controller design, provides good control performance, and address the challenges of differential explosion. Compared with [12], [16], [17], [18], the novel control strategy has a more compact and concise structure.

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